EWMA CONTROL CHARTS FOR SKEWED DISTRIBUTIONS

ÇARPIK DAĞILIMLARI İÇİN EWMA KONTROL KARTLARI

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To the Most Precious Being in My Life,

MY SWEET MOTHER

ABSTRACT

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The classic Shewhart control charts are generally used for monitoring the process mean and variability in the characteristics of a random quality variable of interest and are based on the normality assumptions. For skewed distributions, in order to demonstrate the changes in the population, non-symmetric control limits need to be used. Methods such as the Weighted Variance (WV) Weighted Standard Deviation (WSD) and Skewness Correction (SC) are used with skewed distributions.

The classic $\bar{X}$ and R control charts and all their derivatives are generally used to detect large shifts in the process mean hence making them not too reliable in situations where in small shifts are of interest. To solve such problems, the Exponentially Weighted Moving Average (EWMA) control charts is used in this work.

The main aim of this thesis is to apply the Skewness Correction method to the EWMA chart and propose a control limit called Skewness Correction EWMA (SC-EWMA) for skewed distributions. The performances of the newly proposed method are compared and contrasted with those of the Weighted Variance EWMA (WV-EWMA) which was developed by Khoo and Atta (2008), Weighted Standard Deviation EWMA (WSD-EWMA) which was developed by Atta and Ramli (2011) and the classic EWMA control limits based on the degree of skewness and varying smoothing parameters. The comparison is made with respect to their type-I errors by using the Monte Carlo simulation technique with data generated from the lognormal, Gamma and Weibull distributions.

Keywords: Skewed Distributions, EWMA Control Charts, WV method, WSD method, SC Method.

ÖZET

ÇARPIK DAĞILIMLARI İÇİN EWMA KONTROL KARTLARI

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Klasik Shewhart kontrol kartları genellikle ilgilenilen rastgele kalite değişkeninin özelliklerindeki değişkenliği izlemek için kullanılır ve normallik varsayımlarına dayanır. Çarpık dağılımlarda kitledeki değişiklikleri göstermek için simetrik olmayan kontrol limitlerinin kullanılması gerekmektedir. Ağırlıklı Varyans (WV) Ağırlıklı Standart Sapma (WSD) ve Düzeltilmiş Çarpıklık (SC) gibi yöntemler çarpık dağılımlarda kullanılır.

Klasik $\bar{X}$ ve R kontrol kartları ve bunların tüm türevleri genellikle süreçteki büyük değişimleri tespit etmek için kullanılır, bu nedenle küçük değişimler söz konusu olduğu durumlarda bu kartlar çok güvenilir değildir. Bu tür sorunları çözmek için, bu çalışmada Üstel Ağırlıklı Hareketli Ortalama (EWMA) kontrol kartları kullanılmıştır.

Bu tezin temel amacı, çarpık dağılımlar için Düzeltilmiş Çarpıklık yöntemini EWMA control kartlarına uygulamak ve Düzeltilmiş Çarpıklık EWMA (SC-EWMA) kontrol limitlerini önermektir. Çarpıklık derecesine ve değişen ağırlıklandırma parametrelerine göre, önerilen yeni yöntemin performansı, Khoo ve Atta (2008) tarafından geliştirilen Ağırlıklı Varyans EWMA (WV-EWMA), Atta ve Ramli (2011) tarafından geliştirilen Ağırlıklı Standart Sapma

EWMA (WSD-EWMA) ve klasik EWMA yöntemleri ile karşılaştırılmıştır ve farkları ortaya .çikarilmiştir. Karşılaştırma, lognormal, gamma ve Weibull dağılımlarından üretilen verilerle Monte Carlo simülasyon tekniği kullanılarak 1. Tip hatalarına göre yapılmıştır.

Anahtar Kelimeler: EWMA Kontrol Kartlarrı, WV yöntemi, WSD yöntemi, SC yöntemi, Çarpık Dağılımları

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MOUSTAPHA AMINOU TUKUR

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ABREVIATIONS

UCL	Upper Control Limit
LCL	Lower Control Limit
CL	Centre Line
SC	Skewness Correction
WV	Weighted Variance
WSD	Weighted Standard Deviation
EWMA	Exponentially Weighted Moving Average
ARL	Average Run Length
DIST	Distribution

1 GENERAL INTRODUCTION

Nowadays, technology has increased drastically that consumers do not take any mistakes in the products they consume. Due to the competitive nature of the markets and the demanding nature of consumers, producers tend to use the best techniques available at their disposal to produce the best quality of goods. Quality is a very complex term to define due to its subjective nature but according to the Oxford dictionary quality can be defined as a degree of excellence or better still as a characteristic or distinctive attribute of a person, good or service. In our context, quality can be defined vaguely as one or more desirable characteristics that a product or service must have.

Quality is one of the most important factors of decision making for consumers when it comes to choosing among competing products or services. This concept is noticed in all fields regardless of the sector, individual or institutions involved. Therefore, understanding the concept of quality is a key factor of survival in the competitive markets for producers.

As earlier mentioned, the quality of a product can be described or evaluated in several different ways according to the parties involved. However, general dimensions of quality were given by Garven in the year 1987 and was summarized as performance which checks if the product does the intended job or not, reliability which checks the failure rate of the product, durability which determines how long the product lasts, serviceability which checks how easily the product can be repaired, aesthetics which give an idea on what the product looks like, features which give an idea on what the product does, perceived quality which checks the reputation of the producer or its products and conformance to standards which checks if the product is made exactly as the designer intended (Montgomery, 2013). In service providing sectors such as banking and finance, healthcare and tourism, responsiveness, professionalism and attentiveness can be added to the above features in describing the quality of a good or service.

Due to its subjective nature quality needs to be improved all the time. Quality can be noticed to be inversely proportional to variability. Due to this relationship, statistical methods must be applied in improving the qualities of a product. This is also because variability can only be described using statistical methods.

1.1 Statistical Methods for Quality Control and Improvement

An industrial production process can be described as a process or method which involves the transformation of input materials using well selected and precise techniques to obtain an output good or service with all the desired features. Statistical process control (SPC), statistical experimental design and acceptance sampling are the most used techniques in controlling and improving the quality of a good or service. Statistical experimental design involves the variation of some input factors to determine the effects these changes can get on the desirable output. Acceptance sampling involves the testing of the quality of a good at some point in the production line. This is the oldest quality control method and helps to adjust the production process before the goods are produced and sent to the market. Statistical Process control is a process used in the industrial setting to conceive, control and analyse the steps involved in the production of a good or service.

1.2 Statistical Process Control

Statistical Process Control is a powerful collection of problems-solving tools which are very useful in obtaining process stability and improving capability through the reduction of variability (Montgomery, 2013). Statistical process control is a very convenient method in monitoring all kinds of production processes because it is based on experimentally proven statistical principles and methods. The main reasons of creating a process, monitoring it and then improving it when the need arises is to produce best quality goods hence maximizing profit and surviving competition in the free market. Statistical Process Control uses seven major tools namely; the histogram (stem-and-leaf plot), check sheet, pareto chart, cause-and-effect diagram, defect concentration diagram, scatter diagram and most importantly control charts (Montgomery, 2013).

A control chart is a statistical device which is used basically for studying and controlling repetitive processes. This concept was brought forward by Walter A. Shewhart of the Bell Telephone Laboratories in the 1920s. Shewhart suggests that control charts may serve firstly, to define the goal or standard for a process that the management would like to attain; secondly, it may be used as an instrument for attaining that goal and thirdly, it may serve as a means of judging whether the goal set by the management has been attained or not (Duncan, 1974). Therefore, we can rightly say that control charts are instruments used for specification, production and inspection in an industrial setting.

In any production setting, no matter the amount of care taken or how well the process is designed, there will always exist some natural variability which may result from minor unavoidable causes. These natural variabilities are generally termed "stable system of change causes" (Montgomery, 2013). A process operating under such conditions is said to be in statistical control. However, there exist other kinds of variations which are not natural. Such variations are said to be "assignable causes of variation" in a process (Montgomery, 2013). As the name implies, assignable causes of variation may result from human error, machine errors or raw-materials error. A process running under assignable causes of variation is said to be an "out-of-control" process. Generally, processes are operated in the stable state but as time goes by, they tend to move towards the out of control state. Knowing full well that the main objective of statistical process control is to remove variability in a process, the production engineer involved is always interested in detecting shifts, attributing causes to them and again ensuring the process goes back into a statically in control state.

1.2.1 Features of Control Charts

As earlier stated, a control chart is a graph or diagram which enables a producer to set attainable goals, control these goals and make sure these goals are attained. It comprises basically of three parts namely the Upper Control Limit (UCL), the Central Line (CL) and the Lower Control Limit (LCL). The central line is the targeted goal of production while the lower and upper control limits are the maximum boundaries for a process considered to be in control. Points are distributed randomly around the central line but not above or below the upper and lower control limits respectively as shown in Figure 1.1 below. Whenever points fall out of the upper or lower control limits or when points start following a particular pattern in the stable state, the process now is considered to be out of control and measures should be taken to assign causes to the change and also to make sure the process goes back into a normal stable state.

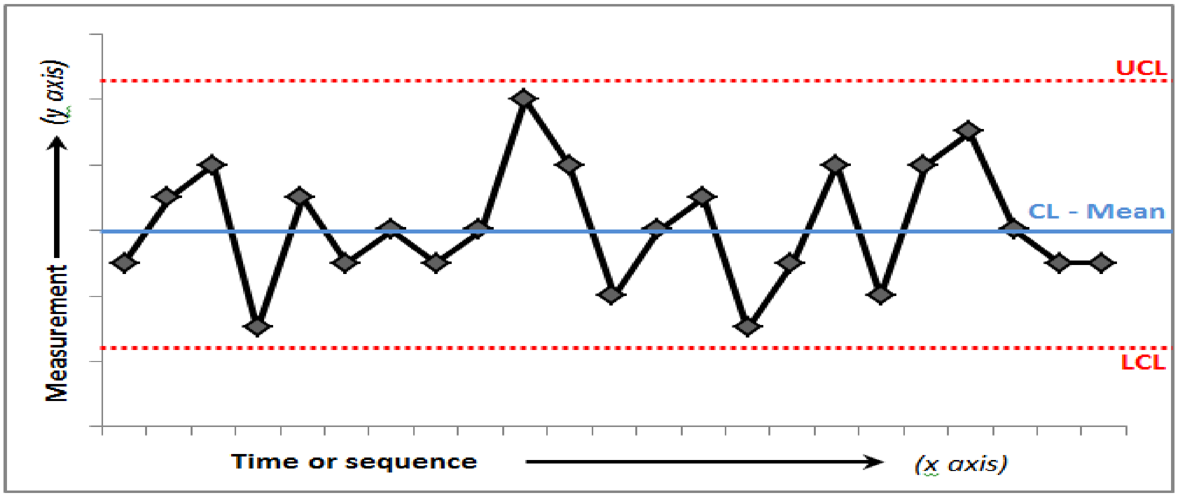

Figure 1.1 Theoretical Basis of a control chart

Let θ be a quality characteristic of interest, $\hat{\theta}$ be its unbiased estimate, $E(\hat{\theta})$ be the expected mean and $\sigma(\hat{\theta})$ be the standard deviation of the estimator $\hat{\theta}$. The central line, upper and lower class limits of the quality in question can be derived as follows from the normal distribution and the Z transformation below (Mitra, 2008):

$$Z = \frac{E(\hat{\theta}) - \hat{\theta}}{\sigma(\hat{\theta})} \quad (1.1)$$

The probability Z can be calculated within a given interval say K as follows:

$$-K \leq \frac{E(\hat{\theta}) - \hat{\theta}}{\sigma(\hat{\theta})} \leq K$$
$$E(\hat{\theta}) - K\,\sigma(\hat{\theta}) \leq \hat{\theta} \leq E(\hat{\theta}) + K\,\sigma(\hat{\theta}) \quad (1.2)$$

From Eq. (1.2):

$$CL = E(\hat{\theta})$$
$$UCL = E(\hat{\theta}) + K\,\sigma(\hat{\theta}) \qquad (1.3)$$
$$LCL = E(\hat{\theta}) - K\,\sigma(\hat{\theta})$$

K in the above Equations represents the number of standard deviations of the sample statistic that the control limits are placed from the central line (Mitra, 2008).

1.3 Distributions Commonly Used in Quality Control

There are many types of distributions both discrete and continuous used in quality control depending on the variable to be investigated. The most used discrete distributions are the hypergeometric, the binomial, the Poisson, the negative Binomial and the geometric distributions. However, in the scope of this work we are going to focus mainly on the continuous distributions used in quality control.

1.3.1 The Normal Distribution

The normal distribution also called Gauss distribution is the most used distribution in statistics. This distribution was brought forward by the German mathematician Carl Friedrich Gauss and later developed by the French mathematician Marquis de Laplace. Since then most statistical developments and attributes are made with respect to this powerful distribution.

Assuming that X is a random variable which is normally distributed with population mean μ and variance σ^2 ($X \sim N(\mu, \sigma^2)$), its probability density function is shown below.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\ ;\ -\infty \le x \le +\infty \qquad (1.4)$$

The graphical appearance of any normal distribution curve is a symmetric, unimodal bell-shaped curve. Knowing that the mean of a population is the measure of the central tendency or location, changing the values of the mean moves the central location of the curve either to the left or to the right depending on the value of the change. On the other hand, variance or standard deviation which is the square root of the variance is a measure of dispersion from the mean. Hence changing its value changes the height and steepness of the curve. 68.28% of the population lies between the limits defined by the mean plus or minus one standard deviation $(\mu \pm 1\sigma)$, 95.46% lies in the interval $(\mu \pm 2\sigma)$ and 99.73% lies in the interval $(\mu \pm 3\sigma)$ respectively as shown in Figure 2 below (Montgomery, 2013).

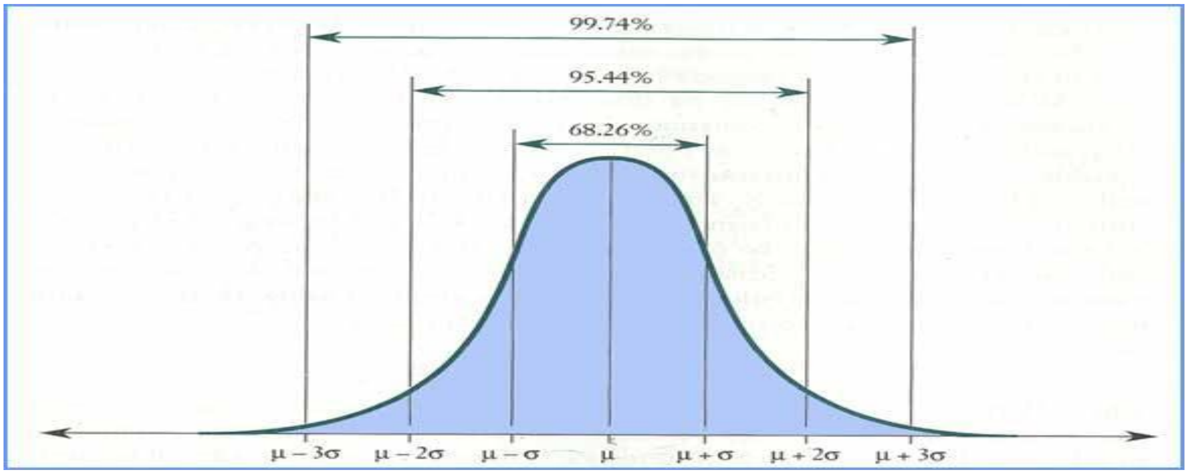

Figure 1.2 Areas Under the Normal Distribution Curve

The area under the normal distribution curve cannot be computed analytically under the closed form due to the complex nature of the density function. To find the cumulative distribution function of any value under the normal distribution, a transformation to the standard normal distribution is used. Tables of values of the standard normal distributions are very common in the appendix potions of all statistics books. The famous transformation used is $Z = \frac{X-\mu}{\sigma}$ which has as $\mu = 0$ and $\sigma = 1$ $(Z \sim N(0,1))$. The probability density function of the standard normal distribution is given below.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \;\; ; \; -\infty \leq z \leq +\infty \qquad (1.5)$$

1.3.2 The lognormal Distribution

Lognormal distribution is a distribution derived from normally distributed variables. It gets its name because of the natural logarithmic transformation undergone by a normally distributed variable. It is widely used in the modelling and analysis of a product which fades gradually over a time period. It can also be used to model compound stocks in the stock exchange markets. Its logarithmic nature makes its range only to differ on the positive axes.

Let w be a normally distributed random variable with mean μ and variance σ^2 then the lognormal distribution function can be obtained as follows (Montgomery, 2013):

$$\begin{gathered} x = exp\,(w) \\ ln\,(x) = w \end{gathered} \tag{1.6}$$

From Eq (1.6), the probability density function of the random variable X which is log normally distributed is given as follows.

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} exp\left[-\frac{(ln\,(x) - \mu)^2}{2\sigma^2}\right]; 0 < x < \infty \tag{1.7}$$

The mean, variance and skewness of the random variable X are given below:

$$E(X) = e^{\mu + \frac{\sigma^2}{2}} \tag{1.8}$$

$$\mathrm{Var(X)} = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) \tag{1.9}$$

$$k_3(X) = \sqrt{e^{\sigma^2} - 1}\,(2 + e^{\sigma^2}) \tag{1.10}$$

1.3.3 The Exponential Distribution

This distribution is mostly used in the fields of reliability analysis to describe the time to the failure of a component or system (Mitra, 2008). An exponential distribution represents a constant failure rate hence it is used to model failures that happen randomly and independently (Mitra, 2008). Another important feature of the exponential distribution is its being memoryless. If a random variable X is exponentially distributed with parameter λ ($X \sim exp\,(\lambda)$), its probability density function is given as follows.

$$f(x) = \lambda e^{-\lambda x}; \quad x \geq 0; \lambda > 0 \tag{1.11}$$

The mean, variance and skewness of the random variable X which is exponentially distributed is given below.

$$E(X) = \frac{1}{\lambda} \tag{1.12}$$

$$Var(X) = \frac{1}{\lambda^2} \tag{1.13}$$

$$k_3(X) = 2 \tag{1.14}$$

In some sources, $\lambda = \frac{1}{\theta}$ is more prevalently used to describe the exponential distribution. The parameter λ is the failure rate of the component or system while the mean $\mu = \frac{1}{\lambda}$ of the distribution is the mean time to failure (Montgomery, 2013).

1.3.4 The Gamma Distribution

This is another important distribution which is used in reliability analysis. It has a shape and scale parameters. Changing the values of these parameters give different shapes of the gamma distribution. In some special cases, precisely changing the shape parameter reduces the gamma distribution to an exponential distribution. If a random variable X is gamma distributed ($X \sim gamma(\alpha, \beta)$), then its probability distribution function is given below:

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}} \; ; x \geq 0; \; \alpha, \beta > 0 \tag{1.15}$$

The mean, variance and skewness of the gamma distributed random variable X is given thus.

$$\begin{aligned} E(X) &= \alpha\beta \\ Var(X) &= \alpha\beta^2 \\ k_3(X) &= \frac{2}{\sqrt{\alpha}} \end{aligned} \tag{1.16}$$

Note also that $\beta = \frac{1}{\lambda}$ is commonly used in some sources.

1.3.5 The Weibull Distribution

This distribution was first brought forward by the Swedish mathematician Waloddi Weibull who described it detailly in 1951. It is also a distribution used in reliability analysis basically to describe the time to failure of mechanical and electrical components (Mitra, 2008). Just like the gamma distribution, this distribution is also very flexible and hence by altering the values of its shape and scale parameters, it tends to take different shapes. It can also reduce to either the exponential distribution or the normal distribution by specifically choosing the scale and shape parameters. It is a three-parameter distribution which can also be reduced to two depending on the work to be done. If a random variable X is Weibull distributed ($X \sim Weibull(\gamma, \alpha, \beta)$), then its probability density function is given below.

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x-\gamma}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}\right]; x \geq \gamma \tag{1.17}$$

The mean and the variance of the random variable X is given below.

$$E(X) = \gamma + \alpha\Gamma\left(\frac{1}{\beta}+1\right) \tag{1.18}$$

$$Var(X) = \alpha^2\left\{\Gamma\left(\frac{2}{\beta}+1\right) - \left[\Gamma\left(\frac{1}{\beta}+1\right)\right]^2\right\} \tag{1.19}$$

As seen in Eq. (1.17) above, Weibull distribution uses a location parameter γ $(-\infty < \gamma < \infty)$, a scale parameter α $(\alpha > 0)$ and a shape parameter $\beta(\beta > 0)$ respectively (Mitra, 2008). Taking the location parameter as $\gamma = 0$ and the scale parameter as $\alpha = \frac{1}{\lambda}$ reduces the Weibull density function to the more prevalently used two parameter Weibull density function as follows.

$$f(x) = \beta\lambda^{\beta}x^{\beta-1}\exp(-x\lambda)^{\beta} \tag{1.20}$$

The mean, variance and skewness in this case not forgetting $\alpha = \frac{1}{\lambda}$ is given below.

$$E(X) = \alpha\Gamma\left(\frac{1}{\beta} + 1\right) \tag{1.21}$$

$$Var(X) = \alpha^2 \left\{\Gamma\left(\frac{2}{\beta} + 1\right) - \left[\Gamma\left(\frac{1}{\beta} + 1\right)\right]^2\right\} \tag{1.22}$$

$$k_3(X) = \frac{\Gamma\left(1 + \frac{3}{\beta}\right)\left(\frac{1}{\lambda}\right)^3 - 3\mu\sigma^2 - \mu^3}{\sigma^3} \tag{1.23}$$

The classic Shewhart control charts are generally used for monitoring the process mean and variability in the characteristics of a random quality variable of interest and are based on the normality assumptions. However, real life problems in most cases are not normally distributed. Using the $\bar{X}$ and R charts in such cases produce misleading results because, the increase in the skewness of the distributions will obviously lead to a relative increase in the type-I error produced due to the changes within the population (Karagöz & Hamurkaroğlu, Control Charts For Skewed Distribution: Weibull, Gamma, Lognormal, 2012). For skewed distributions, in order to demonstrate the changes in the population, non-symmetric control limits need to be used (Bai & Choi, 1995). Methods such as the Weighted Variance (WV) proposed by Choobineh and Ballard (1987), Weighted Standard Deviation (WSD) proposed by Chang and Bai (2001) and Skewness Correction (SC) proposed by Chan and Cui (2003) can be used. Using these methods help the experimenter to avoid misinterpretation of results and hence produce a better output.

Burr (1967) studied the effect of non-normality on constants for $\bar{X}$ and R charts and came up with a wide range of different Tables of values for the constants under different distributions. Nelson 1979) studied the control charts for Weibull Processes when specific standards are given. Choobineh and Ballard (1987) came up with a new heuristic method called WV for setting the limits of a control chart, compared it with the Shewhart's control limits when the underlying population is symmetric and when it is skewed. Cheng and Xie (2000) developed a new approach in controlling lognormal data using its normal counterpart when a specific interval for the lognormal mean is given. Khoo, Atta, and Chen (2003) proposed a WV method to compute and effectively monitor the limits of the $\bar{X}$ and S charts for skewed distributions with moderate and large sample sizes. Chan and Cui (2003) proposed a method of SC for skewed distributions, compared it with the Shewhart's approach and WV through their type-1 errors. Their findings showed that the type-I error of the SC, WV and Shewhart's methods are compatible for approximately symmetric distributions (Chan & Cui, 2003). Karagöz and Hamurkaroğlu (2012) studied the control charts methods used for skewed distributions under Lognormal, Gamma and Weibull distributions. They found out that the SC method produced better results than the other methods as the skewness is increased (Karagöz & Hamurkaroğlu, Control Charts For Skewed Distribution: Weibull, Gamma, Lognormal, 2012). Karagöz (2018) proposed new robust $\bar{X}$ control Charts methods for studying skewed and contaminated processes under non-normality using trimmed mean and interquartile range.

The classic $\bar{X}$ and R control charts and all their derivatives are generally used to detect large shifts in the process mean hence making them not too reliable in situations where in small shifts are of interest. To solve such problems, the Exponentially Weighted Moving Average (EWMA) control charts which was proposed by Roberts (1957) can be used.

Researchers have developed an interest in the EWMA after its discovery and a lot of work was done in its regards. Robinson and Ho (1978); Crowder (1989) and Lucas and Saccucci (1990) all studied the properties of the EWMA numerically and proposed vital relations between the parameters of the EWMA which are discussed in greater details in the subsequent chapters.

The main aim of this thesis is to apply the Skewness Correction method to the EWMA chart and propose a control limit called Skewness Correction EWMA (SC-EWMA) for skewed distributions. The performances of the newly proposed method are compared and contrasted with those of the Weighted Variance EWMA (WV-EWMA) which was developed by Khoo and Atta (2008), Weighted Standard Deviation EWMA (WSD-EWMA) which was developed by Atta and Ramli (2011) and the classic EWMA control limits based on the degree of skewness and varying smoothing parameters. The comparison is made with respect to their type-I errors by using the Monte Carlo simulation technique with data generated from the lognormal, Gamma and Weibull distributions.

This work starts with a chapter dedicated only to general information about quality control, statistical process control and the distributions commonly used in quality control. Chapter two concentrates mostly on control charts for variables, the different types of charts which exist and how they are applied in daily life. The Exponential Weighted Moving Average (EWMA) chart which is the epicentre of this study is also introduced in this chapter. Chapter three focuses on the methods used when the quality variable to be investigated is skewed. Here, the theoretical basis of the methods used throughout this work is outlined. Chapter four is where all the necessary Monte Carlo simulations are made, and the results obtained are tabulated and analysed. Chapter 5 which is the last chapter is where a general review of the work is done, and necessary suggestions made based on the obtained results and the theoretical expectations of the work. In this chapter, a general conclusion of the work is arrived.

2 CONTROL CHARTS

As earlier mentioned in the previous chapter, control charts are basically graphs used to investigate, maintain or improve a production process. It consists of a centreline, the lower and upper control limits. In this chapter we are going to speak on the various types of charts which exist, how to construct these charts and above all the statistical theories behind these charts.

2.1 Control Limits

The limits on the control charts are probability limits which are determined so that if chance causes alone were at work, the probability of a point falling above the upper control limit or below the lower control limit would be considerably small say one out of a thousand (Duncan, 1974). Consequently, when a point falls outside the desirable limits, there is going to be a search for an assignable cause. These said probabilities determine the risk of making such a search when there are no assignable causes of variation involved. Since these probabilities are very small, it therefore implies that if a point falls outside them then there are assignable causes involved in the variation. In other words, they are used as an assurance for the search. It is also important to note that the values of these probabilities are arbitrary and may differ with respect to the final objectives of the management.

If the system of chance causes produce a variation in the random variable investigated which follows a normal distribution, then generally the 3σ limits are used which is equivalent to a probability of one out of a thousand (0.001) (Duncan, 1974). Under a normal curve, the probability that a deviation from the mean exceeds the 3σ limit in both directions is 0.0027 (Duncan, 1974).

2.2 Variables Control Charts

A variable in the context of quality control can be termed as any quality characteristic which can be expressed or measured on a numerical scale. Examples of variables can be length, width, height, volume, time to achieve a given result, diameter, thickness, breaking strength or even the viscosity of a liquid. Variable control charts are generally preferred to those of attributes because generally attributes do not show the exact extend to which a quality characteristic is nonconforming. For changes to be detected in any process, it is vital to control the mean and variability of the quality variable investigated. In the production of even the simplest product, there are many quality variables which exist. Due to time factor and the limited resources involved in production, a comprehensive scale of preference of the variables is constructed from which only the most important variables' control charts are constructed. The importance or preference criteria used in selection depends on either available quality obligations or demands from the management.

2.2.1 Control charts for The Mean and Range

As earlier stated Shewhart control charts are the mother charts of all the existing variables' control charts. These charts are mainly used for monitoring the process average and its variability. The most prevalently used Shewhart charts are the control charts for mean ($\bar{X}$ control chart), the control chart for the standard deviation (S control chart) and the control chart for the range (*R* control chart). In this section, only the control chart used for monitoring the process mean ($\bar{X}$ control chart) and that for monitoring the process variability using the range (R control chart) are discussed.

Supposing that a quality characteristic under investigation say X is normally distributed with mean μ and standard deviation σ where both μ and σ are known and if a sample of size n say $X_1, X_2, \dots, X_n$ are taken, then the mean of this sample is given as follows.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \qquad (2.1)$$

It is clearly known from probability theory that the mean of the samples $\bar{X}$ is normally distributed with mean μ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (Nasirova and others, 2009). Therefore, the probability that any sample mean will fall between the confidence level $1 - \alpha$ is given below.

$$\mu \pm Z_{\frac{\alpha}{2}} \sigma_{\bar{x}} = \mu \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \qquad (2.2)$$

Conclusively, from Eq (2.2) the centerline, upper and lower control limits for the $\bar{X}$ control chart when μ and σ are known are given as follows.

$$UCL = \mu + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$
$$CL = \mu \qquad (2.3)$$
$$LCL = \mu - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

In practice and by convention, the $Z_{\frac{\alpha}{2}}$ in Eq (2.3) is replaced by 3 to maintain the three-sigma rule (Montgomery, 2013).

Control charts for the mean and range when the values of the mean μ and standard deviation σ are unknown are obtained using the steps below (Mitra, 2008).

- Step 1: Measurements of the quality characteristic to be investigated are recorded using a selected sampling technique and size.
- Step 2: The sample mean $\bar{X}$ and range R are calculated for each sample using the formulae below.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \qquad (2.4)$$

$$R = X_{max} - X_{min} \qquad (2.5)$$

Here, n is the sample size, X_i is the i^{th} observation, X_{max} is the largest observation and X_{min} is the smallest observation.

- Step 3: The centreline and the trial control limit which are the initial control limits to be used for later measurements for each chart are obtained.

For the $\bar{X}$-chart, the centreline $\bar{\bar{X}}$ is given as follows;

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{m} \bar{X}_i}{m} \qquad (2.6)$$

Here, m represents the number of samples used.

Similarly, the centreline $\bar{R}$ for the R-chart is obtained as follows;

$$\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m} \tag{2.7}$$

Knowing the relationship which exists between the range of a sample from a normal distribution and the standard deviation of that distribution, the relative range of the variable in question say W with mean d_2 and whose parameters of distribution are functions of the sample size n is given below (Montgomery, 2013).

$$W = \frac{R}{\sigma} \tag{2.8}$$

From above, an estimate for the standard deviation σ can be obtained as follows given $\bar{R}$ to be the average range of the m samples (Montgomery, 2013).

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \tag{2.9}$$

Using $\bar{\bar{X}}$ as an estimate of μ and $\frac{\bar{R}}{d_2}$ as an estimate for σ, the upper and lower control limits and the centreline for the $\bar{X}$-chart are given below:

$$\begin{aligned} UCL &= \bar{\bar{X}} + \frac{3}{d_2\sqrt{n}}\,\bar{R} \\ CL &= \bar{\bar{X}} \\ LCL &= \bar{\bar{X}} - \frac{3}{d_2\sqrt{n}}\,\bar{R} \end{aligned} \tag{2.10}$$

For simplicity, A_2 is used in place of $\frac{3}{d_2\sqrt{n}}$ because A_2 is a standard constant whose values are found in the appendix sections of most statistics books. Similarly, d_2 is also a standard constant and its values are also found in the appendix section of statistics books.

From Eq (2.8) and assuming the standard deviation of W to be a constant say d_3 then

$$R = W\sigma \tag{2.11}$$

$$\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2} \tag{2.12}$$

Consequently, the upper and lower control limits and the centreline of the R-chart are given below (Montgomery, 2013).

$$\begin{aligned} UCL &= \bar{R} + 3d_3 \frac{\bar{R}}{d_2} \\ CL &= \bar{\bar{R}} \\ LCL &= \bar{R} - 3d_3 \frac{\bar{R}}{d_2} \end{aligned} \tag{2.13}$$

Generally, the constants $D_3 = 1 - 3\frac{d_3}{d_2}$ and $D_4 = 1 + 3\frac{d_3}{d_2}$ are used and their values and those of d_3 are found in the appendix sections of most statistics books.

The above charts are all generally called classic Shewhart's charts. They are the most widely used charts in quality control, but they do have their limitations like any other scientifically derived method. They are generally insensitive to small shifts in the process mean (Mitra, 2008). Cumulative Sum Charts, Moving-Averages and the EWMA control charts can be used as alternatives to effectively solve the insensitive nature of the Shewhart Charts in the detection of shifts of small magnitudes in the process mean (Borror, Montgomery, & Runger, 1999).

2.2.2 Moving-Average Control Charts

In situations whereby the product characteristics are measured automatically or when the time to produce a unit is too long, the best chart to be used is the moving average control chart (Mitra, 2008). Moving average values are correlated by nature. Supposing that samples of size n are collected from a given process and the first t sample means given by $\bar{X}_1, \bar{X}_2, \bar{X}_3, \dots, \bar{X}_t$, then the moving average of width *m*, at time step t is given as follows (Mitra, 2008).

$$M_t = \frac{\bar{X}_t + \bar{X}_{t-1} + \bar{X}_{t-2} + \bar{X}_{t-3} + \cdots + \bar{X}_{t-w+1}}{m} \tag{2.14}$$

At any time t, in order to update the moving average by adding the newest mean, the oldest mean is dropped. The variance of each sample mean is

$$Var(\bar{X}_t) = \frac{\sigma^2}{n} \tag{2.15}$$

Here, σ^2 is the population variance of the individual values.
The variance of the moving average M_t can be derived thus

$$\begin{aligned} Var(M_t) &= \frac{1}{m^2} \sum_{i=t-m+1}^{t} Var(\bar{X}_i) \\ &= \frac{1}{m^2} \sum_{i=t-m+1}^{t} \frac{\sigma^2}{n} \\ &= \frac{\sigma^2}{nm} \end{aligned} \tag{2.16}$$

The centreline, upper and lower control limits for the moving-average chart are given below.

$$\begin{aligned} UCL_t &= \bar{\bar{X}} + 3\frac{\sigma}{\sqrt{nm}} \\ CL_t &= \bar{\bar{X}} \\ UCL_t &= \bar{\bar{X}} - 3\frac{\sigma}{\sqrt{nm}} \end{aligned} \tag{2.17}$$

It is clearly noticed from Eq. (2.17) that the control limits are inversely proportional to the width w, i.e. as the width w increases, the values of the control limits decrease accordingly. To detect shifts of extremely small magnitudes, larger values of w should be chosen. In this case, the moving average M_t can be reformulated as follows.

$$M_t = \frac{\sum_{i=1}^{t} \bar{X}_i}{t} \tag{2.18}$$

Hence, the control limits for such a start-up period are given below.

$$\begin{aligned} UCL_t &= \bar{\bar{X}} + 3\frac{\sigma}{\sqrt{nt}} \\ CL_t &= \bar{\bar{X}} \\ UCL_t &= \bar{\bar{X}} - 3\frac{\sigma}{\sqrt{nt}} \end{aligned} \tag{2.19}$$

2.2.3 Exponentially Weighted Moving-Average control charts

The EWMA is a control chart used in detecting very small shifts in the process mean just like the moving-average control charts but are more sensitive (Roberts, 2000). It is also used as a control tool. It is generally used with individual observations (Crowder, 1989).

The EWMA chart is constructed based on varying weights from previous observations. EWMA control charts are also called geometric moving average charts. The exponentially weighted moving average when the starting parameters are unknown is defined as follows (Mitra, 2008).

$$G_t = \lambda \bar{X}_t + (1 - \lambda) G_{t-1} \tag{2.20}$$

Here, λ which lies in the interval $0 < \lambda \leq 1$ is a weighting or smoothing constant. The starting value which is also the process target is given such that $G_0 = \bar{\bar{X}}$.
From Eq (2.20), EWMA G_t which is a weighted average of all the previous sample means can be illustrated as follows (Montgomery, 2013).

$$G_t = \lambda\bar{X}_t + (1-\lambda)[\lambda\bar{X}_{t-1} + (1-\lambda)G_{t-2}]$$
$$= \lambda\bar{X}_t + \lambda(1-\lambda)\bar{X}_{t-1} + \lambda(1-\lambda)^2\bar{X}_{t-2} + \cdots + (1-\lambda)^t G_0$$
$$G_t = \lambda\sum_{j=0}^{t-1}(1-\lambda)^j\bar{X}_{t-j} + (1-\lambda)^t G_0 \qquad (2.21)$$

The weight $\lambda(1-\lambda)^j$ in Eq (2.21) above decreases geometrically with the age of the sample mean as it becomes less recent (Montgomery, 2013). The weights sum up to unity according to the relation below obtained from the properties of the geometric distribution.

$$\lambda\sum_{j=0}^{t-1}(1-\lambda)^j = \lambda\left[\frac{1-(1-\lambda)^t}{1-(1-\lambda)}\right] = 1-(1-\lambda)^t \qquad (2.22)$$

Assuming the sample means $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_{t-1}$ to be independent of each other and the population standard deviation to be σ, the variance of G_t can be given by (Montgomery, 2013);

$$Var(G_t) = \left(\frac{\sigma^2}{n}\right)\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}] \qquad (2.23)$$

The centreline, upper and lower control limits for the EWMA control chart when the parameters are known are given below (Montgomery, 2013).

$$UCL_{EWMA_t} = \mu + K\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]}$$
$$CL_{EWMA_t} = \mu \qquad (2.24)$$
$$LCL_{EWMA_t} = \mu - K\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]}$$

Here, K is a number chosen by the experimenter which is directly related to the smoothing constant. The centreline, upper and lower control limits for the EWMA control chart when the parameters are unknown are given below (Mitra, 2008).

$$UCL_{EWMA_t} = \bar{\bar{X}} + K\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]}$$
$$CL_{EWMA_t} = \bar{\bar{X}} \tag{2.25}$$
$$LCL_{EWMA_t} = \bar{\bar{X}} - K\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]}$$

For large values of *t* the standard deviation of G_t is given asymptotically by

$$\sigma_G = \sqrt{\frac{\sigma^2}{n}\left(\frac{\lambda}{2-\lambda}\right)} \tag{2.26}$$

Therefore, the centreline, upper and lower control limits of the EWMA control chart in this case when the parameters are known are given below (Mitra, 2008).

$$UCL_{EWMA} = \mu + K\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}$$
$$CL_{EWMA} = \mu \tag{2.27}$$
$$LCL_{EWMA} = \mu - K\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}$$

Similarly, the centreline, upper and lower control limits of the EWMA control chart in this scenario when the parameters are unknown are given below (Mitra, 2008).

$$UCL_{EWMA} = \bar{\bar{X}} + K\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$CL_{EWMA} = \bar{\bar{X}} \tag{2.28}$$

$$LCL_{EWMA} = \bar{\bar{X}} - K\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}$$

Generally in a wide range of applications where in individual measurements are involved, a well -designed EWMA as a control chart is highly recommended particularly during the process monitoring phase (Montgomery, 2013).

2.2.3.1 Design and Choice of Parameters for the EWMA

Crowder (1989) studied the properties of the EWMA numerically and came up with a concise method comprising of 4 steps to determine the best ordered pair of (λ, K) satisfying conditions for setting up of the EWMA control chart. The main objective of this method is to find the best set of (λ, K) which for a given in-control average run length (ARL) minimizes the out-of-control ARL for a specific shift in the process mean. ARL can easily be defined for type-1 error as the total number of observations before an out-of-control signal is noticed when a process is in-control. To find the ordered pair to be used for the design of an EWMA control chart the following steps are followed (Crowder, 1989).

- For the case in which the process shift is assumed to be zero, the smallest ARL is chosen based on the requirements of the management. In other words, the false alarm rate (type-I error) is determined.
- The magnitude of shift in the process which must be detected quickly is decided. The value of the smoothing parameter λ which produces an optimal (minimum) ARL for that size shift is chosen. This choice is facilitated by the graph in Figure 2.1 below which was reproduced from (Crowder, 1989).

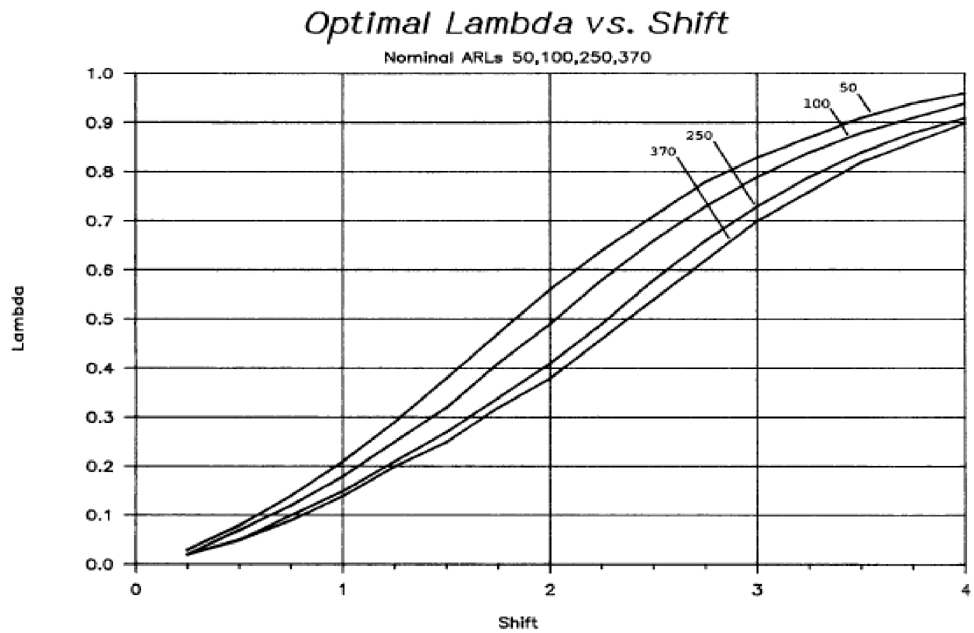

Figure 2.1 Optimal λ for the EWMA

- The control limit constant K which satisfies the in-control ARL in step 1 is found using the values of λ in the previous step. The value of K can easily be obtained from the graph in Figure 2.2 below which was reproduced from (Crowder, 1989).

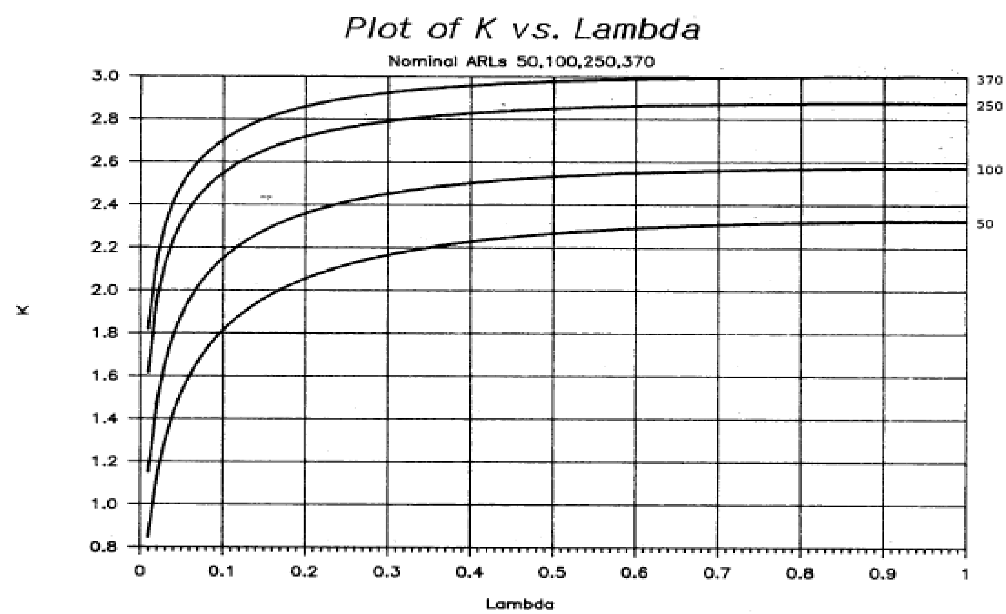

Figure 2.2 Combinations of λ and K

- Lastly, a sensitivity analysis by comparing the out-of-control ARLs (Type-II error) for the optimal combination of (λ, K) to other combinations of (λ, K) which produces the same in-control ARL (type-I error) is performed. The best combination of (λ, K) which produces the desired results based on the type-II errors is chosen.

3 CONTROL CHARTS FOR SKEWED DISTRIBUTIONS

As earlier mentioned, the classic Shewhart control charts which are used for monitoring the process mean and variability in the characteristics of a random quality variable of interest is based on the normality assumption. The normality assumption simply means that the distribution of the data extracted from the characteristics of the quality variable show either a relative normal distribution with a value of the mean and standard deviation or can easily be made normal using the central limit theorem. This chapter focuses basically on situations wherein the distribution of the random quality variable under investigation is skewed.

In situations wherein the distribution of the random variable in question is not normal or can't directly be made normal using the central limit theorem, using the classic Shewhart charts for the mean and range produces misleading results because, the increase in the skewness of the distribution leads to a relative increase in the type-I error produced due to the changes within the population (Karagöz & Hamurkaroğlu, Control Charts For Skewed Distribution: Weibull, Gamma, Lognormal, 2012). This problem can simply be solved by either increasing the sample size used or by transforming the data. However, doing so takes a lot of cost and time hence, making experimenters not to prefer it. Therefore, for skewed distributions, in order to demonstrate the changes in the population, non-symmetric control limits need to be used (Bai & Choi, 1995). Methods such as the Weighted Variance (WV), Weighted Standard Deviation (WSD) and Skewness Correction (SC) are generally more reliable Using these methods help the experimenter to avoid misinterpretation of results and hence produce a better output.

3.1 Weighted Variance method

This method works basically by dividing the area under a probability density function of a random variable under investigation into two portions with respect to the mean of the distribution. Each portion has the same mean but different standard deviations and are separate symmetric distributions (curves). These new portions can be identical if the mother curve is symmetric. In cases where the mother distribution is skewed, one curve is longer than the other depending on the side of the skewness. The control limits for the mean and the range are then obtained using these new distributions. In other words, one of the two distributions is used for the upper control limit while the other one is used for the lower control limit (Choobineh & Ballard, 1987).

3.1.1 $\bar{X}$ and R Control Limits for the Weighted Variance Method

Just like the Shewhart's approach, the WV method uses the standard deviation to establish the control limits for the mean and range charts. The only difference between the two methods lies at the level of the two multiplication factors added to the WV method (Karagöz & Hamurkaroğlu, Control Charts For Skewed Distribution: Weibull, Gamma, Lognormal, 2012). Therefore, the control limits obtained from the WV method are also considered to be the Shewhart type charts.

The two multiplication factors used in the establishment of the control limits for the mean ($\bar{X}$) and range (R) charts are as follows (Bai & Choi, 1995).

$$UCL : \sqrt{2P_X} \qquad (3.1)$$

$$LCL: \sqrt{2(1 - P_X)}$$

Assuming the random quality characteristic under investigation to be X, the term P_X in Eq. (3.1) is a probability value and is given below.

$$P_X = P(X \leq \mu_X) \qquad (3.2)$$

It is vital to note that μ_X used in Eq. (3.2) is the mean of the quality process.

If the parameters of the quality process under investigation are known, the control limits of the $\bar{X}$ chart obtained using the WV method are as follows.

$$UCL_{WV} = \mu_X + 3\frac{\sigma_X}{\sqrt{n}}\sqrt{2P_X}$$

$$CL_{WV} = \mu_X \qquad (3.3)$$

$$LCL_{WV} = \mu_X - 3\frac{\sigma_X}{\sqrt{n}}\sqrt{2(1 - P_X)}$$

From Eq. (3.3), σ_X is the process standard deviation and n is the sample size used in the subgroups.

The control limits obtained with respect to the WV method for the R chart when the process parameters are known are given below.

$$UCL_{WV_R} = \mu_R + 3\sigma_R\sqrt{2P_X}$$
$$CL_{WV_R} = \mu_R \quad (3.4)$$
$$LCL_{WV_R} = \left[\mu_R - 3\sigma_R\sqrt{2(1-P_X)}\,\right]^+$$

From Eq. (3.4), μ_R is the mean of the range of the distribution, σ_R is the standard deviation of the range and $[a]^+ = \left[\mu_R - 3\sigma_R\sqrt{2(1-P_X)}\,\right]^+$ can be expressed as $[a]^+ = Max[0, a]$ (Choobineh & Ballard, 1987). For relatively small or medium sizes of n, the WV method gives better results than the classic Shewhart method (Bai & Choi, 1995).

In practice, the process parameters are unknown and are generally predicted. For the WV method, the probability value P_X is predicted as follows (Bai & Choi, 1995).

$$\hat{P}_X = \frac{\sum\sum\delta\left(\bar{\bar{X}} - X_{nm}\right)}{nm} \quad (3.5)$$

From Eq. (3.5), $\delta(x)$ can be expantiated as follows.

$$\delta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (3.6)$$

The $\bar{X}$ control limits obtained with respect to the WV method when the process parameters are unknown are given below.

$$UCL_{WV_{\overline{X}}} = \bar{\bar{X}} + 3\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{2\hat{P}_X}$$

$$CL_{WV_{\bar{X}}} = \bar{\bar{X}} \tag{3.7}$$

$$LCL_{WV_{\bar{X}}} = \bar{\bar{X}} - 3\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{2\left(1-\hat{P}_X\right)}$$

Taking $W_U = \frac{3\sqrt{2\hat{P}_X}}{d_2'\sqrt{n}}$ and $W_L = \frac{3\sqrt{2(1-\hat{P}_X)}}{d_2'\sqrt{n}}$, Eq. (3.7) can be readjusted to the more prevalently used form as follows.

$$UCL_{WV_{\bar{X}}} = \bar{\bar{X}} + W_U\bar{R}$$

$$CL_{WV_{\bar{X}}} = \bar{\bar{X}} \tag{3.8}$$

$$LCL_{WV_{\bar{X}}} = \bar{\bar{X}} - W_L\bar{R}$$

The values of the constants W_U and W_L were calculated for various values of n by (Bai & Choi, 1995) and can be found in the appendix sections of most quality control books and articles.

The R control limits obtained with respect to the WV method when the process parameters are unknown are given below.

$$UCL_{WV_R} = \bar{R}\left[1 + 3\frac{d_3'}{d_2'}\sqrt{2\hat{P}_X}\right]$$

$$CL_{WV_R} = \bar{R} \tag{3.9}$$

$$LCL_{WV_R} = \bar{R}\left[1 - 3\frac{d_3'}{d_2'}\sqrt{2(1-\hat{P}_X)}\right]$$

Taking $V_U = [1 + 3\frac{d_3'}{d_2'}\sqrt{2\hat{P}_X}\,]$ and $V_L = [1 - 3\frac{d_3'}{d_2'}\sqrt{2(1-\hat{P}_X)}\,]$, Eq. (3.9) can be readjusted to the more commonly used form as follows.

$$UCL_{WV_R} = V_U\bar{R}$$

$$CL_{WV_R} = \bar{R} \tag{3.10}$$

$$LCL_{WV_R} = V_L\bar{R}$$

3.2 Weighted Standard Deviation method

It is also a very easy method to understand and it has almost the same basic principle as that of the WV method. This method depends on the level and direction of skewness of the distribution of a random variable. This method works basically by dividing the area under a probability density function of a random variable under investigation into two portions with respect to the mean of the distribution. Each portion has the same mean but different standard deviations and are separate symmetric distributions (curves). Here, the sum of the newly obtained standard deviations is equal to the original standard deviation of the distribution ($\sigma = \sigma_U + \sigma_L$).

Considering the quality random variable under investigation to be X, the main logic of this method is described graphically in Figure 3.1 below which was reproduced from (Chang & Bai, 2001).

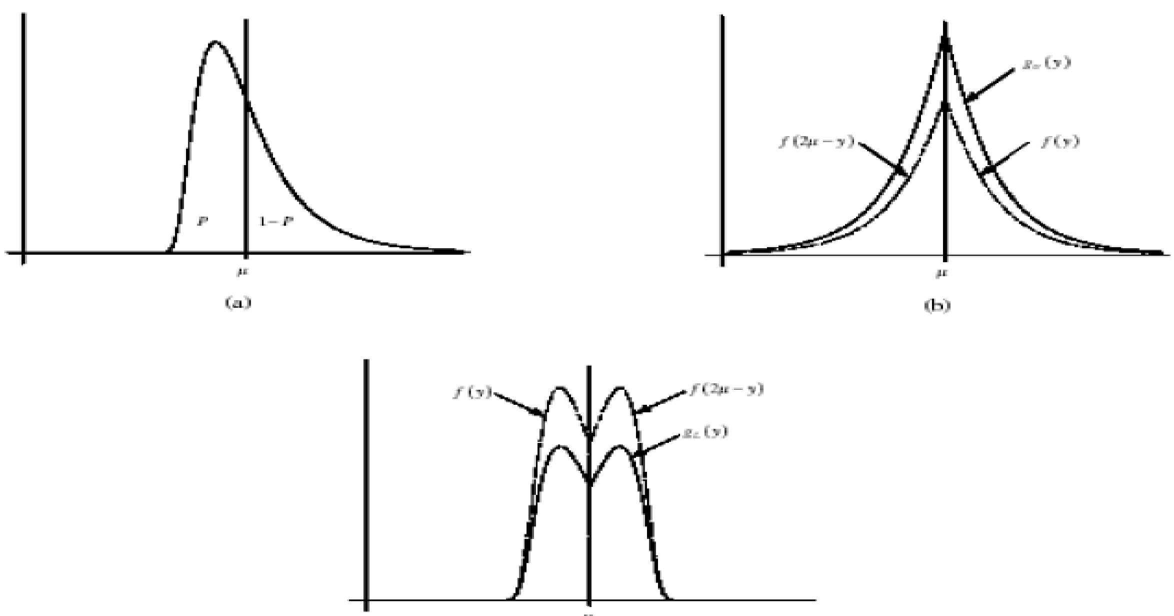

Figure 3.1 WSD method

From Figure 3.1, if the symmetry of the right hand side of the function $f(x)$ in (a) is taken with respect to the mean and in order to abide by all the properties of probability density functions, the symmetric probability density function $g_U(y)$ is obtained as shown by graph (b). Similarly, if the symmetry of the left hand side of the function $f(x)$ is taken about the mean and in order to abide by all the properties of probability density functions, the symmetric probability density function $g_L(y)$ is obtained as shown by graph (c). It is important to emphasize on the fact that the newly obtained symmetric density functions have the same mean but different standard deviations viz σ_U and σ_L. The probability density functions $g_U(y)$ and $g_L(y)$ are shown below (Chang & Bai, 2001).

$$g_U(y) = \begin{cases} \frac{1}{2(1-P)} f(2\mu - y), & y \leq \mu \\ \frac{1}{2(1-P)} f(y), & y > \mu \end{cases} \tag{3.11}$$

$$g_L(y) = \begin{cases} \frac{1}{2P} f(y), & y \leq \mu \\ \frac{1}{2P} f(2\mu - y), & y > \mu \end{cases} \tag{3.12}$$

From graph (a) in Figure 3.1 above, the relationships below are true.

$$P(X \leq \mu) = P \ \Rightarrow F(\mu) = P$$
$$P(X > \mu) = 1 - P \Rightarrow 1 - F(\mu) = 1 - P$$

Using the linear transformation $y = 2\mu - X$ gives the following result.

$$f(y) = f\left(g^{-1}(y)\right)\left|\frac{dy}{dx}\right| \Rightarrow f(y) = f(2\mu - y)$$

The weighting or loading factors A and B which satisfy the probability density function are obtained as follows (Karagöz & Hamurkaroğlu, Control Charts For Skewed Distribution: Weibull, Gamma, Lognormal, 2012).

$$A\int_{-\infty}^{\mu} f(2\mu - y)dy + A\int_{\mu}^{\infty} f(y)dy = 1$$
$$A * (1 - P) + A * (1 - P) = 1$$

$$\therefore A = \frac{1}{2(1-P)}$$

$$B\int_{-\infty}^{\mu} f(y)dy + B\int_{\mu}^{\infty} f(2\mu - y)\, dy = 1$$

$$B * P + B * P = 1$$

$$\therefore B = \frac{1}{2P}$$

The upper standard deviation σ_U and lower standard deviation σ_L are obtained from Eq. (3.11) and (3.12) respectively as follows (Chang & Bai, 2001).

$$\sigma_U^2 = \int_{-\infty}^{\mu} \frac{1}{2(1-P)}(y-\mu)^2 f(2\mu - y)dy + \int_{\mu}^{\infty} \frac{1}{2(1-P)}(y-\mu)^2 f(y)dy$$

$$= \int_{\mu}^{\infty} \frac{1}{1-P}(y-\mu)^2 f(y)dy$$

$$\therefore \sigma_U = \sqrt{\frac{1}{1-P}\int_{\mu}^{\infty}(y-\mu)^2 f(y)dy}$$

$$\sigma_L^2 = \int_{-\infty}^{\mu} \frac{1}{2P}(y-\mu)^2 f(y)dy + \int_{\mu}^{\infty} \frac{1}{2P}(y-\mu)^2 f(2\mu - y)dy$$

$$= \int_{-\infty}^{\mu} \frac{1}{P}(y-\mu)^2 f(y)dy$$

$$\therefore \sigma_L = \sqrt{\frac{1}{P}\int_{-\infty}^{\mu}(y-\mu)^2 f(y)dy}$$

Using $\int_{\mu}^{\infty}(y-\mu)^2 f(y)dy \cong P\sigma^2$ obtained from the semi-variance approach which was developed by Choobineh and Branting (1986), the final approximation for the upper and lower variances (σ_U and σ_L) are given as follows.

$$\sigma_U \cong \sqrt{\frac{P}{1-P}}\,\sigma \tag{3.13}$$

$$\sigma_L \cong \sqrt{\frac{1-P}{P}}\,\sigma \tag{3.14}$$

Assuming the standard deviation σ is divided into the upper standard deviation σ_U^W and the lower standard deviation σ_L^W respectively, then the expression $\sigma = \sigma_U^W + \sigma_L^W$ can easily be written. If the ratio of σ_U^W to σ is assumed to be equal to that of σ_U to $\sigma_U + \sigma_L$, then the relation $\frac{\sigma_U^W}{\sigma} = \frac{\sigma_U}{\sigma_U+\sigma_L}$ can be written. Similarly, using the same ideology, the relation $\frac{\sigma_L^W}{\sigma} = \frac{\sigma_L}{\sigma_U+\sigma_L}$ can be given. From these two relations, if $\frac{\sigma_U}{\sigma_U+\sigma_L} \cong P$ and $\frac{\sigma_L}{\sigma_U+\sigma_L} \cong 1-P$ then, the equations below are true (Chang & Bai, 2001).

$$\sigma_U^W \equiv P\sigma \tag{3.15}$$

$$\sigma_L^W \equiv (1-P)\sigma \tag{3.16}$$

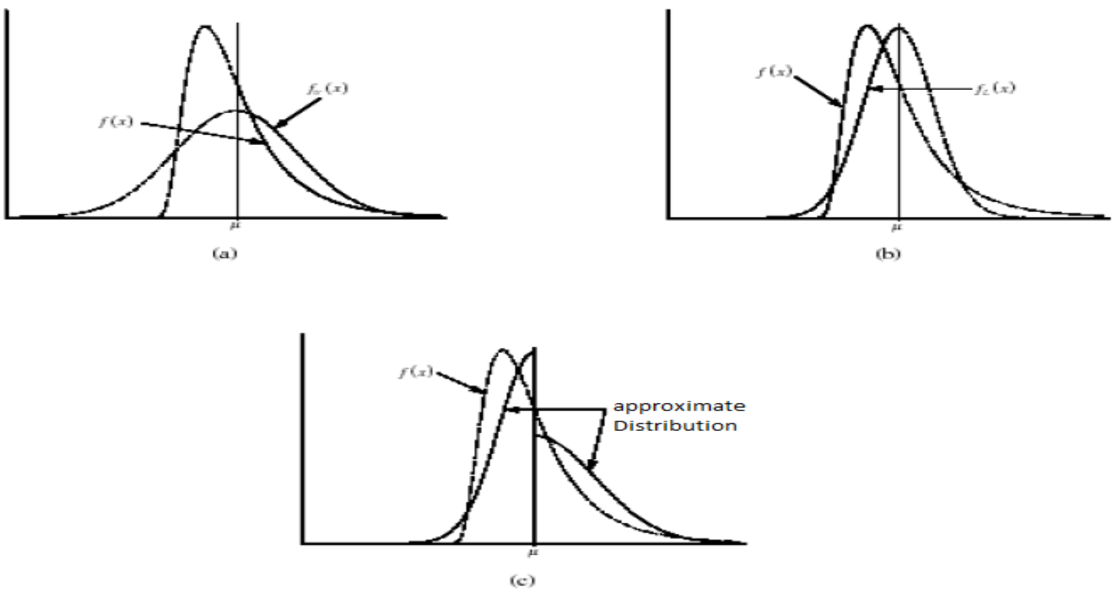

Figure 3.2 Probability Density Functions for the WSDs

The upper part of the original probability density function's density function is given as follows and is represented graphically in (a) in Figure 3.2 above.

$$f_U(x) = \frac{1}{2\sigma_U^W}\phi\left(\frac{x-\mu}{2\sigma_U^W}\right) \tag{3.17}$$

Similarly, the lower part of the original probability density function's density function is given as follows and is represented graphically in (b) in Figure 3.2 above.

$$f_L(x) = \frac{1}{2\sigma_L^W}\phi\left(\frac{x-\mu}{2\sigma_L^W}\right) \tag{3.18}$$

If the distribution of the random variable under investigation is symmetric then $P = \frac{1}{2}$ and $\sigma_U^W = \sigma_L^W = \frac{1}{2}$. If the distribution is skewed to the right, then $P > \frac{1}{2}$ and $\sigma_U^W > \sigma_L^W$. If the distribution is skewed to the left then $P < \frac{1}{2}$ and $\sigma_U^W < \sigma_L^W$ (Chang & Bai, 2001).

3.2.1 $\bar{\bar{X}}$ chart Under the Weighted Standard Deviation Method

When the parameters of the random variable under investigation are known, the control limits for the mean chart under the WSD method are given as follows (Chang & Bai, 2001).

$$\begin{aligned} UCL_{WSD_{\bar{X}}} &= \mu + Z_{\frac{\alpha}{2}}\frac{2\sigma_U^W}{\sqrt{n}} \\ UCL_{WSD_{\bar{X}}} &= \mu + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}2P \\ LCL_{WSD_{\bar{X}}} &= \mu - Z_{\frac{\alpha}{2}}\frac{2\sigma_L^W}{\sqrt{n}} \\ LCL_{WSD_{\bar{X}}} &= \mu - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}2(1-P) \end{aligned} \tag{3.19}$$

When the parameters of the random variable under investigation are unknown, the control limits for the mean chart under the WSD method are given as follows (Chang & Bai, 2001).

$$UCL_{WSD_{\bar{X}}} = \bar{\bar{X}} + 3\frac{\bar{R}}{d_2^{**}\sqrt{n}}2\hat{P} \tag{3.20}$$

$$LCL_{WSD_{\bar{X}}} = \bar{\bar{X}} - 3\frac{\bar{R}}{d_2^{**}\sqrt{n}}2\left(1-\hat{P}\right)$$

The estimates of the parameters and the constants used in Eq. (3.20) were studied and explicitly explained by (Karagöz & Hamurkaroğlu, Control Charts For Skewed Distribution: Weibull, Gamma, Lognormal, 2012).

3.3 Skewness Correction (SC) Method

This method works generally for skewed distributions. The control limits obtained by this method are based on the level of skewness of the distribution estimated from the subgroups. Also, no prior parameters assumption is made on the nature and shape of the distribution of the random variable under investigation. If the process distribution is symmetric, the control limits obtained by the SC method turn to the classic Shewhart control charts.

3.3.1 Fundamental Principle of the Skewness Correction method

Let the random variable under investigation say X be standardised with mean $\mu = 0$, standard deviation $\sigma = 1$ and skewness k_3. If k_3 is known, the centreline, upper and lower control limits of the aforementioned process for an individual observation based on the SC method are given as follows (Chan & Cui, 2003).

$$UCL_{SC} = 3 + \frac{\frac{4}{3}k_3(\bar{X})}{1 + 0.2k_3(\bar{X})^2}$$

$$CL_{SC} = 0 \qquad (3.21)$$

$$UCL_{SC} = -3 + \frac{\frac{4}{3}k_3(\bar{X})}{1 + 0.2k_3(\bar{X})^2}$$

These control limits are based on the skewness of the process estimated from the subgroups. These control limits differ from the classic Shewhart control charts by just $\frac{\frac{4}{3}k_3(\bar{X})}{1+0.2k_3(\bar{X})^2}$. The SC method is based on the Cornish-Fisher expansion, which is discussed detailly by Çalışkan, (2006).

3.3.2 The Mean and Range Control Charts Under the Skewness Correction method

The control charts for the mean ($\bar{X}$-Chart) and the range (R -chart) obtained by the SC method when the process distribution and parameters are known are given below (Chan & Cui, 2003).

$$
\begin{aligned}
UCL_{SC_{\bar{X}}} &= \mu_X + (3 + c_4^*)\frac{\sigma_X}{\sqrt{n}} \\
CL_{SC_{\bar{X}}} &= \mu_X \\
LCL_{SC_{\bar{X}}} &= \mu_X + (-3 + c_4^*)\frac{\sigma_X}{\sqrt{n}}
\end{aligned} \tag{3.22}
$$

$$
\begin{aligned}
UCL_{SC_R} &= \mu_R + (3 + d_4^*)\sigma_R \\
CL_R &= \mu_R \\
LCL_{SC_R} &= \mu_R + (-3 + d_4^*)\sigma_R
\end{aligned} \tag{3.23}
$$

The Lower control limit, LCL_R is set to be zero if its original value is negative. The constants c_4^* and d_4^* are the SC constants. When the distribution of the random variable under investigation is symmetric, the constant $c_4^* = 0$, the skewness coefficent $k_3 = 0$ and hence the $\bar{X}$ chart tends to the classic Shewhart chart (Chan & Cui, 2003).

Similarly, the control charts for the mean ($\bar{X}$-Chart) obtained by the SC method when the process distribution and parameters are unknown are given below (Chan & Cui, 2003).

$$
\begin{aligned}
UCL_{SC_{\bar{X}}} &= \bar{\bar{X}} + \left(3 + \frac{4\frac{k_3(\bar{X})}{(3\sqrt{n})}}{1 + 0.2\frac{k_3(\bar{X})^2}{n}} \right) \frac{\bar{R}}{d_2^*\sqrt{n}} \equiv \bar{\bar{X}} + A_U^*\bar{R} \\
CL_{\bar{X}} &= \bar{\bar{X}} \\
LCL_{SC_{\bar{X}}} &= \bar{\bar{X}} + \left(-3 + \frac{4\frac{k_3(\bar{X})}{(3\sqrt{n})}}{1 + 0.2\frac{k_3(\bar{X})^2}{n}} \right) \frac{\bar{R}}{d_2^*\sqrt{n}} \equiv \bar{\bar{X}} - A_L^*\bar{R}
\end{aligned} \tag{3.24}
$$

The values of the constants used in Eq. (3.24) were studied and given by Chan and Cui (2003).

The control charts for the range (R-chart) obtained by the SC method when the process distribution and parameters are unknown are given as follows.

$$UCL_{SC_R} = \left[1 + (3 + d_4^*)\frac{d_3^*}{d_2^*}\right]\bar{R} \equiv D_4^*\bar{R}$$
$$CL_R = \bar{R} \tag{3.25}$$
$$UCL_{SC_R} = \left[1 + (-3 + d_4^*)\frac{d_3^*}{d_2^*}\right]^+ \bar{R} \equiv D_3^*\bar{R}$$

From Eq. (3.25), given $a = \left[1 + (-3 + d_4^*)\frac{d_3^*}{d_2^*}\right]$, then $a^+ = a$ for $a \geq 0$ and $a^+ = 0$ for $a < 0$. The constants used in this equation were studied and given by Chan and Cui (2003).

In most situations, the skewness k_3 needs to be estimated as follows (Chang & Bai, 2001).

$$k_3^* = \frac{1}{nr - 3}\sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{X_{ij} - \bar{\bar{X}}}{S_{nr}}\right)^3 \tag{3.26}$$

Here, $\bar{\bar{X}} = \frac{1}{nr}\sum_{i=1}^{r}\sum_{j=1}^{n}X_{ij}$ and $S_{nr} = \sqrt{\frac{1}{nr-1}\sum_{i=1}^{r}\sum_{j=1}^{n}\left(X_{ij} - \bar{\bar{X}}\right)^2}$. It is worth important to note that S_{nr} in Eq. (3.26) above can be replaced by $\frac{\bar{R}}{d_2^*}$. It is also clear that the sample skewness, k_3^* is the third moment estimator. If $|k_3^*|$ is very small or large, the correction amount $\frac{4\frac{k_3(\bar{X})}{(3\sqrt{n})}}{1+0.2\frac{k_3(\bar{X})^2}{n}}$ would be relatively small. In other words, it can simply be said that the correction amount is more stable than the estimator of skewness k_3 itself (Chan & Cui, 2003).

3.3.3 Constants Estimation Under the Skewness Correction Method

Like those of the classic Shewhart control charts, the control chart constants under the skewness correction method are $d_2^*, d_3^*, C_4^*\ and\ d_4^*$. The first two constants are obtained in the same way as those of the WV method. The remaining two constants C_4^* and d_4^* are given as follows (Chan & Cui, 2003).

$$C_4^* = \frac{\frac{4}{3}k_3(\bar{X})}{1 + 0.2k_3(\bar{X})^2} \tag{3.27}$$

The term $k_3(\bar{X})$ in Eq. (3.27) is the skewness of the mean $\bar{X}$ of the subgroup in question.

$$d_4^* = \frac{\frac{4}{3}k_3(R)}{1 + 0.2k_3(R)^2} \tag{3.28}$$

The term $k_3(R)$ in Eq. (3.28) is the skewness of the relative range.

Provided the process distribution and subgroup's size n are known, the values of the constants d_2^*, d_3^* and d_4^* can be obtained directly through numerical integration. However, in practice these values are unknown.

3.4 The EWMA for Skewed Distributions

As earlier mentioned, the classic EWMA control chart which is used in detecting small shifts in the process mean assumes that the distribution of the process is symmetrical. Whenever the distribution of the process is skewed, using the classic EWMA control chart might not produce desired and reliable results. Hence methods such as the WV, WSD and SC used for skewed distributions can be used also with the EWMA to give better results. These methods are expatiated below.

3.4.1 The WV-EWMA control Chart

The WV-EWMA method was developed by (Khoo & Atta, 2008). Given the exponentially weighted moving average control chart defined in Eq. (2.27), the control limits for the WV-EWMA control chart when the parameters are known is given below (Khoo & Atta, 2008).

$$UCL_{WV_{EWMA}} = \mu + K\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{2P_X}$$

$$CL_{WV_{EWMA}} = \mu \tag{3.29}$$

$$LCL_{WV_{EWMA}} = \mu - K\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{2(1-P_X)}$$

The values of the constants K and λ are obtained depending on the desired conditions of the average run length as described in the previous chapter. The control limits in Eq.(3.29) tends to the standard EWMA control limits if the value of the probability P_X is $P_X = 0.5$ (Khoo, Atta, & Chen, 2009).

When the parameters are unknown, the control limits for the WV-EWMA control chart are computed using the following formulations (Khoo & Atta, 2008).

$$UCL_{WV_{EWMA}} = \bar{\bar{X}} + K\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{2\hat{P}_X}$$

$$CL_{WV_{EWMA}} = \bar{\bar{X}} \tag{3.30}$$

$$LCL_{WV_{EWMA}} = \bar{\bar{X}} - K\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{2(1-\hat{P}_X)}$$

3.4.2 The WSD-EWMA Control Chart

This method was proposed by (Atta & Ramli, 2011). Given the exponentially weighted moving average defined in Eq. (2.27), the control limits for the WSD-EWMA control chart when the parameters are known is given below (Atta & Ramli, 2011).

$$UCL_{WSD_{EWMA}} = \mu + K\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\ (2P_X)$$

$$CL_{WSD_{EWMA}} = \mu \tag{3.31}$$

$$LCL_{WSD_{EWMA}} = \mu - K\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\ [2(1-P_X)]$$

When the parameters are unknown, the control limits for the WSD-EWMA control chart are computed using the following formulations (Atta & Ramli, 2011).

$$UCL_{WSD_{EWMA}} = \bar{\bar{X}} + K\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\ (2\hat{P}_X)}$$

$$CL_{WSD_{EWMA}} = \bar{\bar{X}} \tag{3.32}$$

$$LCL_{WSD_{EWMA}} = \bar{\bar{X}} - K\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\ \left[2\left(1-\hat{P}_X\right)\right]}$$

Here, the constant $d_2' = P_X d_2\big(2n(1-P_X)\big) + (1-P_X)d_2(2P_X)$ where $d_2(n)$ is d_2 for the normal distribution when the sample size is equal to n (Khoo, Atta, & Chen, 2009). The value of this constant is computed by Chang and Bai (2001).

3.4.3 The Proposed SC-EWMA Control Chart

Given the exponentially weighted moving average defined in Eq. (2.27), the control limits for the SC-EWMA control chart when the parameters are known is proposed as follows.

$$UCL_{SC_{EWMA}} = \mu + \left(K + \frac{\frac{4}{3}k_3(\bar{X})}{1+0.2k_3^2(\bar{X})}\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\ \frac{\sigma}{\sqrt{n}}$$

$$CL_{SC_{EWMA}} = \mu \tag{3.33}$$

$$LCL_{SC_{EWMA}} = \mu + \left(-K + \frac{\frac{4}{3}k_3(\bar{X})}{1+0.2k_3^2(\bar{X})}\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\ \frac{\sigma}{\sqrt{n}}$$

When the parameters are unknown, the control limits for the SC-EWMA control chart are computed using the following formulations.

$$UCL_{SC_{EWMA}} = \bar{\bar{X}} + \left(K + \frac{4k_3/3\sqrt{n}}{1+0.2k_3^2/n}\right)\ \sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\frac{\bar{R}}{d_2'\sqrt{n}}$$

$$CL_{SC_{EWMA}} = \bar{\bar{X}} \tag{3.34}$$

$$LCL_{SC_{EWMA}} = \bar{\bar{X}} + \left(-K + \frac{4k_3/3\sqrt{n}}{1 + 0.2k_3^2/n}\right) \sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \frac{\bar{R}}{d_2'\sqrt{n}}$$

As earlier mentioned, the term $C_4^* = \frac{\frac{4}{3}k_3(\bar{X})}{1+0.2k_3^2(\bar{X})}$ is computed for various values of n and given by (Karagöz,2018).

4 SIMULATION STUDY AND DISCUSSIONS

In this chapter, the experimental design used for this work and the Monte Carlo simulation technique are discussed. The newly developed SC-EWMA method is compared with existing heuristic methods of WV-EWMA, WSD-EWMA and the classic EWMA using a Monte Carlo simulation under the lognormal, Gamma and Weibull distributions. The comparison is made based on their type-I errors which is the probability of detecting an out-of-control situation even though the process is in control. Type-I error is also called the false alarm detection rate. The Monte Carlo simulation program is written in MATLAB 2016b 64-bit version 9.1.0. The experimental design, the simulation study and the results obtained from the simulation studies are discussed subsequently.

4.1 Experimental Design

Throughout the scope of this work, the quality variable under investigation is assumed to either be gamma distributed with shape parameter (α) and scale parameter (β), Weibull distributed with scale parameter (λ) and shape parameter (β) or lognormal distributed with location parameter (μ) and scale parameter (σ). For the gamma distribution, the skewness is only affected by the shape parameter (α) hence, throughout this work the scale parameter (β) is given a constant value of 1. For the Weibull distribution, the skewness is affected only by the shape parameter (β) hence, the scale parameter (λ) is given a constant value of 1. Lastly, for the lognormal distribution, the skewness is affected only by the scale parameter (σ) hence, the location parameter (μ) is given a constant value of 0. The skewness values, the varying parameters and their corresponding probabilities are given in Table 4.1 below. The probabilities are taken to two decimal places and are also obtained from the Monte Carlo simulation.

Table 4.1 Varying Parameters and their probabilities for Gamma, Weibull and Lognormal Distributions

k_3	Gamma		Weibull		Lognormal	
	α	P_X	β	P_X	σ	P_X
0.50	16.00	0.53	2.15	0.54	0.16	0.53
1.00	4.00	0.57	1.57	0.57	0.32	0.56
1.50	1.80	0.60	1.20	0.61	0.44	0.59
2.00	1.00	0.63	1.00	0.63	0.54	0.61
2.50	0.64	0.66	0.86	0.66	0.66	0.63
3.00	0.44	0.69	0.77	0.68	0.72	0.64

The EWMA statistic is set up following the steps provided in chapter 2. The in-control ARL used for this study is 370 which is equivalent to a type-I error (α) of 0.0027 for the classic Shewhart method when the process is in-control. In the coding however, type-I error of α is used because of the following relation.

$$ARL = \frac{1}{\alpha} \tag{4.1}$$

The optimal ordered pairs of (λ, K) used for this study are given below. These values are obtained from the graphs in Figures 2.1 and 2.2. Some of these optimal pairs are found in the interval $0.05 < \lambda < 0.25$ which was studied by Borror, Montgomery and Runger (1999) and found to be the best smoothing parameter interval corresponding to the in-control type-I error of 0.0027 or ARL of 370 for the classic Shewhart charts.

Table 4.2 Values of the smoothing and control chart constants

λ	0.10	0.20	0.30	0.40	0.70
K	2.6952	2.8537	2.9286	2.9614	3.0000

The subgroup sizes used for this work are $n = \{3,5,7,10\}$ which are chosen to represent small, medium and large group sizes respectively. The WV constant (d_2^{WV}) and WSD constant (d_2^{WSD}) reproduced from (Chang and Bai, 2001), the d_2 and C_4^* constants used for the various subgroup sizes when the distribution is normal are given in the Tables below. Not that the $\hat{P}_X$ used for the WV and WSD methods are computed simultaneously within the simulation program.

Table 4.3 Values of the smoothing and control chart constants

Gamma	k_3	d_2	C_4^*	d_2^{WV}	d_2^{WSD}
3	0.50	1.6791	0.3414	1.681	1.670
	1.00	1.6406	0.6515	1.634	1.623
	1.50	1.5804	0.9012	1.578	1.577
	2.00	1.5001	1.1033	1.505	1.524
	2.50	1.4102	1.2429	1.421	1.454
	3.00	1.3157	1.3386	1.327	1.373
5	0.50	2.3089	0.2669	2.311	2.313
	1.00	2.2595	0.5163	2.251	2.282
	1.50	2.1827	0.7362	2.180	2.249
	2.00	2.0827	0.9281	2.090	2.184
	2.50	1.9758	1.0811	1.987	2.110
	3.00	1.8621	1.2011	1.874	2.023
7	0.50	2.6858	0.2253	2.688	2.694
	1.00	2.6328	0.4413	2.625	2.667
	1.50	2.5531	0.6368	2.550	2.629
	2.00	2.4502	0.8146	2.456	2.570
	2.50	2.3417	0.9647	2.354	2.511
	3.00	2.2306	1.0931	2.243	2.426
10	0.50	3.0587	0.1881	3.061	3.070
	1.00	3.0050	0.3726	2.997	3.044
	1.50	2.9258	0.5423	2.923	3.011
	2.00	2.8287	0.7032	2.836	2.926
	2.50	2.7323	0.8450	2.742	2.901
	3.00	2.6348	0.9715	2.646	2.824

Table 4.4 Constants Values for Weibull Distributions

Weibull	k_3	d_2	C_4^*	d_2^{WV}	d_2^{WSD}
3	0.50	1.6880	0.3702	1.685	1.660
	1.00	1.6447	0.6537	1.644	1.623
	1.50	1.5726	0.9223	1.560	1.559
	2.00	1.4995	1.1017	1.505	1.521
	2.50	1.4221	1.2355	1.410	1.454
	3.00	1.3552	1.3162	1.338	1.402
5	0.50	2.3088	0.2879	2.306	2.307
	1.00	2.2559	0.5173	2.255	2.282
	1.50	2.1702	0.7529	2.154	2.228
	2.00	2.0831	0.9283	2.090	2.184
	2.50	1.9903	1.0764	1.977	2.110
	3.00	1.9102	1.1819	1.889	2.055
7	0.50	2.6721	0.2415	2.668	2.689
	1.00	2.6172	0.4418	2.617	2.667
	1.50	2.5340	0.6522	2.519	2.613
	2.00	2.4499	0.8185	2.457	2.579
	2.50	2.3601	0.9660	2.346	2.511
	3.00	2.2808	1.0758	2.260	2.456
10	0.50	3.0213	0.2044	3.018	3.066
	1.00	2.9709	0.3708	2.971	3.044
	1.50	2.8990	0.5531	2.887	2.996
	2.00	2.8301	0.7046	2.834	2.962
	2.50	2.7530	0.8464	2.741	2.901
	3.00	2.6842	0.9586	2.666	2.851

Table 4.5 Constants Values for lognormal Distribution

Lognormal	k_3	d_2	C_4^*	d_2^{WV}	d_2^{WSD}
3	0.50	1.6776	0.3337	1.679	1.670
	1.00	1.6352	0.6547	1.642	1.637
	1.50	1.5860	0.8784	1.577	1.593
	2.00	1.5335	1.0381	1.522	1.559
	2.50	1.4587	1.1940	1.455	1.521
	3.00	1.4174	1.2529	1.419	1.454
5	0.50	2.3092	0.2602	2.309	2.313
	1.00	2.2575	0.5223	2.265	2.291
	1.50	2.1974	0.7207	2.188	2.261
	2.00	2.1346	0.8787	2.120	2.228
	2.50	2.0423	1.0527	2.038	2.184
	3.00	1.9911	1.1274	1.994	2.161
7	0.50	2.6877	0.2229	2.688	2.694
	1.00	2.6381	0.4480	2.646	2.677
	1.50	2.5790	0.6268	2.569	2.643
	2.00	2.5159	0.7773	2.501	2.613
	2.50	2.4231	0.9512	2.418	2.579
	3.00	2.3696	1.0323	2.373	2.561
10	0.50	3.0640	0.1826	3.064	3.070
	1.00	3.0225	0.3787	3.028	3.053
	1.50	2.9701	0.5368	2.958	3.023
	2.00	2.9145	0.6748	2.901	2.978
	2.50	2.8300	0.8432	2.825	2.926
	3.00	2.7806	0.9246	2.783	2.908

4.2 Simulation Algorithm

As earlier mentioned, the Monte Carlo simulation technique is used throughout the scope of this work. The algorithmic steps used for the simulation are as follows.

- N independent identically distributed (i.i.d) pseudo random numbers are generated from gamma $(gamma(\alpha, 1))$, Weibul($weibull(1, \lambda)$) and lognormal($lognormal(0, \sigma)$) distributions for each of the subgroup sample sizes $n = \{3,5,7,10\}$.
- The above process is repeated 100 times and their mean, grand mean, range and the mean of the range computed.
- The $\hat{P}_X$ value is then obtained from the randomly generated data in the previous step.

- The lower and upper control limits for the classic EWMA, SC-EWMA, WV-EWMA and WSD-EWMA are computed.
- The EWMA test statistics G_t is computed for all the 100 subgroups of size n generated above.
- The EWMA test statistics G_t are checked to be within the control limits computed above or not for each of the methods and their type-I errors are obtained and recorded.
- The above steps are repeated 10.000 times which is the simulation size and the mean of the 10.000 type-I errors for each method is computed and recorded.

It is vitally important to note that small changes are made to the program to suit the various smoothing parameters.

4.3 Results and Discussions

After successfully executing the programs, the results obtained are tabulated as follows in hierarchical order of the subgroup sizes n.

Table 4.6 Table of Results for n=3 and lambda=0.10

	λ=0.10					
n	**Dist**	**k_3**	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
3		0.50	0.0033	0.0043	0.0030	0.0032
		1.00	0.0042	0.0089	0.0029	0.0037
		1.50	0.0061	0.0173	0.0033	0.0057
		2.00	0.0092	0.0273	0.0034	0.0089
		2.50	0.0137	0.0379	0.0039	0.0147
		3.00	0.2080	0.0472	0.0044	0.0240
	Weibull					
3		0.50	0.0031	0.0045	0.0029	0.0029
		1.00	0.0041	0.0090	0.0031	0.0037
		1.50	0.0065	0.0177	0.0031	0.0056
		2.00	0.0093	0.0273	0.0035	0.0090
		2.50	0.0137	0.0378	0.0036	0.0129
		3.00	0.0176	0.0448	0.0032	0.0172
	Lognormal					
3		0.50	0.0031	0.0042	0.0029	0.0030
		1.00	0.0042	0.0090	0.0031	0.0040
		1.50	0.0058	0.0158	0.0029	0.0049
		2.00	0.0083	0.0236	0.0031	0.0065
		2.50	0.0117	0.0332	0.0033	0.0095
		3.00	0.0142	0.0378	0.0036	0.0096

From Table 4.6 above, it is clearly seen that the WV-EWMA produces the smallest type-I errors for all the distributions. When the skewness is less than or equal to 1.50, WSD-EWMA also performs very well and can be used as an alternative for the WV-EWMA for all the distributions. Another important remark which can be made from this Table is the fact that the classic EWMA produces very good results for all the distributions when the skewness is extremely small and almost symmetrical. The SC-EWMA is noticed not to produce good results for all the distributions as the skewness is increased.

Table 4.7 Table of Results for n=3 and lambda=0.20

	λ=0.20					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
3		0.50	0.0034	0.0038	0.0030	0.0030
		1.00	0.0047	0.0065	0.0029	0.0032
		1.50	0.0069	0.0113	0.0029	0.0040
		2.00	0.0107	0.0172	0.0033	0.0061
		2.50	0.0159	0.0228	0.0034	0.0096
		3.00	0.0231	0.0272	0.0038	0.0160
	Weibull					
3		0.50	0.0032	0.0040	0.0029	0.0027
		1.00	0.0045	0.0067	0.0029	0.0032
		1.50	0.0071	0.0117	0.0027	0.0039
		2.00	0.0109	0.0180	0.0032	0.0063
		2.50	0.0154	0.0236	0.0033	0.0084
		3.00	0.0204	0.0264	0.0034	0.0111
	Lognormal					
3		0.50	0.0034	0.0038	0.0031	0.0030
		1.00	0.0050	0.0069	0.0034	0.0036
		1.50	0.0070	0.0110	0.0032	0.0041
		2.00	0.0099	0.0156	0.0035	0.0051
		2.50	0.0142	0.0215	0.0040	0.0073
		3.00	0.0164	0.0232	0.0043	0.0066

From Table 4.7 above, it is clearly seen that the WV-EWMA produces the smallest type-I error for all the distributions. When the skewness is less than or equal to 1.50, WSD-EWMA also performs very well and can be used as an alternative for the WV-EWMA for all the distributions. Another important remark which can be made from this Table is the fact that the classic EWMA produces very good results for all the distributions when the skewness is extremely small and almost symmetrical. The SC-EWMA is noticed not to produce good results for all the distributions as the skewness is increased.

Table 4.8 Table of Results for n=3 and lambda=0.30

	λ=0.30					
n	**Dist**	**k_3**	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
3		0.50	0.0034	0.0035	0.0030	0.0029
		1.00	0.0048	0.0051	0.0028	0.0027
		1.50	0.0076	0.0080	0.0031	0.0031
		2.00	0.0121	0.0118	0.0036	0.0045
		2.50	0.0177	0.0139	0.0040	0.0061
		3.00	0.0248	0.0149	0.0044	0.0094
	Weibull					
3		0.50	0.0030	0.0031	0.0025	0.0022
		1.00	0.0047	0.0050	0.0028	0.0026
		1.50	0.0079	0.0080	0.0028	0.0030
		2.00	0.0121	0.0116	0.0035	0.0043
		2.50	0.0170	0.0141	0.0038	0.0052
		3.00	0.0220	0.0153	0.0041	0.0068
	Lognormal					
3		0.50	0.0034	0.0034	0.0030	0.0029
		1.00	0.0054	0.0055	0.0034	0.0034
		1.50	0.0076	0.0085	0.0034	0.0036
		2.00	0.0105	0.0111	0.0038	0.0043
		2.50	0.0154	0.0142	0.0048	0.0059
		3.00	0.0180	0.0151	0.0051	0.0052

From Table 4.8, it is noticed that for the gamma distribution the WSD-EWMA method produces the smallest type-I errors when the skewness is smaller or equal to 1.50. However, as the skewness increases beyond 1.50, the WV-EWMA produces the best results. This clearly shows us that these two methods could be used as alternatives of one another under these conditions. For the Weibull distribution, the WV-EWMA produces the overall best results. However, for small values of the skewness up to 1.50, the WSD-EWMA produces very good results too. Once more, it can clearly be stated that the WV-EWMA and the WSD-EWMA produce the best results and could be substituted for one another. Lastly, for the lognormal distribution, a similar scenario is observed where in the WV-EWMA produces the overall best results but when the skewness is smaller or equal to 1.50, the WSD-EWMA produces very similar results hence making it an alternative to the WV-EWMA.

Table 4.9 Table of Results for n=3 and lambda=0.40

	λ=0.40					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
3		0.50	0.0034	0.0032	0.0030	0.0027
		1.00	0.0053	0.0043	0.0030	0.0026
		1.50	0.0085	0.006	0.0035	0.0028
		2.00	0.0131	0.0077	0.0041	0.0034
		2.50	0.0195	0.0086	0.0048	0.0044
		3.00	0.0269	0.0075	0.0054	0.0055
	Weibull					
3		0.50	0.0029	0.0029	0.0025	0.0021
		1.00	0.0050	0.0040	0.0029	0.0023
		1.50	0.0087	0.0060	0.0032	0.0025
		2.00	0.0134	0.0078	0.0042	0.0035
		2.50	0.0190	0.0091	0.0048	0.0040
		3.00	0.0239	0.0087	0.0052	0.0046
	Lognormal					
3		0.50	0.0034	0.0032	0.0029	0.0028
		1.00	0.0056	0.0045	0.0034	0.0030
		1.50	0.0085	0.0065	0.0038	0.0034
		2.00	0.0121	0.0086	0.0046	0.0041
		2.50	0.0167	0.0105	0.0055	0.0053
		3.00	0.0194	0.0107	0.0061	0.0048

From Table 4.9, it is noticed that the WSD-EWMA produces the smallest overall type-I errors for all the distributions. It is closely followed by the WV-EWMA which also produces good results but not as small as the former. Under this experimental condition, when the skewness is smaller or equal to 1.50, the SC-EWMA also produces considerable results hence could be used as an alternative to the WSD-EWMA method.

Table 4.10 Table of Results for n=3 and lambda=0.70

	λ=0.70					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
3	**Gamma**					
		0.50	0.0038	0.0028	0.0031	0.0026
		1.00	0.0064	0.0030	0.0036	0.0025
		1.50	0.0107	0.0030	0.0048	0.0029
		2.00	0.0166	0.0026	0.0059	0.0034
		2.50	0.0234	0.0025	0.0069	0.0039
		3.00	0.0313	0.0029	0.0080	0.0044
	Weibull					
3		0.50	0.0033	0.0022	0.0025	0.0019
		1.00	0.0060	0.0022	0.0033	0.0021
		1.50	0.0112	0.0025	0.0045	0.0025
		2.00	0.0166	0.0027	0.0060	0.0034
		2.50	0.0228	0.0029	0.0068	0.0041
		3.00	0.0278	0.0031	0.0074	0.0047
	Lognormal					
3		0.50	0.0039	0.0030	0.0032	0.0028
		1.00	0.0069	0.0035	0.0042	0.0033
		1.50	0.0107	0.0044	0.0053	0.0038
		2.00	0.0144	0.0048	0.0061	0.0045
		2.50	0.0198	0.0050	0.0076	0.0058
		3.00	0.0225	0.0049	0.0084	0.0057

From Table 4.10, for the gamma distribution, the overall smallest type-I errors are produced by the SC-EWMA which is closely followed by the WSD-EWMA. When the skewness is smaller or equal to 1.50, the WSD-EWMA produces very good results, in some cases even smaller than the former. For the Weibull distribution, the overall best results are produced by the SC-EWMA. Here, just like that of the gamma distribution, the WSD-EWMA closely follows the former. When the skewness is smaller or equal to 1.50, the WSD-EWMA also performs very well producing smaller type 1 errors as compared to the SC-EWMA. For the lognormal distribution, similar observations are made where in the overall best type-I errors are produced by the SC-EWMA method followed by the WSD-EWMA. It can therefore be stated that under this experimental condition the SC-EWMA and the WSD-EWMA are alternatives of each other.

Table 4.11 Table of Results for n=5 and lambda=0.10

	λ=0.10					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
5		0.50	0.0029	0.0035	0.0028	0.0031
		1.00	0.0041	0.0065	0.0031	0.0047
		1.50	0.0056	0.0108	0.0033	0.0074
		2.00	0.0083	0.0180	0.0039	0.0129
		2.50	0.0118	0.0258	0.0042	0.0205
		3.00	0.0171	0.0351	0.0048	0.0332
	Weibull					
5		0.50	0.0031	0.0038	0.0028	0.0032
		1.00	0.0037	0.0062	0.0029	0.0047
		1.50	0.0057	0.0113	0.0031	0.0079
		2.00	0.0079	0.0179	0.0038	0.0126
		2.50	0.0117	0.0261	0.0038	0.0184
		3.00	0.0149	0.0332	0.0037	0.0243
	Lognormal					
5		0.50	0.0032	0.0036	0.0029	0.0032
		1.00	0.0039	0.0062	0.0030	0.0044
		1.50	0.0052	0.0105	0.0030	0.0063
		2.00	0.0068	0.0151	0.0031	0.0085
		2.50	0.0099	0.0238	0.0034	0.0126
		3.00	0.0119	0.0286	0.0036	0.0157

From Table 4.11, it is clearly seen that the WV-EWMA produces the best overall type-I errors for all the distributions. It is seconded by the classic EWMA when the skewness is less than or equal to 1.50. The WSD-EWMA also produces similar results to those of the classic EWMA though not as small as the former when the skewness is less than or equal to 1.50.

Table 4.12 Table of Results for n=5 and lambda=0.20

	λ=0.20					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
5	**Gamma**					
		0.50	0.0032	0.0034	0.0029	0.0032
		1.00	0.0042	0.0050	0.0028	0.0040
		1.50	0.0059	0.0076	0.0029	0.0060
		2.00	0.0089	0.0116	0.0032	0.0097
		2.50	0.0129	0.0167	0.0033	0.0163
		3.00	0.0187	0.0217	0.0037	0.0266
	Weibull					
5		0.50	0.0032	0.0034	0.0028	0.0031
		1.00	0.0041	0.0047	0.0028	0.0040
		1.50	0.0063	0.0082	0.0028	0.0066
		2.00	0.0090	0.0117	0.0032	0.0098
		2.50	0.0129	0.0166	0.0031	0.0138
		3.00	0.0166	0.0206	0.0030	0.0185
	Lognormal					
5		0.50	0.0032	0.0033	0.0029	0.0031
		1.00	0.0043	0.0050	0.0031	0.0041
		1.50	0.0058	0.0075	0.0030	0.0053
		2.00	0.0079	0.0106	0.0030	0.0068
		2.50	0.0114	0.0158	0.0035	0.0101
		3.00	0.0139	0.0187	0.0038	0.0124

From Table 4.12, it is clearly seen that the WV-EWMA method produces the overall best type-I errors for all the distributions. It is closely followed by the classic EWMA for the gamma distribution, both the WSD-EWMA and the classic EWMA for the Weibull distribution and lastly by the WSD-EWMA for the lognormal distribution when the skewness is less than or equal to 1.50.

Table 4.13 Table of Results for n=5 and lambda=0.30

	λ=0.30					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
5		0.50	0.0030	0.0030	0.0027	0.0029
		1.00	0.0041	0.0039	0.0026	0.0034
		1.50	0.0065	0.0057	0.0028	0.0049
		2.00	0.0096	0.0079	0.0030	0.0074
		2.50	0.0144	0.0111	0.0032	0.0124
		3.00	0.0203	0.0130	0.0035	0.0198
	Weibull					
5		0.50	0.0030	0.0030	0.0026	0.0028
		1.00	0.0041	0.0038	0.0026	0.0035
		1.50	0.0064	0.0057	0.0024	0.0048
		2.00	0.0096	0.0081	0.0030	0.0075
		2.50	0.0137	0.0108	0.0030	0.0103
		3.00	0.0181	0.0132	0.0033	0.0137
	Lognormal					
5		0.50	0.0031	0.0030	0.0027	0.0029
		1.00	0.0044	0.0042	0.0029	0.0036
		1.50	0.0063	0.0059	0.0030	0.0047
		2.00	0.0086	0.0077	0.0032	0.0056
		2.50	0.0121	0.0111	0.0037	0.0078
		3.00	0.0147	0.0128	0.0042	0.0096

From Table 4.13, it is seen that the WV-EWMA produces the overall best type-I error for all the distributions. It is closely followed by the WSD-EWMA which produces good results when the skewness is less than or equal to 1.50 for all the distributions. The SC-EWMA also produces considerably good results for all the distributions when the skewness is less than or equal to 1.50. The latter can therefore be used as an alternative to the former under such experimental conditions.

Table 4.14 Table of Results for n=5 and lambda=0.40

	λ=0.40					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
5		0.50	0.0030	0.0028	0.0026	0.0027
		1.00	0.0044	0.0035	0.0027	0.0032
		1.50	0.0068	0.0046	0.0028	0.0043
		2.00	0.0107	0.0059	0.0032	0.0060
		2.50	0.0154	0.0070	0.0035	0.0089
		3.00	0.0218	0.0080	0.0038	0.0145
	Weibull					
5		0.50	0.0030	0.0027	0.0024	0.0025
		1.00	0.0043	0.0034	0.0026	0.0032
		1.50	0.0070	0.0044	0.0026	0.0041
		2.00	0.0106	0.0059	0.0032	0.0059
		2.50	0.0150	0.0076	0.0034	0.0079
		3.00	0.0192	0.0089	0.0038	0.0103
	Lognormal					
5		0.50	0.0031	0.0029	0.0027	0.0028
		1.00	0.0048	0.0037	0.0030	0.0034
		1.50	0.0070	0.0050	0.0032	0.0043
		2.00	0.0095	0.0066	0.0036	0.0052
		2.50	0.0136	0.0085	0.0044	0.0069
		3.00	0.0159	0.0095	0.0048	0.0081

From Table 4.14, it is seen that the WV-EWMA produces the overall best type-I error for all the distributions. It is closely followed by the WSD-EWMA which produces good results when the skewness is less than or equal to 1.50 for all the distributions. The SC-EWMA also produces considerably good results which are very close to those of the WSD-EWMA for all the distributions when the skewness is less than or equal to 1.50. The latter can therefore be used as an alternative to the former under such experimental conditions.

Table 4.15 Table of Results for n=5 and lambda=0.70

	λ=0.70					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
5	**Gamma**					
		0.50	0.0033	0.0027	0.0028	0.0027
		1.00	0.0054	0.0027	0.0030	0.0028
		1.50	0.0084	0.0028	0.0035	0.0032
		2.00	0.0132	0.0027	0.0044	0.0038
		2.50	0.0188	0.0024	0.0051	0.0043
		3.00	0.0258	0.0024	0.0058	0.0053
	Weibull					
5		0.50	0.0029	0.0022	0.0023	0.0021
		1.00	0.0051	0.0024	0.0027	0.0025
		1.50	0.0089	0.0025	0.0033	0.0029
		2.00	0.0133	0.0028	0.0044	0.0038
		2.50	0.0183	0.0029	0.0050	0.0043
		3.00	0.0232	0.0031	0.0056	0.0050
	Lognormal					
5		0.50	0.0034	0.0028	0.0029	0.0029
		1.00	0.0056	0.0031	0.0035	0.0033
		1.50	0.0084	0.0035	0.0039	0.0038
		2.00	0.0117	0.0042	0.0048	0.0046
		2.50	0.0162	0.0048	0.0059	0.0057
		3.00	0.0189	0.0050	0.0066	0.0064

From Table 4.15, it is noticed that the SC-EWMA produces the smallest and hence best type-I errors for all the distributions. It was seconded by both the WV-EWMA and the WSD-EWMA which produce results that are very closely related for all the distributions. Any one of these two methods could be used as alternatives for the SC-EWMA under this experimental condition.

Table 4.16 Table of Results for n=7 and lambda=0.10

	λ=0.10					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
7	**Gamma**					
		0.50	0.0030	0.0033	0.0029	0.0033
		1.00	0.0038	0.0051	0.0029	0.0046
		1.50	0.0049	0.0081	0.0032	0.0075
		2.00	0.0071	0.0131	0.0038	0.0132
		2.50	0.0096	0.0189	0.0043	0.0220
		3.00	0.0138	0.0264	0.0052	0.0346
	Weibull					
7		0.50	0.0031	0.0034	0.0028	0.0034
		1.00	0.0041	0.0051	0.0031	0.0052
		1.50	0.0054	0.0088	0.0032	0.0085
		2.00	0.0070	0.0130	0.0037	0.0133
		2.50	0.0100	0.0196	0.0040	0.0195
		3.00	0.0123	0.0253	0.0041	0.0257
	Lognormal					
7		0.50	0.0031	0.0034	0.0029	0.0033
		1.00	0.0034	0.0051	0.0029	0.0044
		1.50	0.0045	0.0080	0.0029	0.0062
		2.00	0.0059	0.0120	0.0031	0.0086
		2.50	0.0082	0.0187	0.0036	0.0130
		3.00	0.0098	0.0220	0.0036	0.0155

From Table 4.16, it is seen that the WV-EWMA produces the overall best type-I error for all the distributions. It was followed by the classic EWMA which produces good results for all the distributions when the skewness is smaller than or equal to 1.50. Therefore, the latter can be used as an alternative for the former under this experimental condition.

Table 4.17 Table of Results for n=7 and lambda=0.20

	λ=0.20					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
7		0.50	0.0031	0.0032	0.0029	0.0031
		1.00	0.0041	0.0043	0.0029	0.0043
		1.50	0.0054	0.0062	0.0030	0.0066
		2.00	0.0079	0.0092	0.0032	0.0110
		2.50	0.0111	0.0125	0.0035	0.0183
		3.00	0.0154	0.0169	0.0039	0.0296
	Weibull					
7		0.50	0.0031	0.0030	0.0027	0.0032
		1.00	0.0041	0.0042	0.0029	0.0047
		1.50	0.0059	0.0063	0.0030	0.0071
		2.00	0.0077	0.0090	0.0032	0.0111
		2.50	0.0106	0.0129	0.0032	0.0158
		3.00	0.0137	0.0164	0.0033	0.0210
	Lognormal					
7		0.50	0.0029	0.0031	0.0028	0.0030
		1.00	0.0038	0.0043	0.0029	0.0041
		1.50	0.0052	0.0063	0.0030	0.0055
		2.00	0.0068	0.0088	0.0031	0.0073
		2.50	0.0095	0.0127	0.0034	0.0107
		3.00	0.0115	0.0151	0.0038	0.0130

From Table 4.17, it is seen that the WV-EWMA produces the overall best type-I error for all the distributions. It was followed by the classic EWMA which produces good results for all the distributions when the skewness is smaller than or equal to 1.50. Therefore, the latter can be used as an alternative for the former under this experimental condition.

Table 4.18 Table of Results for n=7 and lambda=0.30

	λ=0.30					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
7		0.50	0.0030	0.0029	0.0027	0.0030
		1.00	0.0040	0.0036	0.0027	0.0037
		1.50	0.0054	0.0047	0.0026	0.0053
		2.00	0.0082	0.0064	0.0028	0.0087
		2.50	0.0118	0.0087	0.0030	0.0147
		3.00	0.0162	0.0111	0.0032	0.0240
	Weibull					
7		0.50	0.0029	0.0027	0.0024	0.0028
		1.00	0.0041	0.0034	0.0027	0.0039
		1.50	0.0062	0.0049	0.0027	0.0059
		2.00	0.0084	0.0067	0.0031	0.0091
		2.50	0.0114	0.0089	0.0028	0.0125
		3.00	0.0148	0.0110	0.0030	0.0167
	Lognormal					
7		0.50	0.0028	0.0027	0.0026	0.0028
		1.00	0.0040	0.0037	0.0028	0.0037
		1.50	0.0055	0.0048	0.0029	0.0046
		2.00	0.0073	0.0064	0.0030	0.0061
		2.50	0.0104	0.0091	0.0035	0.0086
		3.00	0.0122	0.0108	0.0037	0.0105

From Table 4.18, it is clearly seen that the WV-EWMA produces the best overall results for all the distributions. For the gamma distribution, it is closely followed by both the SC-EWMA and the WSD-EWMA which produce results that are very close to one another when the skewness of the distributions is less than or equal to 1.50. For the Weibull distribution, the former is followed by both the WSD-EWMA and the SC-EWMA with the SC-EWMA having smaller results when the skewness of the distribution is less than or equal to 1.50. Lastly, for the lognormal distribution, the former is seconded by both the WSD-EWMA and the SC-EWMA with the WSD-EWMA producing smaller results when the skewness of the distribution is less than or equal to 1.50. It can therefore be deducted that the SC-EWMA and the WSD-EWMA can be used as alternatives to one another under this experimental condition.

Table 4.19 Table of Results for n=7 and lambda=0.40

	λ=0.40					
n	**Dist**	**k_3**	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
7	**Gamma**					
		0.50	0.0029	0.0027	0.0026	0.0027
		1.00	0.0041	0.0032	0.0026	0.0034
		1.50	0.0059	0.0039	0.0025	0.0046
		2.00	0.0090	0.0050	0.0029	0.0073
		2.50	0.0127	0.0061	0.0031	0.0119
		3.00	0.0176	0.0075	0.0034	0.0193
	Weibull					
7		0.50	0.0027	0.0026	0.0023	0.0024
		1.00	0.0044	0.0033	0.0026	0.0031
		1.50	0.0071	0.0045	0.0026	0.0041
		2.00	0.0107	0.0059	0.0032	0.0059
		2.50	0.0151	0.0076	0.0034	0.0079
		3.00	0.0198	0.0087	0.0039	0.0103
	Lognormal					
7		0.50	0.0032	0.0030	0.0028	0.0029
		1.00	0.0046	0.0037	0.0029	0.0033
		1.50	0.0069	0.0050	0.0032	0.0043
		2.00	0.0093	0.0064	0.0035	0.0052
		2.50	0.0136	0.0087	0.0044	0.0070
		3.00	0.0158	0.0095	0.0047	0.0081

From Table 4.19, it is clearly seen that the WV-EWMA produces the best overall results for all the distributions. For the gamma distribution, it is closely followed by both the SC-EWMA and the WSD-EWMA which produce results that are very close to one another when the skewness of the distributions is less than or equal to 1.50. For the Weibull distribution, the former is followed by both the WSD-EWMA and the SC-EWMA with the SC-EWMA having smaller results when the skewness of the distribution is less than or equal to 1.50. Lastly, for the lognormal distribution, the former is seconded by both the WSD-EWMA and the SC-EWMA with the WSD-EWMA producing smaller results when the skewness of the distribution is less than or equal to 1.50. It can therefore be deducted that the SC-EWMA and the WSD-EWMA can be used as alternatives to one another under this experimental condition.

Table 4.20 Table of Results for n=7 and lambda=0.70

	λ=0.70					
n	**Dist**	**k_3**	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
		0.50	0.0033	0.0027	0.0028	0.0027
		1.00	0.0052	0.0027	0.0029	0.0028
7		1.50	0.0087	0.0029	0.0036	0.0033
		2.00	0.0132	0.0027	0.0043	0.0036
		2.50	0.0191	0.0026	0.0052	0.0044
		3.00	0.0259	0.0025	0.0060	0.0053
	Weibull					
		0.50	0.0031	0.0023	0.0024	0.0025
		1.00	0.0046	0.0024	0.0026	0.0029
7		1.50	0.0077	0.0026	0.0029	0.0036
		2.00	0.0110	0.0026	0.0034	0.0044
		2.50	0.0154	0.0030	0.0041	0.0055
		3.00	0.0194	0.0032	0.0046	0.0066
	Lognormal					
		0.50	0.0033	0.0028	0.0028	0.0028
		1.00	0.0054	0.0030	0.0033	0.0031
7		1.50	0.0084	0.0035	0.0038	0.0037
		2.00	0.0118	0.0042	0.0049	0.0047
		2.50	0.0164	0.0050	0.0061	0.0059
		3.00	0.0190	0.0051	0.0068	0.0066

From Table 4.20, it is seen that the SC-EWMA produces the overall best results for the type-I error for all the distributions. For the gamma distribution, it was followed by both the WV-EWMA and the WSD-EWMA which produce very good results for all the skewness. The results produced by these two distributions were close to each other with the WSD-EWMA having smaller values. For the Weibull distribution, the SC-EWMA was followed once more by both the WV-EWMA and the WSD-EWMA with the WV-EWMA producing smaller results for all the skewness. For the lognormal distribution, the SC-EWMA was followed by both the WV-EWMA and the WSD-EWMA. Here, the WSD-EWMA emerged with the smaller results for all the skewness.

Table 4.21 Table of Results for n=10 and lambda=0.10

	λ=0.10					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
10	**Gamma**					
		0.50	0.0028	0.0030	0.0027	0.0031
		1.00	0.0034	0.0042	0.0028	0.0046
		1.50	0.0045	0.0065	0.0033	0.0080
		2.00	0.0063	0.0099	0.0041	0.0138
		2.50	0.0084	0.0142	0.0049	0.0222
		3.00	0.0106	0.0190	0.0057	0.0334
	Weibull					
10		0.50	0.0033	0.0031	0.0027	0.0036
		1.00	0.0041	0.0046	0.0033	0.0058
		1.50	0.0048	0.0066	0.0034	0.0088
		2.00	0.0063	0.0097	0.0040	0.0135
		2.50	0.0078	0.0144	0.0043	0.0196
		3.00	0.0098	0.0186	0.0045	0.0251
	Lognormal					
10		0.50	0.0028	0.0030	0.0027	0.0031
		1.00	0.0034	0.0044	0.0030	0.0043
		1.50	0.0042	0.0063	0.0031	0.0061
		2.00	0.0051	0.0092	0.0034	0.0081
		2.50	0.0066	0.0141	0.0036	0.0113
		3.00	0.0077	0.0175	0.0040	0.0140

From Table 4.21, it is clearly seen that the WV-EWMA produces the smallest type-I errors for all the distributions. When the skewness is less than or equal to 1.50, the classic EWMA also performs very well and can be used as an alternative for the WV-EWMA for all the distributions. Another important remark which can be made from this Table is the fact that the WSD-EWMA and the SC-EWMA produce very good results for all the distributions when the skewness was extremely small and almost symmetrical.

Table 4.22 Table of Results for n=10 and lambda=0.20

	λ=0.20					
n	**Distribution**	**k_3**	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
10		0.50	0.0031	0.0031	0.0029	0.0033
		1.00	0.0037	0.0038	0.0028	0.0044
		1.50	0.0049	0.0050	0.0030	0.0069
		2.00	0.0067	0.0069	0.0033	0.0115
		2.50	0.0092	0.0100	0.0041	0.0195
		3.00	0.0123	0.0129	0.0046	0.0298
	Weibull					
10		0.50	0.0031	0.0029	0.0025	0.0034
		1.00	0.0040	0.0036	0.0028	0.0048
		1.50	0.0054	0.0053	0.0031	0.0078
		2.00	0.0066	0.0069	0.0033	0.0116
		2.50	0.0086	0.0099	0.0034	0.0166
		3.00	0.0111	0.0127	0.0037	0.0216
	Lognormal					
10		0.50	0.0030	0.0030	0.0028	0.0031
		1.00	0.0036	0.0038	0.0029	0.0041
		1.50	0.0045	0.0050	0.0030	0.0053
		2.00	0.0055	0.0069	0.0030	0.0067
		2.50	0.0075	0.0099	0.0034	0.0093
		3.00	0.0092	0.0118	0.0036	0.0114

From Table 4.22, it is noticed that the WV-EWMA produces the overall best results for the type-I errors for all the distributions. It was closely followed by both the classic EWMA and the SC-EWMA which produce relatively similar results for all the distributions when the skewness is less than or equal to 1.50.

Table 4.23 Table of Results for n=10 and lambda=0.30

	λ=0.30

n	Dist	k_3	EWMA	EWMA_SC	EWMA_WV	EWMA_WSD
	Gamma					
10		0.50	0.0028	0.0026	0.0026	0.0028
		1.00	0.0037	0.0032	0.0026	0.0039
		1.50	0.0050	0.0041	0.0027	0.0059
		2.00	0.0070	0.0051	0.0029	0.0096
		2.50	0.0097	0.0069	0.0033	0.0159
		3.00	0.0127	0.0086	0.0035	0.0249
	Weibull					
10		0.50	0.0031	0.0027	0.0025	0.0032
		1.00	0.0039	0.0031	0.0026	0.0043
		1.50	0.0053	0.0041	0.0026	0.0066
		2.00	0.0070	0.0052	0.0029	0.0097
		2.50	0.0092	0.0069	0.0029	0.0132
		3.00	0.0118	0.0087	0.0031	0.0176
	Lognormal					
10		0.50	0.0029	0.0028	0.0027	0.0029
		1.00	0.0039	0.0034	0.0029	0.0037
		1.50	0.0046	0.0041	0.0027	0.0046
		2.00	0.0061	0.0053	0.0029	0.0057
		2.50	0.0087	0.0075	0.0035	0.0077
		3.00	0.0102	0.0088	0.0037	0.0092

From Table 4.23, it is observed that the WV-EWMA produces the best overall results for the type-I error for all the distributions. The SC-EWMA comes second and produces very good results for all the distributions when the skewness is less than or equal to 1.50. It can therefore be used as an alternative to the WV-EWMA for this interval. The classic EWMA and the WSD-EWMA also produce considerable results for all the distributions when the skewness is smaller than or equal to 1.50.

Table 4.24 Table of Results for n=10 and lambda=0.40

	λ=0.40					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
10	**Gamma**					
		0.50	0.0028	0.0026	0.0025	0.0027
		1.00	0.0037	0.0028	0.0024	0.0034
		1.50	0.0053	0.0034	0.0026	0.0051
		2.00	0.0074	0.0042	0.0027	0.0081
		2.50	0.0102	0.0053	0.0030	0.0135
		3.00	0.0137	0.0062	0.0031	0.0213
	Weibull					
10		0.50	0.0031	0.0026	0.0024	0.0030
		1.00	0.0040	0.0028	0.0025	0.0040
		1.50	0.0058	0.0034	0.0025	0.0056
		2.00	0.0077	0.0044	0.0029	0.0085
		2.50	0.0101	0.0054	0.0028	0.0112
		3.00	0.0124	0.0066	0.0030	0.0147
	Lognormal					
10		0.50	0.0027	0.0025	0.0025	0.0026
		1.00	0.0036	0.0030	0.0027	0.0033
		1.50	0.0050	0.0038	0.0028	0.0043
		2.00	0.0064	0.0045	0.0029	0.0050
		2.50	0.0091	0.0061	0.0035	0.0067
		3.00	0.0106	0.0070	0.0038	0.0079

From Table 4.24, it is seen that the WV-EWMA produces the best overall results for the type-I error for all the distributions. It is seconded by the SC-EWMA method which gives good results for all the distributions. It can conveniently be used as an alternative to the WV-EWMA under these experimental conditions. The classic EWMA and the WSD-EWMA also produce reasonable results for all the distributions when the skewness is less than or equal to 1.50.

Table 4.25 Table of Results for n=10 and lambda=0.70

	λ=0.70					
n	**Dist**	k_3	**EWMA**	**EWMA_SC**	**EWMA_WV**	**EWMA_WSD**
	Gamma					
10		0.50	0.0028	0.0025	0.0025	0.0026
		1.00	0.0041	0.0026	0.0026	0.0031
		1.50	0.0062	0.0027	0.0028	0.0040
		2.00	0.0091	0.0027	0.0029	0.0056
		2.50	0.0126	0.0028	0.0033	0.0081
		3.00	0.0166	0.0027	0.0037	0.0121
	Weibull					
10		0.50	0.0032	0.0024	0.0024	0.0029
		1.00	0.0044	0.0024	0.0025	0.0033
		1.50	0.0065	0.0026	0.0025	0.0041
		2.00	0.0090	0.0027	0.0030	0.0056
		2.50	0.0121	0.0029	0.0033	0.0068
		3.00	0.0154	0.0032	0.0037	0.0084
	Lognormal					
10		0.50	0.0029	0.0026	0.0026	0.0027
		1.00	0.0041	0.0028	0.0029	0.0031
		1.50	0.0059	0.0031	0.0031	0.0036
		2.00	0.0078	0.0036	0.0035	0.0041
		2.50	0.0111	0.0042	0.0043	0.0050
		3.00	0.0129	0.0047	0.0048	0.0058

From Table 4.25, it is clearly seen that the SC-EWMA produces the overall best results for the type-I errors for all the distributions. It is closely followed by the WV-EWMA for all the distributions. The latter can be used as an alternative to the former. The WSD-EWMA also produces reasonable results when the skewness is smaller or equal to 1.50. However, for the lognormal distribution it produces good results for all the skewness.

4.4 Graphical Representation of Results

The results obtained, tabulated and analysed above can also be represented graphically as follows.

4.4.1 Gamma Distribution

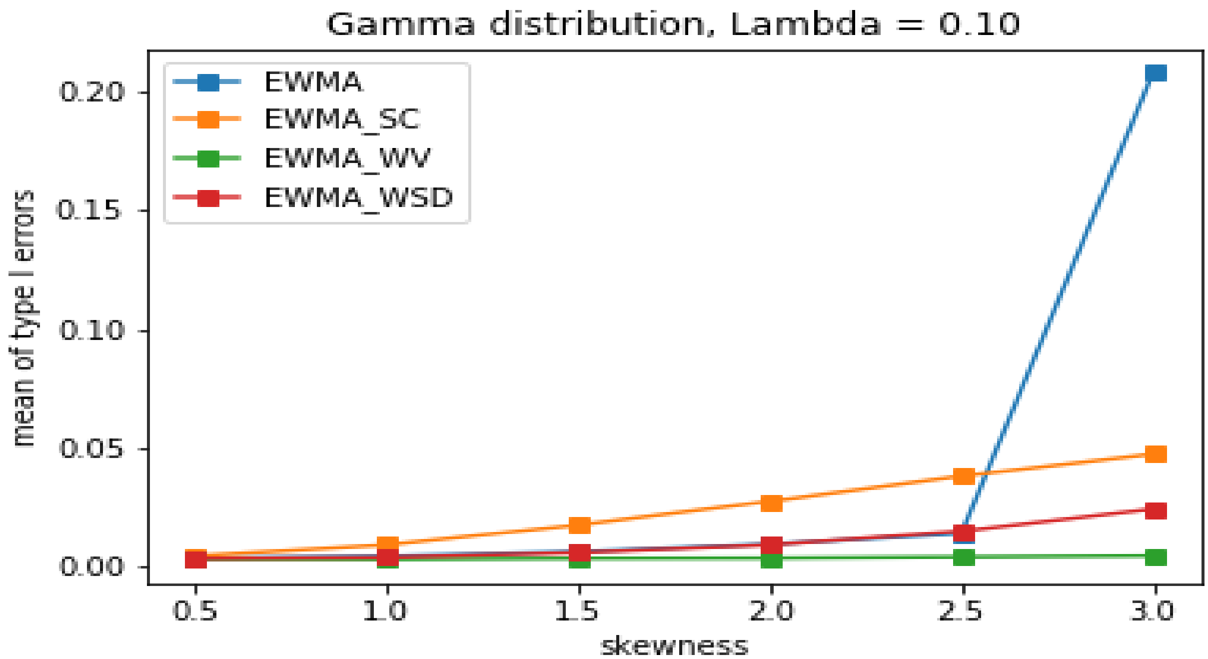

Figure 4.1 Graph of the Gamma distribution when n=3 and λ=0.10

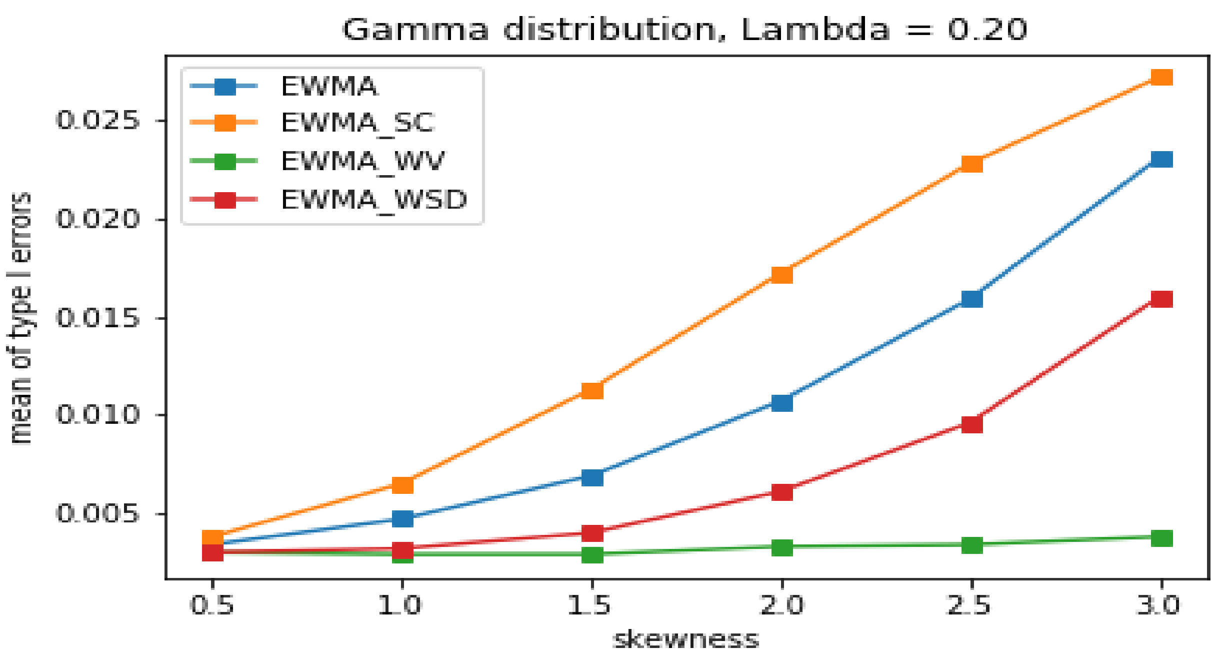

Figure 4.2 Graph of the Gamma distribution when n=3 and λ=0.20

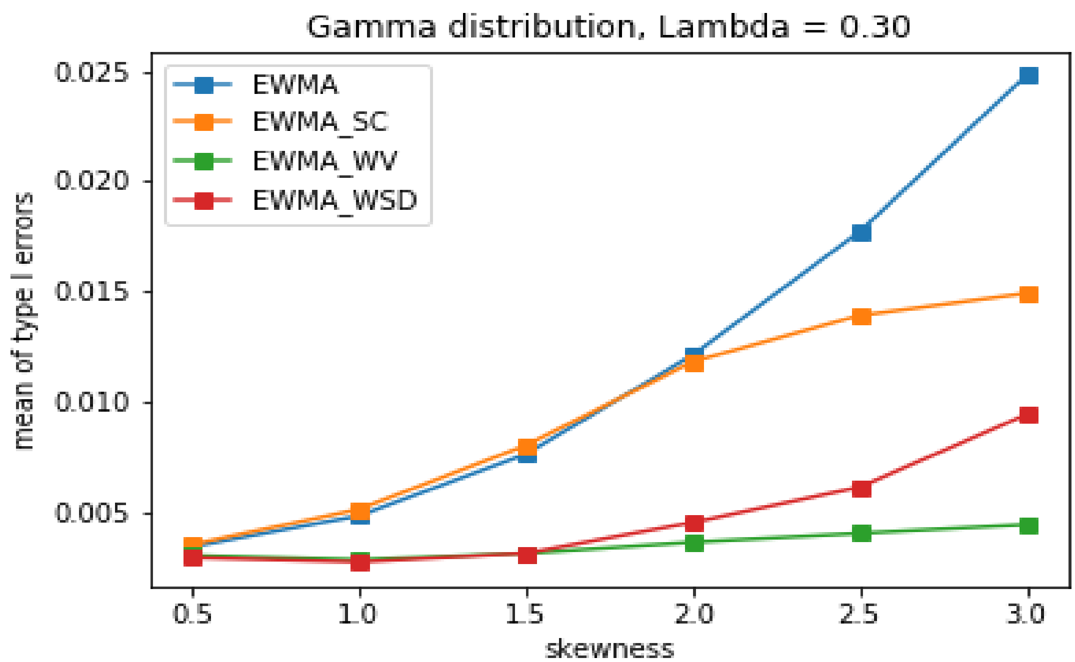

Figure 4.3 Graph of the Gamma distribution when n=3 and λ=0.30

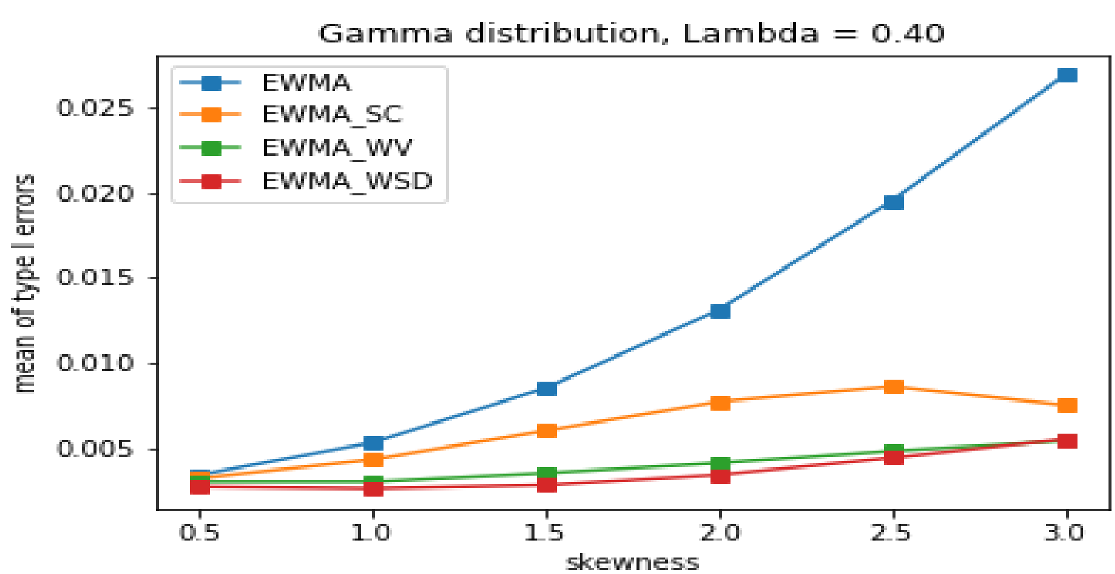

Figure 4.4 Graph of the Gamma distribution when n=3 and λ=0.40

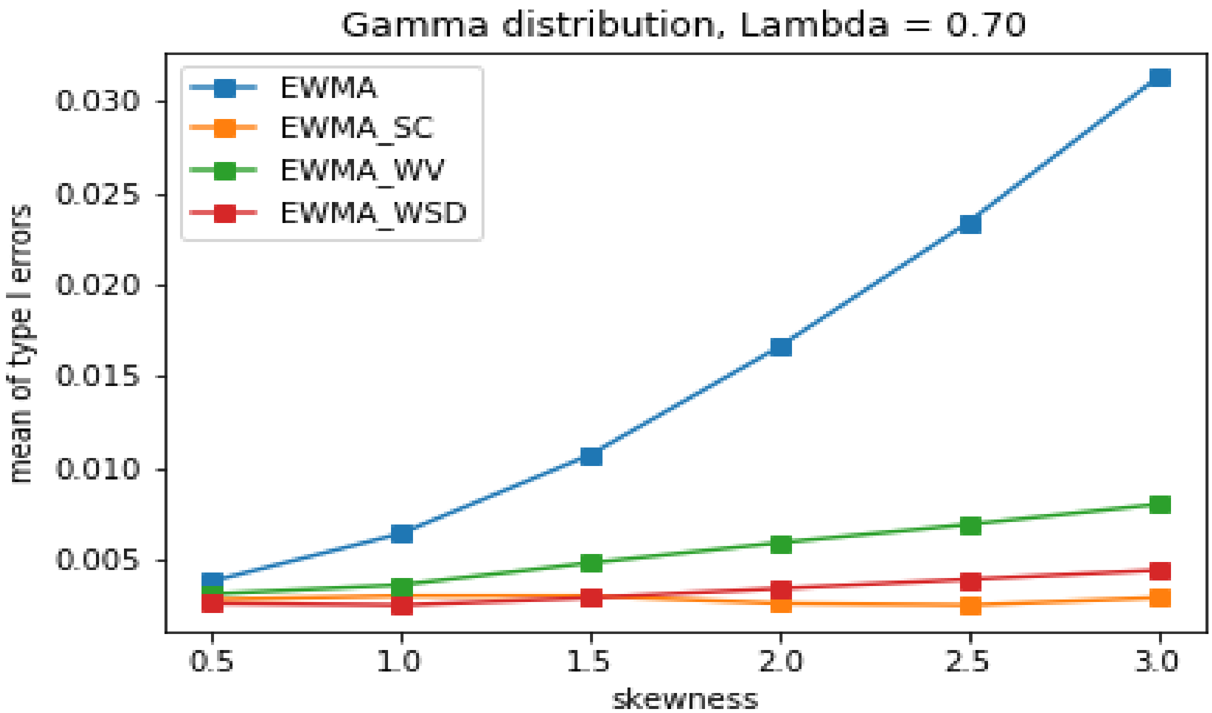

Figure 4.5 Graph of the Gamma distribution when n=3 and λ=0.70

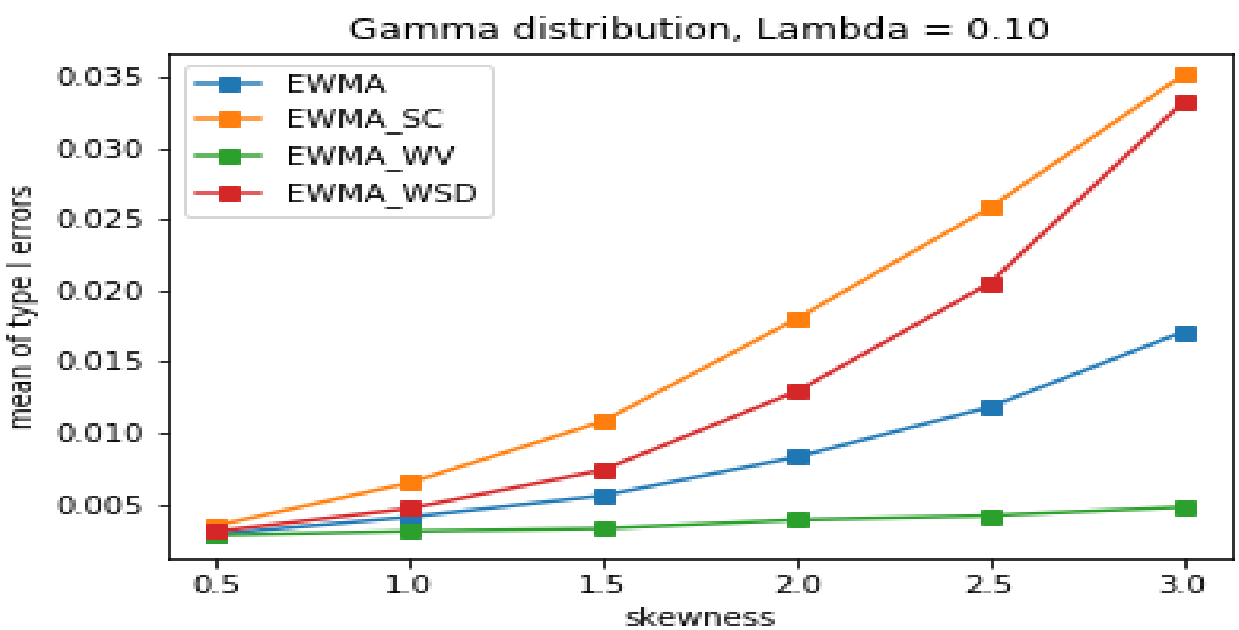

Figure 4.6 Graph of the Gamma distribution when n=5 and λ=0.10

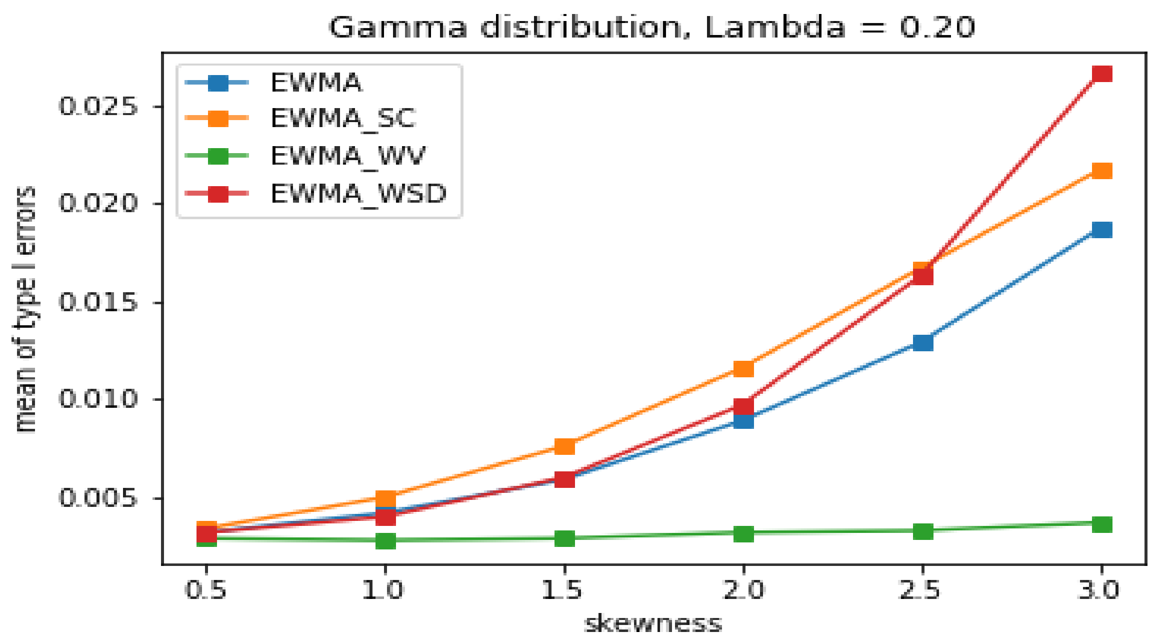

Figure 4.7 Graph of the Gamma distribution when n=5 and λ=0.20

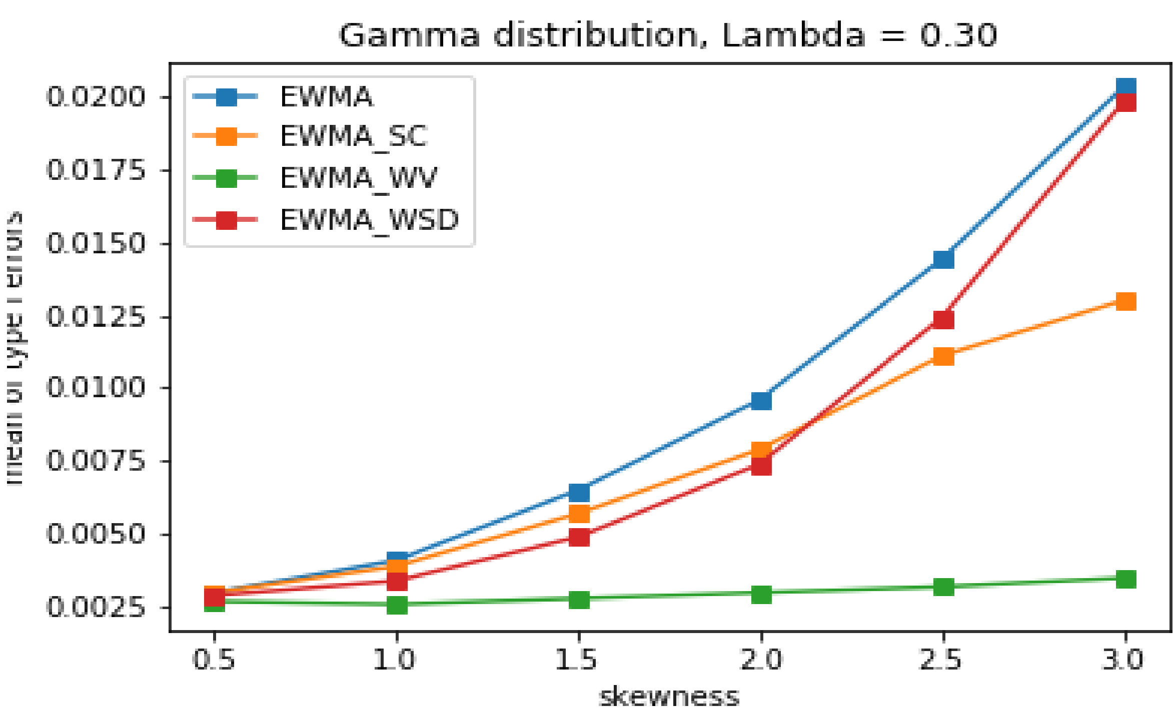

Figure 4.8 Graph of the Gamma distribution when n=5 and λ=0.30

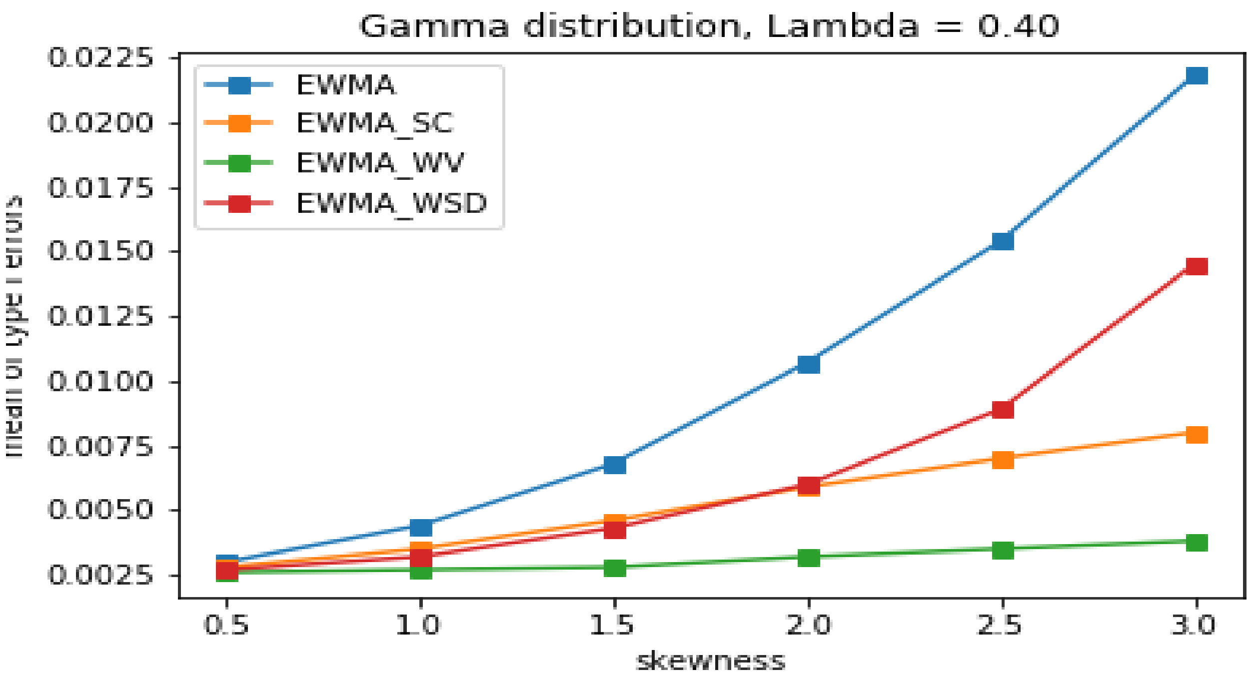

Figure 4.9 Graph of the Gamma distribution when n=5 and λ=0.40

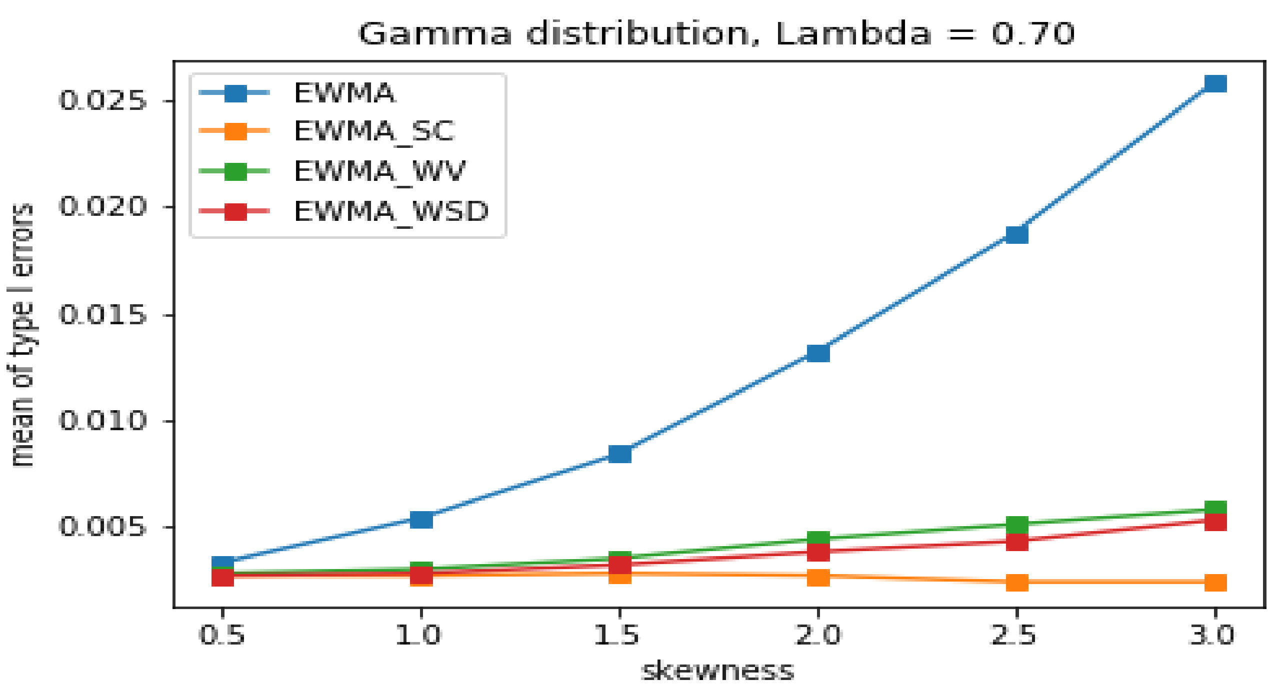

Figure 4.10 Graph of the Gamma distribution when n=5 and λ=0.70

4.4.2 Weibull Distribution

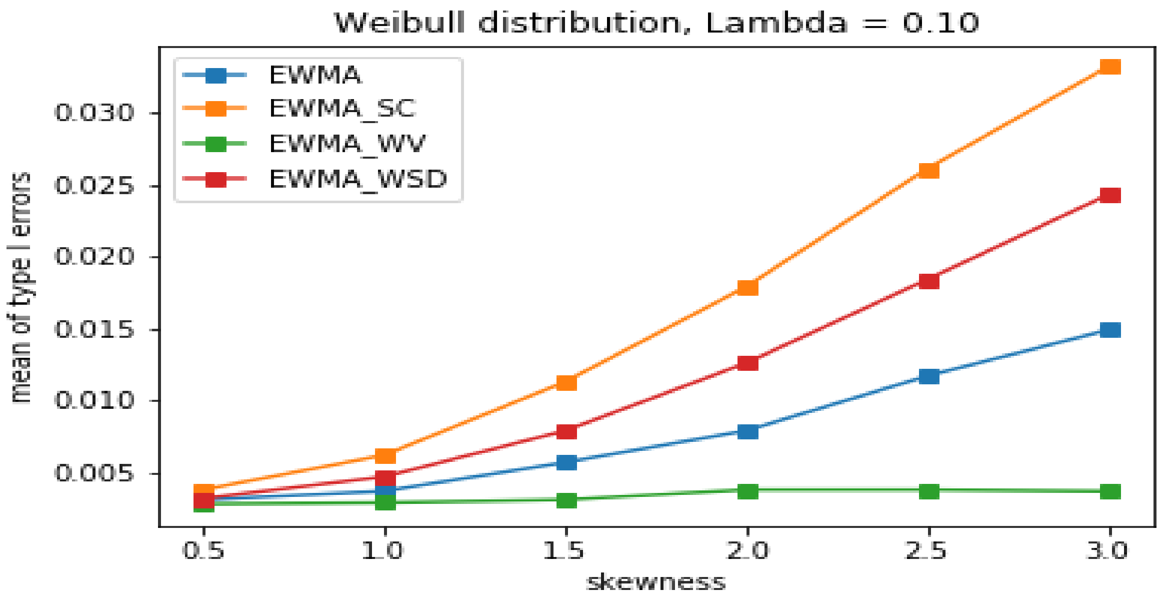

Figure 4.11 Graph of the Weibull distribution when n=5 and λ=0.10

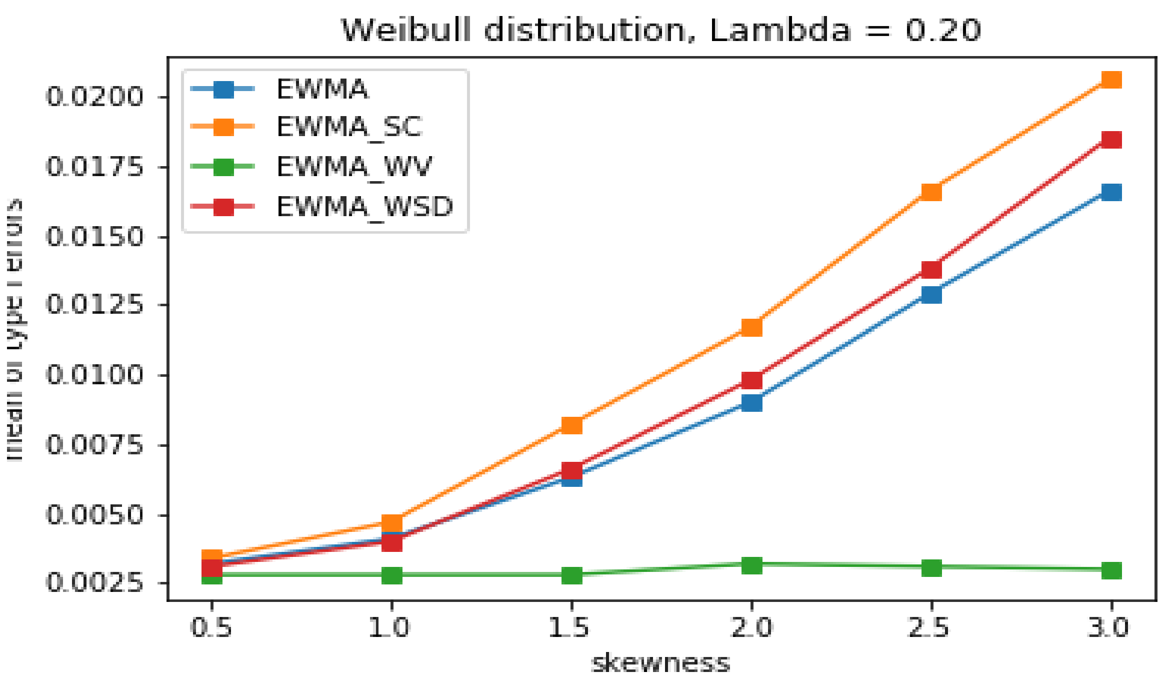

Figure 4.12 Graph of the Weibull distribution when n=5 and λ=0.20

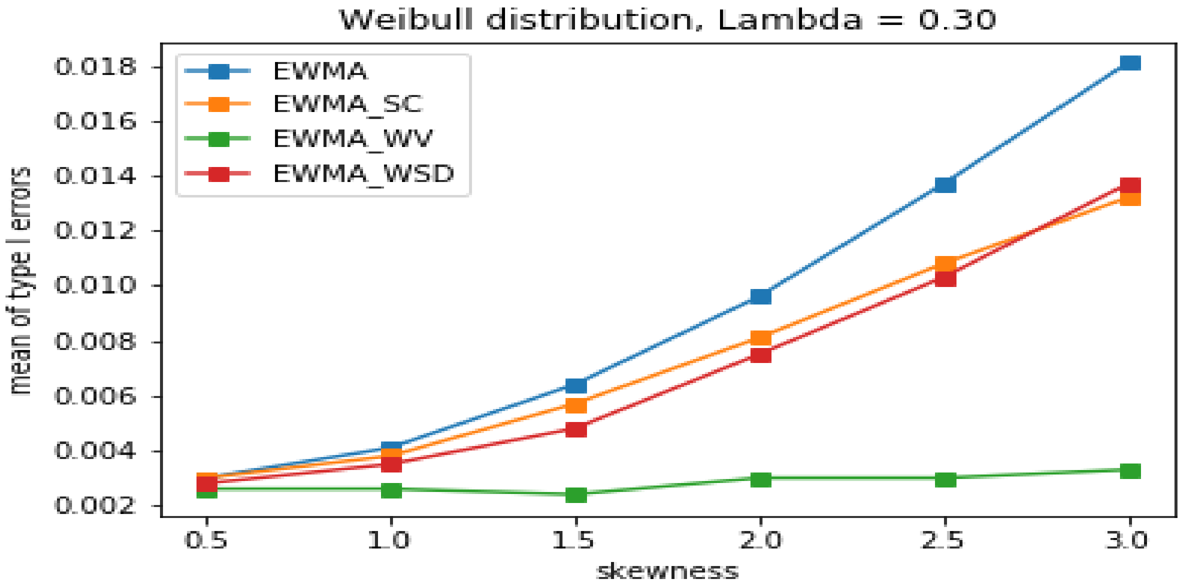

Figure 4.13 Graph of the Weibull distribution when n=5 and λ=0.30

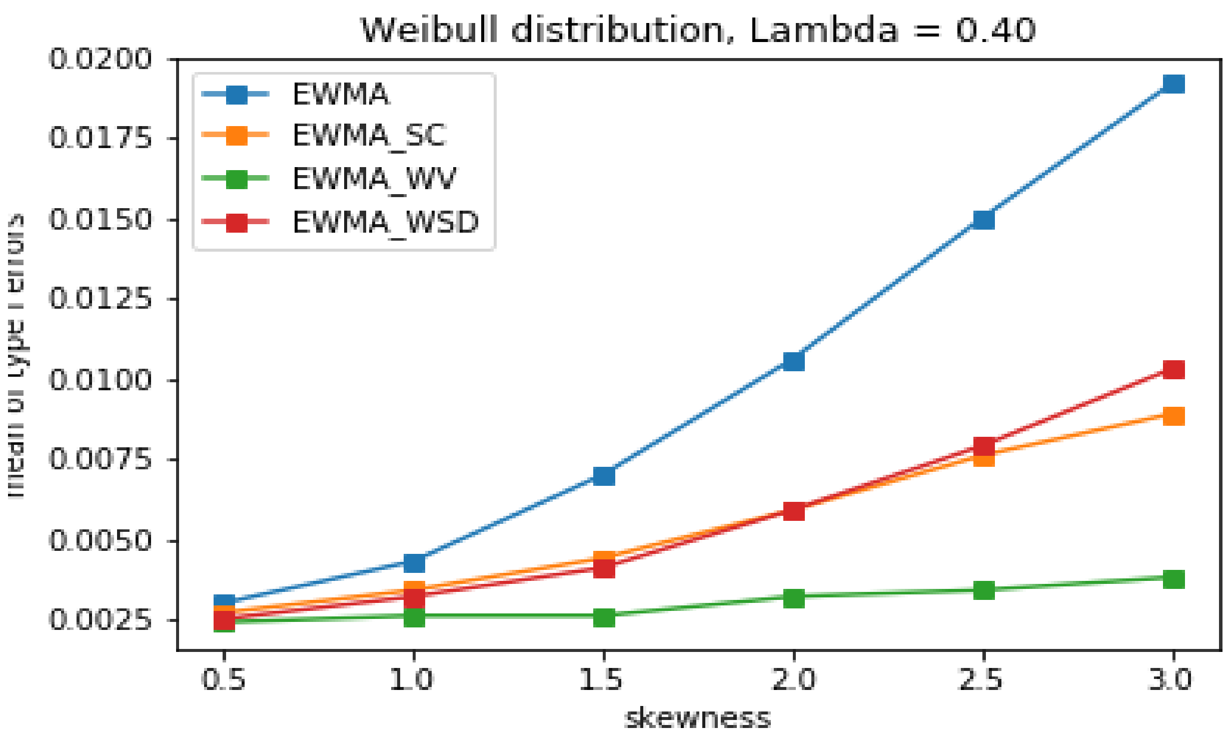

Figure 4.14 Graph of the Weibull distribution when n=5 and λ=0.40

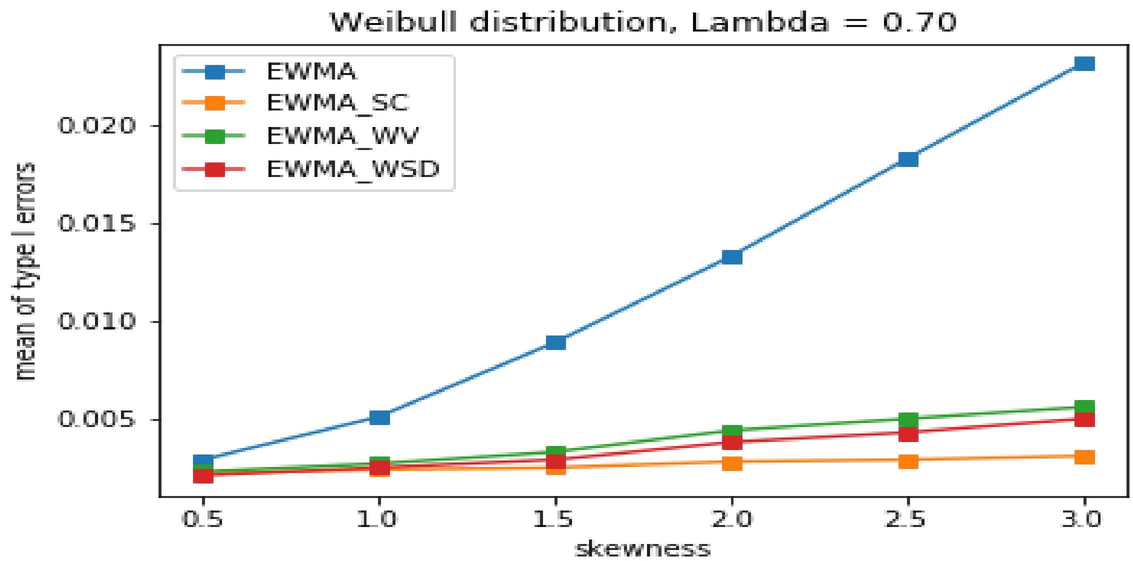

Figure 4.15 Graph of the Weibull distribution when n=5 and λ=0.70

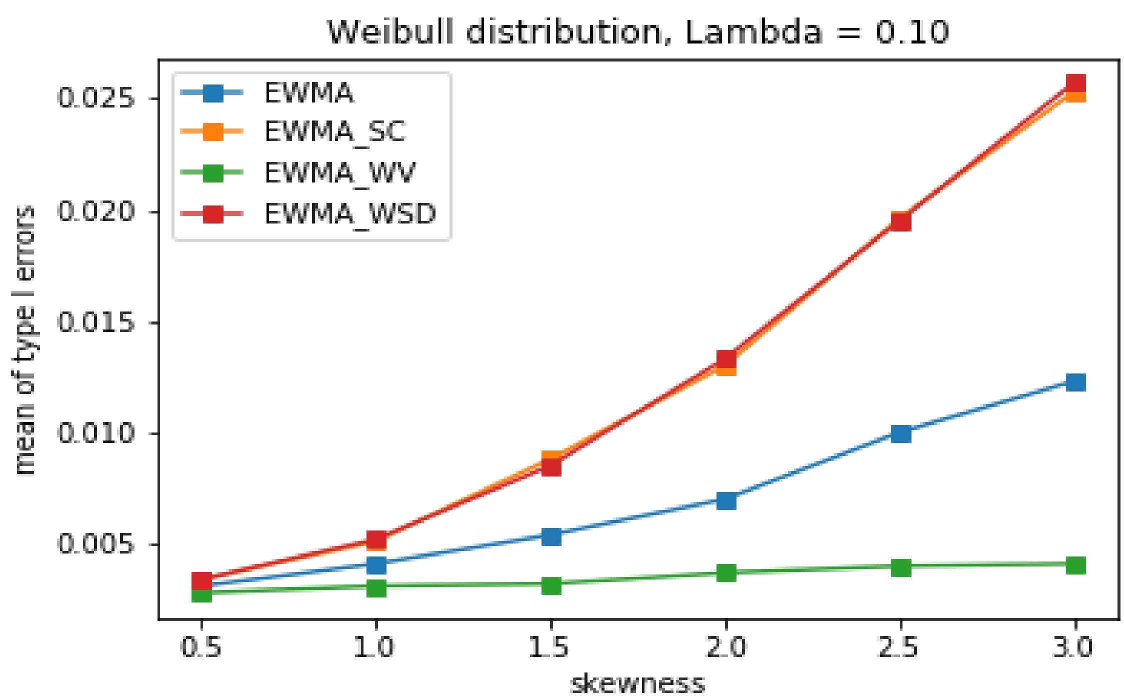

Figure 4.16 Graph of the Weibull distribution when n=7 and λ=0.10

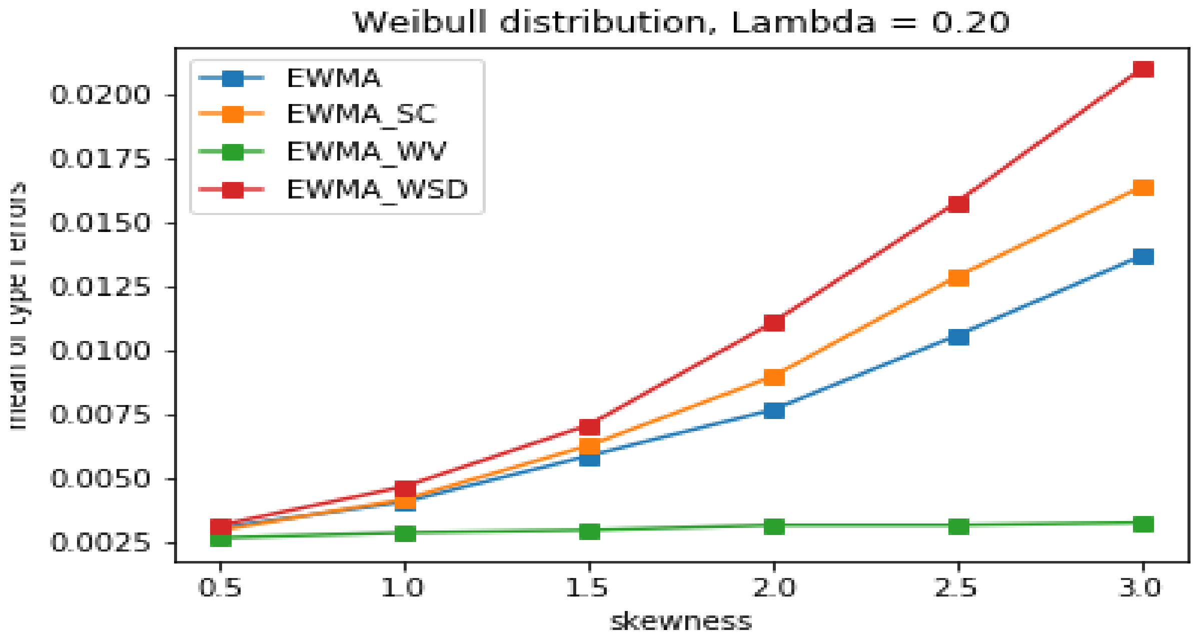

Figure 4.17 Graph of the Weibull distribution when n=7 and λ=0.20

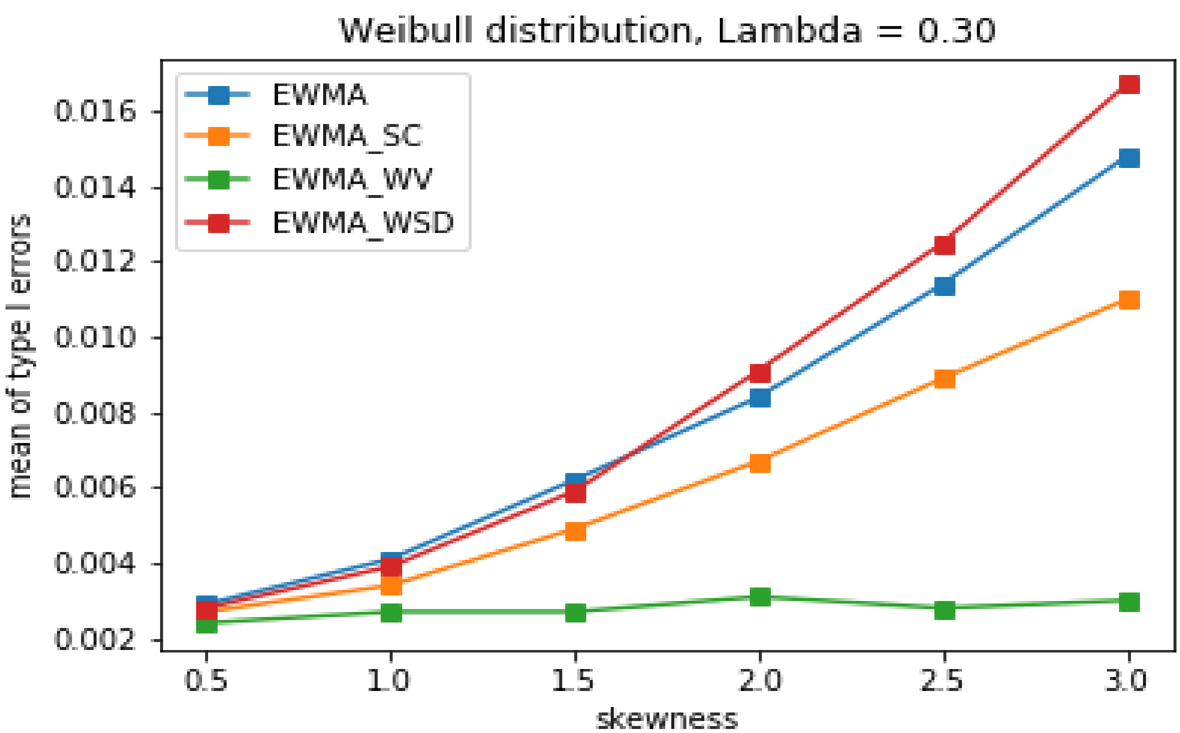

Figure 4.18 Graph of the Weibull distribution when n=7 and λ=0.30

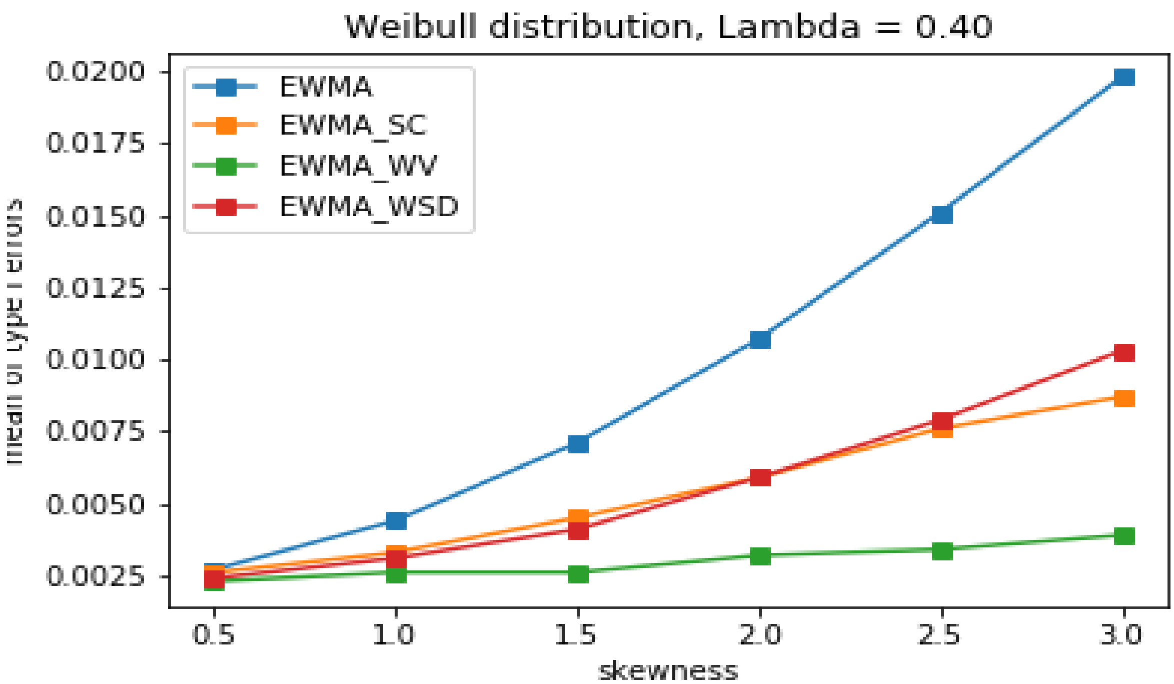

Figure 4.19 Graph of the Weibull distribution when n=7 and λ=0.40

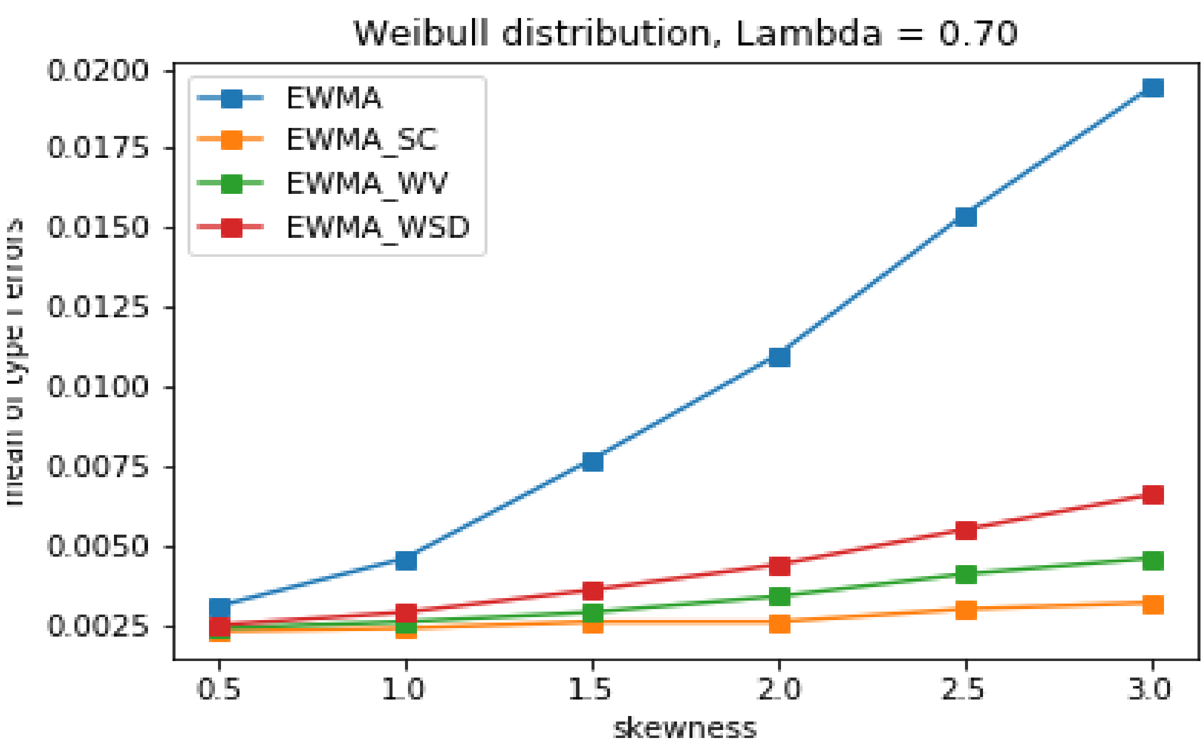

Figure 4.20 Graph of the Weibull distribution when n=7 and λ=0.70

4.4.3 Lognormal Distribution

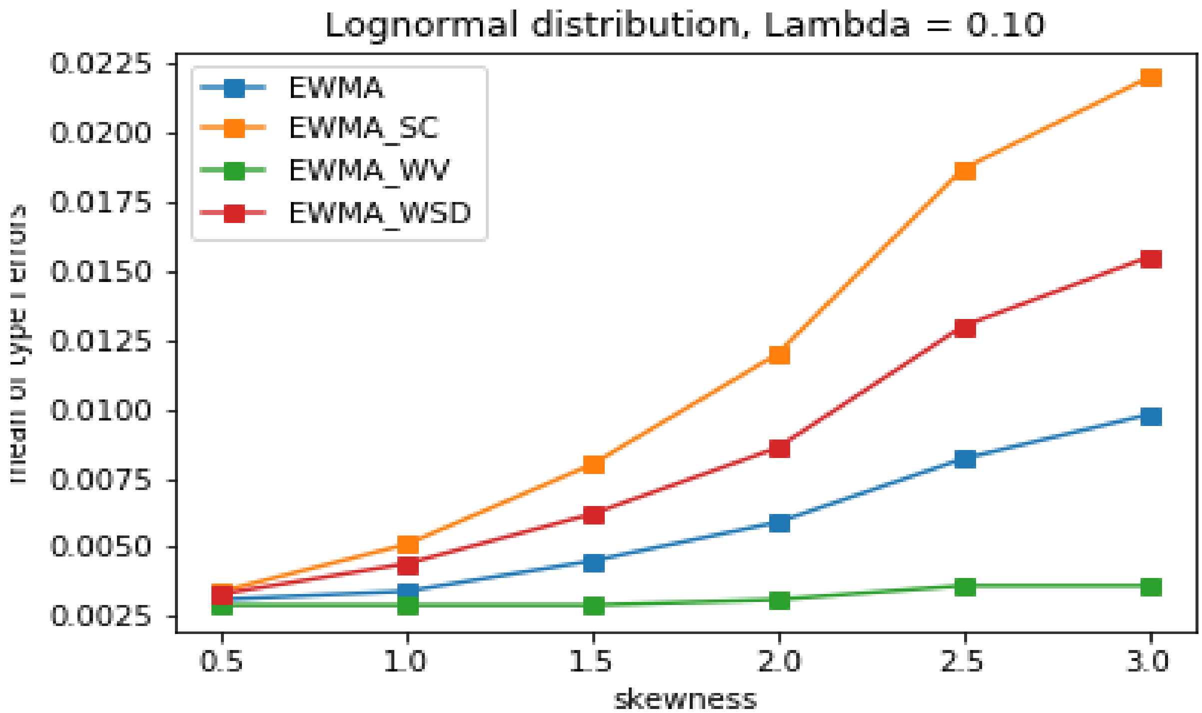

Figure 4.21 Graph of the Lognormal distribution when n=7 and λ=0.10

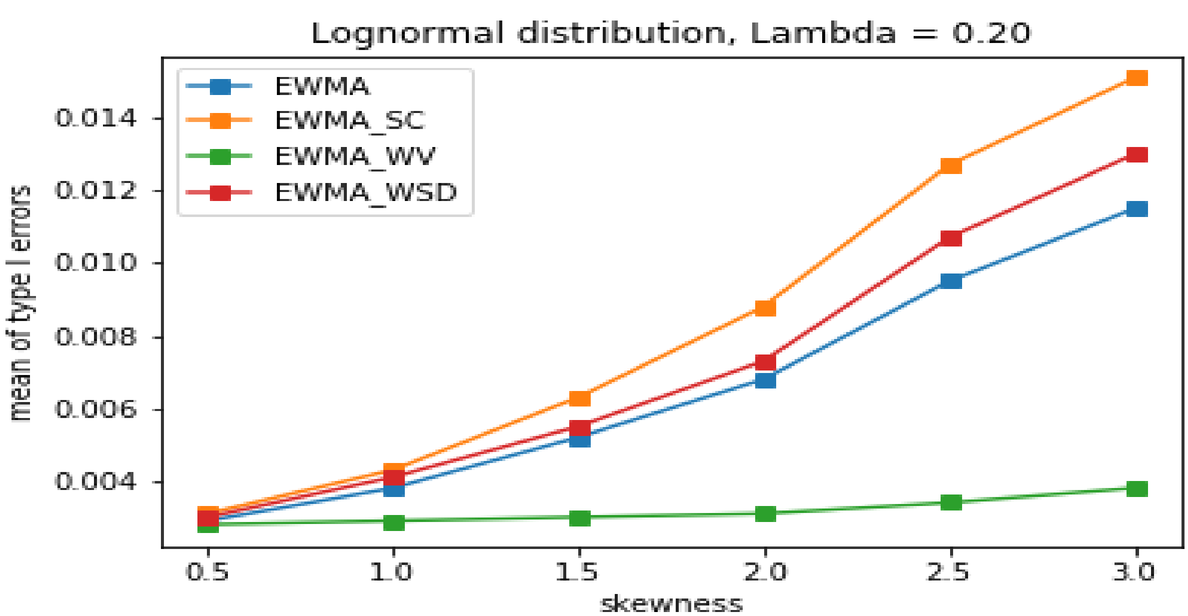

Figure 4.22 Graph of the Lognormal distribution when n=7 and λ=0.20

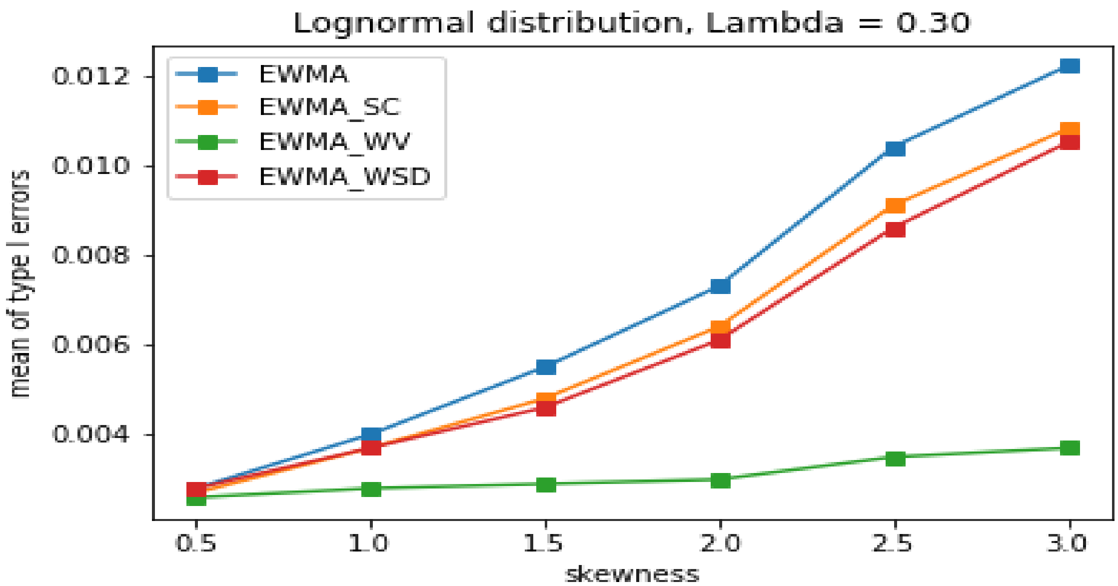

Figure 4.23 Graph of the Lognormal distribution when n=7 and λ=0.30

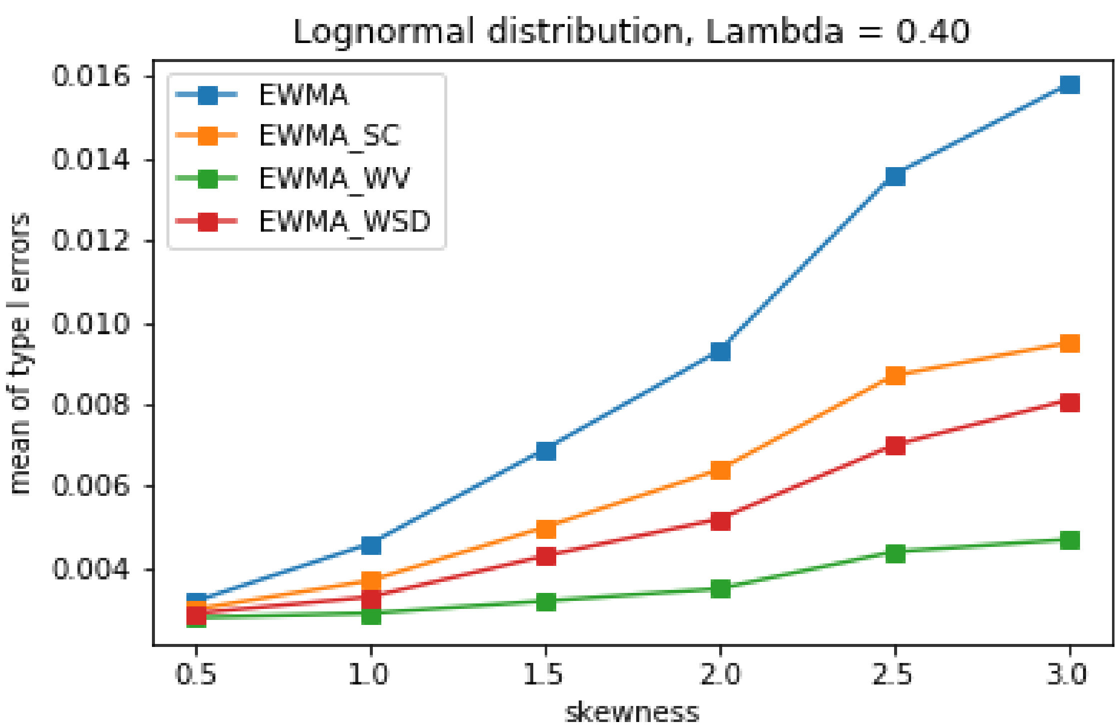

Figure 4.24 Graph of the Lognormal distribution when n=7 and λ=0.40

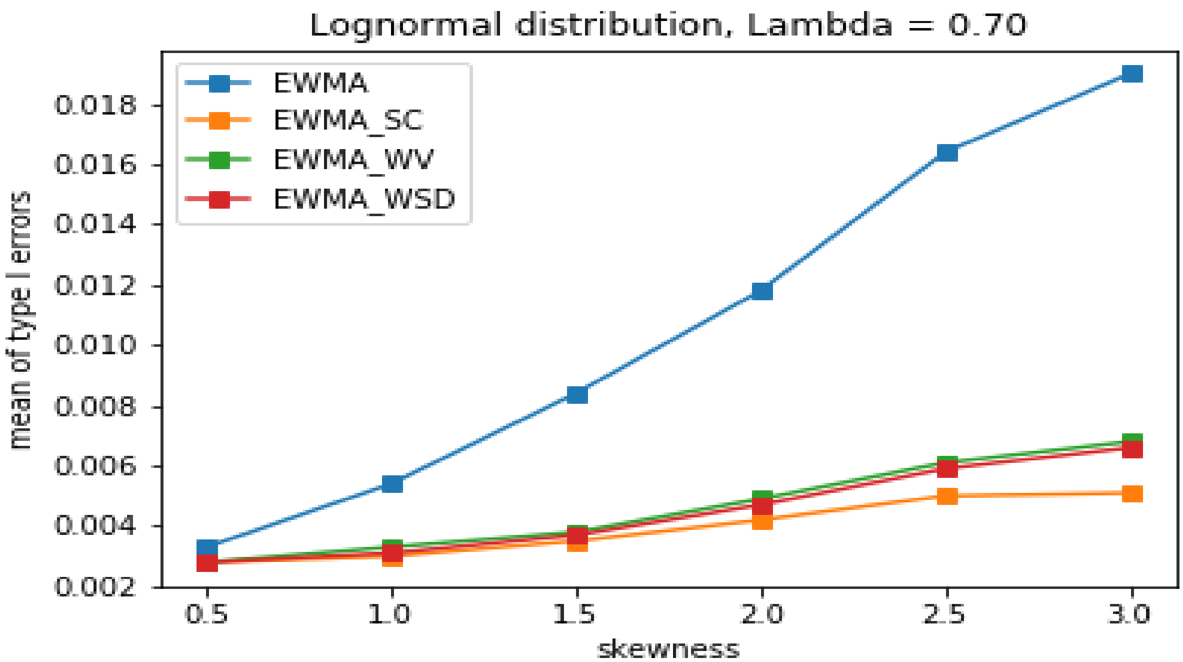

Figure 4.25 Graph of the Lognormal distribution when n=7 and λ=0.70

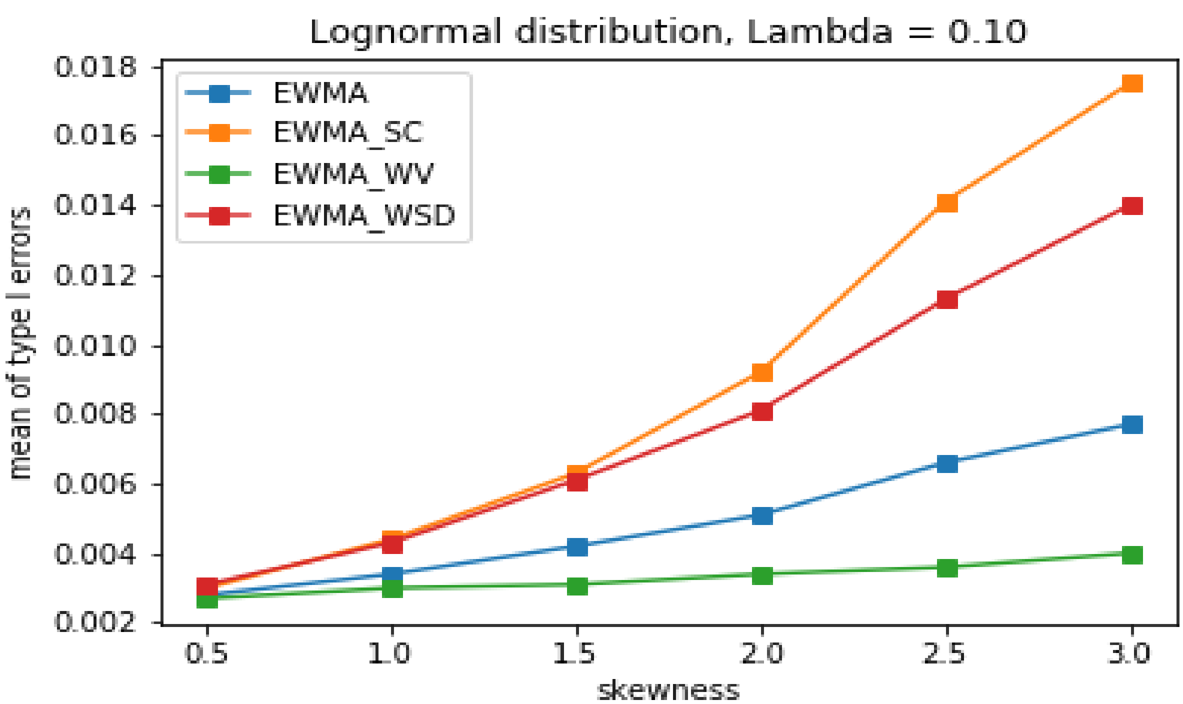

Figure 4.26 Graph of the Lognormal distribution when n=10 and λ=0.10

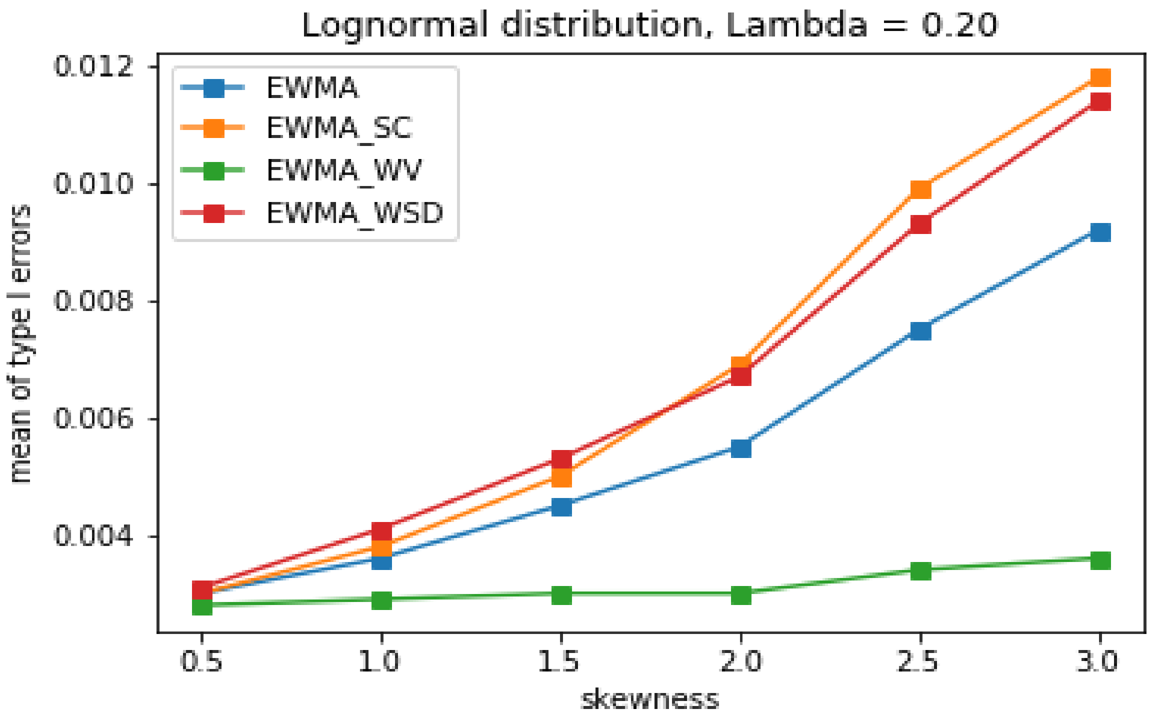

Figure 4.27 Graph of the Lognormal distribution when n=10 and λ=0.20

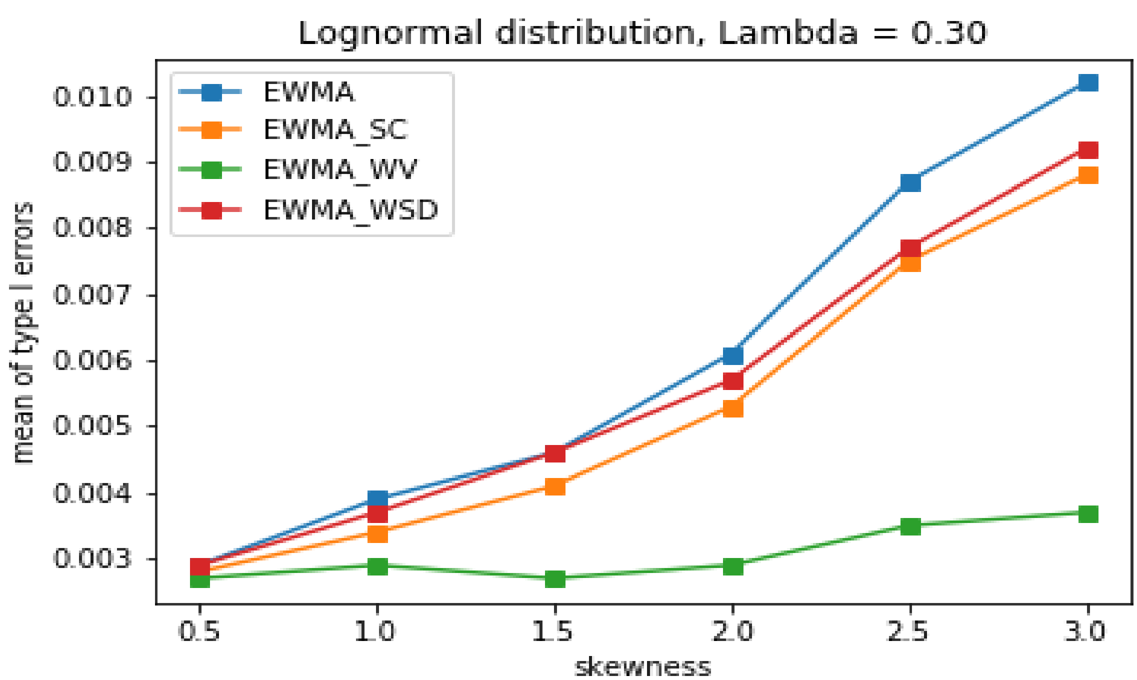

Figure 4.28 Graph of the Lognormal distribution when n=10 and λ=0.30

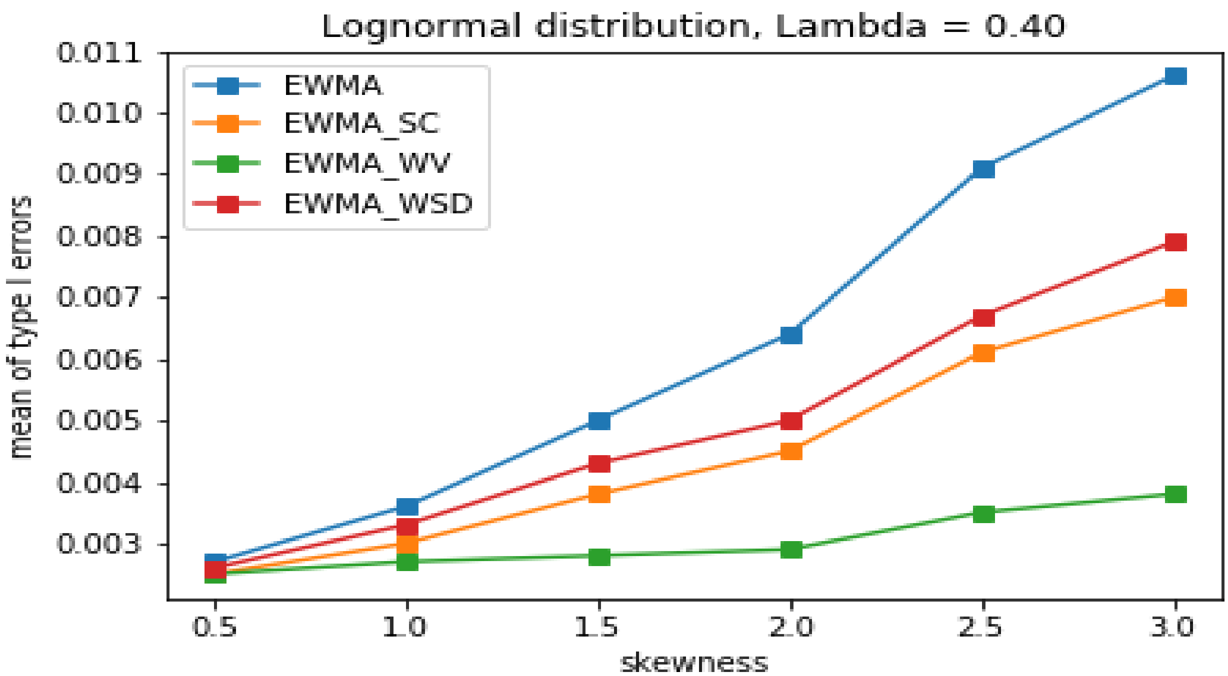

Figure 4.29 Graph of the Lognormal distribution when n=10 and λ=0.40

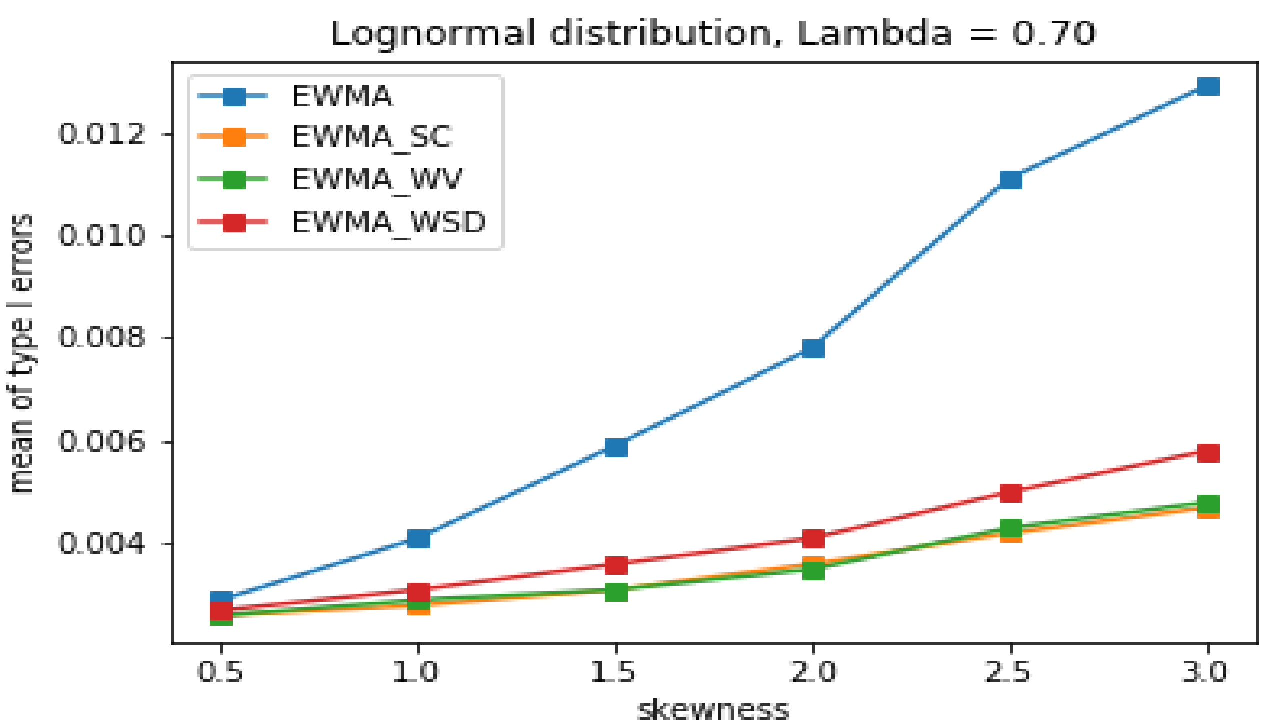

Figure 4.30 Graph of the Lognormal distribution when n=10 and λ=0.70

5 CONCLUSION

This work was mainly concentrated on the exponentially weighted moving average (EWMA) control chart which is also termed the geometric moving average charts. This control chart type was of interest because of its ability to detect smaller shifts in the process mean faster than the classic Shewhart chart. Works from Bai and Choi (1995); Choobineh and Ballard (1987), and Nelson (1979) showed that most of the time under industrial conditions, the variable under investigation has a skewed distribution. Because of this reason, existing methods of control charts for skewed distributions was studied and a new approach using the skewness correction technique was proposed. The performances of these methods for the EWMA were compared among each other for specifically chosen skewed distributions. The distributions chosen for this work were the gamma, Weibull and lognormal because of their wide range of skewness. The comparison was done with the help of a Monte Carlo simulation technique which was preferred to other simulation techniques like the Markov's approximation and the integral methods because of its simplicity and numerical nature. The comparison was done based on the average type-I errors of the methods for a wide range of skewness of the various distributions involved.

This work was structured into 5 chapters. Chapter 1 focused mainly on the general introduction of statistical process controls, experimental design as a sampling technique in statistics and numerous continuous distributions such as the normal distribution, the exponential distribution, the Weibull distribution, the lognormal distribution and the gamma distribution. Chapter 2 focused mainly on control charts as a technique used in industrial statistics and quality control. Here, the classic Shewhart's charts which is considered as the mother of all control charts for variables was discussed. The EWMA was also discussed in all aspects with its design which is very complex to understand broken down and simplified. Chapter 3 also spoke about control chart for variables, but this time went into those used when the distribution of the variable under investigation is skewed. The EWMA was also adopted for use when the distribution of the variable under investigation is skewed. Here, a new method using the skewness correction technique was developed and adopted for the circumstance. Chapter 4 was where the Monte Carlo simulation was introduced. Here, the algorithm used to attain our objectives was discussed in detail. The results obtained were tabulated and discussed subsequently.

5.1 Observations, Remarks and Proposals

Sequel to the analyses of the results made in the previous chapter, the following conclusions and proposals can be made.

- For n=3, when the smoothing parameter was in the interval $0.10 \leq \lambda \leq 0.20$, the Weibull distribution produced the smallest results which were very close to the expected in-control value of 0.0027. The WV-EWMA emerged as the most suitable method to be used. As an alternative the WSD-EWMA could also be used if the experimenter feels like doing so because it also produced good readings.
- For n=3, as the smoothing parameter is increased to the interval $0.30 \leq \lambda \leq 0.40$, the Weibull distribution produced the smallest results for the type-I error. The WSD-EWMA gave better readings in most cases as compared to the WV-EWMA especially when the skewness was extremely small. These two methods could be used as alternatives to each other depending on the obligations put in place by the governing body of the place where the experiment is being carried out.
- For n=3, as the smoothing parameter was further increased to a higher value of $\lambda = 0.70$, the Weibull distribution produced the smallest readings when the skewness was less than or equal to 1.50. As the skewness is increased beyond 1.50, the gamma distribution overtook the former and produced the smallest results. The WSD-EWMA produced the smallest readings when the skewness was smaller than or equal to 1.50. The SC-EWMA which was the proposed method emerged as the one with the smallest type-I errors for all the distributions when the skewness was greater than 1.50. These two methods could be used as alternatives to each other under this circumstance.
- For n=5 and 7, when the smoothing parameter was $\lambda = 0.10$, the lognormal distribution produced the smallest results. When the smoothing parameter was increased to $\lambda = 0.20$, the Weibull distribution produced the smallest results. The WV-EWMA emerged as the most suitable method to be used. As an alternative, the classic EWMA could also be used if the experimenter feels like doing so because it also produced good readings when the skewness was smaller than or equal to 1.50.

- For n=5 and 7, as the smoothing parameter was increased to the interval $0.30 \leq \lambda \leq 0.40$, the Weibull distribution produced the smallest results for the type-I error. The WV-EWMA gave best results for the type-I error. The WSD-EWMA produced good readings when the skewness was less than or equal to 1.50 for the first situation while the SC-EWMA produced better results for the latter. This method could be used as an alternative to the former when the skewness is less than or equal to 1.50.
- For n=5 and 7, as the smoothing parameter is increased to $\lambda = 0.70$, the gamma distribution produced the smallest results for the type-I error. The SC-EWMA which was the proposed method gave the smallest readings for the type-I error. It was closely followed by the WSD-EWMA which could be used as an alternative to the SC-EWMA.
- For n=10, when the smoothing parameter was in the interval $0.10 \leq \lambda \leq 0.20$, the lognormal distribution produced the smallest overall results for the type-I error. The WV-EWMA also produced the best readings for the type-I error. It was followed by the classic EWMA method when the skewness was less than or equal to 1.50. Under this skewness, the latter could be used as an alternative to the former.
- For n=10, when the smoothing parameter was in the interval $0.30 \leq \lambda \leq 0.40$, the Weibull distribution produced the overall smallest readings for the type-I error. The WV-EWMA also produced the best results for the type-I error. It was closely followed by the SC-EWMA method which produced good results when the skewness was less than or equal to 1.50. The latter can therefore be used as an alternative to the former under such skewness.
- For n=10, when the smoothing parameter was further increased to $\lambda = 0.70$, the gamma distribution produced the overall smallest readings for the type-I error. The SC-EWMA which was the proposed method performed best and produced the best results for the type-I error. It was followed by the WV-EWMA which produced considerably good results. Under these experimental conditions, the WV-EWMA could be used as an alternative to the SC-EWMA.

In a nutshell, it can be deducted from above that the EWMA methods for skewed distributions are highly affected by the value of the smoothing parameter λ and the sample size n used for the subgroups. It can also be suggested that if extremely small or medium shifts in the process mean are of interest, the WV-EWMA should be preferred to the others. If larger shifts are of interest, then the SC-EWMA should be preferred for optimal solutions to be obtained.

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