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**ÇOK ÇEŞİTLİ/AZ HACİMLİ ELEKTRONİK KART ÜRETİMİ YAPAN BİR
FİRMANIN ÜRETİM ÇİZELGELEMESİNİ GERÇEKLEŞTİREN KARMA-
TAMSAYI PROGRAMLAMASI**

Emre Özgenç EKİCİ

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AN OPTIMAL MIXED INTEGER PROGRAM FOR SCHEDULING IN
A HIGH MIX-LOW VOLUME PRINTED WIRING BOARD
PRODUCTION COMPANY

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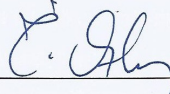
Production Management and Quantitative Methods

Master's Thesis

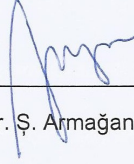
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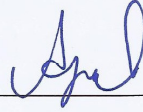
Emre Özgenç Ekici tarafından hazırlanan "Çok Çeşitli/Az Hacimli Elektronik Kart Üretimi Yapan bir Firmanın Üretim Çizelgelemesini Gerçekleştiren Karma-Tamsayı Programlaması" başlıklı bu çalışma, [16.07.2013] tarihinde yapılan savunma sınavı sonucunda başarılı bulunarak jürimiz tarafından Yüksek Lisans Tezi olarak kabul edilmiştir.



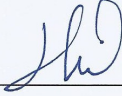
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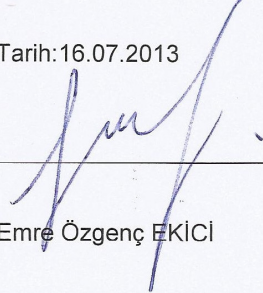
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TEŐEKKÜR

Öncelikle eğitimim konusunda beni her zaman destekleyen ve yüksek lisans fikrini hep aklımda tutan anneme, tez yazılımı hazırlamak ve tez yazmak için zaman yaratmam konusunda bana yardımcı olan ve beni hep mutlu eden eşime, ayrıca tez konusu seçimi, tez yazım ve sonuçlandırma aşamasında beni yönlendiren, destekleyen, ve hep özgür bırakarak yaratıcı olmamı sağlayan sayın hocalarım Prof. Dr. Ő. Armağın TARIM ve Dr. Roberto ROSSI'ye çok teşekkür ediyorum.

ÖZET

EKİCİ, Emre Özgenç. Çok Çeşitli/Az Hacimli Elektronik Kart Üretimi Yapan bir Firmanın Üretim Çizelgelemesini Gerçekleştiren Karma-Tamsayı Programlaması, Yüksek Lisans Tezi, Ankara, 2013.

Çok Çeşitlilik/Az hacimli üretim yapan, birden fazla üretim safhası bulunan, her safhada birden fazla paralel kaynağı olan Hibrit Akış Atölye (HAA) alanlarında optimum üretim çizelgesi yapmak gerçek hayatta rutin ve zor bir problemdir. Bu tür üretim alanlarının optimum üretim çizelgeleri için literatürde bir çok özel karma-tamsayı programları mevcuttur. Bu tezde ise, çok değişkenlik/az hacimli üretim yapan, birden fazla safhası bulunan, her safhada paralel kaynakları bulunan, safhalar arasında bekleme zaman/hacim sınırı olmayan, parti bölmeye izin veren, her işin birbirinden bağımsız kurulum süresinin olduğu gerçek bir firmanın Elektronik Kart Üretim Çizelgelemesi için geliştirilen Karma-Tamsayı Modeli yer almaktadır. Bu modelde, amaç fonksiyonu; geç kalan işlerin maliyeti ve her safhadaki kapasite artırımı maliyet toplamını minimum hale getirmek olarak belirlenmiştir.

Anahtar Sözcükler

Hibrit Akış Atölye, Karma-Tamsayı Programlama, Çizelgeleme, Elektronik Kart Üretim

ABSTRACT

EKİCİ, Emre Özgenç. An Optimal Mixed Integer Program for Scheduling in a High Mix - Low Volume Printed Wiring Board Production Company, Master's Thesis, Ankara, 2013.

Finding optimum schedules for High Mix / Low Volume, multistage Hybrid Flow Shop (HFS) areas, is regular and hard problem. For these kinds of production areas, there are many specific Mixed Integer Programs in literature. In this thesis, another Mixed Integer Program is demonstrated for a real Printed Wiring Board Production Company with specific features: High Mix / Low Volume, multistage production with parallel units, Unlimited Interstate Buffer, Allowed Lot-Splitting, Independent set-up times of each job. Objective Function of the model is minimizing sum of costs of tardy jobs and capacity increment at each stage.

Key Words

Hybrid Flow Shop, Printed Wiring Board Production, Scheduling, Mixed Integer Programming

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INTRODUCTION

The company that this thesis is about is ASELSAN A.Ş. Defense Systems Technologies Division Electronics Production Department and it is working according to High Mix/low Volume Production Type.

4 main stages exist for Printed Circuit/Wiring Board (PCB) assembly at Electronics Production Department;

1. SMD Assembly

At this stage, SMD components such as resistors, diodes, capacitors, transistors, integrated circuits are mounted.

2. Through Hole Component Assembly and Rework

All electronic through-hole components and electromechanical components such as connectors, heat sinks, screws, RF screens are fixed and soldered on PCB's by using different kind of soldering methods.

3. Functional Test

PCB's are checked if they are functioning properly in accordance with their documented specifications by using special test setups, and test equipments.

4. Conformal Coating and Gluing

The conformal coating process is applied to protect PCBs from degradation caused by dust, dirt, moisture, thermal cycling and exposure to solvents.

At each stage, there are parallel sources, and owing to nature of electronic production, each production order follows same routing. (From stage1 to Stage 4) In literature, these kinds of productions types are called Hybrid Flow Shop (HFS) (Gupta, 1988; VoX, 1993). This production type is frequently-used in electronic industry and it has multi-stage, parallel resources in each stage and jobs having same production flow. Therefore, this scheduling problem easily can be regarded as Hybrid Flow Shop scheduling problem with its own

additional special features. Each job has its own due-dates; each stage has its own capacity. And there are costs that company pays for tardy jobs and capacity increment.

Also, because of high variety, various Printed Wiring Board Assembly production orders come to Electronics Production Department. All of these production orders includes different release date, due date, quantity, material name, routing, documents needed, sub-materials needed. After that, capacity constraints are included manually by Shop Floor Engineer. During Manufacturing Execution stage, in line with due-dates, release-dates, and capacity min/max borders, Shop Floor Engineer schedules each production order by splitting/not splitting the batches, increasing/decreasing capacity of each stage. This process is not cost affective and totally manual. Therefore, in this thesis mainly this scheduling problem is examined, a Mixed Integer Model is developed, solved and demonstrated.

Company Information

As a developer of state of the art Land, Air, Naval and Space technology, ASELSAN is Turkey's leading defense industry company. Since the day it was founded, ASELSAN has attached the utmost importance to R&D activities as well as the advancement of technology, and endeavors to achieve its goal of increasing its share in the international market by taking part in international projects.

For the last four years, ASELSAN sustained its position in the world as being in the list of Top 100 Defense Companies. ASELSAN's objective is to become one of the top 50 companies in the world through the development of original and national opportunities and talents of the highest level.

ASELSAN operates under four divisions:

Communications and Information Technologies,

Radar, Electronic Warfare and Intelligence Systems

Defense Systems Technologies

Microelectronics, Guidance & Electro-Optics Division.

ASELSAN operates in the fields of the design, development, production, system integration, and after-sales services of

1. Military Communications Systems
2. Radar Systems
3. Electronic Warfare Systems
4. Electro-Optic Systems
5. Navigation and Avionic Systems
6. Weapons Systems
7. Command Control Communication Computer Intelligence, Reconnaissance and Surveillance Systems
8. Naval Systems
9. Unmanned Systems
10. Traffic and Toll Collection Systems.

Production and engineering activities are realized in Macunköy and Akyurt facilities, all of them are located in Ankara.

The Communications and Information Technologies Division, Defense Systems Technologies and Radar, Electronic Warfare and Intelligence System Divisions and the headquarter are situated in Macunköy Facilities which is established on a total area of 187.000 m², of which 110.000 m² is closed area.

Microelectronics, Guidance and Electro optics Division is situated in Akyurt Facilities, which is nearby Ankara Esenboğa Airport, is established on a total area of 232.000 m², of which 54.000 m² is closed area.

Some part of the overall R&D activities with respect to Communications and Information Technologies Division are carried out in Aselsan Teknokent, a premise in Middle East Technical University, which is established on a total area of 4,500 m², of which 4,000 m² is closed area, which is also located in Ankara.

This research is done in Defense Systems Technologies Division and focused on Electronics Production Department Scheduling Problems.

Due to defense systems market conditions, ASELSAN Defense Systems Technologies Division is an Engineer-To-Order Company and it has High Mix/Low Volume Production type.

After ASELSAN Defense Systems Technologies Division has got sales contracts, engineering activities begins. For each "Sales contract", a "Project Name" is given. During and after engineering activities, Bill of Materials is stabilized and fixed. Master Production Schedules are created and Material Resource Planning algorithms are run according to master plans. Consequently, purchase orders and production orders are created in batches. Lot-Sizing procedure is made by ERP system. All of materials are procured or produced during these stages. For instance, during this research, company has got approximately totally different 25 projects running.

Electronic Production Department and Printed Wiring Board Assembly

In Electronic Production Department, Printed Circuit/Wiring Boards (PCBs), modules, LRU's and cables are produced by certified technicians. All personnel are trained in ESD and safety transportation. According to working shops, respected operators are certified periodically by authorized trainer regarding IPC-A-610, J-STD-001, IPC-A-620, IPC-7721/7711 standards. The production area with air conditioning and the floor completely covered with conductive dissipative material against ESD. The internal atmospheric pressure is held above that of outside to insure a dust free work place. All the work benches are covered with ESD dissipative mats.

All of multi-layer, double-sided PCBs which have surface mounted (SMD) and through-hole (leaded) components are produced according to IPC-A-610 standard.

In addition, the work benches in PCB assembly, visual inspection and repair touch up areas are equipped with overhead ionizing neutralizers. The effectiveness of the ESD preventive hardware is periodically checked using electrostatic field tester, surface resistivity tester, ground tester and wrist strap tester.

Additionally; According to the volume of production and test measurement complexity, different types of functional test methods are applied to the cable, harness, PCB, LRU and subsystems as known as manual tests methods or automated test methods. Automated test methods can be performed with general purpose or special to type automated test equipments. There are many standard test devices working between DC and up to 50 GHz frequency.

Briefly, 4 main stages exist for Printed Circuit/Wiring Board (PCB) assembly;

1. SMD (Surface Mounted Devices) assembly
2. Through-hole (leaded) components assembly and rework
3. Functional Test
4. Conformal Coating and Gluing

Therefore, this scheduling problem easily can be regarded as Hybrid Flow Shop scheduling problem with its own additional special features. As it is mentioned, each job has its own due-dates; each stage has its own capacity. And there are costs that company pays for tardy jobs and capacity increment.

Research Objectives:

The aim of this thesis is to study real company scheduling restrictions and Hybrid Flow Shop scheduling models and combine them together to design a

decision support tool for scheduling jobs. Therefore objectives of this thesis can be regarded as following:

1. Analyzing Hybrid Flop Shop Scheduling Problem and special features of the company.
2. Developing a suitable model for scheduling problem of the company.
3. Evaluating the research results and making this model a part of the decision support tools of the company.

1. PROBLEM STATEMENT

1.1. PRINTED WIRING BOARD ASSEMBLY ENVIRONMENT IN THE COMPANY

In Electronic Production Area, all electronic through-hole and SMD components such as resistors, diodes, capacitors, transistors, integrated circuits are mounted and electromechanical components such as heat sinks, screws, and RF screens are fixed on PCB's.

All of these assemblies are made simply through 4 main stages.

1. SMD Assembly
2. Through Hole Component Assembly and Rework
3. Functional Test
4. Conformal Coating and Gluing

Because of High Mix/Low Volume production structure, each stage is not fully automated and some stage usually encounters labor-intensive jobs. Except SMD assembly, remaining 3 stages can be regarded as labor intensive. Regarding this situation; for 3 stages, scheduling and capacity planning is done due to labor restrictions and for SMD Assembly stage scheduling and capacity planning is done due to machine restrictions. Each stage has parallel labor and machine resources. Because of High Mix, set-up times are inevitable for this case.

Additionally, approximately 1.000 PCBs of 80 different types are produced per month in this production area.

1. SMD (Surface Mounted Devices) Assembly

SMD components such as resistors, diodes, capacitors, transistors, integrated circuits are mounted. There are 2 parallel SMD machines in production line. Materials are prepared and put into feeders, trays and tubes before assembly and before machine run, an assembly program which is showing the exact

locations of materials on PCB's, should be prepared. Except setup activities, this stage can be considered as machine-intensive stage. Capacity planning, scheduling is done due to SMD Machines restrictions.



Figure -1: One of SMD Machine used in SMD Assembly Process

2. Through Hole Component Assembly and Rework

All electronic through-hole components and electromechanical components such as connectors, heat sinks, screws, RF screens are fixed and soldered on PCB's by using different kind of soldering methods such as wave soldering system, Selective Soldering System or manual soldering.

Additionally there is an extra stage called "Component Forming, Potting and Preparing" which can be regarded as setup of this stage. In this extra stage, Component leads are cut and formed, some small mechanical parts such as pins, rivets, carriers are formed and placed on pcb, peel able solder mask material applied to PCB's prior to PCB assembly. Therefore, this stage is a setup for "Through Hole Component Assembly and Rework" stage.

This stage can be considered as labor-intensive stage. Capacity planning, scheduling is done due to labor resource restrictions.



Figure-2: Wave-Soldering Machine



Figure-3: Selective-Soldering Machine

3. Functional Test

PCB's are checked if they are functioning properly in accordance with their documented specifications by using special test setups, and test equipments. The PCB's that pass the test, go on the consequent stage. But the PCB's that cannot pass, are sent to previous stage for repair/rework.

This stage can be considered as labor-intensive stage. Capacity planning, scheduling is done due to labor resource restrictions.

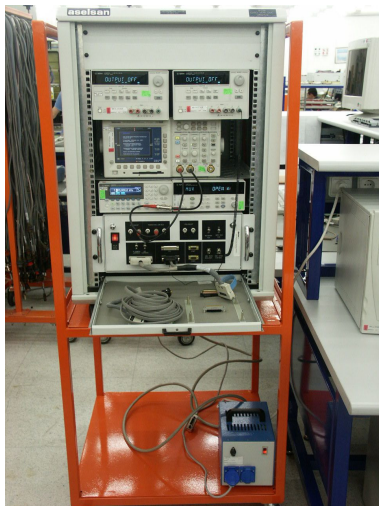


Figure-4: Sample Test Setup (1)



Figure-5: Sample Test Setup (2)

4. Conformal Coating and Gluing

The conformal coating process is applied to protect PCBs from degradation caused by dust, dirt, moisture, thermal cycling and exposure to solvents. This process is applied manually or by special automatic machine

Also, In order to stabilize the large components subject to mechanical shock in operation, gluing process is applied to fix the components to each other or prevent the leakage.

This stage also can be considered as labor-intensive stage. Capacity planning, scheduling is done due to labor resource restrictions.



Figure-6: Gluing Process

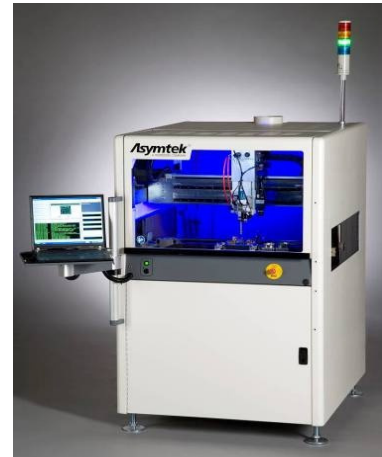


Figure-7: Conformal Coating Machine

The flow of PCBs can be demonstrated briefly as at Figure-8:

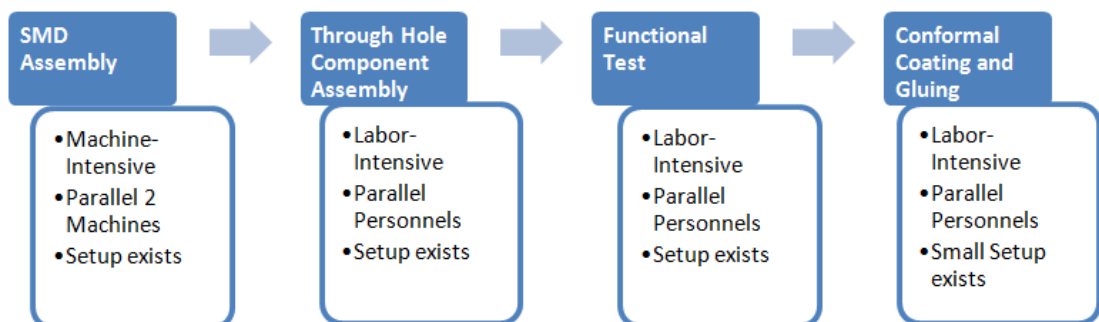


Figure-8: Flow of PCB Production

1.2. PRINTED WIRING BOARD ASSEMBLY PLANNING AND SCHEDULING PROBLEM

Due to various Project Plans and Master Production Schedules (MPSs), Engineering Departments fix and freeze Bill of Materials (BOMs) data of Materials. Afterward, Production Planning Department runs Material Resource Planning, explodes BOMs and creates production and purchase orders regarding Project Plans and MPSs.

Lots and batches are determined during this activity. There is not a methodic lot-sizing strategy in the company. "Lot-for-Lot" Lot-Sizing method is used usually. For the cases, "Lot for Lot" method is not used, Production Planning Engineers decide the lots for each material type without regarding a common algorithm.

After material procurement activities, various production orders come to Electronics Production Department in High Mix/low Volume type.

Each production order includes a release date, due date, quantity, material name, routing, documents needed, sub-materials needed. Subsequent to production orders arrival, capacity constraints are included manually by Shop Floor Engineer. During Manufacturing Execution stage, in line with due-dates, release-dates, and capacity min/max borders, Shop Floor Engineer schedules each production order by splitting/not splitting the batches, increasing/decreasing capacity of each stage.

As it's seen, this scheduling problem has its own additional special features and can be regarded as Hybrid Flow Shop scheduling problem.

Therefore, this scheduling problem has;

Assumptions:

1. For each stage, setup time of each job is independent from other jobs' setup times. (Sequence – independent).

Because of Engineer to Order and High Mix/low Volume production type of company, there is no product line and grouping of products, so one type of

product do not resemble another. This causes totally different setups for each product and totally different and independent setup times for each product.

2. Some obligatory wait-times just like oven, cleaning etc. are not managed.

Wait-times are assumed that they are occurred after work-hours. These kinds of waiting times for each product make the schedules totally unmanageable. So as to manage these wait-times, the company put them often after work-hours.

3. Material handling times are not managed.

In the ASELSAN Defense Systems Technologies Division PCB Production Area, there is at most 15 meters distance and it takes at most 2 minute walk between stages. Therefore, the company neglects these material handling times in their schedules.

4. All of parallel labor and machine resources that are at the same stage are identical.

In order to formulize and calculate the capacity of each stage, company makes their schedules by neglecting differences of each resource.

5. Processing times of each job is deterministic.

In scheduling studies hold during and before production in company, process times are determined previously and if they are changed during production, all of scheduling studies are redone.

6. Lot splitting is allowed in each stage.

Lots for each production order are not determined via common algorithm. Frequently, while scheduling production orders, lots of production orders are divided into sub-lots because of capacity constraints of stages or different urgency conditions of sub-lots.

7. Raw material, tooling or infrastructure availability is not managed.

As in all scheduling problems, in order to take healthy solutions from scheduling methods, these kinds of availability problems are neglected by the company.

8. Processed units are always satisfactory.

In order to handle Reworks/Fail Processes for jobs, the company chooses to re-schedule them as another job.

9. Some sub-lots of customer order can have different due date than remaining part of customer order.

The company uses the way of re-scheduling these Sub-lots with different due dates as another job.

10. Unlimited buffer between stages.

The company has no determined space limit between and in stages.

Given Data:

1. Release date of each job.

Every production order is coming to production area after kitting process at warehouse. Therefore, each production order has a release date. On the other hand, same date can be named as completion date of kitting process of that production order at warehouse. These release dates are determined by Production Planning Engineers in the company.

2. Due date of each job.

Production Planning Engineers determine due date of each job according to Master Production Plan and MRP.

3. Min/max capacity borders in each stage during a time period.

As a result of labor-intense stages, capacity of each stage can be changed by managers due to capacity constraints. Minimum and maximum level of

capacity for a time period is determined and capacity decisions are made due to these borders.

4. Process time and setup time of each job.

Every production order has pre-determined, deterministic process time and setup time for each stage.

5. Costs of each tardy jobs and capacity increment.

In order to make meaningful schedules, unit costs for tardy jobs and capacity increment are predetermined by managers.

Constraints:

1. Capacity of each stage has minimum and maximum borders for labor and machine resources.
2. Every job or part of the batch has due date and release date.

Decision Variables:

1. Start time of each stage for each job
2. Finish time of each stage for each job
3. Can batches be divided to sub-lots? Or will whole batch be processed?
4. How many capacity units used in each stage for a time-period

Objectives:

1. Minimizing total cost of tardy jobs.
2. Minimizing total cost of capacity increments at each stage.

A small Example of Problem:

In order to illustrate ASELSAN Defense Systems Technologies Division PCB Production Scheduling Problem, a small example with 6 production order can be examined.

Production Order #	Stock Number	Amount (Unit)	Release Date	Due Date
1	A	10	01.01.2014	08.01.2014
2	B	5	02.01.2014	08.01.2014
3	C	1	03.01.2014	06.01.2014
4	D	12	01.01.2014	08.01.2014
5	E	8	01.01.2014	07.01.2014
6	F	4	01.01.2014	08.01.2014

These 6 production orders come to PCB production area, and then process times and setup times of these production orders are determined such as:

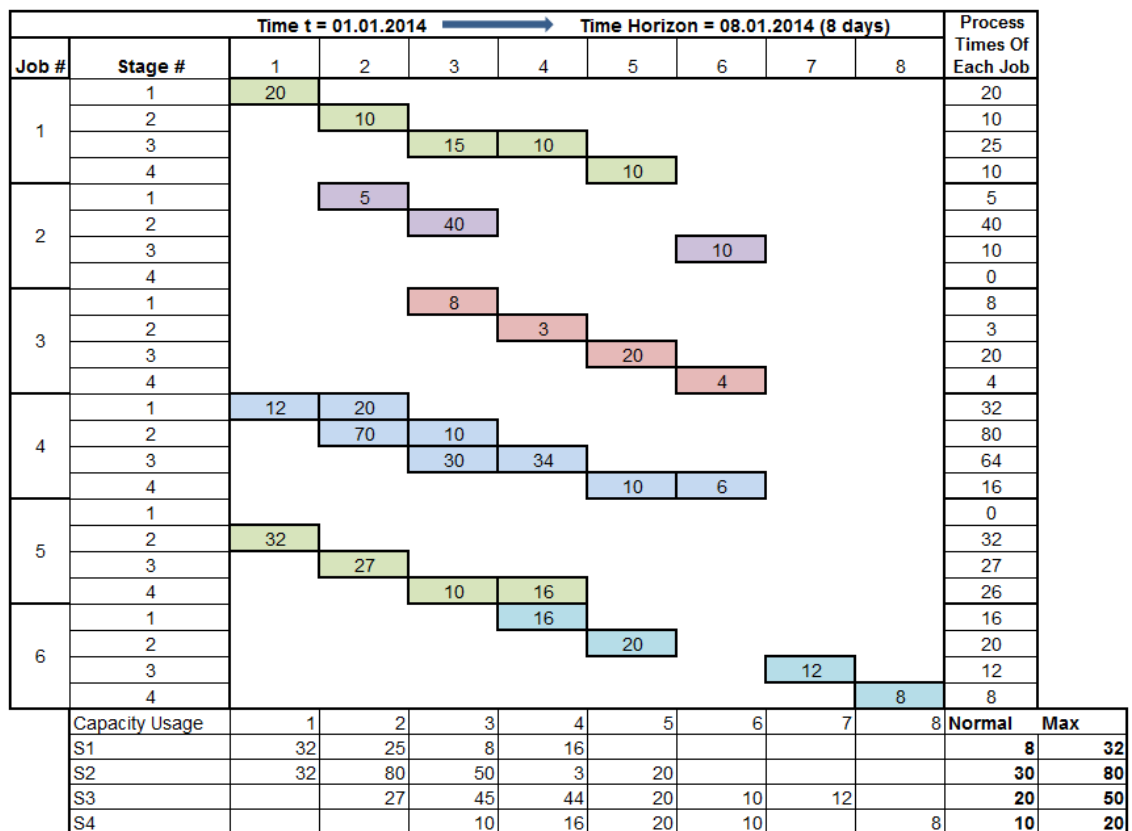
Process Time Matr	Stage #			
	1	2	3	4
	20	10	25	10
	5	40	10	0
	8	3	20	4
	32	80	64	16
	0	32	27	26
	16	20	12	8

Setup Time Matrix	Stage #			
	1	2	3	4
	8	2	4	1
	4	3	3	0
	4	1	2	1
	12	3	4	2
	0	1	2	3
	8	2	1	1

Then minimum and maximum borders for capacity of each stage for each time period are determined.

Stage #	Capacity Borders (hr)	
	Minimum	Maximum
1	8	32
2	30	80
3	20	50
4	10	20

After all, according to capacity, due date, release date constraints, a feasible schedule is found and executed manually by Shop Floor Control Engineer as following: (Please note that setup times are not included in this demonstration.)



Although it is a small example, you can see capacity-usage imbalance over time periods in the schedule. Regarding that the company schedules from 80 to 110 jobs daily, it is not hard to see the complexity of this problem. During scheduling and manufacturing execution, company faces periodically many due-date and capacity violations.

This research focuses on this complex problem and aims to develop a decision support tool for scheduling from 80 to 110 jobs for ASELSAN Defense Systems Technologies Division.

2. LITERATURE REVIEW

2.1. HYBRID FLOW SHOP SCHEDULING PROBLEM

Hybrid Flow Shop (HFS) (Gupta, 1988; VoX, 1993) is a frequently-used production type with multi-stage, parallel resources in each stage and jobs having same production flow. The queues between the various stages may be managed by various disciplines (First In First Out, Earliest Due Date etc.). HFS is named also as “Flexible Flow Shop” or “Multi-Processors Flow Shop” in literature. (M. Pinedo, 2002) A machine/labor can process at most one job at a time and a job can be processed by at most one machine/labor at a time. (Tamás Kis, Erwin Pesch, 2005) In real world, this special production type has been used generally by the companies that have Low Volume/High Mix production. In Electronics, Foods, Textile Industries, this type can be seen broadly.

HFS production type can be demonstrated mathematically as: (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

3. There should be more than one stage for production. $s > 1$
4. Each stage can have more than 1 parallel resource. $K > 0$
5. Each job should follow same routing. stage1-> stage2 -> stage3-> ...

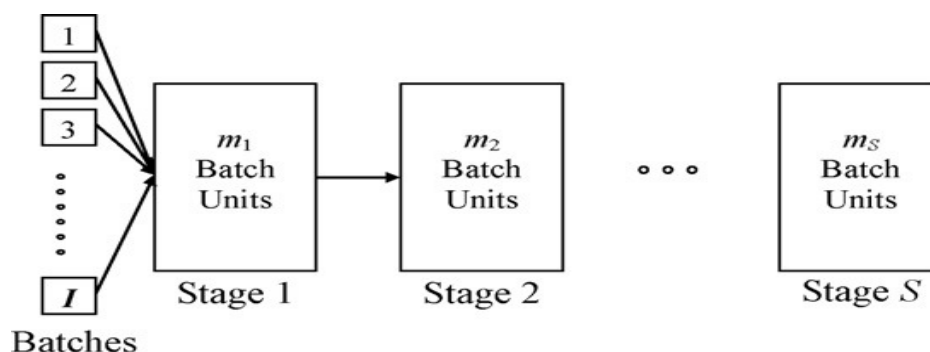


Figure – 9: Schematic of Hybrid Flow Shop with identical parallel units. (Yu Liu, I.A. Karimi, 2008)

HFS scheduling problem can be defined as; finding an optimal or near-optimal schedule of all jobs due to a given objective. However, HFS scheduling problem in most cases NP-Hard. (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

HFS scheduling problems vary because of different industry applications and manufacturing environments. This carries different assumptions, constraints, variables, parameters and objectives to HFS scheduling problem.

In literature, for solving different kinds of HFS scheduling problem, several exact methods and heuristics methods are proposed. Additionally, for stochastic type of this problem, there are some special heuristics (metaheuristics) just like Simulated Annealing, Genetic Algorithms and Tabu Searches etc.

Commonly, scheduling problems are classified according to shop-configuration (α), set of constraints and assumptions (β), objective functions (γ) and are denoted simply as $\alpha | \beta | \gamma$. (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

Shop-configuration (α)

Shop-configuration (α) identifies the number of stages, number of machines in each stage and characteristics of machines. Parameter (α) is composed of 4 characteristics that are defined in Table 1.

Set of constraints and assumptions (β)

This parameter described in Table 1, demonstrates special features, constraints and assumptions of manufacturing area.

Objective functions (γ)

This parameter depicts the objective function type that is used for that specific scheduling problem.

Parameters of Scheduling Problems
Shop-configuration (α)

α_1	<p>General type of Manufacturing Area. Flow-Shop, Job-Shop, Open-Shop etc.</p> <p>Example: FH = Hybrid Flow Shop</p>
α_2	Number of Stages
α_3 and α_4	<p>Properties of Machines in each stage</p> <p>Example: $\alpha_3 \in \{\emptyset, P, Q, R\}$</p> <p>P = identical parallel machines</p> <p>Q = uniform parallel machines</p> <p>R = unrelated parallel machines</p> <p>\emptyset = single machine</p>
Set of constraints and assumptions (β) (Most common parameters)	
r_j	job j cannot start processing before its release date r_j
prmu	jobs are processed in every stage in the same order
prec	there are precedence constraints between operations from different jobs.
M_j	the processing of job j is restricted to the set of machines M_j at stage k. This is known as eligibility.
S_{sd}	the setup times are dependent on the sequence of operations.
prmp	preemptions are permitted.
block	the buffer capacities between stages are limited. The jobs must wait in the previous

	stage until sufficient space is released.
recrc	jobs are allowed/required to be processed more than once in the same stage.
unavail	machines are not available at all times.
no – wait	jobs are not allowed to wait between two successive stages.
$p_j = p$	all processing times are equal to p.
Split	if batches of jobs can be divided into parts. (Larysa Burtseva, Rainier Romero, Salvador Ramirez, Victor Yaurima, Félix F. González-Navarro and Pedro Flores Perez (2012))
Objective functions (γ)	
$C_{\max} = \max_j C_j$	maximum completion time
$F_{\max} = \max_j(C_j - r_j)$	maximum flow time
$L_{\max} = \max_j(L_j)$	maximum lateness
$T_{\max} = \max_j(T_j)$	maximum tardiness
$E_{\max} = \max_j(E_j)$	maximum earliness
$C = \sum C_j$	total/average completion time
$C^w = \sum w_j C_j$	total/average weighted completion time
$F = \sum F_j$	total/average flow time
$F^w = \sum w_j F_j$	total/average weighted flow time
$T = \sum T_j$	total/average tardiness
$T^w = \sum w_j T_j$	total/average weighted tardiness
$U = \sum U_j$	number of late jobs
$U^w = \sum w_j U_j$	total/average weighted number of late

	jobs
$E = \sum E_j$	number of earliness
$E^w = \sum w_j E_j$	total/average weighted earliness

Table-1 (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

Therefore, as an example, a HFS scheduling problem that has;

- m stage
- parallel uniform machines at each stage
- unlimited buffer between stages
- preemption-forbidden
- release dates of all jobs
- set-up times independent from each other
- allowed-lot-splitting
- objective function that is sum of number of earliness and number of tardy jobs

can be denoted simply as $\alpha_1 \alpha_2 \alpha_3 \alpha_4 \beta \gamma$ **FHm (PM^(k))_{k=1}^m | r_j, split, S_{nsd} | U + E. (Table-1)**

For solving different kind of HFS scheduling problems, there are two ways proposed in literature;

1. Exact methods such as; Integer Programming and Branch & Bound
2. Heuristics and metaheuristics such as; Complicated Dispatching Rules, Simulated Annealing, Genetic Algorithms and Tabu Search Algorithms etc.

2.2. SOLVING METHODS FOR HFS SCHEDULING PROBLEM

2.2.1. Exact Methods:

In literature, for solving HFS Scheduling problem, different type Branch and Bound (B&B) Based Algorithms and Integer Programming Models are used widely.

B&B Based Algorithms are briefly concentrates on canceling non-promising solutions as far as possible during the run. This decreases the branches of search tree and it helps to find optimum much easier (Ahmet Bolat, Ibrahim Al-Harkan, Bandar Al-Harbi , 2005). In literature, from easier version of HFS scheduling problem (2 or 3 stages and 5-10 jobs) to hardest ones (n stages and m jobs), various B&B based Algorithms can be found.

Integer programming is also often used to formulate and solve this HFS scheduling problem. Again, there are different types of Integer Programming Models for different types of HFS's. However, general HFS scheduling problem that has negligible setups, no release dates, not allowed preemption, unlimited buffer between stages, identical machines in each stage, can be formulated such as: (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

Index:

j = job number

k = stage number

l = machine number

Parameters:

p_{jk} = processing time required by job j in stage k

Q = Big Number

Decision Variables:

c_{jk} = completion time of job j in stage k . $c_{j0} = 0$.

$Y_{jkl} = 1$ if job j on stage k is assigned at machine l

0 otherwise

$X_{jrk} = 1$ if job j precedes job r on stage k

0 otherwise

$$\text{minimize } Z \tag{1}$$

s.t.

$$Z \geq c_{jm}, \forall j \tag{2}$$

$$\sum_{l=1}^{M^{(k)}} Y_{jkl} = 1, \forall(j, k) \tag{3}$$

$$c_{jk} - c_{j,k-1} \geq \sum_{l=1}^{M^{(k)}} Y_{jkl} p_{jk}, \forall(j, k) \tag{4}$$

$$Q(2 - Y_{jkl} - Y_{qkl} + X_{jqk}) + c_{jk} - c_{qk} \geq p_{jk}, \forall(j, k, l, q) \text{ such that } j < r \tag{5}$$

$$Q(3 - Y_{jkl} - Y_{qkl} - X_{jqk}) + c_{qk} - c_{jk} \geq p_{qk}, \forall(j, k, l, q) \text{ such that } j < r \tag{6}$$

$$Y_{jkl} \in \{0, 1\}, \forall(j, k, l) \tag{7}$$

$$X_{jrk} \in \{0, 1\}, \forall(j, q, k) \tag{8}$$

$$c_{jk} \geq 0, \forall(j, k). \tag{9}$$

Figure – 10: Basic of Hybrid Flow Shop Mixed Integer Programming Model (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

Z is restricted to be greater or equal to the completion time of the last operation to finish its processing, i.e., the makespan. Several other criteria will be discussed in Section 3. The set of constraints (3), guarantees that all operations are assigned strictly to one machine at each stage. Constraint set (4), restrict the starting time of operation o_{jk} to be greater or equal to its release time from the previous stage. Constraint sets (5) and (6) prevent any two operations from overlapping in a common machine. Constraint sets (7), (8) and (9) define the domains of the decision variables (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009).

Furthermore, in literature, there are Mixed Integer Programming applications for PCB Production Companies. In (Tadeusz Sawik, Andreas Schaller, Thomas M. Tirpak, 2002 and Tadeusz Sawik, 2001), there are convenient MIP applications for HFSs which have no storage or limited-storage between stages, no setup, no lot-splitting, medium to high volume and SMT (Surface Mount Technology) machines-intense features. In (Tadeusz Sawik, 2000), there is a suitable MIP application for HFS that has jobs having different routes, limited-storage between stages, no setup, no lot-splitting, medium to high volume and SMT machines-intense features. In (Tadeusz Sawik, 2002), there is a fitting MIP application for HFS that has batches of different type of units, limited-storage between stages, no setup, no batch-splitting, medium to high volume and SMT machines-intense features. Additionally, a hierarchically MIP approach to HFS scheduling problem exists in (Tadeusz Sawik, 2006). Two MIP models are demonstrated, by first MIP model, which sub production lots of customer orders has to be scheduled in the periods of time-horizon, is determined, by second MIP model, which machines in each stage are assigned to each sub production lot of customer orders, is determined. Over again, this model is created for HFSs that have no setup times, predetermined production lots for each customer order (therefore no lot-splitting via MIP modeling), medium to high volume and SMT machines-intense characteristics. Another MIP Approach and reactive scheduling algorithms exists in (Tadeusz Sawik, 2007) for HFSs that have predetermined production lots, limited-lot-splitting (lot splitting allowed only for specific pre-determined customer orders and only for two subsequent periods), no setup, often-rescheduling environment, make to order, medium to high volume traits. Different rescheduling algorithms (REALL, REMANT, RENON etc.) are compared via computational results.

Additional to PCB Production Application, there are other MIP applications for different HFS types. In (Yu Liu, I.A. Karimi, 2008), there are slot-based and sequence based MIP approaches for HFSs that have Zero wait between stages/No Inter-stage buffer, no setup times, parallel non-identical resources/units, weighted Just In time objectives, sequence-dependent batches/jobs, no lot-splitting traits.

A convenient MIP approach for real steel company applications exists in (Stefan Voss, Andreas Witt, 2007). Similar to ASELN HFS Problem, this MIP application models a steel company HFS problem with characteristics; parallel identical machines, unlimited inter-stage buffer, sequence-dependent setup times, lot-splitting, minimum weighted set-up times and tardiness objectives.

Special lot-splitting MIP model for reed switch manufacturing exists in (Larysa Burtseva, Rainier Romero, Salvador Ramirez, Victor Yaurima, Félix F. González-Navarro and Pedro Flores Perez (2012)). This MIP application has traits; two stations with parallel identical machines (FH2), at first station allowed lot-splitting, sequence independent-setup times, unlimited inter-stage buffer, high-volume. Besides MIP application, lot-splitting issue is comprehensively examined.

2.2.2. Heuristics and Meta-Heuristics

For every type of scheduling problem in different type of industries, -from simplest to complex- heuristics and algorithms can be found in literature. Simplest heuristics in literature are dispatching rules for HFS problems. These rules are easy to use and do not need hard computational effort. They are designed for finding near-optimal solutions. As example, Johnson's rule, Kusiak's Rule, Gupta's Heuristic, Palmer's Heuristic, CDS and RA Heuristics for Flow Shop Scheduling, Earliest Due Date, Shortest Process Time Dispatching Rules are simplest ones.

By development of computer technology, some stochastic and variable elements are started to be included in algorithms. These kinds of algorithms and heuristics are called "Metaheuristics". Complicated Dispatching Rules, Simulated Annealing, Genetic Algorithms and Tabu Search Algorithms can be given as examples for this kind. (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009)

3. APPROACH AND MODEL PROPOSED

3.1. APPROACH

In order to formulate and develop a HFS Mixed Integer Programming Model for ASELSAN Defense Systems Technologies Division Production Area, the model that is proposed should have assumptions, constraints, given data and objective function such as:

Assumptions:

1. HFS should have 4 stages. However, it is good to remember that an m-stage model can handle this problem.
 - a. SMD Assembly
 - b. Through Hole Component Assembly and Rework
 - c. Functional Test
 - d. Conformal Coating and Gluing
2. Each stage has parallel labor and machine resources
3. Because of High Mix/low Volume, set-up times are inevitable and setup time of each job is independent from other jobs' setup times. (sequence – independent)
4. Material handling times are not managed
5. Some obligatory wait-times just like oven, cleaning etc. are not managed. Wait-times are assumed that they are occurred after work-hours.
6. Material handling times are not managed.
7. All of labor and machine resources that are at the same stage are identical.
8. Processing times of each job is deterministic.

9. Lot splitting is allowed in each stage.
10. Raw material, tooling or infrastructure availability is not managed.
11. Processed units are always satisfactory. Reworks/Fail Processes for jobs should be re-scheduled as another job.
12. Some sub-lots of customer order can have different due date than remaining part of customer order. Sub-lots that have different due dates should be re-scheduled as another job.
13. Unlimited buffer between stages.

Given Data:

1. Release date of each job
2. Due date of each job
3. Min/max capacity borders in each stage during a time period
4. Process time and setup time of each job
5. Unit Costs of each tardy job and capacity increment

Constraints:

1. Capacity of each stage has minimum and maximum borders for labor and machine resources.
2. Every job or part of the batch has due date and release date.

Decision Variables:

1. Start time of each stage for each job
2. Finish time of each stage for each job
3. Can batches be divided to sub-lots? Or will whole batch be processed?
4. How many capacity units used in each stage for a time-period

Objectives:

Minimizing total cost of tardy jobs.

Minimizing total cost of capacity increments at each stage.

Therefore for simple notation, this kind of HFS scheduling problem, can be denoted simply as;

FHm (PM^(k))_{k=1}^m | r_j, split, S_{nsd} | U + Total Capacity

And ASELSAN Defense Systems Technologies Division simply waits from this research to adopt a decision support tool that is finding optimal schedules in line with due dates and release dates of all jobs (**from 80 up to 110**) at four stages via using lot-splitting and capacity increase/decrease decisions.

MIP model that is included in (Stefan Voss, Andreas Witt, 2007) can be considered as a fit model for ASELSAN's HFS Scheduling Problem because the model has traits; parallel identical machines, unlimited inter-stage buffer, sequence-dependent setup times, lot-splitting, minimum weighted set-up times and tardiness objectives. The MIP model proposed in (Stefan Voss, Andreas Witt, 2007) includes sequence-dependent setup times and lot-splitting by using model approach in (Haase, K., 1994), minimum weighted set-up times and tardiness objectives different to the MIP model that is proposed in (Talbot, F.B., 1982). This model is prepared for a steel company HFS problem.

On the other hand, it is good to remember that, different from the model in (Stefan Voss, Andreas Witt, 2007), ASELSAN Defense Systems Technologies Division HFS Scheduling problem has sequence-independent setup times and allowed-capacity decrease/increase attribute.

The model in (Stefan Voss, Andreas Witt, 2007) is;

FHm (PM^(k))_{k=1}^m | split, S_{sd} | T^w (Stefan Voss, Andreas Witt, 2007)

Index:

j = job number

j = 1,.....J+1

t	= each time period	$t = 1, \dots, T$
m	= stage number	$m = 1, \dots, M$
r	= resource number in each stage	$r = 1, \dots, R$
a	= setup state of resource r	$a = 1, \dots, A_r$

Parameters (Given Data):

$d_{j,m}$	= process time of job j in stage m
d_j	= total process time of job j
P_j	= immediate predecessors of a job j
S_j	= immediate successors of a job j
T	= Time Horizon
dd_j	= due date of job j
$wt_{h,m}$	= waiting time for predecessor job h of job j in mode m

Earliest Start Time

$$EST_j = \begin{cases} 0 & \text{if } P_j = \{0\}; \\ \text{Max}((EST_h + d_h) \mid h \in P_j) & \text{otherwise} \end{cases}$$

Latest Start Time

$$LST_j = \begin{cases} T - d_j & \text{if } j \in P_{j+1} \\ \text{Min}((LST_k - d_j) \mid k \in S_j) & \text{otherwise} \end{cases}$$

$c_{j,m,r,a}$ = capacity consumption of job j in stage m at resource r of setup state a

$scr_{r,b,a}$ = setup cost matrix from b setup state to a setup state for resource r

scr_r^{\max} = maximum setup cost that is incurred at resource r

$st_{r,a}$ = minimal time that setup state a requires at resource r

Decision Variables:

$C_{r,t}$	=	1	if r resource is working at period t
		0	not-working or break-down

$x_{j,m,t}$ = 1; if job j is completed in period t in mode m ;
0; otherwise

$SC_{r,t}$ = setup cost incurred at resource r in time period t

$Y_{r;a,t}$ = 1; if resource r is setup in state a in period t ;
0; otherwise

Total Model: (Stefan Voss, Andreas Witt, 2007)

$$\text{Minimize } \sum_{r=1}^R \sum_{t=1}^T SC_{r,t} + \lambda * \sum_{t=EFT_{j+1}}^{LFT_{j+1}} t * x_{j+1,1,t} + (1-\lambda) * \sum_{j \in P_{j+1}} \sum_{m=1}^{M_j} \sum_{t=\max(EFT_{j+1}, dd_j)}^{LFT_j} w_j * (t - dd_j) * x_{j,m,t}$$

Subject to

$$\sum_{m=1}^{M_j} \sum_{t=EFT_j}^{LFT_j} x_{j,m,t} = 1 \quad j=1, \dots, J+1 \quad (1)$$

$$\sum_{m=1}^{M_h} \sum_{t=EFT_h}^{LFT_h} (t + wt_{h,m}) * x_{h,m,t} \leq \sum_{m=1}^{M_j} \sum_{t=EFT_j}^{LFT_j} (t - d_{j,m}) * x_{j,m,t} \quad j=1, \dots, J+1, h \in P_j \quad (2)$$

$$\sum_{j=1}^J \sum_{m=1}^{M_j} \sum_{\tau=\max(t, EFT_j)}^{\min(t+d_{j,m}-1, LFT_j)} c_{j,m,r,a} * x_{h,m,\tau} \leq C_{r,t} * y_{r,a,t} \quad r=1, \dots, R \quad t=1, \dots, T \quad a=1, \dots, A_r \quad (3)$$

$$\sum_{a=1}^{A_r} y_{r,a,t} = 1 \quad r=1, \dots, R \quad t=1, \dots, T \quad (4)$$

$$SC_{r,t} - sc_r^{\max} * y_{r,a,t} + \sum_{b=1}^{A_r} (sc_r^{\max} * y_{r,b,t-1} - sc_{r,b,a} * y_{r,b,t-1}) \geq 0 \quad r=1, \dots, R \quad a=1, \dots, A_r \quad t=1, \dots, T \quad (5)$$

$$\sum_{v=t}^{t+st_{r,a}-1} y_{r,a,v} \geq st_{r,a} (y_{r,a,t} - y_{r,a,t-1}) \quad r=1, \dots, R \quad a=1, \dots, A_r \quad t=1, \dots, T \quad (6)$$

$$SC_{r,a,t} \geq 0 \quad r=1, \dots, R \quad a=1, \dots, A_r \quad t=1, \dots, T \quad (7)$$

$$y_{r,a,t} \in \{0,1\} \quad r=1, \dots, R \quad a=1, \dots, A_r \quad t=1, \dots, T \quad (8)$$

$$x_{j,m,t} \in \{0,1\} \quad j=1, \dots, J+1 \quad m=1, \dots, M_j \quad t=1, \dots, T \quad (9)$$

$$0 \leq \lambda \leq 1$$

It's seen that; objective function of the model composes of three elements. First one is, total setup change cost in each resource at each time period. Second one is, weighted total completion time of each job therefore makespan. Third one is weighted total tardy jobs. (Stefan Voss, Andreas Witt, 2007)

First constraint (1) is providing each job is completed exactly once during time horizon (T). Second constraint (2) ensures precedence relation of each job with other jobs and adding a waiting time to each job's process time for cooling time of heated materials. Third constraint (3) prohibits capacity violations in each time period for each resource and each setup state. Fourth constraint (4) assures every resource to have only one setup state in each period. Fifth constraint (5) provides that a variable setup cost occurs if a resource (r) changes its setup state from a to b from the period t-1 to t. Sixth constraint (6) is about a special case for steel company which is; "if a resource change its setup state from 1 to 3 than there is no job of transition 2 state available. Therefore, whenever this kind of case occurs, this constraint implies setup times with duration of zero or minimum $st_{r,a}$." (Stefan Voss, Andreas Witt, 2007)

As a result, this model decides each time period that each job is completed and each time period that each resource at each stage changes its setup state by minimizing sum of total setup cost, weighted total completion time and weighted total tardy jobs. (Stefan Voss, Andreas Witt, 2007)

This model is most approximate proposed-model in literature for ASELSAN Defense Systems Technologies Division HFS Scheduling problem. **Also, it should be mentioned that the model in (Stefan Voss, Andreas Witt, 2007) is more comprehensive than model that is demonstrated in this thesis. And it includes a complicated solution for a harder scheduling problem than ASELSAN has.** However, regarding two primary points, it is clear that this model cannot be directly applied to ASELSAN Defense Systems Technologies Division. It should be converted for special case of the company. First thing to change is, the company's HFS problem has setup-independent trait which means each job has independent setup time from other jobs in each stage because of company's High Mix / Low Volume issue. Second one is; the company has three labor-intensive stages (Through Hole Component Assembly and Rework, Functional Test and Conformal Coating and Gluing) and one machine-intensive stage (SMD Assembly). Therefore, ASELSAN Defense Systems Technologies Division wants to schedule all jobs due to labor-intense

constraints too. And capacity increase/decrease in each stage is allowed, probable and frequently used.

For that reason, a new model approach is needed for this special case and this is what this research is all about.

3.2. MODEL PROPOSED

As mentioned before, simply ASELSAN Defense Systems Technologies Division waits from this research to adopt a decision support tool that is finding optimal schedules in line with due dates and release dates of all jobs (**from 80 up to 110**) at four stages via using lot-splitting and capacity increase/decrease decisions.

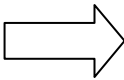
This Hybrid Flow Shop Scheduling Problem starts with arrival of various production orders to Electronics Production Department in High Mix/low Volume type. Each production order includes a release date, due date, quantity, material name, routing, documents needed, sub-materials needed. Subsequent to production orders arrival, capacity constraints are included manually. During Manufacturing Execution stage, in line with due-dates, release-dates, and capacity min/max borders, each production order in time horizon (T) is scheduled by splitting/not splitting the batches, increasing/decreasing capacity of each stage.

In order to comprehend all these assumptions, constraints and objectives of ASELSAN Defense Systems Technologies Division Hybrid Flow Shop Scheduling Problem, we start with dividing scheduling time horizon (T) to equal time periods as it is mentioned in (Stefan Voss, Andreas Witt, 2007). This equal time periods can be regarded as 1 work-day, 1 work-hour or 4 work-hours etc. and it should be determined before. For ASELSAN's case, 4 work-hours time period and $60 \times 4 \text{ Work - Hours} = 30 \text{ Days Time Horizon}$ will be used. This will ease to handle entire HFS problem.

Additionally, for the company's case, it will be good to think of process times of each job for all stages in terms of pre-determined equal time periods. Therefore, process time matrix is converted from hour or second based process time to this pre-determined time periods based process times. For instance;

First matrix is converted to second one by rounding up values as it is demonstrated below.

Process Time Matrix (Hour-Based)	Stage #			
Job #	1	2	3	4
1	8	12	16	4
2	4	2	4	6
3	8	9	6	4



Process Time Matrix (Pre-determined Time Period = 4-hours)	Stage #			
Job #	1	2	3	4
1	2	3	4	1
2	1	1	1	2
3	2	2	2	1

This process-time conversion and pre-determined time periods provides our model to handle the capacity and lot-splitting notion. In order to understand this special scheduling problem easily, the Figure-11 can be used. As it's seen, there are four important assumptions to make the problem easier.

1. Every job has process times at each stage in terms of pre-determined time-periods.
2. In each period, there is capacity limits for all stages and if these capacity limits are exceeded, capacity-cost occurs.
3. At all stages, batches of jobs can be divided into minimum sub-lots and sub-batches which's total process time at that stage should be equal to pre-determined time period.
4. At every lot-split occasion, setup time occurs again. Therefore, for our new model, setup cost will balance lot-splitting occasions.

It is important to add that same should be applied to setup times and setup times should be converted from hour or second based setup time to this pre-determined time periods based setup times.

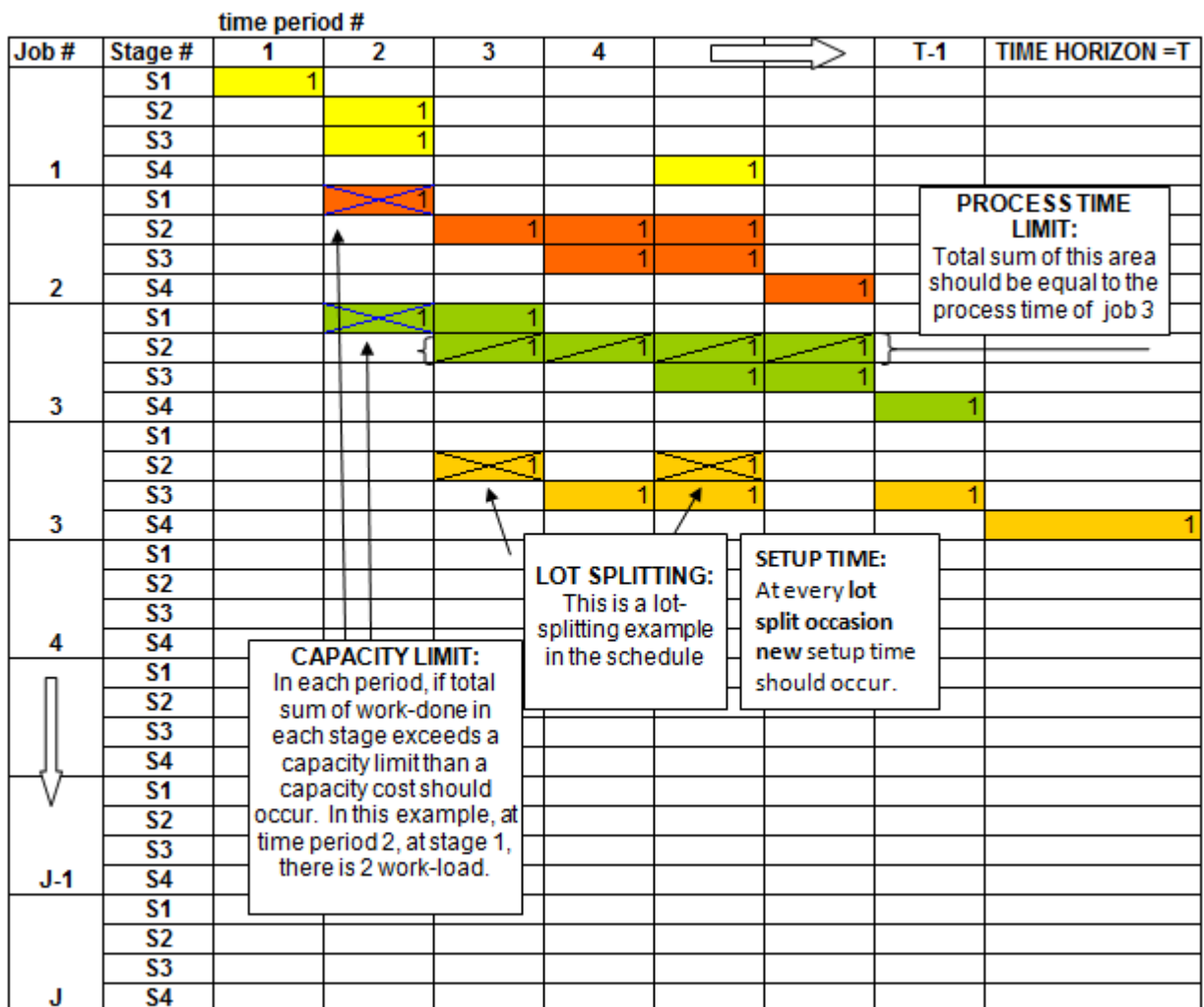


Figure- 11: Easy schematic of ASELSAN Defense Systems Technologies Division Hybrid Flow Shop Scheduling Problem

It is easy to see that these assumptions make it easier to develop a Mixed Integer Programming Model for ASELSAN Defense Systems Technologies Division Hybrid Flow Shop Scheduling Problem, solve it and obtain optimal schedules of all jobs (from 80 to 110 jobs).

Considering ASELSAN Defense Systems Technologies Division Hybrid Flow Shop Scheduling Problem as whole, major indexes in the proposed model are:

j	= job number	$j = 1, \dots, J+1$
t	= each time period	$t = 1, \dots, T$
s	= stage number	$s = 1, \dots, S$

There are parameters (given-data) given by the company and used in the model and they are:

$p_{j,s}$	= process time of job j in stage s
$ST_{j,s}$	= setup time of job j in stage s
T	= Time Horizon
rd_j	= release date of job j
dd_j	= due date of job j
$\min C_{s,t}$	= minimum capacity level at each stage and in each time period
$\max C_{s,t}$	= maximum capacity level at each stage and in each time period
$pbin_{j,s}$	= 1 if there is a process time of job j at stage s 0 otherwise

TH = Time Horizon

CC = Capacity cost for each increment

TJC = Cost paid for each Tardy job

$BigM$ = Big number

The model that is proposed in this research begins with finding decision variables. Most important decision variable is;

$X_{j,s,t} = 1$; if s th stage of job j is being done in period t ;

0; otherwise

This decision variable is used mainly for capacity limitations, process time limitations, setup costs determination and lot-splitting. If $X_{j,s,t}=1$, this means “**one time period part**” of process time of s th stage of job j is done during the period t. If $X_{j,s,t}=0$, then there is not work-done in the period t for sth stage of job j.

Another important decision variable is;

$S_{j,s,t} = 1$; if a setup occurs at s th stage of job j is completed in period t
0; otherwise

This acts like balancing variable for lot-splitting or not decisions. By adding setup cost for each lot-splitting event to objective function will force the model to balance these lot-splitting events. For each stage of each job, If $X_{j;s;t-1}$ equals to 0 and $X_{j;s;t}$ equals to 1 then $S_{j;s;t}$ will be 1. Else $S_{j;s;t}$ will be 0 in the model.

Capacity decision variable is another positive integer decision variable;

$C_{t,s}$ = capacity at each stage in t th time period.

In each time period, there is a minimum level of capacity for each stage, and maximum level of capacity of each stage. This variable cannot exceed maximum level for each stage and each time period. And if this variable takes a value bigger than minimum level, company is paying capacity increment cost for the amount of $(C_{t,s} - \text{minimum level}_{t,s})$.

Finish time positive integer decision variable is:

F_j = Completion time of j th job.

This is exact time of completion time of each job.

Tardy Job decision variables are:

$TA_j = 1$; if Finish time is greater than due date of each job. ($F_j > dd_j$)

0; otherwise

These binary variables act for balancing costs of tardy jobs.

As mentioned before ASELSAN Defense Systems Technologies Division aims to minimize total cost of tardy jobs and minimize total cost of capacity increments at each stage. Therefore, proposed model in this research has an objective function contains three of them but weights them using different costs for them. (CC and TJC.)

First one is for total cost of tardy jobs;

$$\sum_{j=1}^J TJC * TA_j$$

Second one is for total cost of capacity increments;

$$\sum_{j=1}^J CC * (C_{t,s} - \min C_{t,s})$$

For third one, this part also forces the model for using 0 resources (below the minimum capacity border) as much as possible at each stage and in each time period.

Therefore; our exact objective function is:

$$Z = \sum_{j=1}^J TJC * TA_j + \sum_{j=1}^J CC * (C_{t,s} - \min C_{t,s})$$

In order to achieve real-life assumptions and obstacles of ASELSAN Defense Systems Technologies Division Hybrid Flow Shop Scheduling Problem, model has important constraints such as following.

1. Every job has to be finished before time horizon $T+1$.

This is handled easily by;

$$F_j \leq T \quad j = 1, \dots, J \quad (1)$$

2. Completion time of each job is calculated by finding biggest time period which has the value of $X_{j,s,t} = 1$. And also if there is a setup time (if $S_{j,s,t}=1$) at that time period, it should be added to found value.

This is formulated as;

$$(X_{j,t,s} * t) + (S_{j,t,s} * ST_{j,s}) \leq F_j \quad j = 1, \dots, J \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (2)$$

3. Process times of each job at each stage should be matched with given process time values. For each job and each stage, this is ensured by equalizing sum of $X_{j,s,t}$ from beginning of time period ($t=1$) to time horizon(T) to given process times.

$$\sum_{t=1}^T X_{j,t,s} = p_{j,s} \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (3)$$

4. If a lot-splitting event occurs in the schedule, a setup time should occur in that time period when a divided sub-lot begins its work. Assume that there is a work in a period of a job's stage, $X_{j,s,t} = 1$. If there is not a work in one period before that time period ($t-1$) of that job's stage ($X_{j,s,t-1} = 0$.) then there is a setup cost in that period (t) $S_{j,s,t}=1$. For all other cases, setup time will not occur. (Otherwise, $S_{j,s,t}=0$)

So as to understand easily, the following table can be examined.

$X_{j,s;t-1}$	$X_{j,s;t}$	$S_{j,s;t}$
1	1	0
1	0	0
0	1	1
0	0	0

Table 2: Setup Cost Table

And this is handled by the model as following:

$$X_{j,t,s} - X_{j,t-1,s} \leq S_{j,t,s} \quad j = 1, \dots, J \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (4)$$

5. Given release dates of all jobs should be met. If there is a process time at j th job's s th stage ($pbin_{j,s}=1$), all of $X_{j,s,t}$'s should be 1 after release dates of jobs ($t \geq rd_j$).

This is assured by:

$$pbin_{j,s} * X_{j,t,s} * rd_j \leq t * X_{j,t,s} \quad j = 1, \dots, J \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (5)$$

6. Maximum border of capacity ($\max C_{t,s}$) for each stage (s) and in each time interval (t) should be met.

This is assured by:

$$C_{t,s} \leq \max C_{t,s} \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (6)$$

7. If finish time of a job (F_j) is bigger than due date of a job (dd_j), then a tardy job occurs and for that job, tardy job variable (TA_j) should be 1. Due to this is a minimization problem, this is handled by using Big M method as following:

$$F_j - dd_j \leq BigM * TA_j \quad j = 1, \dots, J \quad (7)$$

8. Variable Capacity of each stage in each period should be bigger than or equal to work-load of total jobs on that stage during that period. Work-load is calculated with total sum of $X_{j,s,t}$'s (Process Time-Load) and $S_{j,s,t} * X_{j,s,t}$ (if setup occurs ($S_{j,s,t}=1$) then setup-time load occurs) for every job at each stage and in each period.

This is ensured by:

$$\sum_{j=1}^J (X_{j,t,s} + (S_{j,t,s} * ST_{j,s})) \leq C_{t,s} \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (8)$$

9. Because of lot-splitting issue, stages of jobs have different prerequisite conditions. There are S-1 different conditions and constraints for a start of a stage. In this research for ASELSAN Defense Systems Technologies Division, it is considered that S=4 is enough. Therefore, mainly 3 different conditions are examined and added.

- a. If a job has process time at current stage (s) and following stage (s+1). (If $p_{bin_{j,s}}=1$ and $p_{bin_{j,s+1}}=1$) The subsequent stage of current stage cannot start before a sub-lot of job finishes its work at current stage. Therefore, subsequent stage looks for only if some part of preceding stage finishes. In order to see easily, figure in the below demonstrates that (s+1)st stages (in red marks) do not wait for accomplishment of total work at preceding stage and do not start before preceding stage.

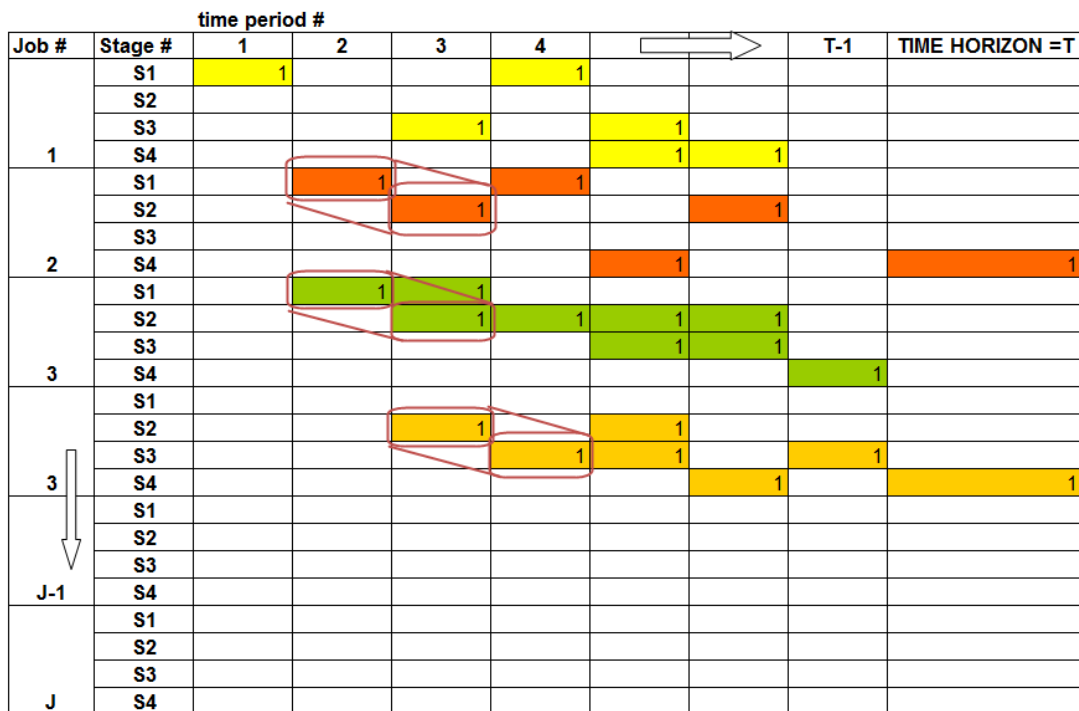


Figure-12: Simple Examples for Constraint 9.a

In order to formulate this condition ($p_{bin_{j,s}}=1$ and $p_{bin_{j,s+1}}=1$ case), for each job, partial work at preceding stage (s) should be done at least one period before

$s+1^{st}$ stage starts. For instance, imagine that at time period $t+1$, in order to start $s+1^{st}$ stage ($X_{j,s+1,t+1} = 1$), at least one of $X_{j,s,1}, X_{j,s,2}, X_{j,s,3}, \dots, X_{j,s,t-1}, X_{j,s,t}$ should be equal to 1. Therefore this can be visualized as following:

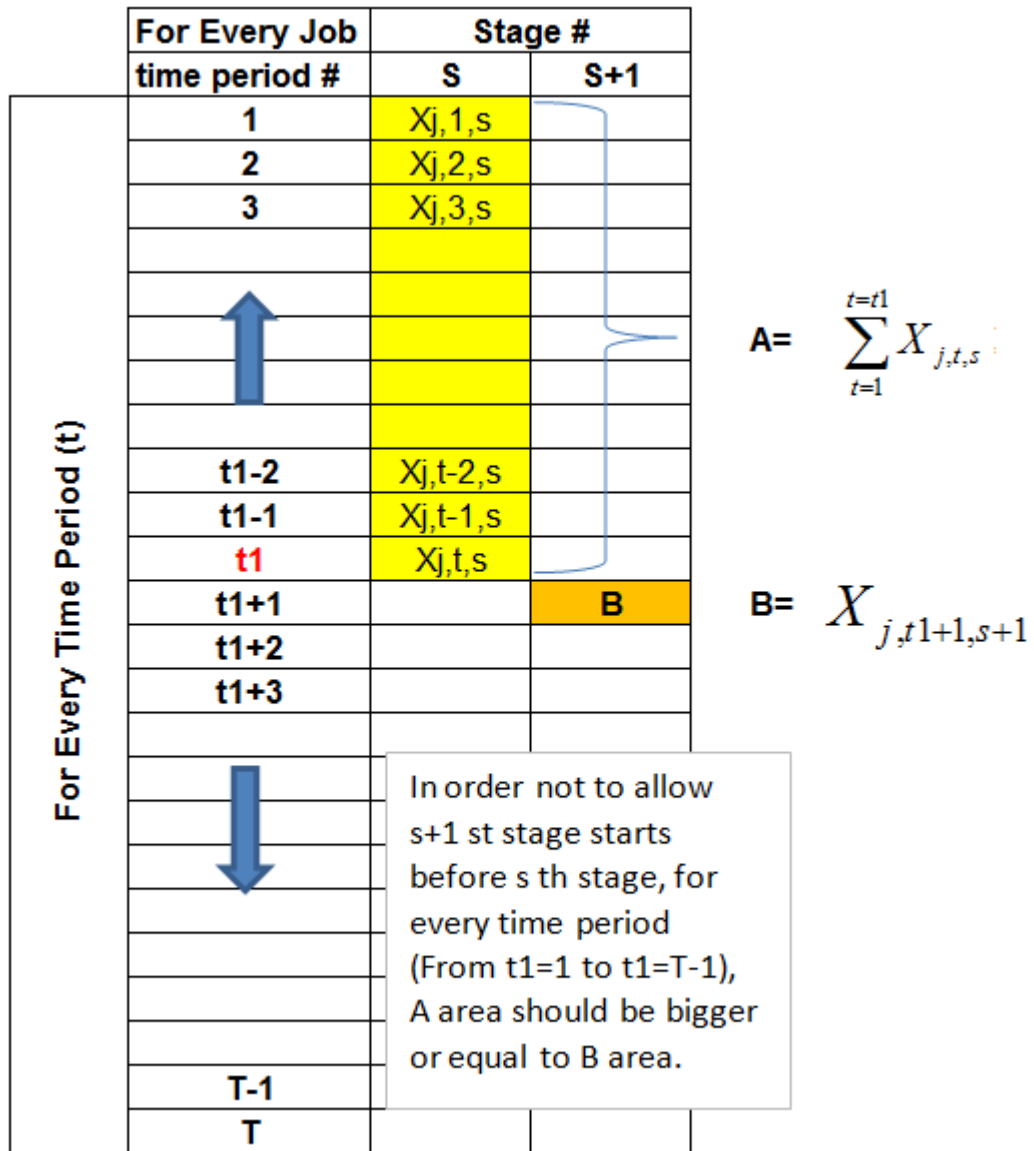


Figure-13: Visualization Of Constraint 9.a

Consequently, this constraint can be added to the model easily as following:

$$\sum_{t=1}^{t=t1} X_{j,t,s} \geq pbin_{j,s} * X_{j,t1+1,s+1} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (9.a)$$

- b. If a job has process time at current stage (s) and following (s+2)nd stage. (If $p_{bin_{j,s}}=1$ and $p_{bin_{j,s+2}}=1$). This constraint is added in order to handle a special case that if (s+1)st stage has not a process time. By this constraint, start of (s+2)nd stage before a sub-lot of job finishes its work at sth stage is forbidden. In order to see easily, figure in the below demonstrates that (s+2)nd stages (in red marks) do not start before partial finish of sth stage and do not wait for accomplishment of total work at sth stage.

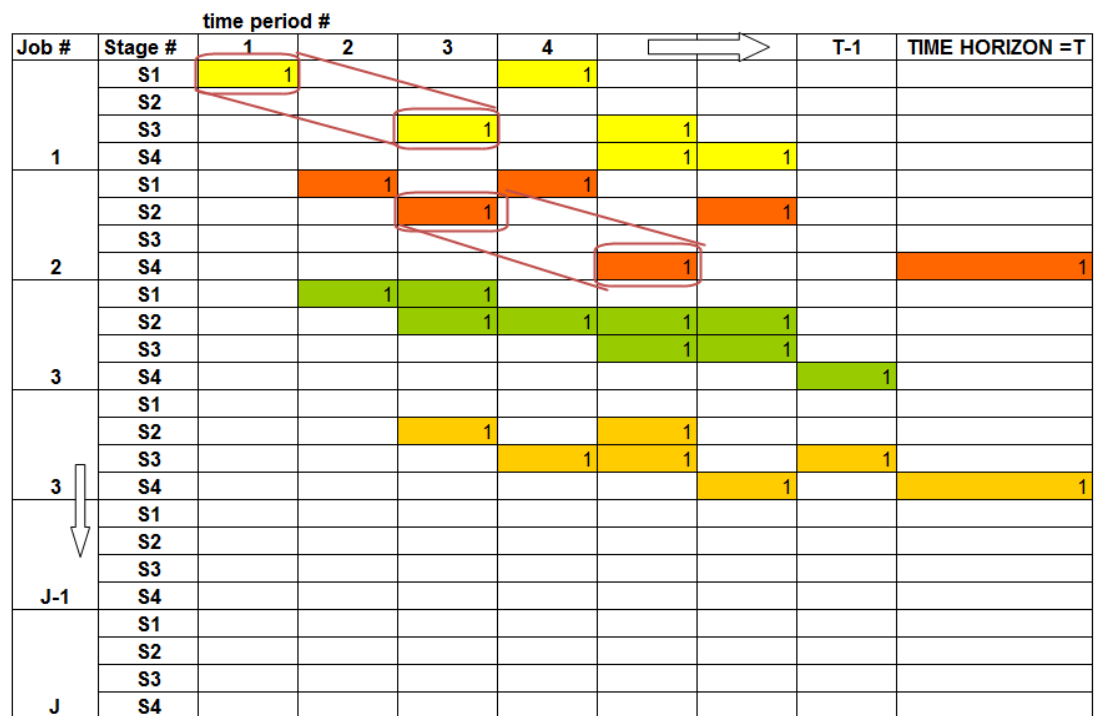


Figure-14: Simple Examples for constraint 9.b

As visualized in Figure 13 for Constraint 9.a, similarly this constraint is constructed such as:

$$\sum_{t=1}^{t=t1} X_{j,t,s} \geq p_{bin_{j,s}} * X_{j,t1+1,s+2} \quad j = 1, \dots, J \quad t1 = 1, \dots, T - 1 \quad s = 1, \dots, S \quad (9.b)$$

- c. If a job has process time at current stage (s) and following (s+3)rd stage. (If $p_{bin_{j,s}}=1$ and $p_{bin_{j,s+3}}=1$). This constraint is added in order to handle a special case that if (s+1)st and (s+2)nd stage

have not process times. By this constraint, start of $(s+3)^{rd}$ stage before a sub-lot of job finishes its work at s^{th} stage is forbidden. In order to see easily, figure in the below demonstrates that $(s+3)^{rd}$ stages (in red marks) do not start before partial finish of s^{th} stage and do not wait for accomplishment of total work at s^{th} stage.

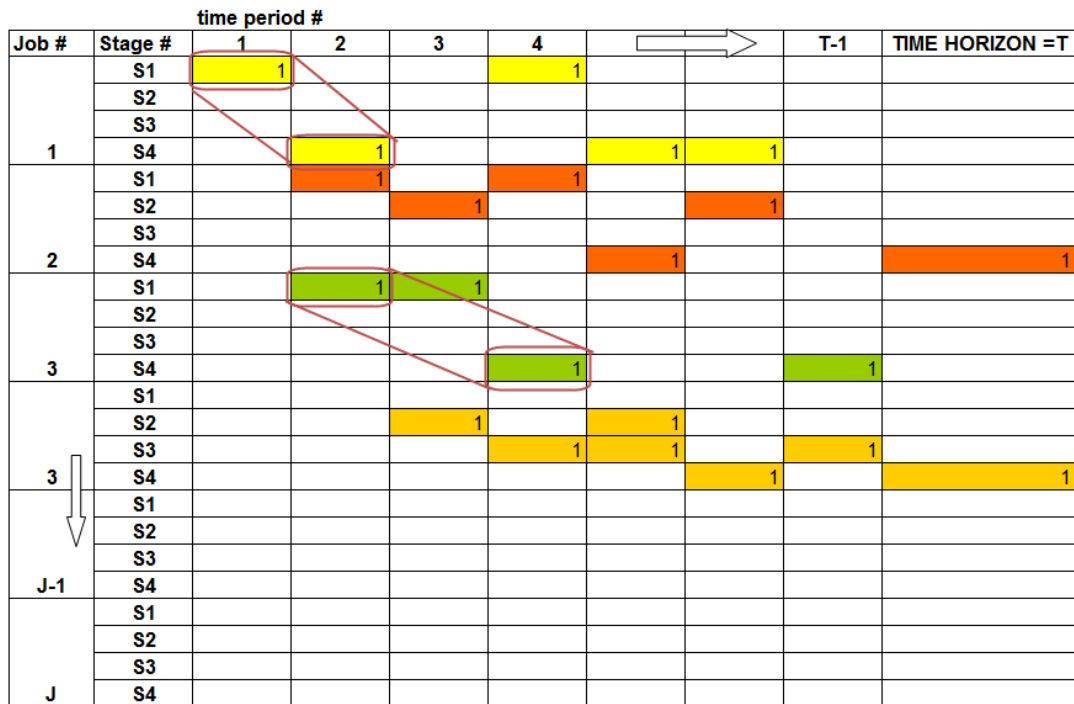


Figure-15: Simple Examples for constraint 9.c

As visualized in Figure 13 for Constraint 9.a, similarly this constraint is constructed such as:

$$\sum_{t=1}^{t-tl} X_{j,t,s} \geq pbin_{j,s} * X_{j,tl+1,s+3} \quad j = 1, \dots, J \quad tl = 1, \dots, T - 1 \quad s = 1, \dots, S \quad (9.c)$$

These three constraints strictly arrange starting periods of each stage of every job. In order to visualize it, these constraints force the schedule to be like in Figure 16. Constraints construct an imaginary line such as red lines in the Figure 16 for each stage of every job which works as a minimum border of starting points of each stage.

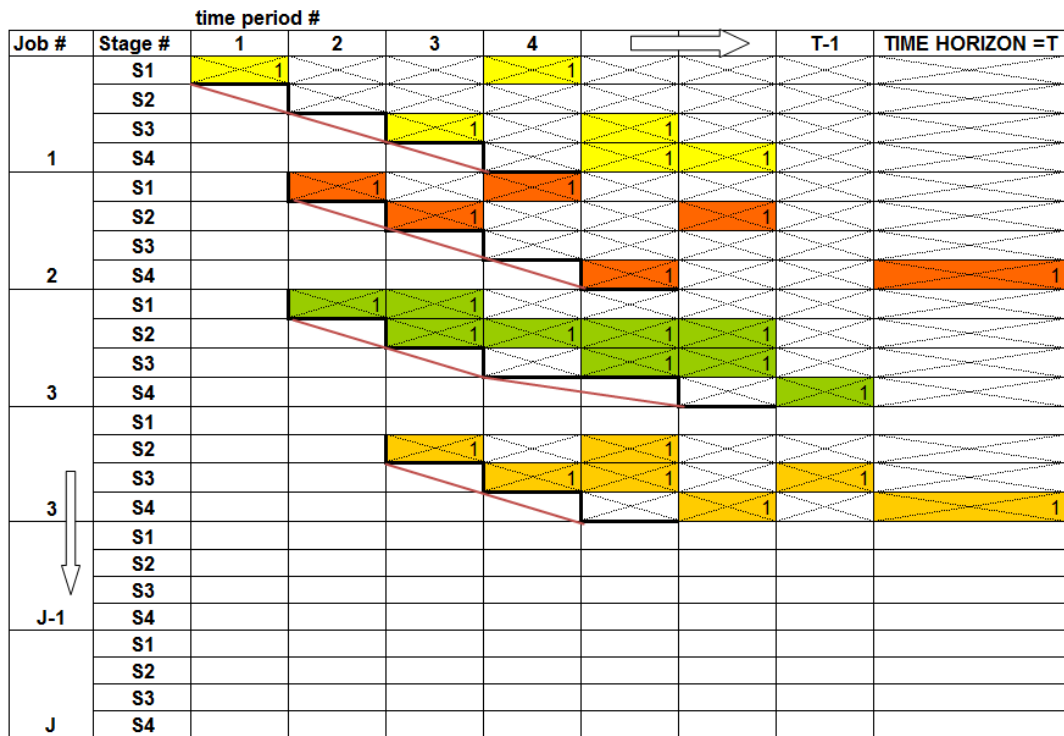


Figure-16: Schedule Visualization after Constraints 9.a, 9.b and 9.c.

10. For finishing of process times at each stage, there are also S-1 different conditions and constraints. And, in this research for ASELSAN Defense Systems Technologies Division, it is considered that S=4 is enough. Therefore, mainly 3 different conditions are added.

- a. If a job has process time at current stage (s) and following stage (s+1). (If $p_{bin_{j,s}}=1$ and $p_{bin_{j,s+1}}=1$) The subsequent stage cannot end before current stage finishes all of job's work. Therefore, subsequent stage ends after all work of preceding stage finishes. In order to see easily, figure in the below demonstrates that (s+1)st stages (in red marks) do waits for accomplishment of total work at preceding stage and do not end before preceding stage.

Job #	Stage #	time period #				T-1	TIME HORIZON =T
		1	2	3	4		
1	S1	1			1		
	S2						
	S3			1	1		
	S4				1	1	
2	S1		1		1		
	S2			1		1	
	S3						
	S4				1		1
3	S1		1	1			
	S2			1	1	1	1
	S3					1	1
	S4						1
3	S1			1	1		
	S2				1	1	1
	S3						1
	S4					1	1
J-1	S1						
	S2						
	S3						
	S4						
J	S1						
	S2						
	S3						
	S4						

Figure-17: Simple Examples for constraint 10.a

In order to formulize this condition ($p_{bin_{j,s}}=1$ and $p_{bin_{j,s+1}}=1$ case), for each job, all of work at preceding stage (s) should be done at least one period before $s+1^{st}$ stage ends. For instance, think that at time period $t+1$, in order to finish $s+1^{st}$ stage (total sum of $X_{j,s+1,1}, X_{j,s+1,2}, X_{j,s+1,3}, \dots, X_{j,s+1,t}, X_{j,s+1,t+1} = P_{j,s+1}$, total sum of $X_{j,s,1}, X_{j,s,2}, X_{j,s,3}, \dots, X_{j,s,t-1}, X_{j,s,t}$ should be equal to $P_{j,s}$) at least one period before. Therefore, in order to finish s^{th} stage at t^{th} time period, at least one of $X_{j,s+1,t+1}, X_{j,s+1,t+2}, X_{j,s+1,t+3}, \dots, X_{j,s+1,T-1}, X_{j,s+1,T}$ equals to 1. Because $s+1^{th}$ stage always should finish after than s^{th} stage. This can be visualized as following:

	For Every Job	Stage #	
	time period #	S	S+1
For Every Time Period (t)	1		
	2		
	3		
	↑		
	t1-2		
	t1-1		
	t1	$X_{j,t1,s}=A$	
	t1+1		$X_{j,t1+1,s+1}$
	t1+2		$X_{j,t1+2,s+1}$
	t1+3		$X_{j,t1+3,s+1}$
	↓		
	T-1		$X_{j,T-1,s+1}$
	T		$X_{j,T,s+1}$

In order not to allow s th stage ends before s+1 st stage, for every time period (From t1=2 to t1=T), B area should be bigger or equal to A area.

$$A = X_{j,t1,s}$$

$$B = \sum_{t=t1+1}^{t=T} X_{j,t,s+1}$$

Figure-18: Visualization Of Constraint 10.a

Consequently, this constraint can be added to the model easily as following:

$$\sum_{t=t1+1}^{t=T} X_{j,t,s+1} \geq pbin_{j,s+1} * X_{j,t1,s} \quad j = 1, \dots, J \quad t1 = 1, \dots, T - 1 \quad s = 1, \dots, S \quad (10.a)$$

- b. If a job has process time at current stage (s) and following stage (s+2). (If $pbin_{j,s}=1$ and $pbin_{j,s+2}=1$) This constraint is added in order to handle a special case that if (s+1)st stage has not a process time. The (s+2)nd stage cannot end before current stage (sth stage) finishes all of job's work. Therefore, (s+2)nd stage ends after all

work of s^{th} stage finishes. In order to see easily, figure in the below demonstrates that $(s+2)^{nd}$ stages (in red marks) do waits for accomplishment of total work at s^{th} stages and do not end before s^{th} stages.

Job #	Stage #	time period #				→	T-1	TIME HORIZON =T
		1	2	3	4			
1	S1	1			1			
	S2							
	S3							
	S4		1			1	1	
2	S1		1		1			
	S2			1			1	
	S3							
	S4					1		1
3	S1		1	1				
	S2							
	S3							
	S4				1		1	
3	S1							
	S2			1		1		
	S3				1	1	1	
	S4						1	1
J-1	S1							
	S2							
	S3							
	S4							
J	S1							
	S2							
	S3							
	S4							

Figure-19: Simple Examples for constraint 10.b

As visualized in Figure 18 for Constraint 10.a, similarly this constraint is constructed such as:

$$\sum_{t=t+1}^{t=T} X_{j,t,s+2} \geq pbin_{j,s+2} * X_{j,t1,s} \quad j = 1, \dots, J \quad t1 = 1, \dots, T - 1 \quad s = 1, \dots, S \quad (10.b)$$

- c. If a job has process time at current stage (s) and following stage (s+3). (If $pbin_{j,s}=1$ and $pbin_{j,s+3}=1$) This constraint is added in order to handle a special case that if $(s+1)^{st}$ and $(s+2)^{nd}$ stages have not process times. The $(s+3)^{rd}$ stage cannot end before current stage (s^{th} stage) finishes all of job's work. Therefore, $(s+3)^{rd}$ stage ends after all work of s^{th} stage finishes. In order to see easily, figure in the below demonstrates that $(s+3)^{rd}$ stages (in red marks) do waits

for accomplishment of total work at s^{th} stages and do not end before s^{th} stages.

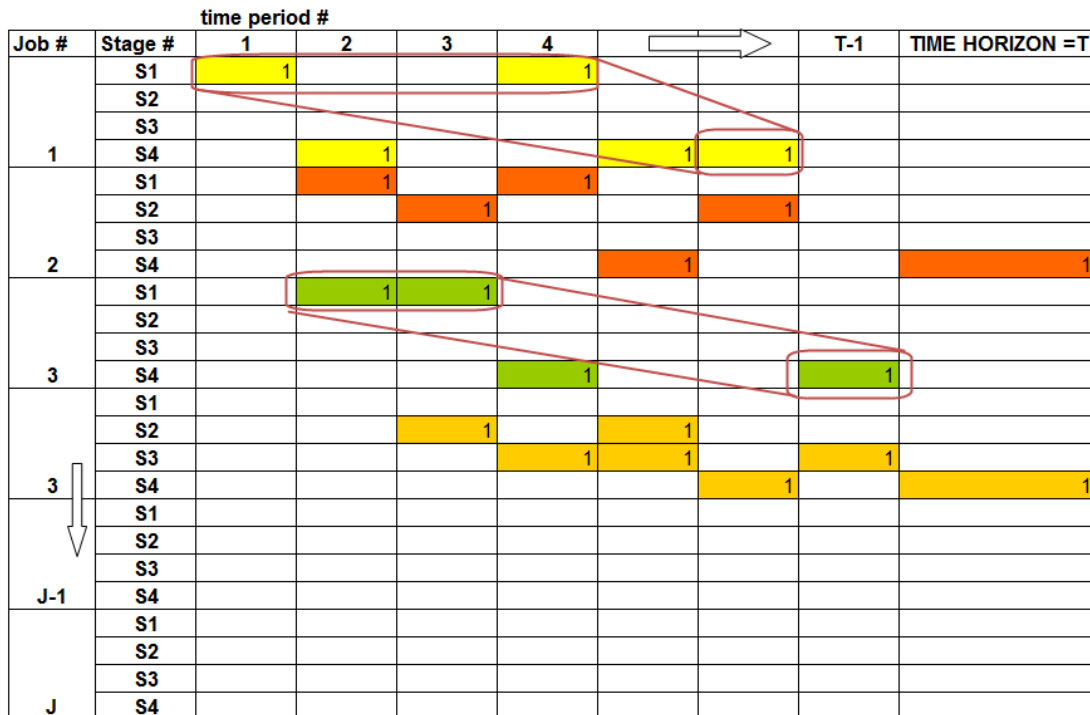


Figure-20: Simple Examples for constraint 10.c

As visualized in Figure 18 for Constraint 10.a, similarly this constraint can be added as:

$$\sum_{t=l+1}^{t=T} X_{j,t,s+3} \geq pbin_{j,s+3} * X_{j,t1,s} \quad j = 1, \dots, J \quad t1 = 1, \dots, T - 1 \quad s = 1, \dots, S \quad (10.c)$$

These six constraints (9.a, 9.b, 9.c, 10.a, 10.b and 10.c) strictly arrange starting and finishing periods of each stage of every job. In order to visualize it, these constraints force the schedule to be like in Figure 21. Constraints construct two imaginary lines such as red lines in the Figure 21 for each stage of every job. One works as a minimum border of starting points of stages and the other works as a maximum border of finishing points of each stage. By these constraints, model forces the schedules to be between these imaginary lines.

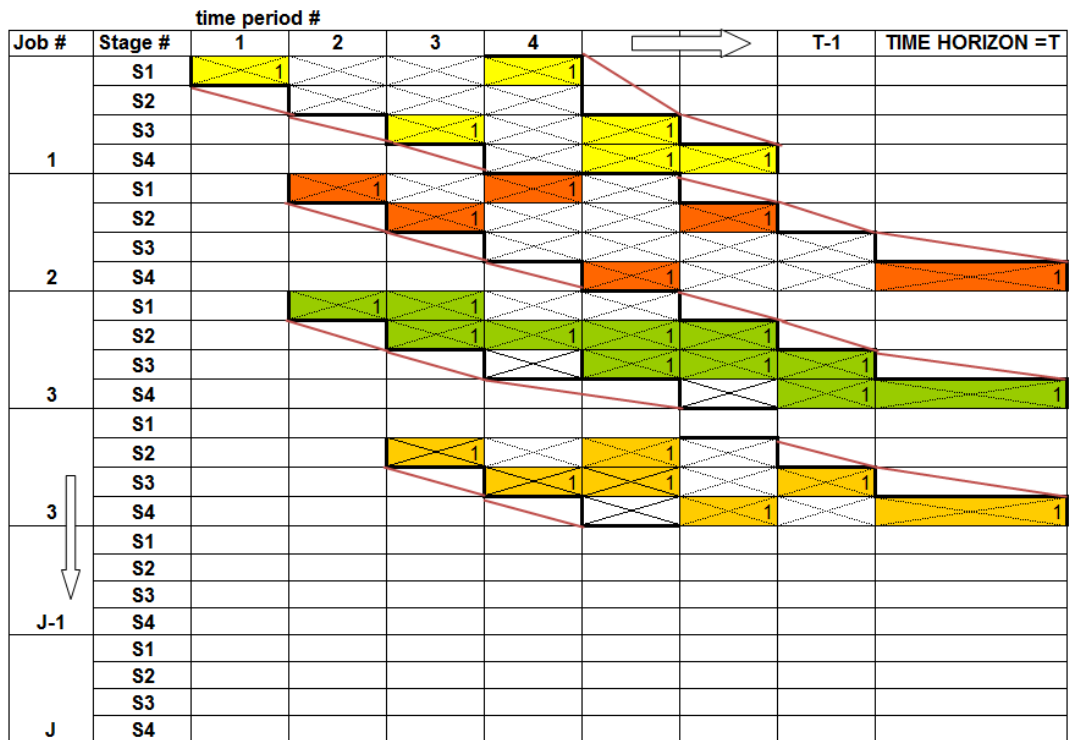


Figure-21: Schedule Visualization after Constraints 9.a, 9.b, 9.c, 10.a, 10.b and 10.c.

11. Additionally, three special cases for time period =1 should be handled. These constraints ensures not to start $s+1^{\text{st}}$, $s+2^{\text{nd}}$ and $s+3^{\text{rd}}$ stages of each job at time period =1 if there is a process time at stage s . They are easily added to model as following:

$$pbin_{j,s} * X_{j,1,s+1} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (11.a)$$

$$pbin_{j,s} * X_{j,1,s+2} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (11.b)$$

$$pbin_{j,s} * X_{j,1,s+3} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (11.c)$$

12. Also, there are three special cases for time period =60 that should be handled. These three constraints ensures s^{th} stage of each job not to end at time period = 60 before $s+1^{\text{st}}$, $s+2^{\text{nd}}$ and $s+3^{\text{rd}}$ stages if there are process times at $s+1^{\text{st}}$, $s+2^{\text{nd}}$ or $s+3^{\text{rd}}$ stages. Here are these constraint:

$$pbin_{j,s+1} * X_{j,60,s} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (12.a)$$

$$pbin_{j,s+2} * X_{j,60,s} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (12.b)$$

$$pbin_{j,s+3} * X_{j,60,s} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (12.c)$$

After detailed demonstration of parameters, decision variables, constraints and objective function, whole proposed model for ASELSAN Defense Systems Technologies Division HFS Scheduling Problem seems like following:

Index:

j	= job number	$j = 1, \dots, J+1$
t	= each time period	$t = 1, \dots, T$
s	= stage number	$s = 1, \dots, S$

Parameters (Given-Data)

$p_{j,s}$	= process time of job j in stage s
$ST_{j,s}$	= setup time of job j in stage s
T	= Time Horizon
rd_j	= release date of job j
dd_j	= due date of job j
$\min C_{s,t}$	= minimum capacity level at each stage and in each time period
$\max C_{s,t}$	= maximum capacity level at each stage and in each time period
$pbin_{j,s}$	= 1 if there is a process time of job j at stage s 0 otherwise
TH	= Time Horizon
CC	= Capacity cost for each increment
TJC	= Cost paid for each Tardy job

BigM = Big number

Decision Variables:

$X_{j,s,t}$ = 1; if sth stage of job j is being done in period t ;
0; otherwise

$S_{j,s,t}$ = 1; if a setup occurs at sth stage of job j is completed in period t
0; otherwise

$C_{t,s}$ = capacity at each stage in tth time period.

F_j = Completion time of jth job.

TA_j = 1; if Finish time is greater than due date of each job. ($F_j > dd_j$)
0; otherwise

Complete Proposed Model for T=60 and S=4:

$$\text{Minimize } Z = \sum_{j=1}^J TJC * TA_j + \sum_{j=1}^J CC * (C_{t,s} - \min C_{t,s})$$

Subject to :

$$F_j \leq T \quad j = 1, \dots, J \quad (1)$$

$$(X_{j,t,s} * t) + (S_{j,t,s} * ST_{j,s}) \leq F_j \quad j = 1, \dots, J \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (2)$$

$$\sum_{t=1}^T X_{j,t,s} = p_{j,s} \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (3)$$

$$X_{j,t,s} - X_{j,t-1,s} \leq S_{j,t,s} \quad j = 1, \dots, J \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (4)$$

$$pbin_{j,s} * X_{j,t,s} * rd_j \leq t * X_{j,t,s} \quad j = 1, \dots, J \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (5)$$

$$C_{t,s} \leq \max C_{t,s} \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (6)$$

$$F_j - dd_j \leq BigM * TA_j \quad j = 1, \dots, J \quad (7)$$

$$\sum_{j=1}^J (X_{j,t,s} + (S_{j,t,s} * ST_{j,s})) \leq C_{t,s} \quad t = 1, \dots, T \quad s = 1, \dots, S \quad (8)$$

$$\sum_{t=1}^{t1} X_{j,t,s} \geq pbin_{j,s} * X_{j,t1+1,s+1} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (9.a)$$

$$\sum_{t=1}^{t1} X_{j,t,s} \geq pbin_{j,s} * X_{j,t1+1,s+2} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (9.b)$$

$$\sum_{t=1}^{t1} X_{j,t,s} \geq pbin_{j,s} * X_{j,t1+1,s+3} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (9.c)$$

$$\sum_{t=t1+1}^{t=T} X_{j,t,s+1} \geq pbin_{j,s+1} * X_{j,t1,s} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (10.a)$$

$$\sum_{t=t1+1}^{t=T} X_{j,t,s+2} \geq pbin_{j,s+2} * X_{j,t1,s} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (10.b)$$

$$\sum_{t=t1+1}^{t=T} X_{j,t,s+3} \geq pbin_{j,s+3} * X_{j,t1,s} \quad j = 1, \dots, J \quad t1 = 1, \dots, T-1 \quad s = 1, \dots, S \quad (10.c)$$

$$pbin_{j,s} * X_{j,1,s+1} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (11.a)$$

$$pbin_{j,s} * X_{j,1,s+2} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (11.b)$$

$$pbin_{j,s} * X_{j,1,s+3} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (11.c)$$

$$pbin_{j,s+1} * X_{j,60,s} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (12.a)$$

$$pbin_{j,s+2} * X_{j,60,s} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (12.b)$$

$$pbin_{j,s+3} * X_{j,60,s} = 0 \quad j = 1, \dots, J \quad s = 1, \dots, S \quad (12.c)$$

$$X_{j,t,s} = \{0,1\} \quad S_{j,t,s} = \{0,1\} \quad TA_j = \{0,1\} \quad EA_j = \{0,1\}$$

$$F_j \geq 0$$

$$C_{t,s} \geq 0$$

4. COMPUTATIONAL RESULTS AND DISCUSSION

The MIP model, developed in this research for ASELSAN Defense Systems Technologies Division HFS Scheduling Problem, is programmed at GAMS Integrated Development Environment 23.7.3 version. With real-data acquired from the company, it is ran in PC that has Windows 7 Home Premium 64-Bit Operating System, Intel Core i3 2.53 GHz CPU, 3 GB RAM and it is solved with CPLEX Solver in GAMS. The written MIP program is updatable from Microsoft Excel therefore many real scenarios developed for the company are tried and solved easily in the program. And also program is able to screen results to Microsoft Excel.

In order to evaluate the scalability of the program, from easiest to hardest 9 numerical examples has been created and tried to solve such as:

	Stage #	Job #	Time Horizon
1	2	4	4
2	3	6	8
3	4	6	8
4	4	15	20
5	4	30	20
6	4	60	60
7	4	110	60
8	4	152	60
9	4	160	60

Table 3: Properties of Solved Examples

During solving hard examples, best possible solutions so far at time 5, 10 and 20 minutes are taken and demonstrated in this research in order to evaluate at what time and at which percentage that proposed MIP model get closer to objective function. Also, characteristics and run-times of examples are demonstrated in this research so as to test speed of the proposed MIP model.

After solving these imaginary cases for testing proposed-MIP model, a real scheduling case is solved depending on real-data taken from ASELSAN Defense Systems Technologies Division.

For all of these examples, in order to simulate realistic cases, coordinated with the company, tardy job cost has been determined as $TJC = 20.000$ \$, and capacity increment cost has been determined as $CC = 100$ \$. Additionally, capacity level and maximum capacity borders for all periods have been determined such as:

Time Period	Stage 1		Stage 2		Stage 3		Stage 4	
	Normal Limit	Max.	Normal Limit	Max.	Normal Limit	Max.	Normal Limit	Max.
1 → t	8	12	20	40	5	11	2	8

But as mentioned before, for the proposed model, different capacity level for each period and for each stage can be determined due to different production environments and conditions.

This is a minimization problem therefore all of objective values can be less than 0 (-) too. That negative amount can be regarded as positive profit that is gained from the optimal schedules.

Run-Results of 10 Numerical Examples:

1. Numerical Example:

For 2 stages, 4 jobs and 4 time horizon; process times and setup times are determined such as:

Pr. Orders	Release Dates	Due Dates	Process Times		Setup Times	
			Stage 1	Stage 2	Stage 1	Stage 2
j1	0	5	0	2	0	2
j2	1	5	3	1	2	0
j3	0	4	0	3	0	1
j4	0	4	2	1	3	1

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS	17	SINGLE EQUATIONS	169
BLOCKS OF VARIABLES	6	SINGLE VARIABLES	81
NON ZERO ELEMENTS	423	DISCRETE VARIABLES	68

OBJECTIVE VALUE = -9.100

Run Time = 0.05 Sec

Proposed MIP model results an optimal schedule just like following:

OPTIMAL SCHEDULE			
Job#	Time #	Stage 1	Stage 2
j1	t1	0	1
j1	t2	0	1
j2	t1	1	0
j2	t2	1	0
j2	t3	1	0
j2	t4	0	1
j3	t2	0	1
j3	t3	0	1
j3	t4	0	1
j4	t1	1	0
j4	t2	1	0
j4	t3	0	1

Completion times and due dates of each job are like this:

Job #	Completion Time	Due Dates
j1	3	5
j2	4	5
j3	4	4
j4	4	4

As it is seen no tardy jobs occur in optimal schedule.

For optimal schedule, capacity level for each stage at each time period has been like this:

CAPACITY LEVEL AT EACH TIME PERIOD		
Time #	Stage 1	Stage 2
t1	7	3
t2	2	3
t3	1	3
t4	0	2

2. Numerical Example:

For 3 stages, 6 jobs and 8 time horizon; process times and setup times are determined such as:

Pr. Orders	Release Dates	Due Dates	Process Times			Setup Times		
			Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
j1	0	8	0	0	2	0	0	0
j2	1	7	0	0	5	0	0	1
j3	0	8	0	3	3	0	1	1
j4	0	9	2	1	0	3	1	0
j5	0	9	2	0	1	1	0	1
j6	1	9	1	3	2	2	1	1

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS 57 SINGLE EQUATIONS 1,033

BLOCKS OF VARIABLES 6 SINGLE VARIABLES 325

NON ZERO ELEMENTS 3,579 DISCRETE VARIABLES 290

OBJECTIVE VALUE = -7.900

Run Time = 0.02 Sec

Proposed MIP model results an optimal schedule just like following:

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t1	0	0	1	0
j1	t2	0	0	1	0
j2	t1	0	0	1	0

j2	t2	0	0	1	0
j2	t3	0	0	1	0
j2	t4	0	0	1	0
j2	t5	0	0	1	0
j3	t1	0	1	0	0
j3	t2	0	1	1	0
j3	t3	0	1	1	0
j3	t4	0	0	1	0
j4	t2	1	0	0	0
j4	t3	1	0	0	0
j4	t8	0	1	0	0
j5	t5	1	0	0	0
j5	t7	1	0	0	0
j5	t8	0	0	1	0
j6	t1	1	0	0	0
j6	t2	0	1	0	0
j6	t5	0	1	1	0
j6	t6	0	1	0	0
j6	t7	0	0	1	0

Completion times and due dates of each job are like this:

Job #	Completion Time	Due Dates
j1	2	8
j2	5	7
j3	4	8
j4	9	9
j5	9	9
j6	8	9

As it is seen no tardy jobs occur in optimal schedule.

For optimal schedule, capacity level for each stage at each time period has been like this:

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	3	2	3	0
t2	4	3	4	0
t3	1	1	2	0
t4	0	0	2	0

t5	2	2	3	0
t6	0	1	0	0
t7	2	0	2	0
t8	0	2	2	0

3. Numerical Example:

For 4 stages, 6 jobs and 8 time horizon; process times and setup times are determined such as:

Pr. Orders	Release Dates	Due Dates	Process Times				Setup Times			
			Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
j1	0	8	3	3	5	3	2	1	1	1
j2	1	7	1	3	2	1	1	1	1	0
j3	0	8	1	1	1	1	1	1	1	1
j4	0	9	0	1	0	1	3	1	1	1
j5	0	9	0	0	1	1	0	1	1	1
j6	1	9	1	3	2	1	2	1	1	1

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS 57 SINGLE EQUATIONS 1,485

BLOCKS OF VARIABLES 6 SINGLE VARIABLES 429

NON ZERO ELEMENTS 5,481 DISCRETE VARIABLES 390

OBJECTIVE VALUE = -8.700

Run Time = 0.01 Sec

Proposed MIP model results an optimal schedule just like following:

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t1	1	0	0	0
j1	t2	1	1	0	0
j1	t3	1	1	1	0

j1	t4	0	1	1	0
j1	t5	0	0	1	0
j1	t6	0	0	1	1
j1	t7	0	0	1	1
j1	t8	0	0	0	1
j2	t1	1	0	0	0
j2	t2	0	1	0	0
j2	t3	0	1	0	0
j2	t4	0	1	0	0
j2	t5	0	0	1	0
j2	t6	0	0	1	0
j2	t7	0	0	0	1
j3	t3	1	0	0	0
j3	t5	0	1	0	0
j3	t6	0	0	1	0
j3	t7	0	0	0	1
j4	t1	0	1	0	0
j4	t6	0	0	0	1
j5	t1	0	0	1	0
j5	t5	0	0	0	1
j6	t1	1	0	0	0
j6	t2	0	1	0	0
j6	t3	0	1	0	0
j6	t4	0	1	0	0
j6	t6	0	0	1	0
j6	t7	0	0	1	0
j6	t8	0	0	0	1

Completion times and due dates of each job are like this:

Job #	Completion Time	Due Dates
j1	8	8
j2	7	7
j3	8	8
j4	7	9
j5	6	9
j6	9	9

As it is seen no tardy jobs occur in optimal schedule.

For optimal schedule, capacity level for each stage at each time period has been like this:

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	8	2	2	0
t2	1	6	0	0
t3	3	3	2	0
t4	0	3	1	0
t5	0	2	3	2
t6	0	0	6	4
t7	0	0	2	4
t8	0	0	0	3

4. Numerical Example:

For process times and setup times of 4 stages, 15 jobs and 20 time horizon, please look at Appendix-1 Table 1- Process times and Setup times of 4th Numerical Example

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS	129	SINGLE EQUATIONS	9,116
BLOCKS OF VARIABLES	6	SINGLE VARIABLES	2,511
NON ZERO ELEMENTS	64,795	DISCRETE VARIABLES	2,415
OBJECTIVE VALUE	=	-59.000	

Run Time = 0.23 Sec

For optimal schedule output, please look at Appendix-1 Table 2- Optimal Schedule Output

For Completion times and due dates of each job, please look at Appendix-1 Table 3- Completion times and Due Dates Output

As it is seen no tardy jobs occur in optimal schedule.

For capacity level for each stage at each time period, please look at Appendix-1 Table 4- Capacity Level for Each Stage Output

5. Numerical Example:

For process times and setup times of 4 stages, 30 jobs and 20 time horizon, please look at Appendix-2 Table 1- Process times and Setup times of 5th Numerical Example

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS 129 SINGLE EQUATIONS 18,064

BLOCKS OF VARIABLES 6 SINGLE VARIABLES 4,941

NON ZERO ELEMENTS 129,485 DISCRETE VARIABLES 4,830

OBJECTIVE VALUE = -48.400

Run Time = 0.48 Sec

For optimal schedule output, please look at Appendix-2 Table 2- Optimal Schedule Output

For Completion times and due dates of each job, please look at Appendix-2 Table 3- Completion times and Due Dates Output

As it is seen no tardy jobs occur in optimal schedule.

For capacity level for each stage at each time period, please look at Appendix-2 Table 4- Capacity Level for Each Stage Output

6. Numerical Example:

For process times and setup times of 4 stages, 60 jobs and 60 time horizon; please look at Appendix-3 Table 1- Process times and Setup times of 6th Numerical Example

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS 369 SINGLE EQUATIONS 108,316

BLOCKS OF VARIABLES 6 SINGLE VARIABLES 29,161

NON ZERO ELEMENTS 2,073,541 DISCRETE VARIABLES 28,860

OBJECTIVE VALUE = -164.400

Total Run-Time = 47,42 Seconds

For optimal schedule output, please look at Appendix-3 Table 2- Optimal Schedule Output

For Completion times and due dates of each job, please look at Appendix-3 Table 3- Completion times and Due Dates Output

As it is seen no tardy jobs occur in optimal schedule.

For capacity level for each stage at each time period, please look at Appendix-3 Table 4- Capacity Level for Each Stage Output

7. Numerical Example:

For process times and setup times of 4 stages, 110 jobs and 60 time horizon; please look at Appendix-4 Table 1- Process times and Setup times of 7th Numerical Example

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS	369	SINGLE EQUATIONS	198,076
BLOCKS OF VARIABLES	6	SINGLE VARIABLES	53,261
NON ZERO ELEMENTS	3,800,719	DISCRETE VARIABLES	52,910

OBJECTIVE VALUE = -129.200

Total Run-Time = 169.28 Seconds

For optimal schedule output, please look at Appendix-4 Table 2- Optimal Schedule Output

For Completion times and due dates of each job, please look at Appendix-4 Table 3- Completion times and Due Dates Output

As it is seen no tardy jobs occur in optimal schedule.

For capacity level for each stage at each time period, please look at Appendix-4 Table 4- Capacity Level for Each Stage Output

It is seen that for 110 jobs, model can find optimal schedule in 170 Sec. Therefore, no additional solution is taken during execution.

8. Numerical Example:

For process times and setup times of 4 stages, 152 jobs and 60 time horizon; please look at Appendix-5 Table 1- Process times and Setup times of 8th Numerical Example

Execution Statistics has been just like this:

BLOCKS OF EQUATIONS	369	SINGLE EQUATIONS	273,554
BLOCKS OF VARIABLES	6	SINGLE VARIABLES	73,505
NON ZERO ELEMENTS	5,249,831	DISCRETE VARIABLES	73,110
OBJECTIVE VALUE	= -100.400		

Total Run-Time = 1.289,53 Seconds / 20 Min. 89 Sec.

For optimal schedule output, please look at Appendix-5 Table 2- Optimal Schedule Output

For Completion times and due dates of each job, please look at Appendix-5 Table 3- Completion times and Due Dates Output

As it is seen no tardy jobs occur in optimal schedule.

For capacity level for each stage at each time period, please look at Appendix-5 Table 4- Capacity Level for Each Stage Output

As mentioned before, additional to optimal schedule, at time 5, 10 and 20 minutes best solutions so far are saved. These solutions that are taken from model have been:

	300 Sec	600 Sec.	1200 Sec.	Optimal Solution
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Best Objective So Far	-110.711	-107.420	-104.118	Obj.Value= -100.400 Run Time= 1.290 Sec.
	% 10	% 7	% 3,7	

9. Numerical Example:

For process times and setup times of 4 stages, 160 jobs and 60 time horizon; please look at Appendix-6 Table 1- Process times and Setup times of 9th Numerical Example

After execution, it is seen that model cannot solve such a large problem like this. No optimal schedule is generated. In order to observe the limits of model, it is seen that for 152 jobs, optimal schedule is generated however for 153 jobs, optimal schedule cannot be generated. Therefore, it is observed that for 4 stages and 60 time horizon, proposed model has 152 jobs limit.

Run-Result of a Real Scheduling Case of the Company:

After demonstrating that the proposed model has the ability of solving scheduling problems up to 152 jobs, for 4 stages and for 60 time horizon, model has been tried on a real scheduling problem based on real data of production orders and capacity limits that are given by the company.

In 15.02.2013, a snapshot of all jobs in production is taken from the company for testing our model. There were 124 jobs in this case and company wanted to obtain an optimal schedule for 60 time periods/4 work hours (30 days/1.5 Months). For process times and setup times of real-life example, please look at Appendix-7 Table 1- Process times and Setup times of Real-Life Example

Normal capacity level and max capacity level during periods was like following:

Time Period	Stage 1		Stage 2		Stage 3		Stage 4	
	Normal Limit	Max.	Normal Limit	Max.	Normal Limit	Max.	Normal Limit	Max.
t1	8	12	20	30	10	20	2	8
t2	8	12	20	30	10	20	2	8
t3	8	12	20	30	10	20	2	8
t4	8	12	20	30	10	20	2	8

t5	8	12	20	30	10	20	2	8
t6	8	12	20	30	10	20	2	8
t7	8	12	20	30	10	20	2	8
t8	8	12	20	30	10	20	2	8
t9	8	12	20	30	10	20	2	8
t10	8	12	20	30	10	20	2	8
t11	8	12	20	30	10	20	2	8
t12	8	12	20	30	10	20	2	8
t13	8	12	20	30	10	20	2	8
t14	8	12	20	30	10	20	2	8
t15	8	12	20	30	10	20	2	8
t16	8	12	20	30	10	20	2	8
t17	8	12	20	30	10	20	2	8
t18	8	12	20	30	10	20	2	8
t19	8	12	20	30	10	20	2	8
t20	8	12	20	30	10	20	2	8
t21	8	12	20	30	10	20	2	8
t22	8	12	20	30	10	20	2	8
t23	8	12	20	30	10	20	2	8
t24	8	12	20	30	10	20	2	8
t25	8	12	20	30	10	20	2	8
t26	8	12	20	30	10	20	2	8
t27	8	12	20	30	10	20	2	8
t28	8	12	20	30	10	20	2	8
t29	8	12	20	30	10	20	2	8
t30	8	12	20	30	10	20	2	8
t31	8	12	20	30	10	20	2	8
t32	8	12	20	30	10	20	2	8
t33	8	12	20	30	10	20	2	8
t34	8	12	20	30	10	20	2	8
t35	8	12	20	30	10	20	2	8
t36	8	12	20	30	10	20	2	8
t37	8	12	20	30	10	20	2	8
t38	8	12	20	30	10	20	2	8
t39	8	12	20	30	10	20	2	8
t40	8	12	20	30	10	20	2	8
t41	8	12	20	30	10	20	2	8
t42	8	12	20	30	10	20	2	8
t43	8	12	20	30	10	20	2	8
t44	8	12	20	30	10	20	2	8
t45	8	12	20	30	10	20	2	8
t46	8	12	20	30	10	20	2	8
t47	8	12	20	30	10	20	2	8
t48	8	12	20	30	10	20	2	8
t49	8	12	20	30	10	20	2	8
t50	8	12	20	30	10	20	2	8
t51	8	12	20	30	10	20	2	8
t52	8	12	20	30	10	20	2	8
t53	8	12	20	30	10	20	2	8

t54	8	12	20	30	10	20	2	8
t55	8	12	20	30	10	20	2	8
t56	8	12	20	30	10	20	2	8
t57	8	12	20	30	10	20	2	8
t58	8	12	20	30	10	20	2	8
t59	8	12	20	30	10	20	2	8
t60	8	12	20	30	10	20	2	8

Model has been run and execution statistics has been just like this:

BLOCKS OF EQUATIONS 369 SINGLE EQUATIONS 222,963

BLOCKS OF VARIABLES 6 SINGLE VARIABLES 60,009

NON ZERO ELEMENTS 4,258,716 DISCRETE VARIABLES 59,64

OBJECTIVE VALUE = - 116.800

Run Time = 237.73 Seconds

For optimal schedule output, please look at Appendix-7 Table 2- Optimal Schedule Output

For Completion times and due dates of each job, please look at Appendix-7 Table 3- Completion times and Due Dates Output

As it is seen, all due date constraints are met.

For capacity level for each stage at each time period, please look at Appendix-7 Table 4- Capacity Level for Each Stage Output

Discussion

As it is demonstrated in 9 numerical examples and a real scheduling case of ASELSAN Defense Systems Technologies Division, proposed model has ability of solving real life Hybrid Flow Shop Scheduling problems of the company up to 153 production orders.

For 9 numerical examples and real life case, following results has been gathered:

Problem Type				SOLUTION RESULTS				
	Stage #	Job #	Time Horizon	Discrete Var.	Equations	Nonzero Elements	Objective Value	Run Time (Sec.)
1	2	4	4	68	169	423	-9.100	0,05
2	3	6	8	290	1.033	3.579	-7.900	0,02
3	4	6	8	390	1.485	5.481	-8.700	0,01
4	4	15	20	2.415	9.116	64.795	-59.000	0,23
5	4	30	20	4.830	18.064	129.485	-48.400	0,48
6	4	60	60	28.860	108.316	2.073.541	-164.400	47,42
7	4	110	60	52.910	198.076	3.800.719	-129.200	169,28
8	4	152	60	73.110	273.554	5.249.831	-100.400	1.289,53
9	4	160	60	Unavalible	Unavalible	Unavalible	Unavalible	Unavalible
Real Case	4	124	60	59,64	222,963	4,258,716	- 116.800	237.73

For 152 jobs, 4 stages and 60 time horizons, it takes ~20 min. to solve the problem but it can find an optimal. According to real life experiences of ASELSAN Defense Systems Technologies Division Production Department, maximum load of production is average 140-150 production orders. Also, it is seen from the results of the model, there are no tardy jobs in the optimal schedules. There are early jobs because of not including "Cost for early jobs". Model arranges all of the jobs due to tardy-job costs and capacity increment costs. Additionally, run-time of the model is short enough for fast re-scheduling and taking action immediately. It takes from 2 to 5 minutes for solving an average real-life case of ASELSAN Defense Systems Technologies Division. Therefore, proposed model in this research can be used as a decision support tool, for optimizing the schedules and deciding lot-splitting, capacity increment and due date strategies for each production order.

5. CONCLUSION AND FUTURE RESEARCH

In this research, for ASELSAN Defense Systems Technologies Division Production Scheduling Problem is examined and a Mixed Integer Program is proposed, demonstrated, and tested for numerical and real-life examples. As it is mentioned before, briefly ASELSAN Defense Systems Technologies Division Production Department Printed Wiring Board Production area has main 4 stages:

- a. SMD Assembly
- b. Through Hole Component Assembly and Rework
- c. Functional Test
- d. Conformal Coating and Gluing

Each stage has parallel labor and machine resources. All of labor and machine resources that are at the same stage are identical. Lot splitting is allowed in each stage. There are unlimited buffer between stages. Because of High Mix/low Volume, set-up times are inevitable and setup time of each job is independent from other jobs' setup times. (sequence – independent). And whenever lot of a production order is split to sub-lots, new setup time occurs for that stage.

Each production order has its own release date, due date, process and setup times for each stage. During manufacturing execution, capacity of each stage can be changed and lots of production orders can be divided to sub-lots due to scheduling strategies in line with the release date, due-date and capacity borders. ASELSAN Defense Systems Technologies Division Production Department mainly wants to minimize sum of total cost of tardy jobs and total cost of capacity increments. Therefore, briefly company has needed a decision support tool for its scheduling problem that gives optimal or near-optimal schedules corresponding to cost constraints.

The scheduling problem of ASELSAN Defense Systems Technologies Division is named as Hybrid Flow Shop Scheduling problem in literature. As it mentioned in (Gupta, 1988; VoX, 1993), it has m stage, each stage has parallel resources and each job follows the same routing. Corresponding to notation given in (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009), this specific scheduling problem of the company is denoted as:

FHm (PM^(k))_{k=1}^m | r_j, split, S_{nsd} | U + Total Capacity

It is observed in (Rubén Ruiz, José Antonio Vázquez-Rodríguez, 2009) that this kind of problems can be solved by Exact Methods (such as Branch & Bound Algorithms, Integer Programming) and Heuristics/Meta-heuristics. In order to find exact optimal schedules for the company, Mixed Integer Programming method is chosen and a comprehensive Mixed Integer Program is built in GAMS Integrated Development Environment 23.7.3 version, solved by CPLEX solver and tested for 9 numerical and 1 real-life examples in this research. Summary of the results of examples can be seen at the table below:

Problem Type				SOLUTION RESULTS				
	Stage #	Job #	Time Horizon	Discrete Var.	Equations	Nonzero Elements	Objective Value	Run Time (Sec.)
1	2	4	4	68	169	423	-9.100	0,05
2	3	6	8	290	1.033	3.579	-7.900	0,02
3	4	6	8	390	1.485	5.481	-8.700	0,01
4	4	15	20	2.415	9.116	64.795	-59.000	0,23
5	4	30	20	4.830	18.064	129.485	-48.400	0,48
6	4	60	60	28.860	108.316	2.073.541	-164.400	47,42
7	4	110	60	52.910	198.076	3.800.719	-129.200	169,28
8	4	152	60	73.110	273.554	5.249.831	-100.400	1.289,53
9	4	160	60	Unavalible	Unavalible	Unavalible	Unavalible	Unavalible
Real Case	4	124	60	59,64	222,963	4,258,716	- 116.800	237.73

It's demonstrated that it gives optimal schedules up to 153 jobs, 4 stages and 60 Time-periods. It should be added that no due-date violations occur in optimal schedules for all of 9 numerical and 1 real-life examples.

According to the information that is taken from the ASELSAN Defense Systems Technologies Division Production Department, company has average 120-130

and maximum 140-150 production orders load. This means that proposed model can be used easily as a scheduling support tool for choosing optimal or near-optimal schedules. Additionally, run-time of the model for 130-140 production orders is average ~5 minutes long and it is small enough to take action immediately and re-schedule if it is needed.

For future research, some additional features can be added to the model just like:

1. Obligatory Wait times (Owen, Washing etc) can be added to each stage.
2. For each stage and each period, minimum capacity border can be determined in order to make optimal schedules more realistic. Because in real-life, in most cases, equalizing capacity level of a stage to 0, is impossible.
3. Early jobs and completion time objectives can be added.
4. Just In Time concept can be included at each stage by new constraints.
5. Advanced models can balance capacity level at each stage at each period by new constraints.

It is obvious that these features make the model more realistic, more accurate and more convenient to schedule real-life Hybrid Flow Shop Scheduling cases.

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APPENDIX-1

Pr. Orders	Release Dates	Due Dates	Process Times				Setup Times			
			Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
j1	1	20	1	6	8	4	4	0	1	0
j2	0	13	1	3	2	1	2	0	1	0
j3	0	20	1	1	1	1	4	0	1	0
j4	0	20	0	1	0	1	0	0	0	0
j5	0	13	0	0	1	1	0	0	1	0
j6	0	20	1	3	2	1	2	0	1	0
j7	0	20	1	1	1	1	2	0	1	0
j8	1	17	0	1	0	1	0	0	0	0
j9	0	20	0	0	1	1	0	0	1	0
j10	0	22	1	2	1	0	2	0	1	0
j11	0	22	1	1	1	1	2	0	1	0
j12	0	13	0	1	0	1	0	0	0	0
j13	0	22	0	0	1	1	0	0	1	0
j14	0	13	1	3	2	1	4	0	1	0
j15	0	20	1	1	1	1	4	0	1	0

Table 1- Process times and Setup times of 4th Numerical Example

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t1	1	0	0	0
j1	t2	0	1	0	0
j1	t3	0	1	0	0
j1	t5	0	1	0	0
j1	t6	0	1	0	0
j1	t7	0	1	0	0
j1	t8	0	1	0	0
j1	t9	0	0	1	0
j1	t10	0	0	1	0
j1	t11	0	0	1	0
j1	t12	0	0	1	0
j1	t13	0	0	1	1
j1	t14	0	0	1	0
j1	t15	0	0	1	1
j1	t16	0	0	1	0
j1	t18	0	0	0	1
j1	t20	0	0	0	1
j2	t1	1	0	0	0
j2	t2	0	1	0	0

j2	t9	0	1	0	0
j2	t10	0	1	0	0
j2	t11	0	0	1	0
j2	t12	0	0	1	0
j2	t13	0	0	0	1
j3	t2	1	0	0	0
j3	t15	0	1	0	0
j3	t17	0	0	1	0
j3	t18	0	0	0	1
j4	t1	0	1	0	0
j4	t3	0	0	0	1
j5	t3	0	0	1	0
j5	t11	0	0	0	1
j6	t1	1	0	0	0
j6	t2	0	1	0	0
j6	t12	0	1	0	0
j6	t13	0	1	1	0
j6	t14	0	0	1	0
j6	t15	0	0	0	1
j7	t13	1	0	0	0
j7	t14	0	1	0	0
j7	t18	0	0	1	0
j7	t20	0	0	0	1
j8	t1	0	1	0	0
j8	t2	0	0	0	1
j9	t5	0	0	1	0
j9	t6	0	0	0	1
j10	t11	1	0	0	0
j10	t13	0	1	0	0
j10	t14	0	1	0	0
j10	t19	0	0	1	0
j11	t3	1	0	0	0
j11	t16	0	1	0	0
j11	t17	0	0	1	0
j11	t20	0	0	0	1
j12	t1	0	1	0	0
j12	t3	0	0	0	1
j13	t6	0	0	1	0
j13	t15	0	0	0	1
j14	t2	1	0	0	0
j14	t3	0	1	0	0
j14	t8	0	1	0	0
j14	t9	0	1	1	0
j14	t10	0	0	1	0
j14	t13	0	0	0	1

j15	t16	1	0	0	0
j15	t18	0	1	0	0
j15	t19	0	0	1	0
j15	t20	0	0	0	1

Table 2- Optimal Schedule Output

j1	20	20
j2	13	13
j3	18	20
j4	3	20
j5	11	13
j6	15	20
j7	20	20
j8	2	17
j9	6	20
j10	20	22
j11	20	22
j12	3	13
j13	15	22
j14	13	13
j15	20	20

Table 3- Completion times and Due Dates Output

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	11	3	0	0
t2	10	3	0	1
t3	3	2	2	2
t5	0	1	2	0
t6	0	1	2	1
t7	0	1	0	0
t8	0	2	0	0
t9	0	2	4	0
t10	0	1	2	0
t11	3	0	3	1
t12	0	1	2	0
t13	3	2	3	3
t14	0	2	2	0
t15	0	1	1	3
t16	5	1	1	0
t17	0	0	4	0
t18	0	1	2	2
t19	0	0	4	0
t20	0	0	0	4

Table 4- Capacity Level for Each Stage Output

APPENDIX-2

Pr. Orders	Release Dates	Due Dates	Process Times				Setup Times			
			Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
j1	1	20	1	6	8	4	4	0	1	0
j2	0	13	1	3	2	1	2	0	1	0
j3	0	20	1	1	1	1	4	0	1	0
j4	0	20	0	1	0	1	0	0	0	0
j5	0	13	0	0	1	1	0	0	1	0
j6	0	20	1	3	2	1	2	0	1	0
j7	0	20	1	1	1	1	2	0	1	0
j8	1	17	0	1	0	1	0	0	0	0
j9	0	20	0	0	1	1	0	0	1	0
j10	0	22	1	2	1	0	2	0	1	0
j11	0	22	1	1	1	1	2	0	1	0
j12	0	13	0	1	0	1	0	0	0	0
j13	0	22	0	0	1	1	0	0	1	0
j14	0	13	1	3	2	1	4	0	1	0
j15	0	20	1	1	1	1	4	0	1	0
j16	0	20	0	1	0	1	0	0	0	0
j17	0	22	0	0	1	1	0	0	1	0
j18	2	22	1	3	2	1	2	1	1	0
j19	0	20	1	1	1	1	1	1	1	0
j20	0	13	0	1	0	1	0	0	0	0
j21	1	13	1	6	6	1	4	0	1	0
j22	0	13	1	1	1	1	1	1	1	0
j23	0	13	0	1	0	1	0	0	0	0
j24	0	20	0	0	1	1	0	0	1	0
j25	0	17	1	3	2	1	2	1	1	0
j26	0	13	1	1	1	1	1	1	1	0
j27	0	13	0	1	0	1	0	0	0	0
j28	1	22	0	0	1	1	0	0	1	0
j29	0	13	1	4	6	1	4	1	1	0
j30	0	20	1	1	1	1	1	1	1	0

Table 1- Process times and Setup times of 5th Numerical Example

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t1	1	0	0	0
j1	t2	0	1	0	0
j1	t4	0	0	1	0
j1	t5	0	1	0	1
j1	t8	0	1	0	0
j1	t10	0	1	0	0

j1	t11	0	0	0	1
j1	t12	0	0	1	0
j1	t13	0	1	1	1
j1	t14	0	0	1	0
j1	t15	0	0	1	0
j1	t16	0	0	1	0
j1	t17	0	1	1	0
j1	t18	0	0	1	0
j1	t20	0	0	0	1
j2	t6	1	0	0	0
j2	t7	0	1	0	0
j2	t8	0	1	1	0
j2	t10	0	1	0	0
j2	t11	0	0	1	0
j2	t12	0	0	0	1
j3	t4	1	0	0	0
j3	t16	0	1	0	0
j3	t17	0	0	1	0
j3	t20	0	0	0	1
j4	t1	0	1	0	0
j4	t14	0	0	0	1
j5	t3	0	0	1	0
j5	t8	0	0	0	1
j6	t2	1	0	0	0
j6	t3	0	1	0	0
j6	t9	0	1	0	0
j6	t11	0	1	1	0
j6	t12	0	0	1	0
j6	t14	0	0	0	1
j7	t7	1	0	0	0
j7	t8	0	1	0	0
j7	t12	0	0	1	0
j7	t13	0	0	0	1
j8	t1	0	1	0	0
j8	t14	0	0	0	1
j9	t11	0	0	1	0
j9	t20	0	0	0	1
j10	t9	1	0	0	0
j10	t13	0	1	0	0
j10	t14	0	1	0	0
j10	t15	0	0	1	0
j11	t6	1	0	0	0
j11	t15	0	1	0	0
j11	t19	0	0	1	0
j11	t20	0	0	0	1

j12	t1	0	1	0	0
j12	t5	0	0	0	1
j13	t19	0	0	1	0
j13	t20	0	0	0	1
j14	t5	1	0	0	0
j14	t6	0	1	0	0
j14	t7	0	1	0	0
j14	t10	0	1	0	0
j14	t11	0	0	1	0
j14	t12	0	0	1	0
j14	t13	0	0	0	1
j15	t9	1	0	0	0
j15	t12	0	1	0	0
j15	t13	0	0	1	0
j15	t17	0	0	0	1
j16	t1	0	1	0	0
j16	t11	0	0	0	1
j17	t3	0	0	1	0
j17	t11	0	0	0	1
j18	t2	1	0	0	0
j18	t3	0	1	0	0
j18	t4	0	1	0	0
j18	t5	0	1	0	0
j18	t18	0	0	1	0
j18	t19	0	0	1	0
j18	t20	0	0	0	1
j19	t6	1	0	0	0
j19	t9	0	1	0	0
j19	t12	0	0	1	0
j19	t13	0	0	0	1
j20	t1	0	1	0	0
j20	t5	0	0	0	1
j21	t4	1	0	0	0
j21	t5	0	1	0	0
j21	t6	0	1	0	0
j21	t7	0	1	1	0
j21	t8	0	1	1	0
j21	t9	0	0	1	0
j21	t10	0	1	1	0
j21	t11	0	1	1	0
j21	t12	0	0	1	0
j21	t13	0	0	0	1
j22	t6	1	0	0	0
j22	t7	0	1	0	0
j22	t8	0	0	1	0

j22	t13	0	0	0	1
j23	t1	0	1	0	0
j23	t2	0	0	0	1
j24	t13	0	0	1	0
j24	t14	0	0	0	1
j25	t2	1	0	0	0
j25	t6	0	1	0	0
j25	t7	0	1	0	0
j25	t8	0	1	0	0
j25	t15	0	0	1	0
j25	t16	0	0	1	0
j25	t17	0	0	0	1
j26	t6	1	0	0	0
j26	t11	0	1	0	0
j26	t12	0	0	1	0
j26	t13	0	0	0	1
j27	t1	0	1	0	0
j27	t5	0	0	0	1
j28	t13	0	0	1	0
j28	t14	0	0	0	1
j29	t1	1	0	0	0
j29	t2	0	1	0	0
j29	t3	0	1	0	0
j29	t4	0	1	1	0
j29	t5	0	1	1	0
j29	t6	0	0	1	0
j29	t7	0	0	1	0
j29	t8	0	0	1	0
j29	t9	0	0	1	0
j29	t13	0	0	0	1
j30	t9	1	0	0	0
j30	t10	0	1	0	0
j30	t11	0	0	1	0
j30	t14	0	0	0	1

Table 2- Optimal Schedule Output

Job #	Completion Time	Due Dates
j1	20	20
j2	12	13
j3	20	20
j4	14	20
j5	8	13
j6	14	20
j7	13	20

j8	14	17
j9	20	20
j10	16	22
j11	20	22
j12	5	13
j13	20	22
j14	13	13
j15	17	20
j16	11	20
j17	11	22
j18	20	22
j19	13	20
j20	5	13
j21	13	13
j22	13	13
j23	2	13
j24	14	20
j25	17	17
j26	13	13
j27	5	13
j28	14	22
j29	13	13
j30	14	20

Table 3- Completion times and Due Dates Output

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	10	7	0	0
t2	9	3	0	1
t3	0	4	4	0
t4	10	2	4	0
t5	5	4	1	4
t6	12	4	1	0
t7	3	6	3	0
t8	0	5	6	1
t9	10	3	2	0
t10	0	6	1	0
t11	0	4	11	3
t12	0	1	11	1
t13	0	2	7	8
t14	0	1	1	6
t15	0	1	5	0
t16	0	1	2	0
t17	0	1	3	2
t18	0	0	3	0

t19	0	0	5	0
t20	0	0	0	6

Table 4- Capacity Level for Each Stage Output

APPENDIX-3

Pr. Orders	Release Dates	Due Dates	Process Times				Setup Times			
			Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
j1	0	60	1	6	8	4	4	0	1	0
j2	0	40	1	3	2	1	2	0	1	0
j3	0	60	1	1	1	1	4	0	1	0
j4	0	60	0	1	0	1	0	0	0	0
j5	0	40	0	0	1	1	0	0	1	0
j6	0	60	1	3	2	1	2	0	1	0
j7	0	60	1	1	1	1	2	0	1	0
j8	0	50	0	1	0	1	0	0	0	0
j9	0	60	0	0	1	1	0	0	1	0
j10	0	65	1	2	1	0	2	0	1	0
j11	0	65	1	1	1	1	2	0	1	0
j12	0	40	0	1	0	1	0	0	0	0
j13	0	65	0	0	1	1	0	0	1	0
j14	0	40	1	3	2	1	4	0	1	0
j15	0	60	1	1	1	1	4	0	1	0
j16	0	60	0	1	0	1	0	0	0	0
j17	0	65	0	0	1	1	0	0	1	0
j18	0	67	1	3	2	1	2	1	1	0
j19	0	60	1	1	1	1	1	1	1	0
j20	0	40	0	1	0	1	0	0	0	0
j21	0	40	1	6	6	1	4	0	1	0
j22	0	40	1	1	1	1	1	1	1	0
j23	0	40	0	1	0	1	0	0	0	0
j24	0	60	0	0	1	1	0	0	1	0
j25	0	50	1	3	2	1	2	1	1	0
j26	0	40	1	1	1	1	1	1	1	0
j27	0	40	0	1	0	1	0	0	0	0
j28	0	65	0	0	1	1	0	0	1	0
j29	0	40	1	4	6	1	4	1	1	0
j30	0	60	1	1	1	1	1	1	1	0
j31	0	40	0	0	1	1	0	0	1	0
j32	0	65	1	3	2	1	2	1	1	0
j33	0	69	1	1	1	1	1	1	1	0
j34	0	65	0	1	0	1	0	0	0	0
j35	0	65	0	0	1	1	0	0	1	0
j36	0	60	1	3	4	1	4	1	1	0
j37	0	69	1	1	1	1	1	1	1	0
j38	0	45	0	1	0	1	0	0	0	1
j39	0	65	0	0	1	1	0	0	1	0
j40	0	60	1	6	2	1	4	1	1	0
j41	0	60	1	6	6	1	4	0	1	1
j42	0	40	1	1	1	1	1	1	1	0
j43	0	40	0	1	0	1	0	0	0	0

j44	0	60	0	0	1	1	0	0	1	0
j45	0	60	1	3	2	1	2	1	1	0
j46	0	25	1	1	1	1	2	1	1	0
j47	0	29	0	1	0	1	0	0	0	0
j48	0	65	0	0	1	1	0	0	1	0
j49	0	65	1	4	6	1	4	1	1	0
j50	0	60	1	1	1	1	2	1	1	0
j51	0	50	0	0	1	1	0	0	1	1
j52	0	65	1	3	2	1	2	1	1	0
j53	0	69	1	1	1	1	2	1	1	0
j54	0	55	0	1	0	1	0	0	0	1
j55	0	65	0	0	1	1	0	0	1	0
j56	0	60	1	3	4	1	4	1	1	0
j57	0	69	1	1	1	1	1	1	1	0
j58	0	60	0	1	0	1	0	0	0	0
j59	0	65	0	0	1	1	0	0	1	0
j60	0	60	1	6	2	1	4	1	1	0

Table 1- Process times and Setup times of 6th Numerical Example

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t4	1	0	0	0
j1	t5	0	1	0	0
j1	t41	0	1	0	0
j1	t42	0	1	0	0
j1	t44	0	1	0	0
j1	t45	0	1	0	0
j1	t48	0	0	1	0
j1	t49	0	0	1	0
j1	t50	0	0	1	0
j1	t51	0	0	1	0
j1	t52	0	0	1	0
j1	t54	0	0	1	0
j1	t55	0	0	0	1
j1	t56	0	0	1	0
j1	t58	0	1	0	1
j1	t59	0	0	1	1
j1	t60	0	0	0	1
j2	t16	1	0	0	0
j2	t21	0	1	0	0
j2	t25	0	1	0	0
j2	t29	0	1	1	0
j2	t30	0	0	1	0
j2	t31	0	0	0	1
j3	t34	1	0	0	0

j3	t38	0	1	0	0
j3	t52	0	0	1	0
j3	t59	0	0	0	1
j4	t1	0	1	0	0
j4	t55	0	0	0	1
j5	t25	0	0	1	0
j5	t39	0	0	0	1
j6	t2	1	0	0	0
j6	t5	0	1	0	0
j6	t6	0	1	0	0
j6	t14	0	1	0	0
j6	t18	0	0	1	0
j6	t37	0	0	1	0
j6	t60	0	0	0	1
j7	t36	1	0	0	0
j7	t40	0	1	0	0
j7	t48	0	0	1	0
j7	t51	0	0	0	1
j8	t32	0	1	0	0
j8	t49	0	0	0	1
j9	t27	0	0	1	0
j9	t49	0	0	0	1
j10	t3	1	0	0	0
j10	t18	0	1	0	0
j10	t19	0	1	0	0
j10	t52	0	0	1	0
j11	t30	1	0	0	0
j11	t32	0	1	0	0
j11	t37	0	0	1	0
j11	t39	0	0	0	1
j12	t1	0	1	0	0
j12	t26	0	0	0	1
j13	t7	0	0	1	0
j13	t48	0	0	0	1
j14	t5	1	0	0	0
j14	t6	0	1	0	0
j14	t10	0	1	0	0
j14	t14	0	1	1	0
j14	t15	0	0	1	0
j14	t33	0	0	0	1
j15	t19	1	0	0	0
j15	t32	0	1	0	0
j15	t36	0	0	1	0
j15	t49	0	0	0	1
j16	t19	0	1	0	0

j16	t29	0	0	0	1
j17	t24	0	0	1	0
j17	t26	0	0	0	1
j18	t9	1	0	0	0
j18	t11	0	1	0	0
j18	t12	0	1	0	0
j18	t14	0	1	0	0
j18	t18	0	0	1	0
j18	t44	0	0	1	0
j18	t55	0	0	0	1
j19	t43	1	0	0	0
j19	t45	0	1	0	0
j19	t46	0	0	1	0
j19	t48	0	0	0	1
j20	t14	0	1	0	0
j20	t26	0	0	0	1
j21	t7	1	0	0	0
j21	t8	0	1	0	0
j21	t9	0	1	0	0
j21	t10	0	1	0	0
j21	t14	0	1	0	0
j21	t17	0	0	1	0
j21	t18	0	1	1	0
j21	t19	0	1	1	0
j21	t20	0	0	1	0
j21	t21	0	0	1	0
j21	t22	0	0	1	0
j21	t40	0	0	0	1
j22	t3	1	0	0	0
j22	t25	0	1	0	0
j22	t36	0	0	1	0
j22	t39	0	0	0	1
j23	t1	0	1	0	0
j23	t29	0	0	0	1
j24	t55	0	0	1	0
j24	t56	0	0	0	1
j25	t11	1	0	0	0
j25	t23	0	1	0	0
j25	t24	0	1	0	0
j25	t25	0	1	0	0
j25	t44	0	0	1	0
j25	t48	0	0	1	0
j25	t50	0	0	0	1
j26	t4	1	0	0	0
j26	t19	0	1	0	0

j26	t27	0	0	1	0
j26	t29	0	0	0	1
j27	t1	0	1	0	0
j27	t31	0	0	0	1
j28	t7	0	0	1	0
j28	t31	0	0	0	1
j29	t1	1	0	0	0
j29	t2	0	1	0	0
j29	t3	0	1	0	0
j29	t4	0	1	1	0
j29	t5	0	1	0	0
j29	t6	0	0	1	0
j29	t7	0	0	1	0
j29	t8	0	0	1	0
j29	t9	0	0	1	0
j29	t10	0	0	1	0
j29	t31	0	0	0	1
j30	t9	1	0	0	0
j30	t32	0	1	0	0
j30	t36	0	0	1	0
j30	t49	0	0	0	1
j31	t25	0	0	1	0
j31	t26	0	0	0	1
j32	t2	1	0	0	0
j32	t3	0	1	0	0
j32	t4	0	1	0	0
j32	t5	0	1	0	0
j32	t58	0	0	1	0
j32	t59	0	0	1	0
j32	t60	0	0	0	1
j33	t34	1	0	0	0
j33	t44	0	1	0	0
j33	t52	0	0	1	0
j33	t55	0	0	0	1
j34	t19	0	1	0	0
j34	t49	0	0	0	1
j35	t18	0	0	1	0
j35	t31	0	0	0	1
j36	t15	1	0	0	0
j36	t45	0	1	0	0
j36	t46	0	1	1	0
j36	t47	0	1	1	0
j36	t48	0	0	1	0
j36	t53	0	0	1	0
j36	t54	0	0	0	1

j37	t1	1	0	0	0
j37	t2	0	1	0	0
j37	t38	0	0	1	0
j37	t60	0	0	0	1
j38	t1	0	1	0	0
j38	t4	0	0	0	1
j39	t2	0	0	1	0
j39	t26	0	0	0	1
j40	t12	1	0	0	0
j40	t34	0	1	0	0
j40	t40	0	1	0	0
j40	t42	0	1	0	0
j40	t45	0	1	0	0
j40	t49	0	1	0	0
j40	t57	0	1	1	0
j40	t58	0	0	1	0
j40	t59	0	0	0	1
j41	t1	1	0	0	0
j41	t2	0	1	0	0
j41	t3	0	1	0	0
j41	t4	0	1	0	0
j41	t21	0	0	1	0
j41	t22	0	0	1	0
j41	t26	0	0	1	0
j41	t27	0	0	1	0
j41	t48	0	1	0	0
j41	t49	0	1	0	0
j41	t50	0	1	0	0
j41	t51	0	0	1	0
j41	t52	0	0	1	0
j41	t59	0	0	0	1
j42	t35	1	0	0	0
j42	t36	0	1	0	0
j42	t39	0	0	1	0
j42	t40	0	0	0	1
j43	t19	0	1	0	0
j43	t37	0	0	0	1
j44	t20	0	0	1	0
j44	t49	0	0	0	1
j45	t3	1	0	0	0
j45	t4	0	1	0	0
j45	t6	0	1	0	0
j45	t7	0	1	0	0
j45	t30	0	0	1	0
j45	t31	0	0	1	0

j45	t60	0	0	0	1
j46	t3	1	0	0	0
j46	t17	0	1	0	0
j46	t23	0	0	1	0
j46	t25	0	0	0	1
j47	t25	0	1	0	0
j47	t26	0	0	0	1
j48	t2	0	0	1	0
j48	t29	0	0	0	1
j49	t2	1	0	0	0
j49	t3	0	1	0	0
j49	t4	0	1	0	0
j49	t5	0	1	0	0
j49	t6	0	1	0	0
j49	t8	0	0	1	0
j49	t9	0	0	1	0
j49	t10	0	0	1	0
j49	t11	0	0	1	0
j49	t12	0	0	1	0
j49	t13	0	0	1	0
j49	t59	0	0	0	1
j50	t34	1	0	0	0
j50	t36	0	1	0	0
j50	t47	0	0	1	0
j50	t60	0	0	0	1
j51	t47	0	0	1	0
j51	t49	0	0	0	1
j52	t4	1	0	0	0
j52	t6	0	1	0	0
j52	t7	0	1	0	0
j52	t8	0	1	0	0
j52	t18	0	0	1	0
j52	t19	0	0	1	0
j52	t21	0	0	0	1
j53	t52	1	0	0	0
j53	t54	0	1	0	0
j53	t59	0	0	1	0
j53	t60	0	0	0	1
j54	t1	0	1	0	0
j54	t8	0	0	0	1
j55	t2	0	0	1	0
j55	t26	0	0	0	1
j56	t16	1	0	0	0
j56	t27	0	1	0	0
j56	t28	0	1	0	0

j56	t29	0	1	0	0
j56	t52	0	0	1	0
j56	t53	0	0	1	0
j56	t54	0	0	1	0
j56	t55	0	0	1	0
j56	t59	0	0	0	1
j57	t12	1	0	0	0
j57	t17	0	1	0	0
j57	t31	0	0	1	0
j57	t48	0	0	0	1
j58	t1	0	1	0	0
j58	t8	0	0	0	1
j59	t44	0	0	1	0
j59	t60	0	0	0	1
j60	t20	1	0	0	0
j60	t49	0	1	0	0
j60	t50	0	1	0	0
j60	t51	0	1	0	0
j60	t54	0	1	0	0
j60	t55	0	1	0	0
j60	t56	0	1	0	0
j60	t57	0	0	1	0
j60	t58	0	0	1	0
j60	t59	0	0	0	1

Table 2- Optimal Schedule Output

Job #	Completion Time	Due Dates
j1	60	60
j2	31	40
j3	59	60
j4	55	60
j5	39	40
j6	60	60
j7	51	60
j8	49	50
j9	49	60
j10	53	65
j11	39	65
j12	26	40
j13	48	65
j14	33	40
j15	49	60
j16	29	60
j17	26	65

j18	55	67
j19	48	60
j20	26	40
j21	40	40
j22	39	40
j23	29	40
j24	56	60
j25	50	50
j26	29	40
j27	31	40
j28	31	65
j29	31	40
j30	49	60
j31	26	40
j32	60	65
j33	55	69
j34	49	65
j35	31	65
j36	54	60
j37	60	69
j38	5	45
j39	26	65
j40	59	60
j41	60	60
j42	40	40
j43	37	40
j44	49	60
j45	60	60
j46	25	25
j47	26	29
j48	29	65
j49	59	65
j50	60	60
j51	50	50
j52	21	65
j53	60	69
j54	9	55
j55	26	65
j56	59	60
j57	48	69
j58	8	60
j59	60	65
j60	59	60

Table 3- Completion times and Due Dates Output

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	12	7	0	0
t2	11	5	6	0
t3	11	6	0	0
t4	10	6	2	2
t5	5	5	0	0
t6	0	7	2	0
t7	5	2	5	0
t8	0	2	3	3
t9	5	1	2	0
t10	0	2	2	0
t11	3	2	1	0
t12	7	1	1	0
t13	0	0	1	0
t14	0	6	2	0
t15	5	0	1	0
t16	8	0	0	0
t17	0	4	2	0
t18	0	2	9	0
t19	5	7	2	0
t20	5	0	3	0
t21	0	1	3	1
t22	0	0	2	0
t23	0	2	2	0
t24	0	1	2	0
t25	0	5	4	1
t26	0	0	2	7
t27	0	2	5	0
t28	0	1	0	0
t29	0	2	2	4
t30	3	0	3	0
t31	0	0	3	5
t32	0	5	0	0
t33	0	0	0	1
t34	10	2	0	0
t35	2	0	0	0
t36	3	4	6	0
t37	0	0	4	1
t38	0	1	2	0
t39	0	0	2	3
t40	0	3	0	2
t41	0	1	0	0
t42	0	3	0	0

t43	2	0	0	0
t44	0	3	6	0
t45	0	7	0	0
t46	0	1	4	0
t47	0	1	5	0
t48	0	1	7	3
t49	0	5	1	8
t50	0	2	1	1
t51	0	1	3	1
t52	3	0	10	0
t53	0	0	3	0
t54	0	4	3	1
t55	0	1	3	4
t56	0	1	2	1
t57	0	2	4	0
t58	0	1	4	1
t59	0	0	5	8
t60	0	0	0	8

Table 4- Capacity Level for Each Stage Output

APPENDIX-4

Pr. Orders	Release Dates	Due Dates	Process Times				Setup Times			
			Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
j1	0	60	1	6	8	4	4	0	1	0
j2	0	40	1	3	2	1	2	0	1	0
j3	0	60	1	1	1	1	4	0	1	0
j4	0	60	0	1	0	1	0	0	0	0
j5	0	40	0	0	1	1	0	0	1	0
j6	0	60	1	3	2	1	2	0	1	0
j7	0	60	1	1	1	1	2	0	1	0
j8	0	50	0	1	0	1	0	0	0	0
j9	0	60	0	0	1	1	0	0	1	0
j10	0	65	1	2	1	0	2	0	1	0
j11	0	65	1	1	1	1	2	0	1	0
j12	0	40	0	1	0	1	0	0	0	0
j13	0	65	0	0	1	1	0	0	1	0
j14	0	40	1	3	2	1	4	0	1	0
j15	0	60	1	1	1	1	4	0	1	0
j16	0	60	0	1	0	1	0	0	0	0
j17	0	65	0	0	1	1	0	0	1	0
j18	0	67	1	3	2	1	2	1	1	0
j19	0	60	1	1	1	1	1	1	1	0
j20	0	40	0	1	0	1	0	0	0	0
j21	0	40	1	6	6	1	4	0	1	0
j22	0	40	1	1	1	1	1	1	1	0
j23	0	40	0	1	0	1	0	0	0	0
j24	0	60	0	0	1	1	0	0	1	0
j25	0	50	1	3	2	1	2	1	1	0
j26	0	40	1	1	1	1	1	1	1	0
j27	0	40	0	1	0	1	0	0	0	0
j28	0	65	0	0	1	1	0	0	1	0
j29	0	40	1	4	6	1	4	1	1	0
j30	0	60	1	1	1	1	1	1	1	0
j31	0	40	0	0	1	1	0	0	1	0
j32	0	65	1	3	2	1	2	1	1	0
j33	0	69	1	1	1	1	1	1	1	0
j34	0	65	0	1	0	1	0	0	0	0
j35	0	65	0	0	1	1	0	0	1	0
j36	0	60	1	3	4	1	4	1	1	0
j37	0	69	1	1	1	1	1	1	1	0
j38	0	45	0	1	0	1	0	0	0	1
j39	0	65	0	0	1	1	0	0	1	0
j40	0	60	1	6	2	1	4	1	1	0
j41	0	60	1	6	6	1	4	0	1	1
j42	0	40	1	1	1	1	1	1	1	0
j43	0	40	0	1	0	1	0	0	0	0

j44	0	60	0	0	1	1	0	0	1	0
j45	0	60	1	3	2	1	2	1	1	0
j46	0	25	1	1	1	1	2	1	1	0
j47	0	29	0	1	0	1	0	0	0	0
j48	0	65	0	0	1	1	0	0	1	0
j49	0	65	1	4	6	1	4	1	1	0
j50	0	60	1	1	1	1	2	1	1	0
j51	0	50	0	0	1	1	0	0	1	1
j52	0	65	1	3	2	1	2	1	1	0
j53	0	69	1	1	1	1	2	1	1	0
j54	0	55	0	1	0	1	0	0	0	1
j55	0	65	0	0	1	1	0	0	1	0
j56	0	60	1	3	4	1	4	1	1	0
j57	0	69	1	1	1	1	1	1	1	0
j58	0	60	0	1	0	1	0	0	0	0
j59	0	65	0	0	1	1	0	0	1	0
j60	0	60	1	6	2	1	4	1	1	0
j61	5	65	1	1	1	1	1	0	1	0
j62	4	69	0	1	0	1	0	0	0	0
j63	0	50	0	0	1	1	0	0	1	0
j64	0	55	1	3	2	1	4	0	1	0
j65	0	50	1	1	1	1	4	0	1	0
j66	2	50	0	1	0	1	0	0	0	0
j67	1	55	0	0	1	1	0	0	1	0
j68	5	67	1	3	2	1	2	1	1	0
j69	6	50	1	1	1	1	1	1	1	0
j70	7	50	0	1	0	1	0	0	0	0
j71	8	60	1	6	6	1	4	0	1	0
j72	0	50	1	1	1	1	1	1	1	0
j73	1	60	0	1	0	1	0	0	0	0
j74	3	60	0	0	1	1	0	0	1	0
j75	8	50	1	3	2	1	2	1	1	0
j76	10	55	1	1	1	1	2	1	1	0
j77	4	59	0	1	0	1	0	0	0	0
j78	0	65	0	0	1	1	0	0	1	0
j79	5	55	1	4	6	1	4	1	1	0
j80	4	60	1	1	1	1	1	1	1	0
j81	0	60	0	0	1	1	0	0	1	0
j82	0	65	1	3	2	1	2	1	1	0
j83	0	69	1	1	1	1	1	1	1	0
j84	0	65	0	1	0	1	0	0	0	0
j85	1	65	0	0	1	1	0	0	1	0
j86	5	60	1	3	4	1	4	1	1	0
j87	6	69	1	1	1	1	2	1	1	0
j88	7	65	0	1	0	1	0	0	0	0
j89	8	65	0	0	1	1	0	0	1	0
j90	0	60	1	6	2	2	4	1	1	0
j91	1	60	1	6	6	2	4	0	1	0

j92	3	60	1	1	1	1	2	1	1	0
j93	8	60	0	1	0	1	0	0	0	0
j94	10	60	0	0	1	1	0	0	1	0
j95	4	50	1	3	2	1	2	1	1	0
j96	0	55	1	1	1	1	2	1	1	0
j97	5	59	0	1	0	1	0	0	0	0
j98	4	65	0	0	1	1	0	0	1	0
j99	0	65	1	4	6	2	4	1	1	0
j100	0	60	1	1	1	1	1	1	1	0
j101	0	60	0	0	1	1	0	0	1	0
j102	0	65	1	3	2	2	2	1	1	0
j103	1	69	1	1	1	1	2	1	1	0
j104	5	65	0	1	0	1	0	0	0	0
j105	6	65	0	0	1	1	0	0	1	0
j106	7	60	1	3	4	2	4	1	1	0
j107	8	69	1	1	1	1	2	1	1	0
j108	0	65	0	1	0	1	0	0	0	0
j109	1	65	0	0	1	1	0	0	1	0
j110	3	35	1	2	1	1	4	1	1	0

Table 1- Process times and Setup times of 7th Numerical Example

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t4	1	0	0	0
j1	t5	0	1	0	0
j1	t10	0	1	0	0
j1	t41	0	1	0	0
j1	t42	0	0	1	0
j1	t43	0	0	1	0
j1	t44	0	1	1	0
j1	t45	0	1	1	1
j1	t46	0	1	1	1
j1	t47	0	0	1	0
j1	t48	0	0	1	0
j1	t49	0	0	1	1
j1	t52	0	0	0	1
j2	t28	1	0	0	0
j2	t29	0	1	0	0
j2	t32	0	1	0	0
j2	t34	0	1	0	0
j2	t37	0	0	1	0
j2	t38	0	0	1	0
j2	t39	0	0	0	1
j3	t28	1	0	0	0
j3	t40	0	1	0	0
j3	t45	0	0	1	0

j3	t53	0	0	0	1
j4	t1	0	1	0	0
j4	t18	0	0	0	1
j5	t34	0	0	1	0
j5	t37	0	0	0	1
j6	t3	1	0	0	0
j6	t4	0	1	0	0
j6	t5	0	1	0	0
j6	t6	0	1	0	0
j6	t12	0	0	1	0
j6	t13	0	0	1	0
j6	t60	0	0	0	1
j7	t7	1	0	0	0
j7	t10	0	1	0	0
j7	t14	0	0	1	0
j7	t19	0	0	0	1
j8	t9	0	1	0	0
j8	t36	0	0	0	1
j9	t14	0	0	1	0
j9	t25	0	0	0	1
j10	t19	1	0	0	0
j10	t29	0	1	0	0
j10	t38	0	1	0	0
j10	t47	0	0	1	0
j11	t29	1	0	0	0
j11	t39	0	1	0	0
j11	t41	0	0	1	0
j11	t45	0	0	0	1
j12	t17	0	1	0	0
j12	t21	0	0	0	1
j13	t8	0	0	1	0
j13	t19	0	0	0	1
j14	t6	1	0	0	0
j14	t9	0	1	0	0
j14	t11	0	1	0	0
j14	t13	0	1	0	0
j14	t20	0	0	1	0
j14	t21	0	0	1	0
j14	t22	0	0	0	1
j15	t37	1	0	0	0
j15	t49	0	1	0	0
j15	t51	0	0	1	0
j15	t53	0	0	0	1
j16	t19	0	1	0	0
j16	t57	0	0	0	1

j17	t3	0	0	1	0
j17	t20	0	0	0	1
j18	t20	1	0	0	0
j18	t24	0	1	0	0
j18	t43	0	1	0	0
j18	t44	0	1	0	0
j18	t54	0	0	1	0
j18	t55	0	0	1	0
j18	t57	0	0	0	1
j19	t7	1	0	0	0
j19	t20	0	1	0	0
j19	t32	0	0	1	0
j19	t36	0	0	0	1
j20	t6	0	1	0	0
j20	t12	0	0	0	1
j21	t8	1	0	0	0
j21	t9	0	1	0	0
j21	t10	0	1	0	0
j21	t20	0	1	0	0
j21	t29	0	1	0	0
j21	t30	0	1	1	0
j21	t31	0	0	1	0
j21	t32	0	0	1	0
j21	t33	0	0	1	0
j21	t34	0	1	1	0
j21	t35	0	0	1	0
j21	t36	0	0	0	1
j22	t28	1	0	0	0
j22	t31	0	1	0	0
j22	t34	0	0	1	0
j22	t36	0	0	0	1
j23	t4	0	1	0	0
j23	t19	0	0	0	1
j24	t24	0	0	1	0
j24	t49	0	0	0	1
j25	t3	1	0	0	0
j25	t4	0	1	0	0
j25	t5	0	1	0	0
j25	t6	0	1	0	0
j25	t7	0	0	1	0
j25	t8	0	0	1	0
j25	t49	0	0	0	1
j26	t11	1	0	0	0
j26	t19	0	1	0	0
j26	t21	0	0	1	0

j26	t26	0	0	0	1
j27	t6	0	1	0	0
j27	t36	0	0	0	1
j28	t14	0	0	1	0
j28	t28	0	0	0	1
j29	t2	1	0	0	0
j29	t3	0	1	0	0
j29	t4	0	1	0	0
j29	t5	0	1	0	0
j29	t6	0	1	0	0
j29	t8	0	0	1	0
j29	t9	0	0	1	0
j29	t10	0	0	1	0
j29	t11	0	0	1	0
j29	t12	0	0	1	0
j29	t13	0	0	1	0
j29	t40	0	0	0	1
j30	t8	1	0	0	0
j30	t47	0	1	0	0
j30	t51	0	0	1	0
j30	t53	0	0	0	1
j31	t28	0	0	1	0
j31	t36	0	0	0	1
j32	t3	1	0	0	0
j32	t33	0	1	0	0
j32	t34	0	1	0	0
j32	t47	0	1	0	0
j32	t48	0	0	1	0
j32	t49	0	0	1	0
j32	t53	0	0	0	1
j33	t25	1	0	0	0
j33	t31	0	1	0	0
j33	t33	0	0	1	0
j33	t36	0	0	0	1
j34	t19	0	1	0	0
j34	t38	0	0	0	1
j35	t55	0	0	1	0
j35	t59	0	0	0	1
j36	t1	1	0	0	0
j36	t2	0	1	0	0
j36	t3	0	1	0	0
j36	t4	0	1	0	0
j36	t44	0	0	1	0
j36	t45	0	0	1	0
j36	t46	0	0	1	0

j36	t48	0	0	1	0
j36	t56	0	0	0	1
j37	t21	1	0	0	0
j37	t24	0	1	0	0
j37	t33	0	0	1	0
j37	t53	0	0	0	1
j38	t1	0	1	0	0
j38	t27	0	0	0	1
j39	t42	0	0	1	0
j39	t52	0	0	0	1
j40	t21	1	0	0	0
j40	t22	0	1	0	0
j40	t23	0	1	0	0
j40	t24	0	1	0	0
j40	t25	0	1	0	0
j40	t26	0	1	0	0
j40	t27	0	1	0	0
j40	t57	0	0	1	0
j40	t58	0	0	1	0
j40	t60	0	0	0	1
j41	t19	1	0	0	0
j41	t33	0	1	0	0
j41	t35	0	1	0	0
j41	t42	0	1	0	0
j41	t46	0	1	0	0
j41	t47	0	1	0	0
j41	t48	0	0	1	0
j41	t49	0	1	1	0
j41	t50	0	0	1	0
j41	t51	0	0	1	0
j41	t52	0	0	1	0
j41	t53	0	0	1	0
j41	t58	0	0	0	1
j42	t16	1	0	0	0
j42	t19	0	1	0	0
j42	t24	0	0	1	0
j42	t39	0	0	0	1
j43	t7	0	1	0	0
j43	t29	0	0	0	1
j44	t8	0	0	1	0
j44	t20	0	0	0	1
j45	t2	1	0	0	0
j45	t7	0	1	0	0
j45	t8	0	1	0	0
j45	t36	0	1	0	0

j45	t40	0	0	1	0
j45	t41	0	0	1	0
j45	t60	0	0	0	1
j46	t6	1	0	0	0
j46	t10	0	1	0	0
j46	t21	0	0	1	0
j46	t25	0	0	0	1
j47	t1	0	1	0	0
j47	t21	0	0	0	1
j48	t14	0	0	1	0
j48	t25	0	0	0	1
j49	t1	1	0	0	0
j49	t3	0	1	0	0
j49	t4	0	1	1	0
j49	t5	0	1	1	0
j49	t6	0	1	1	0
j49	t15	0	0	1	0
j49	t16	0	0	1	0
j49	t17	0	0	1	0
j49	t53	0	0	0	1
j50	t13	1	0	0	0
j50	t29	0	1	0	0
j50	t34	0	0	1	0
j50	t37	0	0	0	1
j51	t29	0	0	1	0
j51	t37	0	0	0	1
j52	t9	1	0	0	0
j52	t12	0	1	0	0
j52	t13	0	1	0	0
j52	t14	0	1	0	0
j52	t16	0	0	1	0
j52	t17	0	0	1	0
j52	t60	0	0	0	1
j53	t30	1	0	0	0
j53	t31	0	1	0	0
j53	t33	0	0	1	0
j53	t37	0	0	0	1
j54	t7	0	1	0	0
j54	t49	0	0	0	1
j55	t2	0	0	1	0
j55	t52	0	0	0	1
j56	t21	1	0	0	0
j56	t22	0	1	0	0
j56	t23	0	0	1	0
j56	t27	0	1	1	0

j56	t28	0	1	1	0
j56	t29	0	0	1	0
j56	t60	0	0	0	1
j57	t19	1	0	0	0
j57	t21	0	1	0	0
j57	t24	0	0	1	0
j57	t52	0	0	0	1
j58	t7	0	1	0	0
j58	t23	0	0	0	1
j59	t33	0	0	1	0
j59	t52	0	0	0	1
j60	t33	1	0	0	0
j60	t34	0	1	0	0
j60	t35	0	1	0	0
j60	t36	0	1	0	0
j60	t37	0	1	0	0
j60	t38	0	1	0	0
j60	t39	0	1	0	0
j60	t57	0	0	1	0
j60	t58	0	0	1	0
j60	t59	0	0	0	1
j61	t53	1	0	0	0
j61	t55	0	1	0	0
j61	t56	0	0	1	0
j61	t58	0	0	0	1
j62	t7	0	1	0	0
j62	t9	0	0	0	1
j63	t31	0	0	1	0
j63	t50	0	0	0	1
j64	t22	1	0	0	0
j64	t27	0	1	0	0
j64	t39	0	1	0	0
j64	t42	0	1	0	0
j64	t44	0	0	1	0
j64	t45	0	0	1	0
j64	t48	0	0	0	1
j65	t17	1	0	0	0
j65	t20	0	1	0	0
j65	t34	0	0	1	0
j65	t39	0	0	0	1
j66	t34	0	1	0	0
j66	t49	0	0	0	1
j67	t9	0	0	1	0
j67	t52	0	0	0	1
j68	t7	1	0	0	0

j68	t8	0	1	0	0
j68	t9	0	1	0	0
j68	t10	0	1	0	0
j68	t49	0	0	1	0
j68	t50	0	0	1	0
j68	t51	0	0	0	1
j69	t6	1	0	0	0
j69	t7	0	1	0	0
j69	t9	0	0	1	0
j69	t39	0	0	0	1
j70	t31	0	1	0	0
j70	t38	0	0	0	1
j71	t9	1	0	0	0
j71	t10	0	1	0	0
j71	t11	0	0	1	0
j71	t12	0	0	1	0
j71	t13	0	0	1	0
j71	t14	0	0	1	0
j71	t15	0	0	1	0
j71	t39	0	1	0	0
j71	t44	0	1	0	0
j71	t47	0	1	0	0
j71	t48	0	1	0	0
j71	t49	0	1	0	0
j71	t50	0	0	1	0
j71	t58	0	0	0	1
j72	t1	1	0	0	0
j72	t2	0	1	0	0
j72	t5	0	0	1	0
j72	t48	0	0	0	1
j73	t10	0	1	0	0
j73	t20	0	0	0	1
j74	t9	0	0	1	0
j74	t16	0	0	0	1
j75	t8	1	0	0	0
j75	t9	0	1	0	0
j75	t10	0	1	0	0
j75	t11	0	1	0	0
j75	t48	0	0	1	0
j75	t49	0	0	1	0
j75	t50	0	0	0	1
j76	t35	1	0	0	0
j76	t37	0	1	0	0
j76	t45	0	0	1	0
j76	t55	0	0	0	1

j77	t29	0	1	0	0
j77	t56	0	0	0	1
j78	t6	0	0	1	0
j78	t19	0	0	0	1
j79	t5	1	0	0	0
j79	t6	0	1	0	0
j79	t7	0	1	0	0
j79	t8	0	1	0	0
j79	t9	0	1	1	0
j79	t10	0	0	1	0
j79	t11	0	0	1	0
j79	t12	0	0	1	0
j79	t13	0	0	1	0
j79	t14	0	0	1	0
j79	t53	0	0	0	1
j80	t24	1	0	0	0
j80	t35	0	1	0	0
j80	t56	0	0	1	0
j80	t59	0	0	0	1
j81	t5	0	0	1	0
j81	t30	0	0	0	1
j82	t17	1	0	0	0
j82	t30	0	1	0	0
j82	t37	0	0	1	0
j82	t44	0	1	0	0
j82	t45	0	1	0	0
j82	t58	0	0	1	0
j82	t59	0	0	0	1
j83	t10	1	0	0	0
j83	t14	0	1	0	0
j83	t23	0	0	1	0
j83	t54	0	0	0	1
j84	t6	0	1	0	0
j84	t9	0	0	0	1
j85	t5	0	0	1	0
j85	t9	0	0	0	1
j86	t5	1	0	0	0
j86	t6	0	1	0	0
j86	t7	0	1	0	0
j86	t8	0	1	1	0
j86	t9	0	0	1	0
j86	t10	0	0	1	0
j86	t11	0	0	1	0
j86	t60	0	0	0	1
j87	t9	1	0	0	0

j87	t14	0	1	0	0
j87	t15	0	0	1	0
j87	t45	0	0	0	1
j88	t7	0	1	0	0
j88	t8	0	0	0	1
j89	t16	0	0	1	0
j89	t18	0	0	0	1
j90	t43	1	0	0	0
j90	t44	0	1	0	0
j90	t45	0	1	0	0
j90	t46	0	1	0	0
j90	t47	0	1	0	0
j90	t48	0	1	0	0
j90	t49	0	1	0	0
j90	t57	0	0	1	0
j90	t58	0	0	1	1
j90	t59	0	0	0	1
j91	t4	1	0	0	0
j91	t5	0	1	0	0
j91	t8	0	0	1	0
j91	t9	0	0	0	1
j91	t12	0	1	0	0
j91	t13	0	1	0	0
j91	t33	0	1	0	0
j91	t34	0	1	0	0
j91	t36	0	1	0	0
j91	t50	0	0	1	0
j91	t51	0	0	1	0
j91	t52	0	0	1	0
j91	t53	0	0	1	0
j91	t54	0	0	1	0
j91	t58	0	0	0	1
j92	t57	1	0	0	0
j92	t58	0	1	0	0
j92	t59	0	0	1	0
j92	t60	0	0	0	1
j93	t19	0	1	0	0
j93	t49	0	0	0	1
j94	t24	0	0	1	0
j94	t25	0	0	0	1
j95	t45	1	0	0	0
j95	t46	0	1	0	0
j95	t47	0	1	0	0
j95	t48	0	1	1	0
j95	t49	0	0	1	0

j95	t50	0	0	0	1
j96	t32	1	0	0	0
j96	t39	0	1	0	0
j96	t54	0	0	1	0
j96	t55	0	0	0	1
j97	t7	0	1	0	0
j97	t52	0	0	0	1
j98	t23	0	0	1	0
j98	t25	0	0	0	1
j99	t52	1	0	0	0
j99	t53	0	1	0	0
j99	t54	0	1	1	0
j99	t55	0	1	1	0
j99	t56	0	1	1	1
j99	t57	0	0	1	0
j99	t58	0	0	1	0
j99	t59	0	0	1	0
j99	t60	0	0	0	1
j100	t31	1	0	0	0
j100	t38	0	1	0	0
j100	t53	0	0	1	0
j100	t57	0	0	0	1
j101	t2	0	0	1	0
j101	t14	0	0	0	1
j102	t54	1	0	0	0
j102	t55	0	1	0	0
j102	t56	0	1	0	0
j102	t57	0	1	1	0
j102	t58	0	0	1	1
j102	t59	0	0	0	1
j103	t24	1	0	0	0
j103	t33	0	1	0	0
j103	t46	0	0	1	0
j103	t59	0	0	0	1
j104	t6	0	1	0	0
j104	t29	0	0	0	1
j105	t24	0	0	1	0
j105	t52	0	0	0	1
j106	t53	1	0	0	0
j106	t54	0	1	0	0
j106	t55	0	1	1	0
j106	t56	0	1	1	0
j106	t57	0	0	1	0
j106	t58	0	0	1	1
j106	t59	0	0	0	1

j107	t12	1	0	0	0
j107	t15	0	1	0	0
j107	t19	0	0	1	0
j107	t23	0	0	0	1
j108	t19	0	1	0	0
j108	t21	0	0	0	1
j109	t2	0	0	1	0
j109	t9	0	0	0	1
j110	t31	1	0	0	0
j110	t32	0	1	0	0
j110	t33	0	1	0	0
j110	t34	0	0	1	0

Table 2- Optimal Schedule Output

Job #	Completion Time	Due Dates
j1	52	60
j2	39	40
j3	53	60
j4	18	60
j5	37	40
j6	60	60
j7	19	60
j8	36	50
j9	25	60
j10	48	65
j11	45	65
j12	21	40
j13	19	65
j14	22	40
j15	53	60
j16	57	60
j17	20	65
j18	57	67
j19	36	60
j20	12	40
j21	36	40
j22	36	40
j23	19	40
j24	49	60
j25	49	50
j26	26	40
j27	36	40
j28	28	65
j29	40	40

j30	53	60
j31	36	40
j32	53	65
j33	36	69
j34	38	65
j35	59	65
j36	56	60
j37	53	69
j38	28	45
j39	52	65
j40	60	60
j41	59	60
j42	39	40
j43	29	40
j44	20	60
j45	60	60
j46	25	25
j47	21	29
j48	25	65
j49	53	65
j50	37	60
j51	38	50
j52	60	65
j53	37	69
j54	50	55
j55	52	65
j56	60	60
j57	52	69
j58	23	60
j59	52	65
j60	59	60
j61	58	65
j62	9	69
j63	50	50
j64	48	55
j65	39	50
j66	49	50
j67	52	55
j68	51	67
j69	39	50
j70	38	50
j71	58	60
j72	48	50
j73	20	60
j74	16	60

j75	50	50
j76	55	55
j77	56	59
j78	19	65
j79	53	55
j80	59	60
j81	30	60
j82	59	65
j83	54	69
j84	9	65
j85	9	65
j86	60	60
j87	45	69
j88	8	65
j89	18	65
j90	59	60
j91	58	60
j92	60	60
j93	49	60
j94	25	60
j95	50	50
j96	55	55
j97	52	59
j98	25	65
j99	60	65
j100	57	60
j101	14	60
j102	59	65
j103	59	69
j104	29	65
j105	52	65
j106	59	60
j107	23	69
j108	21	65
j109	9	65
j110	35	35

Table 3- Completion times and Due Dates Output

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	12	3	0	0
t2	8	4	6	0
t3	9	5	2	0
t4	10	7	2	0

t5	10	6	7	0
t6	10	12	3	0
t7	8	12	2	0
t8	10	5	11	1
t9	11	7	10	5
t10	2	9	3	0
t11	2	2	5	0
t12	3	3	5	1
t13	3	3	4	0
t14	0	5	10	1
t15	0	2	5	0
t16	2	0	5	1
t17	8	1	2	0
t18	0	0	0	2
t19	10	8	2	4
t20	3	4	2	3
t21	12	2	5	3
t22	5	4	0	1
t23	0	1	6	2
t24	5	5	10	0
t25	2	1	0	5
t26	0	1	0	1
t27	0	4	2	2
t28	10	1	3	1
t29	3	6	3	2
t30	3	3	2	1
t31	7	7	3	0
t32	3	3	3	0
t33	5	7	9	0
t34	0	7	11	0
t35	3	4	1	1
t36	0	4	0	7
t37	5	3	4	5
t38	0	4	1	2
t39	0	6	0	4
t40	0	1	2	1
t41	0	1	3	0
t42	0	2	4	0
t43	5	2	1	0
t44	0	7	5	0
t45	3	3	7	3
t46	0	5	4	1
t47	0	8	3	0
t48	0	3	11	2
t49	0	4	7	7

t50	0	0	6	3
t51	0	0	6	1
t52	5	0	2	8
t53	7	2	4	7
t54	3	3	7	1
t55	0	5	6	2
t56	0	3	6	3
t57	3	1	10	3
t58	0	2	8	8
t59	0	0	3	8
t60	0	0	0	8

Table 4- Capacity Level for Each Stage Output

j43	0	40	0	1	0	1	0	0	0	0
j44	0	60	0	0	1	1	0	0	1	0
j45	0	60	1	3	2	1	2	1	1	0
j46	0	25	1	1	1	1	2	1	1	0
j47	0	29	0	1	0	1	0	0	0	0
j48	0	65	0	0	1	1	0	0	1	0
j49	0	65	1	4	6	1	4	1	1	0
j50	0	60	1	1	1	1	2	1	1	0
j51	0	50	0	0	1	1	0	0	1	1
j52	0	65	1	3	2	1	2	1	1	0
j53	0	69	1	1	1	1	2	1	1	0
j54	0	55	0	1	0	1	0	0	0	1
j55	0	65	0	0	1	1	0	0	1	0
j56	0	60	1	3	4	1	4	1	1	0
j57	0	69	1	1	1	1	1	1	1	0
j58	0	60	0	1	0	1	0	0	0	0
j59	0	65	0	0	1	1	0	0	1	0
j60	0	60	1	6	2	1	4	1	1	0
j61	5	65	1	1	1	1	1	0	1	0
j62	4	69	0	1	0	1	0	0	0	0
j63	0	50	0	0	1	1	0	0	1	0
j64	0	55	1	3	2	1	4	0	1	0
j65	0	50	1	1	1	1	4	0	1	0
j66	2	50	0	1	0	1	0	0	0	0
j67	1	55	0	0	1	1	0	0	1	0
j68	5	67	1	3	2	1	2	1	1	0
j69	6	50	1	1	1	1	1	1	1	0
j70	7	50	0	1	0	1	0	0	0	0
j71	8	60	1	6	6	1	4	0	1	0
j72	0	50	1	1	1	1	1	1	1	0
j73	1	60	0	1	0	1	0	0	0	0
j74	3	60	0	0	1	1	0	0	1	0
j75	8	50	1	3	2	1	2	1	1	0
j76	10	55	1	1	1	1	2	1	1	0
j77	4	59	0	1	0	1	0	0	0	0
j78	0	65	0	0	1	1	0	0	1	0
j79	5	55	1	4	6	1	4	1	1	0
j80	4	60	1	1	1	1	1	1	1	0
j81	0	60	0	0	1	1	0	0	1	0
j82	0	65	1	3	2	1	2	1	1	0
j83	0	69	1	1	1	1	1	1	1	0
j84	0	65	0	1	0	1	0	0	0	0
j85	1	65	0	0	1	1	0	0	1	0
j86	5	60	1	3	4	1	4	1	1	0
j87	6	69	1	1	1	1	2	1	1	0
j88	7	65	0	1	0	1	0	0	0	0
j89	8	65	0	0	1	1	0	0	1	0
j90	0	60	1	6	2	2	4	1	1	0

j91	1	60	1	6	6	2	4	0	1	0
j92	3	60	1	1	1	1	2	1	1	0
j93	8	60	0	1	0	1	0	0	0	0
j94	10	60	0	0	1	1	0	0	1	0
j95	4	50	1	3	2	1	2	1	1	0
j96	0	55	1	1	1	1	2	1	1	0
j97	5	59	0	1	0	1	0	0	0	0
j98	4	65	0	0	1	1	0	0	1	0
j99	0	65	1	4	6	2	4	1	1	0
j100	0	60	1	1	1	1	1	1	1	0
j101	0	60	0	0	1	1	0	0	1	0
j102	0	65	1	3	2	2	2	1	1	0
j103	1	69	1	1	1	1	2	1	1	0
j104	5	65	0	1	0	1	0	0	0	0
j105	6	65	0	0	1	1	0	0	1	0
j106	7	60	1	3	4	2	4	1	1	0
j107	8	69	1	1	1	1	2	1	1	0
j108	0	65	0	1	0	1	0	0	0	0
j109	1	65	0	0	1	1	0	0	1	0
j110	3	35	1	2	1	1	4	1	1	0
j111	0	50	1	6	6	1	0	0	0	0
j112	0	50	1	1	1	1	4	0	1	0
j113	0	60	0	1	0	1	0	1	0	0
j114	0	50	0	0	1	1	0	0	0	0
j115	0	60	1	3	2	1	0	0	1	0
j116	0	60	1	1	1	1	2	1	1	0
j117	0	50	0	1	0	1	0	1	0	0
j118	0	55	0	0	1	1	0	0	0	0
j119	0	59	1	4	6	1	0	0	1	0
j120	0	65	1	1	1	1	4	1	1	0
j121	0	55	0	0	1	1	0	0	1	0
j122	0	60	1	3	2	1	0	0	1	0
j123	0	60	1	1	1	1	2	1	1	0
j124	0	65	0	1	0	1	0	1	0	0
j125	0	69	0	0	1	1	0	0	0	0
j126	0	65	1	3	4	1	0	0	1	0
j127	0	65	1	1	1	1	4	1	1	0
j128	0	60	0	1	0	1	0	1	0	0
j129	0	69	0	0	1	1	0	0	0	1
j130	0	50	1	6	2	1	0	0	1	0
j131	0	50	1	6	6	1	0	0	1	0
j132	0	60	1	1	1	1	4	1	1	0
j133	0	50	0	1	0	1	0	1	0	0
j134	0	60	0	0	1	1	0	0	0	1
j135	0	60	1	3	2	1	0	0	1	0
j136	0	50	1	1	1	1	4	1	1	0
j137	0	55	0	1	0	1	4	0	1	1
j138	0	59	0	0	1	1	0	1	0	0

j139	0	65	1	4	6	1	0	0	0	0
j140	0	55	1	1	1	1	0	0	1	0
j141	0	60	0	0	1	1	2	1	1	0
j142	0	60	1	3	2	1	0	1	0	0
j143	0	65	1	1	1	1	0	0	0	0
j144	0	69	0	1	0	1	0	0	1	0
j145	0	65	0	0	1	1	4	1	1	0
j146	0	65	1	3	4	1	0	0	1	0
j147	5	60	1	1	1	1	0	0	1	1
j148	4	69	0	1	0	1	2	1	1	0
j149	0	60	0	0	1	1	0	1	0	0
j150	0	60	1	6	2	1	0	0	0	1
j151	0	55	1	1	1	1	0	0	1	0
j152	5	59	0	1	0	1	4	1	1	0

Table 1- Process times and Setup times of 8th Numerical Example

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t11	1	0	0	0
j1	t13	0	1	0	0
j1	t28	0	0	1	0
j1	t29	0	0	1	0
j1	t30	0	0	1	0
j1	t31	0	0	1	0
j1	t32	0	1	1	1
j1	t33	0	0	1	0
j1	t34	0	0	1	1
j1	t36	0	0	0	1
j1	t41	0	1	0	0
j1	t43	0	1	0	0
j1	t44	0	1	0	0
j1	t45	0	1	0	0
j1	t52	0	0	1	0
j1	t54	0	0	0	1
j2	t6	1	0	0	0
j2	t22	0	1	0	0
j2	t27	0	1	0	0
j2	t28	0	1	0	0
j2	t32	0	0	1	0
j2	t33	0	0	1	0
j2	t36	0	0	0	1
j3	t34	1	0	0	0
j3	t35	0	1	0	0
j3	t38	0	0	1	0
j3	t40	0	0	0	1

j4	t1	0	1	0	0
j4	t18	0	0	0	1
j5	t10	0	0	1	0
j5	t17	0	0	0	1
j6	t12	1	0	0	0
j6	t23	0	1	0	0
j6	t34	0	0	1	0
j6	t40	0	1	0	0
j6	t41	0	1	0	0
j6	t51	0	0	1	0
j6	t53	0	0	0	1
j7	t15	1	0	0	0
j7	t16	0	1	0	0
j7	t17	0	0	1	0
j7	t18	0	0	0	1
j8	t35	0	1	0	0
j8	t36	0	0	0	1
j9	t22	0	0	1	0
j9	t23	0	0	0	1
j10	t38	1	0	0	0
j10	t45	0	1	0	0
j10	t47	0	1	0	0
j10	t52	0	0	1	0
j11	t24	1	0	0	0
j11	t29	0	1	0	0
j11	t32	0	0	1	0
j11	t33	0	0	0	1
j12	t1	0	1	0	0
j12	t23	0	0	0	1
j13	t17	0	0	1	0
j13	t22	0	0	0	1
j14	t16	1	0	0	0
j14	t21	0	1	0	0
j14	t22	0	1	0	0
j14	t23	0	1	0	0
j14	t26	0	0	1	0
j14	t27	0	0	1	0
j14	t28	0	0	0	1
j15	t11	1	0	0	0
j15	t20	0	1	0	0
j15	t22	0	0	1	0
j15	t27	0	0	0	1
j16	t3	0	1	0	0
j16	t14	0	0	0	1
j17	t17	0	0	1	0

j17	t32	0	0	0	1
j18	t19	1	0	0	0
j18	t27	0	1	0	0
j18	t28	0	1	0	0
j18	t29	0	1	0	0
j18	t34	0	0	1	0
j18	t35	0	0	1	0
j18	t59	0	0	0	1
j19	t31	1	0	0	0
j19	t32	0	1	0	0
j19	t33	0	0	1	0
j19	t34	0	0	0	1
j20	t25	0	1	0	0
j20	t27	0	0	0	1
j21	t8	1	0	0	0
j21	t15	0	1	0	0
j21	t17	0	1	0	0
j21	t19	0	1	0	0
j21	t22	0	1	0	0
j21	t26	0	0	1	0
j21	t27	0	1	1	0
j21	t28	0	0	1	0
j21	t29	0	0	1	0
j21	t30	0	1	1	0
j21	t31	0	0	1	0
j21	t32	0	0	0	1
j22	t9	1	0	0	0
j22	t15	0	1	0	0
j22	t25	0	0	1	0
j22	t33	0	0	0	1
j23	t14	0	1	0	0
j23	t33	0	0	0	1
j24	t57	0	0	1	0
j24	t59	0	0	0	1
j25	t3	1	0	0	0
j25	t4	0	1	0	0
j25	t5	0	1	1	0
j25	t6	0	1	0	0
j25	t29	0	0	1	0
j25	t30	0	0	0	1
j26	t25	1	0	0	0
j26	t26	0	1	0	0
j26	t28	0	0	1	0
j26	t32	0	0	0	1
j27	t7	0	1	0	0

j27	t17	0	0	0	1
j28	t22	0	0	1	0
j28	t54	0	0	0	1
j29	t2	1	0	0	0
j29	t3	0	1	0	0
j29	t4	0	1	0	0
j29	t5	0	1	0	0
j29	t6	0	1	0	0
j29	t9	0	0	1	0
j29	t10	0	0	1	0
j29	t11	0	0	1	0
j29	t12	0	0	1	0
j29	t13	0	0	1	0
j29	t14	0	0	1	0
j29	t16	0	0	0	1
j30	t16	1	0	0	0
j30	t18	0	1	0	0
j30	t56	0	0	1	0
j30	t57	0	0	0	1
j31	t21	0	0	1	0
j31	t28	0	0	0	1
j32	t33	1	0	0	0
j32	t35	0	1	0	0
j32	t36	0	1	0	0
j32	t37	0	1	0	0
j32	t46	0	0	1	0
j32	t47	0	0	1	0
j32	t56	0	0	0	1
j33	t27	1	0	0	0
j33	t28	0	1	0	0
j33	t31	0	0	1	0
j33	t41	0	0	0	1
j34	t40	0	1	0	0
j34	t45	0	0	0	1
j35	t25	0	0	1	0
j35	t27	0	0	0	1
j36	t35	1	0	0	0
j36	t47	0	1	0	0
j36	t48	0	1	0	0
j36	t49	0	1	0	0
j36	t55	0	0	1	0
j36	t56	0	0	1	0
j36	t57	0	0	1	0
j36	t58	0	0	1	0
j36	t60	0	0	0	1

j37	t41	1	0	0	0
j37	t43	0	1	0	0
j37	t48	0	0	1	0
j37	t57	0	0	0	1
j38	t7	0	1	0	0
j38	t9	0	0	0	1
j39	t36	0	0	1	0
j39	t44	0	0	0	1
j40	t5	1	0	0	0
j40	t38	0	1	0	0
j40	t39	0	1	0	0
j40	t47	0	1	0	0
j40	t48	0	1	1	0
j40	t49	0	1	0	0
j40	t50	0	1	0	0
j40	t55	0	0	1	0
j40	t59	0	0	0	1
j41	t8	1	0	0	0
j41	t9	0	1	0	0
j41	t10	0	1	1	0
j41	t11	0	1	1	0
j41	t12	0	1	1	0
j41	t13	0	1	1	0
j41	t14	0	1	1	0
j41	t15	0	0	1	0
j41	t18	0	0	0	1
j42	t25	1	0	0	0
j42	t31	0	1	0	0
j42	t33	0	0	1	0
j42	t34	0	0	0	1
j43	t18	0	1	0	0
j43	t23	0	0	0	1
j44	t41	0	0	1	0
j44	t49	0	0	0	1
j45	t12	1	0	0	0
j45	t28	0	1	0	0
j45	t29	0	1	0	0
j45	t30	0	1	0	0
j45	t38	0	0	1	0
j45	t39	0	0	1	0
j45	t60	0	0	0	1
j46	t7	1	0	0	0
j46	t10	0	1	0	0
j46	t11	0	0	1	0
j46	t12	0	0	0	1

j47	t1	0	1	0	0
j47	t17	0	0	0	1
j48	t29	0	0	1	0
j48	t32	0	0	0	1
j49	t2	1	0	0	0
j49	t3	0	1	0	0
j49	t4	0	1	0	0
j49	t5	0	1	0	0
j49	t6	0	1	0	0
j49	t12	0	0	1	0
j49	t13	0	0	1	0
j49	t14	0	0	1	0
j49	t15	0	0	1	0
j49	t16	0	0	1	0
j49	t17	0	0	1	0
j49	t48	0	0	0	1
j50	t30	1	0	0	0
j50	t31	0	1	0	0
j50	t32	0	0	1	0
j50	t36	0	0	0	1
j51	t8	0	0	1	0
j51	t17	0	0	0	1
j52	t6	1	0	0	0
j52	t18	0	1	0	0
j52	t19	0	1	0	0
j52	t20	0	1	0	0
j52	t53	0	0	1	0
j52	t54	0	0	1	0
j52	t59	0	0	0	1
j53	t13	1	0	0	0
j53	t14	0	1	0	0
j53	t15	0	0	1	0
j53	t34	0	0	0	1
j54	t20	0	1	0	0
j54	t23	0	0	0	1
j55	t11	0	0	1	0
j55	t12	0	0	0	1
j56	t35	1	0	0	0
j56	t36	0	1	0	0
j56	t37	0	1	0	0
j56	t38	0	1	0	0
j56	t49	0	0	1	0
j56	t50	0	0	1	0
j56	t51	0	0	1	0
j56	t52	0	0	1	0

j56	t60	0	0	0	1
j57	t24	1	0	0	0
j57	t26	0	1	0	0
j57	t27	0	0	1	0
j57	t28	0	0	0	1
j58	t10	0	1	0	0
j58	t17	0	0	0	1
j59	t32	0	0	1	0
j59	t38	0	0	0	1
j60	t45	1	0	0	0
j60	t47	0	1	0	0
j60	t48	0	1	0	0
j60	t49	0	1	0	0
j60	t50	0	1	0	0
j60	t51	0	1	0	0
j60	t52	0	1	0	0
j60	t57	0	0	1	0
j60	t58	0	0	1	0
j60	t59	0	0	0	1
j61	t33	1	0	0	0
j61	t35	0	1	0	0
j61	t36	0	0	1	0
j61	t42	0	0	0	1
j62	t10	0	1	0	0
j62	t18	0	0	0	1
j63	t41	0	0	1	0
j63	t44	0	0	0	1
j64	t4	1	0	0	0
j64	t5	0	1	0	0
j64	t8	0	1	0	0
j64	t16	0	1	0	0
j64	t19	0	0	1	0
j64	t20	0	0	1	0
j64	t42	0	0	0	1
j65	t26	1	0	0	0
j65	t28	0	1	0	0
j65	t41	0	0	1	0
j65	t44	0	0	0	1
j66	t2	0	1	0	0
j66	t31	0	0	0	1
j67	t23	0	0	1	0
j67	t30	0	0	0	1
j68	t7	1	0	0	0
j68	t12	0	1	0	0
j68	t13	0	1	0	0

j68	t16	0	1	0	0
j68	t27	0	0	1	0
j68	t28	0	0	1	0
j68	t58	0	0	0	1
j69	t24	1	0	0	0
j69	t31	0	1	0	0
j69	t32	0	0	1	0
j69	t33	0	0	0	1
j70	t25	0	1	0	0
j70	t36	0	0	0	1
j71	t10	1	0	0	0
j71	t11	0	1	0	0
j71	t12	0	1	0	0
j71	t13	0	1	0	0
j71	t14	0	1	0	0
j71	t16	0	0	1	0
j71	t17	0	0	1	0
j71	t18	0	1	1	0
j71	t19	0	1	1	0
j71	t20	0	0	1	0
j71	t21	0	0	1	0
j71	t25	0	0	0	1
j72	t33	1	0	0	0
j72	t34	0	1	0	0
j72	t35	0	0	1	0
j72	t36	0	0	0	1
j73	t50	0	1	0	0
j73	t53	0	0	0	1
j74	t33	0	0	1	0
j74	t42	0	0	0	1
j75	t18	1	0	0	0
j75	t19	0	1	0	0
j75	t20	0	1	1	0
j75	t21	0	1	0	0
j75	t22	0	0	1	0
j75	t33	0	0	0	1
j76	t39	1	0	0	0
j76	t40	0	1	0	0
j76	t41	0	0	1	0
j76	t44	0	0	0	1
j77	t20	0	1	0	0
j77	t25	0	0	0	1
j78	t19	0	0	1	0
j78	t22	0	0	0	1
j79	t5	1	0	0	0

j79	t6	0	1	0	0
j79	t7	0	1	0	0
j79	t8	0	1	0	0
j79	t9	0	1	0	0
j79	t10	0	0	1	0
j79	t11	0	0	1	0
j79	t12	0	0	1	0
j79	t13	0	0	1	0
j79	t14	0	0	1	0
j79	t15	0	0	1	0
j79	t49	0	0	0	1
j80	t11	1	0	0	0
j80	t35	0	1	0	0
j80	t36	0	0	1	0
j80	t54	0	0	0	1
j81	t22	0	0	1	0
j81	t32	0	0	0	1
j82	t46	1	0	0	0
j82	t47	0	1	0	0
j82	t50	0	1	1	0
j82	t51	0	1	0	0
j82	t52	0	0	1	0
j82	t60	0	0	0	1
j83	t25	1	0	0	0
j83	t27	0	1	0	0
j83	t33	0	0	1	0
j83	t36	0	0	0	1
j84	t50	0	1	0	0
j84	t58	0	0	0	1
j85	t28	0	0	1	0
j85	t33	0	0	0	1
j86	t44	1	0	0	0
j86	t49	0	1	0	0
j86	t50	0	1	1	0
j86	t51	0	1	1	0
j86	t52	0	0	1	0
j86	t53	0	0	1	0
j86	t54	0	0	0	1
j87	t33	1	0	0	0
j87	t38	0	1	0	0
j87	t39	0	0	1	0
j87	t54	0	0	0	1
j88	t31	0	1	0	0
j88	t56	0	0	0	1
j89	t36	0	0	1	0

j89	t53	0	0	0	1
j90	t38	1	0	0	0
j90	t39	0	1	0	0
j90	t40	0	1	0	0
j90	t41	0	1	0	0
j90	t42	0	1	0	0
j90	t43	0	1	0	0
j90	t44	0	1	0	0
j90	t50	0	0	1	0
j90	t51	0	0	1	1
j90	t53	0	0	0	1
j91	t9	1	0	0	0
j91	t11	0	1	0	0
j91	t15	0	1	1	0
j91	t16	0	1	1	0
j91	t17	0	1	1	0
j91	t18	0	0	1	1
j91	t19	0	1	1	0
j91	t22	0	1	0	0
j91	t42	0	0	1	0
j91	t43	0	0	0	1
j92	t4	1	0	0	0
j92	t10	0	1	0	0
j92	t11	0	0	1	0
j92	t18	0	0	0	1
j93	t10	0	1	0	0
j93	t17	0	0	0	1
j94	t16	0	0	1	0
j94	t17	0	0	0	1
j95	t21	1	0	0	0
j95	t28	0	1	0	0
j95	t29	0	1	0	0
j95	t30	0	1	1	0
j95	t31	0	0	1	0
j95	t50	0	0	0	1
j96	t43	1	0	0	0
j96	t49	0	1	0	0
j96	t51	0	0	1	0
j96	t55	0	0	0	1
j97	t40	0	1	0	0
j97	t42	0	0	0	1
j98	t7	0	0	1	0
j98	t16	0	0	0	1
j99	t1	1	0	0	0
j99	t3	0	1	0	0

j99	t4	0	1	0	0
j99	t5	0	1	0	0
j99	t6	0	1	0	0
j99	t10	0	0	1	0
j99	t11	0	0	1	0
j99	t12	0	0	1	0
j99	t13	0	0	1	0
j99	t14	0	0	1	0
j99	t15	0	0	1	0
j99	t27	0	0	0	1
j99	t32	0	0	0	1
j100	t10	1	0	0	0
j100	t11	0	1	0	0
j100	t24	0	0	1	0
j100	t27	0	0	0	1
j101	t53	0	0	1	0
j101	t54	0	0	0	1
j102	t3	1	0	0	0
j102	t5	0	1	0	0
j102	t6	0	1	0	0
j102	t7	0	1	0	0
j102	t8	0	0	1	0
j102	t9	0	0	1	0
j102	t24	0	0	0	1
j102	t34	0	0	0	1
j103	t56	1	0	0	0
j103	t57	0	1	0	0
j103	t58	0	0	1	0
j103	t60	0	0	0	1
j104	t40	0	1	0	0
j104	t42	0	0	0	1
j105	t43	0	0	1	0
j105	t55	0	0	0	1
j106	t10	1	0	0	0
j106	t12	0	1	0	0
j106	t13	0	1	0	0
j106	t37	0	1	0	0
j106	t51	0	0	1	0
j106	t52	0	0	1	0
j106	t53	0	0	1	0
j106	t54	0	0	1	0
j106	t56	0	0	0	1
j106	t57	0	0	0	1
j107	t31	1	0	0	0
j107	t49	0	1	0	0

j107	t51	0	0	1	0
j107	t57	0	0	0	1
j108	t20	0	1	0	0
j108	t23	0	0	0	1
j109	t36	0	0	1	0
j109	t60	0	0	0	1
j110	t13	1	0	0	0
j110	t14	0	1	0	0
j110	t35	0	0	0	1
j111	t29	1	0	0	0
j111	t31	0	1	0	0
j111	t35	0	0	1	0
j111	t36	0	1	1	0
j111	t37	0	0	1	0
j111	t38	0	0	1	0
j111	t40	0	1	0	0
j111	t41	0	1	1	0
j111	t43	0	1	0	0
j111	t44	0	1	0	0
j111	t45	0	0	1	0
j111	t48	0	0	0	1
j112	t9	1	0	0	0
j112	t10	0	1	0	0
j112	t14	0	0	1	0
j112	t50	0	0	0	1
j113	t51	0	1	0	0
j113	t60	0	0	0	1
j114	t25	0	0	1	0
j114	t33	0	0	0	1
j115	t2	1	0	0	0
j115	t3	0	1	0	0
j115	t4	0	1	1	0
j115	t39	0	1	0	0
j115	t41	0	0	1	0
j115	t51	0	0	0	1
j116	t31	1	0	0	0
j116	t38	0	1	0	0
j116	t58	0	0	1	0
j116	t59	0	0	0	1
j117	t33	0	1	0	0
j117	t34	0	0	0	1
j118	t7	0	0	1	0
j118	t10	0	0	0	1
j119	t3	1	0	0	0
j119	t4	0	1	0	0

j119	t5	0	1	0	0
j119	t6	0	0	1	0
j119	t7	0	1	1	0
j119	t8	0	1	1	0
j119	t9	0	0	1	0
j119	t10	0	0	1	0
j119	t11	0	0	1	0
j119	t12	0	0	0	1
j120	t7	1	0	0	0
j120	t12	0	1	0	0
j120	t13	0	0	1	0
j120	t19	0	0	0	1
j121	t25	0	0	1	0
j121	t30	0	0	0	1
j122	t1	1	0	0	0
j122	t2	0	1	0	0
j122	t9	0	1	0	0
j122	t12	0	1	1	0
j122	t13	0	0	1	0
j122	t58	0	0	0	1
j123	t6	1	0	0	0
j123	t10	0	1	0	0
j123	t47	0	0	1	0
j123	t50	0	0	0	1
j124	t17	0	1	0	0
j124	t32	0	0	0	1
j125	t19	0	0	1	0
j125	t21	0	0	0	1
j126	t54	1	0	0	0
j126	t55	0	1	0	0
j126	t56	0	1	1	0
j126	t57	0	1	1	0
j126	t58	0	0	1	0
j126	t59	0	0	1	0
j126	t60	0	0	0	1
j127	t41	1	0	0	0
j127	t45	0	1	0	0
j127	t47	0	0	1	0
j127	t50	0	0	0	1
j128	t4	0	1	0	0
j128	t18	0	0	0	1
j129	t28	0	0	1	0
j129	t42	0	0	0	1
j130	t16	1	0	0	0
j130	t20	0	1	0	0

j130	t22	0	1	0	0
j130	t25	0	1	0	0
j130	t29	0	1	0	0
j130	t30	0	1	1	0
j130	t32	0	1	0	0
j130	t39	0	0	1	0
j130	t46	0	0	0	1
j131	t1	1	0	0	0
j131	t33	0	1	0	0
j131	t34	0	1	0	0
j131	t35	0	1	0	0
j131	t36	0	1	0	0
j131	t37	0	1	0	0
j131	t38	0	1	1	0
j131	t39	0	0	1	0
j131	t40	0	0	1	0
j131	t42	0	0	1	0
j131	t43	0	0	1	0
j131	t44	0	0	1	0
j131	t45	0	0	0	1
j132	t24	1	0	0	0
j132	t27	0	1	0	0
j132	t31	0	0	1	0
j132	t40	0	0	0	1
j133	t28	0	1	0	0
j133	t33	0	0	0	1
j134	t49	0	0	1	0
j134	t54	0	0	0	1
j135	t1	1	0	0	0
j135	t3	0	1	0	0
j135	t4	0	1	0	0
j135	t35	0	1	0	0
j135	t38	0	0	1	0
j135	t39	0	0	1	0
j135	t58	0	0	0	1
j136	t14	1	0	0	0
j136	t23	0	1	0	0
j136	t28	0	0	1	0
j136	t38	0	0	0	1
j137	t15	0	1	0	0
j137	t23	0	0	0	1
j138	t33	0	0	1	0
j138	t34	0	0	0	1
j139	t51	1	0	0	0
j139	t52	0	1	0	0

j139	t53	0	0	1	0
j139	t54	0	0	1	0
j139	t55	0	1	1	0
j139	t56	0	1	1	0
j139	t57	0	1	1	0
j139	t58	0	0	1	0
j139	t59	0	0	0	1
j140	t26	1	0	0	0
j140	t31	0	1	0	0
j140	t42	0	0	1	0
j140	t55	0	0	0	1
j141	t57	0	0	1	0
j141	t58	0	0	0	1
j142	t5	1	0	0	0
j142	t10	0	1	0	0
j142	t20	0	1	0	0
j142	t39	0	1	0	0
j142	t43	0	0	1	0
j142	t44	0	0	1	0
j142	t58	0	0	0	1
j143	t31	1	0	0	0
j143	t35	0	1	0	0
j143	t37	0	0	1	0
j143	t39	0	0	0	1
j144	t41	0	1	0	0
j144	t54	0	0	0	1
j145	t10	0	0	1	0
j145	t12	0	0	0	1
j146	t4	1	0	0	0
j146	t23	0	1	0	0
j146	t34	0	1	0	0
j146	t37	0	1	0	0
j146	t52	0	0	1	0
j146	t53	0	0	1	0
j146	t54	0	0	1	0
j146	t55	0	0	1	0
j146	t58	0	0	0	1
j147	t56	1	0	0	0
j147	t57	0	1	0	0
j147	t58	0	0	1	0
j147	t59	0	0	0	1
j148	t25	0	1	0	0
j148	t57	0	0	0	1
j149	t38	0	0	1	0
j149	t42	0	0	0	1

j150	t3	1	0	0	0
j150	t4	0	1	0	0
j150	t5	0	1	0	0
j150	t17	0	1	0	0
j150	t20	0	0	1	0
j150	t35	0	1	0	0
j150	t38	0	1	0	0
j150	t41	0	1	0	0
j150	t53	0	0	1	0
j150	t57	0	0	0	1
j151	t17	1	0	0	0
j151	t20	0	1	0	0
j151	t25	0	0	1	0
j151	t31	0	0	0	1
j152	t53	0	1	0	0
j152	t57	0	0	0	1

Table 2- Optimal Schedule Output

Job #	Completion Time	Due Dates
j1	54	60
j2	36	40
j3	40	60
j4	18	60
j5	17	40
j6	53	60
j7	18	60
j8	36	50
j9	23	60
j10	53	65
j11	33	65
j12	23	40
j13	22	65
j14	28	40
j15	27	60
j16	14	60
j17	32	65
j18	59	67
j19	34	60
j20	27	40
j21	32	40
j22	33	40
j23	33	40
j24	59	60
j25	30	50

j26	32	40
j27	17	40
j28	54	65
j29	16	40
j30	57	60
j31	28	40
j32	56	65
j33	41	69
j34	45	65
j35	27	65
j36	60	60
j37	57	69
j38	10	45
j39	44	65
j40	59	60
j41	19	60
j42	34	40
j43	23	40
j44	49	60
j45	60	60
j46	12	25
j47	17	29
j48	32	65
j49	48	65
j50	36	60
j51	18	50
j52	59	65
j53	34	69
j54	24	55
j55	12	65
j56	60	60
j57	28	69
j58	17	60
j59	38	65
j60	59	60
j61	42	65
j62	18	69
j63	44	50
j64	42	55
j65	44	50
j66	31	50
j67	30	55
j68	58	67
j69	33	50
j70	36	50

j71	25	60
j72	36	50
j73	53	60
j74	42	60
j75	33	50
j76	44	55
j77	25	59
j78	22	65
j79	49	55
j80	54	60
j81	32	60
j82	60	65
j83	36	69
j84	58	65
j85	33	65
j86	54	60
j87	54	69
j88	56	65
j89	53	65
j90	53	60
j91	43	60
j92	18	60
j93	17	60
j94	17	60
j95	50	50
j96	55	55
j97	42	59
j98	16	65
j99	32	65
j100	27	60
j101	54	60
j102	34	65
j103	60	69
j104	42	65
j105	55	65
j106	57	60
j107	57	69
j108	23	65
j109	60	65
j110	35	35
j111	48	50
j112	50	50
j113	60	60
j114	33	50

j115	51	60
j116	59	60
j117	34	50
j118	10	55
j119	12	59
j120	19	65
j121	30	55
j122	58	60
j123	50	60
j124	32	65
j125	21	69
j126	60	65
j127	50	65
j128	18	60
j129	43	69
j130	46	50
j131	45	50
j132	41	60
j133	33	50
j134	54	60
j135	58	60
j136	38	50
j137	23	55
j138	34	59
j139	59	65
j140	55	55
j141	58	60
j142	59	60
j143	39	65
j144	54	69
j145	13	65
j146	58	65
j147	60	60
j148	57	69
j149	42	60
j150	57	60
j151	31	55
j152	57	59

Table 3- Completion times and Due Dates Output

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	12	3	0	0
t2	11	2	0	0

t3	8	9	0	0
t4	9	11	2	0
t5	11	9	2	0
t6	9	7	2	0
t7	11	5	4	0
t8	10	3	5	0
t9	12	3	4	2
t10	12	12	11	1
t11	12	5	11	0
t12	6	9	8	5
t13	8	5	8	0
t14	3	7	7	1
t15	3	7	8	0
t16	8	5	6	2
t17	3	5	9	8
t18	3	6	2	8
t19	3	6	7	1
t20	0	10	6	0
t21	3	2	3	1
t22	0	5	10	2
t23	0	5	2	7
t24	12	0	2	1
t25	6	5	9	2
t26	10	4	4	0
t27	2	7	6	5
t28	0	11	11	3
t29	1	5	6	0
t30	3	4	6	3
t31	11	10	7	2
t32	0	4	11	8
t33	10	4	11	8
t34	5	4	7	7
t35	10	12	4	1
t36	0	5	11	7
t37	0	6	3	0
t38	8	9	10	2
t39	3	5	7	1
t40	0	8	1	3
t41	7	7	11	1
t42	0	1	6	8
t43	3	5	5	1
t44	5	3	2	4
t45	5	4	1	2
t46	3	0	2	1
t47	0	9	5	0

t48	0	3	4	2
t49	0	9	3	2
t50	0	7	7	4
t51	1	5	11	2
t52	0	2	11	0
t53	0	1	11	4
t54	1	0	4	8
t55	0	2	6	3
t56	8	2	6	3
t57	0	6	9	7
t58	0	0	10	8
t59	0	0	1	8
t60	0	0	0	8

Table 4- Capacity Level for Each Stage Output

j43	0	40	0	1	0	1	0	0	0	0
j44	0	60	0	0	1	1	0	0	1	0
j45	0	60	1	3	2	1	2	1	1	0
j46	0	25	1	1	1	1	2	1	1	0
j47	0	29	0	1	0	1	0	0	0	0
j48	0	65	0	0	1	1	0	0	1	0
j49	0	65	1	4	6	1	4	1	1	0
j50	0	60	1	1	1	1	2	1	1	0
j51	0	50	0	0	1	1	0	0	1	1
j52	0	65	1	3	2	1	2	1	1	0
j53	0	69	1	1	1	1	2	1	1	0
j54	0	55	0	1	0	1	0	0	0	1
j55	0	65	0	0	1	1	0	0	1	0
j56	0	60	1	3	4	1	4	1	1	0
j57	0	69	1	1	1	1	1	1	1	0
j58	0	60	0	1	0	1	0	0	0	0
j59	0	65	0	0	1	1	0	0	1	0
j60	0	60	1	6	2	1	4	1	1	0
j61	5	65	1	1	1	1	1	0	1	0
j62	4	69	0	1	0	1	0	0	0	0
j63	0	50	0	0	1	1	0	0	1	0
j64	0	55	1	3	2	1	4	0	1	0
j65	0	50	1	1	1	1	4	0	1	0
j66	2	50	0	1	0	1	0	0	0	0
j67	1	55	0	0	1	1	0	0	1	0
j68	5	67	1	3	2	1	2	1	1	0
j69	6	50	1	1	1	1	1	1	1	0
j70	7	50	0	1	0	1	0	0	0	0
j71	8	60	1	6	6	1	4	0	1	0
j72	0	50	1	1	1	1	1	1	1	0
j73	1	60	0	1	0	1	0	0	0	0
j74	3	60	0	0	1	1	0	0	1	0
j75	8	50	1	3	2	1	2	1	1	0
j76	10	55	1	1	1	1	2	1	1	0
j77	4	59	0	1	0	1	0	0	0	0
j78	0	65	0	0	1	1	0	0	1	0
j79	5	55	1	4	6	1	4	1	1	0
j80	4	60	1	1	1	1	1	1	1	0
j81	0	60	0	0	1	1	0	0	1	0
j82	0	65	1	3	2	1	2	1	1	0
j83	0	69	1	1	1	1	1	1	1	0
j84	0	65	0	1	0	1	0	0	0	0
j85	1	65	0	0	1	1	0	0	1	0
j86	5	60	1	3	4	1	4	1	1	0
j87	6	69	1	1	1	1	2	1	1	0
j88	7	65	0	1	0	1	0	0	0	0
j89	8	65	0	0	1	1	0	0	1	0
j90	0	60	1	6	2	2	4	1	1	0

j91	1	60	1	6	6	2	4	0	1	0
j92	3	60	1	1	1	1	2	1	1	0
j93	8	60	0	1	0	1	0	0	0	0
j94	10	60	0	0	1	1	0	0	1	0
j95	4	50	1	3	2	1	2	1	1	0
j96	0	55	1	1	1	1	2	1	1	0
j97	5	59	0	1	0	1	0	0	0	0
j98	4	65	0	0	1	1	0	0	1	0
j99	0	65	1	4	6	2	4	1	1	0
j100	0	60	1	1	1	1	1	1	1	0
j101	0	60	0	0	1	1	0	0	1	0
j102	0	65	1	3	2	2	2	1	1	0
j103	1	69	1	1	1	1	2	1	1	0
j104	5	65	0	1	0	1	0	0	0	0
j105	6	65	0	0	1	1	0	0	1	0
j106	7	60	1	3	4	2	4	1	1	0
j107	8	69	1	1	1	1	2	1	1	0
j108	0	65	0	1	0	1	0	0	0	0
j109	1	65	0	0	1	1	0	0	1	0
j110	3	35	1	2	1	1	4	1	1	0
j111	0	50	1	6	6	1	0	0	0	0
j112	0	50	1	1	1	1	4	0	1	0
j113	0	60	0	1	0	1	0	1	0	0
j114	0	50	0	0	1	1	0	0	0	0
j115	0	60	1	3	2	1	0	0	1	0
j116	0	60	1	1	1	1	2	1	1	0
j117	0	50	0	1	0	1	0	1	0	0
j118	0	55	0	0	1	1	0	0	0	0
j119	0	59	1	4	6	1	0	0	1	0
j120	0	65	1	1	1	1	4	1	1	0
j121	0	55	0	0	1	1	0	0	1	0
j122	0	60	1	3	2	1	0	0	1	0
j123	0	60	1	1	1	1	2	1	1	0
j124	0	65	0	1	0	1	0	1	0	0
j125	0	69	0	0	1	1	0	0	0	0
j126	0	65	1	3	4	1	0	0	1	0
j127	0	65	1	1	1	1	4	1	1	0
j128	0	60	0	1	0	1	0	1	0	0
j129	0	69	0	0	1	1	0	0	0	1
j130	0	50	1	6	2	1	0	0	1	0
j131	0	50	1	6	6	1	4	1	1	0
j132	0	60	1	1	1	1	4	0	1	1
j133	0	50	0	1	0	1	0	1	0	0
j134	0	60	0	0	1	1	0	0	0	0
j135	0	60	1	3	2	1	0	0	1	0
j136	0	50	1	1	1	1	2	1	1	0
j137	0	55	0	1	0	1	0	1	0	0
j138	0	59	0	0	1	1	0	0	0	0

j139	0	65	1	4	6	1	0	0	1	0
j140	0	55	1	1	1	1	4	1	1	0
j141	0	60	0	0	1	1	0	0	1	0
j142	0	60	1	3	2	1	0	0	1	1
j143	0	65	1	1	1	1	2	1	1	0
j144	0	69	0	1	0	1	0	1	0	0
j145	0	65	0	0	1	1	0	0	0	1
j146	0	65	1	3	4	1	0	0	1	0
j147	5	60	1	1	1	1	4	1	1	0
j148	4	69	0	1	0	1	0	1	0	0
j149	0	60	0	0	1	1	0	0	0	0
j150	0	60	1	6	2	1	0	0	1	0
j151	0	55	1	1	1	1	2	1	1	0
j152	5	59	0	1	0	1	0	0	0	0
j153	4	65	0	0	1	1	0	0	1	0
j154	0	65	1	4	6	2	4	1	1	0
j155	0	60	1	1	1	1	1	1	1	0
j156	0	60	0	0	1	1	0	0	1	0
j157	0	65	1	3	2	2	2	1	1	0
j158	1	69	1	1	1	1	2	1	1	0
j159	5	65	0	1	0	1	0	0	0	0
j160	6	65	0	0	1	1	0	0	1	0

Table 1- Process times and Setup times of 9th Numerical Example

APPENDIX-7

Pr. Orders	Release Dates	Due Dates	Process Times				Setup Times			
			Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
j1	0	62	0	9	2	0	0	0	1	0
j2	0	36	0	1	0	0	0	0	0	0
j3	0	24	0	0	0	6	0	0	0	0
j4	0	12	0	1	0	0	0	1	0	0
j5	0	26	0	1	4	0	0	0	0	0
j6	0	34	0	0	5	0	0	0	1	0
j7	0	28	0	0	5	0	0	0	1	0
j8	0	50	0	2	2	0	0	0	2	0
j9	0	52	0	1	1	0	0	0	0	0
j10	0	36	0	1	2	0	0	0	1	0
j11	0	62	0	5	2	1	0	0	2	0
j12	0	62	0	12	8	4	0	0	1	0
j13	0	66	0	3	21	1	0	1	0	0
j14	0	92	0	1	19	0	0	1	0	0
j15	0	26	0	0	1	0	0	0	0	0
j16	0	70	0	9	21	2	0	1	1	0
j17	0	56	0	2	5	0	0	0	0	0
j18	0	80	0	0	59	25	0	0	1	1
j19	0	14	0	0	0	2	0	0	0	0
j20	0	34	0	0	2	2	0	0	1	1
j21	0	64	0	10	6	3	0	0	2	0
j22	0	36	0	0	2	0	0	0	0	0
j23	0	62	0	3	2	1	0	0	1	0
j24	0	120	0	32	8	4	0	1	2	0
j25	0	64	0	0	1	2	0	0	0	1
j26	0	22	0	0	0	6	0	0	0	0
j27	0	84	0	21	20	8	0	5	2	1
j28	0	76	0	4	1	1	0	0	0	0
j29	0	94	0	0	0	2	0	0	0	0
j30	0	52	0	1	1	0	0	0	1	0
j31	0	54	0	10	9	0	0	1	0	0
j32	0	68	0	0	13	8	0	0	1	1
j33	0	50	0	0	8	2	0	0	1	0
j34	0	38	0	0	4	8	0	0	1	1
j35	0	42	0	0	1	1	0	0	1	0
j36	0	6	0	0	0	1	0	0	0	1
j37	0	50	0	3	1	2	0	0	1	0
j38	0	66	0	41	43	0	0	7	2	0
j39	0	76	0	0	1	0	0	0	4	0
j40	0	76	0	0	1	0	0	0	4	0
j41	0	76	0	0	1	0	0	0	4	0
j42	0	36	2	1	0	0	1	0	0	0

j43	0	42	1	3	0	0	1	0	0	0
j44	0	20	0	3	0	0	0	0	0	0
j45	0	20	0	2	0	0	0	0	0	0
j46	0	48	1	1	1	0	1	1	1	0
j47	10	56	0	1	1	1	0	1	1	0
j48	0	20	0	1	0	0	0	0	0	0
j49	0	22	0	1	1	0	0	0	1	0
j50	0	40	0	3	2	1	0	0	1	0
j51	0	38	0	1	2	1	0	1	2	2
j52	0	10	0	0	0	1	0	0	0	1
j53	0	26	0	7	4	0	0	1	1	0
j54	0	40	2	3	0	0	2	1	0	0
j55	0	40	2	3	0	0	2	1	0	0
j56	0	56	0	1	0	0	0	1	0	0
j57	0	34	0	1	1	0	0	1	0	0
j58	0	10	0	0	0	1	0	0	0	1
j59	0	10	0	0	0	1	0	0	0	0
j60	0	20	0	1	0	1	0	0	0	0
j61	0	46	0	7	5	2	0	1	1	1
j62	0	76	4	5	3	0	1	1	1	0
j63	0	20	0	4	3	0	0	1	1	0
j64	0	20	0	4	3	0	0	1	1	0
j65	0	26	0	1	1	0	0	1	1	0
j66	0	20	0	4	3	0	0	1	1	0
j67	0	26	0	7	0	0	0	0	0	0
j68	0	10	0	0	1	3	0	0	1	1
j69	0	10	0	0	0	1	0	0	0	0
j70	0	20	0	0	1	1	0	0	1	1
j71	0	30	0	7	0	0	0	1	0	0
j72	0	42	0	0	15	6	0	0	0	0
j73	0	26	0	5	6	6	0	1	1	0
j74	0	46	0	1	1	0	0	1	1	0
j75	0	26	0	3	1	0	0	1	1	0
j76	0	26	0	1	1	0	0	1	1	0
j77	0	46	0	0	0	1	0	1	1	1
j78	0	66	0	1	1	0	0	1	1	0
j79	0	66	1	1	1	0	1	1	1	0
j80	0	66	1	2	7	1	1	1	1	1
j81	0	28	0	3	2	0	0	0	1	0
j82	0	64	1	2	3	1	1	1	1	0
j83	10	64	1	1	0	0	1	0	0	0
j84	0	66	1	1	1	1	1	1	1	1
j85	0	30	1	1	1	0	1	1	1	0
j86	0	26	0	0	0	1	0	0	0	1
j87	0	56	1	1	1	0	1	1	1	0
j88	0	56	0	1	1	0	0	1	1	0
j89	0	90	1	1	1	1	1	1	1	1
j90	0	46	0	1	3	0	0	1	1	0

j91	0	26	4	2	17	0	1	1	1	0
j92	0	56	0	1	1	1	0	1	0	1
j93	10	66	0	4	0	0	0	1	0	0
j94	0	66	0	5	2	1	0	1	1	0
j95	0	64	0	0	0	2	0	0	0	0
j96	0	144	8	8	15	5	1	1	1	0
j97	0	36	0	1	1	1	0	1	1	0
j98	0	98	0	26	9	9	0	1	0	1
j99	0	86	0	1	1	1	0	1	0	0
j100	0	20	0	3	0	0	0	0	0	0
j101	0	66	2	1	1	0	2	1	0	0
j102	0	36	0	1	2	0	0	1	1	0
j103	10	46	0	2	1	1	0	1	0	1
j104	0	78	0	11	14	3	0	1	0	1
j105	0	10	0	0	2	0	0	0	1	0
j106	0	46	0	2	4	2	0	0	1	0
j107	0	66	2	2	1	1	1	0	1	0
j108	0	76	1	1	1	1	1	0	1	0
j109	0	76	1	2	1	1	1	1	1	0
j110	0	46	0	1	1	1	0	1	1	0
j111	0	46	0	2	1	1	0	1	0	1
j112	0	86	0	1	1	1	0	1	0	1
j113	0	86	0	1	1	1	0	1	0	1
j114	0	90	0	2	1	1	0	1	0	1
j115	0	28	0	1	1	1	0	2	0	0
j116	0	55	0	0	0	5	0	0	0	0
j117	0	60	0	0	1	0	0	0	1	0
j118	0	64	0	0	1	0	0	0	1	0
j119	0	136	0	0	1	0	0	0	1	0
j120	20	160	0	0	1	0	0	0	1	0
j121	30	70	0	2	2	0	0	1	1	0
j122	30	70	0	2	2	0	0	1	1	0
j123	30	70	0	2	2	0	0	1	1	0
j124	30	66	0	0	1	2	0	0	1	1

Table 1- Process times and Setup times of Real-Life Example

OPTIMAL SCHEDULE					
Job#	Time #	Stage 1	Stage 2	Stage 3	Stage 4
j1	t4	0	1	0	0
j1	t13	0	1	0	0
j1	t14	0	1	0	0
j1	t16	0	1	0	0
j1	t18	0	1	0	0
j1	t25	0	1	0	0
j1	t27	0	1	0	0
j1	t32	0	1	0	0
j1	t45	0	1	0	0

j1	t47	0	0	1	0
j1	t48	0	0	1	0
j2	t32	0	1	0	0
j3	t10	0	0	0	1
j3	t14	0	0	0	1
j3	t15	0	0	0	1
j3	t19	0	0	0	1
j3	t20	0	0	0	1
j3	t23	0	0	0	1
j4	t10	0	1	0	0
j5	t9	0	1	0	0
j5	t12	0	0	1	0
j5	t14	0	0	1	0
j5	t15	0	0	1	0
j5	t23	0	0	1	0
j6	t22	0	0	1	0
j6	t23	0	0	1	0
j6	t24	0	0	1	0
j6	t25	0	0	1	0
j6	t26	0	0	1	0
j7	t5	0	0	1	0
j7	t6	0	0	1	0
j7	t7	0	0	1	0
j7	t8	0	0	1	0
j7	t9	0	0	1	0
j8	t1	0	1	0	0
j8	t37	0	1	0	0
j8	t43	0	0	1	0
j8	t44	0	0	1	0
j9	t34	0	1	0	0
j9	t39	0	0	1	0
j10	t1	0	1	0	0
j10	t27	0	0	1	0
j10	t28	0	0	1	0
j11	t1	0	1	0	0
j11	t2	0	1	0	0
j11	t54	0	1	0	0
j11	t55	0	1	0	0
j11	t56	0	1	1	0
j11	t57	0	0	1	0
j11	t59	0	0	0	1
j12	t1	0	1	0	0
j12	t2	0	1	1	0
j12	t3	0	1	1	0
j12	t4	0	1	0	0

j12	t5	0	1	0	0
j12	t6	0	1	0	0
j12	t7	0	1	0	0
j12	t8	0	1	0	0
j12	t29	0	0	0	1
j12	t43	0	1	0	0
j12	t48	0	1	0	1
j12	t50	0	1	1	1
j12	t51	0	0	1	0
j12	t52	0	0	1	0
j12	t53	0	0	1	0
j12	t54	0	1	1	0
j12	t55	0	0	1	0
j12	t56	0	0	0	1
j13	t1	0	1	0	0
j13	t2	0	1	1	0
j13	t3	0	1	0	0
j13	t5	0	0	1	0
j13	t12	0	0	1	0
j13	t13	0	0	1	0
j13	t14	0	0	1	0
j13	t17	0	0	1	0
j13	t19	0	0	1	0
j13	t20	0	0	1	0
j13	t22	0	0	1	0
j13	t23	0	0	1	0
j13	t25	0	0	1	0
j13	t29	0	0	1	0
j13	t33	0	0	1	0
j13	t36	0	0	1	0
j13	t38	0	0	1	0
j13	t39	0	0	1	0
j13	t46	0	0	1	0
j13	t49	0	0	1	0
j13	t53	0	0	1	0
j13	t54	0	0	1	0
j13	t57	0	0	1	0
j13	t59	0	0	0	1
j14	t8	0	1	0	0
j14	t9	0	0	1	0
j14	t18	0	0	1	0
j14	t19	0	0	1	0
j14	t21	0	0	1	0
j14	t22	0	0	1	0
j14	t28	0	0	1	0

j14	t33	0	0	1	0
j14	t35	0	0	1	0
j14	t36	0	0	1	0
j14	t38	0	0	1	0
j14	t40	0	0	1	0
j14	t42	0	0	1	0
j14	t44	0	0	1	0
j14	t47	0	0	1	0
j14	t48	0	0	1	0
j14	t50	0	0	1	0
j14	t52	0	0	1	0
j14	t53	0	0	1	0
j14	t59	0	0	1	0
j15	t19	0	0	1	0
j16	t1	0	1	0	0
j16	t2	0	1	1	0
j16	t3	0	1	1	0
j16	t4	0	1	1	0
j16	t5	0	1	1	0
j16	t6	0	1	1	0
j16	t7	0	1	1	0
j16	t8	0	1	1	1
j16	t9	0	1	1	0
j16	t10	0	0	1	0
j16	t11	0	0	1	0
j16	t12	0	0	1	0
j16	t13	0	0	1	0
j16	t14	0	0	1	0
j16	t15	0	0	1	0
j16	t16	0	0	1	0
j16	t17	0	0	1	0
j16	t18	0	0	1	0
j16	t19	0	0	1	0
j16	t20	0	0	1	0
j16	t21	0	0	1	0
j16	t22	0	0	1	0
j16	t51	0	0	0	1
j17	t1	0	1	0	0
j17	t2	0	0	1	0
j17	t12	0	1	0	0
j17	t20	0	0	1	0
j17	t35	0	0	1	0
j17	t51	0	0	1	0
j17	t55	0	0	1	0
j18	t1	0	0	1	0

j18	t2	0	0	1	0
j18	t3	0	0	1	0
j18	t4	0	0	1	0
j18	t5	0	0	1	0
j18	t6	0	0	1	0
j18	t7	0	0	1	0
j18	t8	0	0	1	0
j18	t9	0	0	1	0
j18	t10	0	0	1	0
j18	t11	0	0	1	0
j18	t12	0	0	1	0
j18	t13	0	0	1	0
j18	t14	0	0	1	0
j18	t15	0	0	1	0
j18	t16	0	0	1	0
j18	t17	0	0	1	0
j18	t18	0	0	1	0
j18	t19	0	0	1	0
j18	t20	0	0	1	0
j18	t21	0	0	1	0
j18	t22	0	0	1	0
j18	t23	0	0	1	0
j18	t24	0	0	1	0
j18	t25	0	0	1	0
j18	t26	0	0	1	0
j18	t27	0	0	1	0
j18	t28	0	0	1	0
j18	t29	0	0	1	0
j18	t30	0	0	1	0
j18	t31	0	0	1	0
j18	t32	0	0	1	0
j18	t33	0	0	1	0
j18	t34	0	0	1	0
j18	t35	0	0	1	0
j18	t36	0	0	1	1
j18	t37	0	0	1	1
j18	t38	0	0	1	1
j18	t39	0	0	1	1
j18	t40	0	0	1	1
j18	t41	0	0	1	1
j18	t42	0	0	1	1
j18	t43	0	0	1	1
j18	t44	0	0	1	1
j18	t45	0	0	1	1
j18	t46	0	0	1	1

j18	t47	0	0	1	1
j18	t48	0	0	1	1
j18	t49	0	0	1	1
j18	t50	0	0	1	1
j18	t51	0	0	1	1
j18	t52	0	0	1	1
j18	t53	0	0	1	1
j18	t54	0	0	1	1
j18	t55	0	0	1	1
j18	t56	0	0	1	1
j18	t57	0	0	1	1
j18	t58	0	0	1	1
j18	t59	0	0	1	1
j18	t60	0	0	0	1
j19	t1	0	0	0	1
j19	t11	0	0	0	1
j20	t1	0	0	1	0
j20	t2	0	0	1	0
j20	t28	0	0	0	1
j20	t29	0	0	0	1
j21	t1	0	1	0	0
j21	t8	0	1	0	0
j21	t18	0	1	0	0
j21	t21	0	1	0	0
j21	t25	0	1	0	0
j21	t26	0	1	0	0
j21	t29	0	1	0	0
j21	t40	0	1	0	0
j21	t45	0	1	0	0
j21	t52	0	0	1	0
j21	t53	0	0	1	0
j21	t54	0	1	1	0
j21	t55	0	0	1	0
j21	t56	0	0	1	1
j21	t57	0	0	1	0
j21	t58	0	0	0	1
j21	t60	0	0	0	1
j22	t1	0	0	1	0
j22	t34	0	0	1	0
j23	t1	0	1	0	0
j23	t2	0	1	0	0
j23	t13	0	1	0	0
j23	t19	0	0	1	0
j23	t20	0	0	1	0
j23	t26	0	0	0	1

j24	t17	0	1	0	0
j24	t18	0	1	0	0
j24	t19	0	1	0	0
j24	t20	0	1	0	0
j24	t21	0	1	0	0
j24	t22	0	1	0	0
j24	t23	0	1	0	0
j24	t24	0	1	0	0
j24	t25	0	1	0	0
j24	t26	0	1	0	0
j24	t27	0	1	0	0
j24	t28	0	1	0	0
j24	t29	0	1	0	0
j24	t30	0	1	0	0
j24	t31	0	1	0	0
j24	t32	0	1	0	0
j24	t33	0	1	0	0
j24	t34	0	1	0	0
j24	t35	0	1	0	0
j24	t36	0	1	0	0
j24	t37	0	1	0	0
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j105	t1	0	0	1	0
j105	t2	0	0	1	0
j106	t3	0	1	0	0
j106	t19	0	1	0	0
j106	t22	0	0	1	0
j106	t23	0	0	1	1
j106	t24	0	0	1	0
j106	t25	0	0	1	0
j106	t41	0	0	0	1
j107	t8	1	0	0	0
j107	t9	1	0	0	0
j107	t27	0	1	0	0
j107	t28	0	1	0	0
j107	t29	0	0	1	0
j107	t60	0	0	0	1
j108	t17	1	0	0	0
j108	t18	0	1	0	0
j108	t30	0	0	1	0
j108	t34	0	0	0	1
j109	t2	1	0	0	0
j109	t22	0	1	0	0
j109	t23	0	1	0	0

j109	t33	0	0	1	0
j109	t60	0	0	0	1
j110	t40	0	1	0	0
j110	t45	0	0	1	0
j110	t46	0	0	0	1
j111	t17	0	1	0	0
j111	t18	0	1	0	0
j111	t44	0	0	1	0
j111	t45	0	0	0	1
j112	t34	0	1	0	0
j112	t41	0	0	1	0
j112	t49	0	0	0	1
j113	t27	0	1	0	0
j113	t30	0	0	1	0
j113	t31	0	0	0	1
j114	t48	0	1	0	0
j114	t54	0	1	0	0
j114	t55	0	0	1	0
j114	t57	0	0	0	1
j115	t10	0	1	0	0
j115	t20	0	0	1	0
j115	t22	0	0	0	1
j116	t26	0	0	0	1
j116	t33	0	0	0	1
j116	t35	0	0	0	1
j116	t46	0	0	0	1
j116	t49	0	0	0	1
j117	t40	0	0	1	0
j118	t40	0	0	1	0
j119	t39	0	0	1	0
j120	t60	0	0	1	0
j121	t41	0	1	0	0
j121	t42	0	1	0	0
j121	t51	0	0	1	0
j121	t52	0	0	1	0
j122	t41	0	1	0	0
j122	t42	0	1	0	0
j122	t44	0	0	1	0
j122	t45	0	0	1	0
j123	t41	0	1	0	0
j123	t42	0	1	0	0
j123	t47	0	0	1	0
j123	t48	0	0	1	0
j124	t40	0	0	1	0
j124	t53	0	0	0	1

j124	t54	0	0	0	1
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Table 2- Optimal Schedule Output

Job #	Completion Time	Due Dates
j1	48	62
j2	32	36
j3	23	24
j4	11	12
j5	23	26
j6	26	34
j7	9	28
j8	45	50
j9	39	52
j10	28	36
j11	59	62
j12	56	62
j13	59	66
j14	59	92
j15	19	26
j16	51	70
j17	55	56
j18	60	80
j19	11	14
j20	29	34
j21	60	64
j22	34	36
j23	26	62
j24	58	120
j25	42	64
j26	22	22
j27	60	84
j28	48	76
j29	51	94
j30	38	52
j31	47	54
j32	37	68
j33	40	50
j34	13	38
j35	40	42
j36	2	6
j37	48	50
j38	46	66
j39	9	76
j40	5	76

j41	43	76
j42	18	36
j43	39	42
j44	13	20
j45	13	20
j46	35	48
j47	49	56
j48	10	20
j49	18	22
j50	39	40
j51	26	38
j52	2	10
j53	26	26
j54	40	40
j55	16	40
j56	13	56
j57	21	34
j58	2	10
j59	8	10
j60	20	20
j61	45	46
j62	60	76
j63	9	20
j64	18	20
j65	13	26
j66	8	20
j67	23	26
j68	8	10
j69	9	10
j70	9	20
j71	13	30
j72	39	42
j73	26	26
j74	25	46
j75	14	26
j76	21	26
j77	35	46
j78	47	66
j79	58	66
j80	59	66
j81	5	28
j82	20	64
j83	38	64
j84	45	66

j85	24	30
j86	24	26
j87	53	56
j88	27	56
j89	24	90
j90	38	46
j91	26	26
j92	41	56
j93	9	66
j94	60	66
j95	40	64
j96	60	144
j97	29	36
j98	44	98
j99	48	86
j100	19	20
j101	5	66
j102	24	36
j103	43	46
j104	60	78
j105	2	10
j106	41	46
j107	60	66
j108	34	76
j109	60	76
j110	46	46
j111	46	46
j112	50	86
j113	32	86
j114	58	90
j115	22	28
j116	49	55
j117	41	60
j118	41	64
j119	40	136
j120	61	160
j121	52	70
j122	45	70
j123	48	70
j124	54	66

Table 3- Completion times and Due Dates Output

CAPACITY LEVEL AT EACH TIME PERIOD				
Time #	Stage 1	Stage 2	Stage 3	Stage 4
t1	9	27	17	7
t2	7	16	13	0
t3	1	16	10	0
t4	2	14	9	0
t5	1	12	18	1
t6	1	10	11	4
t7	3	13	10	2
t8	2	15	7	7
t9	1	16	13	3
t10	2	13	7	3
t11	2	7	9	2
t12	0	7	13	1
t13	0	11	11	2
t14	0	9	8	2
t15	2	8	7	1
t16	2	6	9	0
t17	3	16	11	1
t18	1	16	8	1
t19	1	10	14	2
t20	3	5	12	5
t21	1	11	8	0
t22	1	7	13	3
t23	1	8	14	8
t24	0	4	13	6
t25	0	5	11	1
t26	0	8	11	3
t27	2	8	8	0
t28	0	6	8	2
t29	0	4	7	4
t30	0	4	6	2
t31	0	5	4	3
t32	0	11	2	1
t33	3	4	9	2
t34	1	8	5	4
t35	0	3	5	4
t36	2	3	8	5
t37	0	5	7	3
t38	0	8	10	2
t39	0	8	13	4
t40	0	8	15	7
t41	0	10	7	5
t42	0	6	5	5

t43	0	3	10	3
t44	0	4	9	6
t45	0	4	8	3
t46	0	2	8	3
t47	2	2	10	1
t48	0	5	9	5
t49	0	1	7	5
t50	0	2	9	2
t51	2	1	8	3
t52	0	3	12	2
t53	0	1	8	5
t54	1	5	7	4
t55	2	1	8	2
t56	1	5	6	6
t57	1	1	7	5
t58	0	1	3	8
t59	0	0	4	6
t60	0	0	3	8

Table 4- Capacity Level for Each Stage Output

CURRICULUM VITAE:

Emre Özgenç Ekici



Current Address

Yıldızevler Mah. Duyu Sok. 31/28
Çankaya/ANKARA

E-Mail : emrekare@gmail.com

Cellular Phone: + (90) 505 282 72 92

EDUCATION

1999-2004 Bilkent University
Industrial Engineering (Undergraduate), Ankara
Full Scholarship Awarded
CGPA: 3.17 / 4.0

2010 Hacettepe University
Business Administration Faculty
Production Management (Graduate)
(Still continued)
CGPA: 3.69 / 4.0

WORK EXPERIENCE

(Compulsory Military Service has been accomplished.)

August 2005 – ,ASELSAN A.Ş. Ankara / Workshop Planning Senior Engineer and Lean Team Leader

- Providing production flow dynamically and interfering production problems as soon as possible, scheduling jobs for each workshop, capacity planning and balancing job allocations for different workshops.
- **Additionally, leading Lean applications, preparing VSM and Kaizen Team Leader**

November 2003 – May 2004, MAN Turkey A.Ş., Ankara (Senior Project)

- **Designing and Programming using C#** a Forklift Scheduling Program in order to improve material flow between assembly line and warehouse

August – September 2002, Kale Holding KaleData A.Ş., İstanbul (summer training)

- Marketing Research Project of Company

SKILLS AND ABILITIES

Computer Programming Languages:

Visual Basic (Very Good), C# (good), Java (good), ASP.NET (good), SQL (good), Html (medium),

Application Programs:

Microsoft Office (Excel, Access, Word, MS Project, MS Visio)(Advanced), Excel Macro (Good), ARENA Simulation Software (Advanced), GAMS Integer Programming Software (Advanced)

Language:

Advanced English, Intermediate German

CERTIFICATES

- **Lean 5S ve Visual Plant Applications, 2011**
- **Lean Hoshin Kanri and VSM Applications, 2011**
- Microsoft Office Project , 2009
- **Ranked Turkey 46th in the University Entrance Exam, 1999**
- **Ranked Turkey 200th in the Fen Liseleri Entrance Exam, 1996**
- **Töbitak Secondary School Math Olympiad Turkey 3rd, 1996**

SOCIAL ACTIVITIES

- **Founder and Organizer of ASELSAN Basketball Tournament, 2009-2012**
- **Coordinator and founder of IYEM (International Young Entrepreneurs Meeting) Organization- FIRST in Turkey-, 2002**
- **Young Entrepreneurs Club President, 2002-2003**
- **Student Union Engineering Faculty Representative, 2000-2001 - General Secretary, 2001-2002**

Personal Traits

- Intelligent
- Creative
- Responsible

Interests

- **Lego Robotics designing**
- **Playing Basketball**
- **Playing guitar, listening music**