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Preservice teachers' performances at mathematical modeling process and views on mathematical modeling

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Abstract

In changing world conditions what we expect fundamentally from our students are to cope with the problems they confronted in their lives thinking creatively and analytically. Mathematical modeling is a kind of problem solving in which real life situations are expressed mathematically. The purpose of this study is to evaluate the preservice mathematics teachers' performances in the mathematical modeling process and to determine their views of modeling process. With this aim 4 modeling activities were given to 60 preservice teachers in a teacher training program. Afterwards semi structured interviews were conducted with four of these students. Results were interpreted in the context of teachers' competencies in modeling process.

Keywords: Mathematics education; preservice mathematics teachers; mathematical modeling; modeling performance.

1. Introduction

The things learned in mathematics courses can leave negative impressions on students, unless they are used in daily life or are made part of students' life. This leads to a lack of sufficient development of systematic and advanced thinking abilities in individuals. Whereas, in the changing world conditions, the fundamental characteristic demanded of students is to be active users of knowledge and to be able to solve any kind of problem they are confronted in any stage of life by using such thinking skills.

Modeling is one of the most widely-researched topics by mathematics educators (Lingefjard and Holmquist, 2005; Verschaffel et al, 1997; Lesh and Doerr, 2003; Erturan, 2007; Kertil, 2008). Models are conceptual systems which are expressed by using external representation and which are used in constructing, defining, and explaining other systems (Lesh and Doerr, 2003). Mathematical model is, on the other hand, is a benefiting process from the mathematical representations in order to find a solution to the problems we come across with in real life (Cheng, 2001). According to Cheng (2001), mathematical model is, at the same time, turning real life problems or situations into mathematical forms by simplifying or concretizing them, in other words, it is the transformation of real life problems into mathematical ones.

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It would be very useful to include such activities, which are highly important in order to develop students' cognitive skills, in teacher training programs because, in addition to the cognitive benefits that would come out in such activities, teachers or preservice teachers would be able to have firsthand knowledge as to which thinking phases the students go through and how this process should be evaluated (Lesh and Doerr, 2003). Besides, teachers need environments in which they can express already available thought systems (Lesh and Doerr, 2003). As Cooney (1999) states in his study, it is not really surprising that the teachers have limited understanding of the topics of school mathematics. One of the most important reasons of this is that after the end of school life, the students do not really have a lot of mathematical experience or learning related to these subjects.

It is said that teacher training programs target mostly filling in the deficiencies in field knowledge of preservice teachers and that it is weak in terms of teaching real-life equivalents and applications of mathematical concepts (Cooney, 1999). A teaching done from the view point of modeling is expected to direct the preservice teachers to express their field knowledge in several ways, to transfer this knowledge to different situations, and to use such knowledge for a more effective class teaching in addition to overcome their deficiencies in mathematical knowledge (Doerr and Lesh, 2003).

The aim of this study is to determine the modeling performances of preservice teachers in teacher training programs and to get their views on modeling process.

2. Method

2.1. Participants

The participants of this study included 60 preservice mathematics teachers (aged 20-23) attending a teacher training program in Ankara during the Fall Semester of 2009-2010. Half of these students come from elementary mathematics education program; the other half comes from secondary mathematics education program. All the students in the study are at the last year of their training program.

2.2. Data Collection

The data sources of the study are four modeling activities and semi-structured interviews. First of all, 60 students were responded 4 modeling activities during 60 minutes time. Written responses of the students for these activities were analyzed by the researchers considering five staged modeling process. These five stages are;

1) Observing a phenomenon, delineating the problem situation inherent in the phenomenon.
2) Discerning the important factors that affect the problem
3) Conjecturing the relationships among the factors and interpreting them mathematically to obtain a model for the phenomenon.
4) Applying appopriate mathematical analysis to the model.
5) Obtaining results and interpreting them in the context of the phenomenon under study and drawing conclusions. (Swetz & Hartzler, 1991)

According to the results gathered from this application, depth interviews have been conducted with some of the high and low achiever students. Interview schedule was designed by researchers to identify the preservice teachers' views on modeling. In these interviews which took approximately 40 minutes have been asked some questions like 'According to you, should be the modeling courses involved in teacher training programs?', 'In your opinion, what factors does the modeling performance depend on?', 'In your opinion, how can be the modeling skills improved?'.

2.3. Modeling Activities

In this study four modeling activities, namely; Pack It Them, Delivering the Mail, The Greening of Forest Acres, Getting the Word Out were used. These activites were presented below.

2.3.1. Pack It Them

A manufacturing company needs to find short term storage for some cyclindrical containers. The company wants to do this at minimum expense. Therefore, the job is to pack them in using as little storage space as possible. The containers are right circular cylinders with a radius of 1m and a height of 3m. All 175 containers must be stored in an upright position. Storage is required for two months. Storage units are available for rent in three sizes: 11m×11m for 134TL; 11m×22m for 210TL; 11m×33m for 260TL. All units are 10m high. Pack them in! Which storage can be rent with minimum cost, show on a model. (Swetz & Hartzler, 1991)

2.3.2. Delivering the Mail

A letter carrier needs to deliver the mail to both sides of a street. She can deliver to all boxes on one side, cross the street, and deliver to all the boxes on the other side. Or she can deliver to one box, cross the street, deliver to two boxes, cross again and deliver to two boxes, and so forth, until all the mail has been delivered. Which method is better; show this on a mathematical model? (Swetz & Hartzler, 1991).

2.3.3. The greening of forest acres

A lawn-service contractor wishes to establish his business in the new Forest Acres housing development. All lot sizes are the same, and each property would require approximately the same maintenance services. The contractor estimates the cost of his services for each homeowner would be 275TL a year. Forest Acres has 400 homes. He knows from past experience that in such a situation he can count on about 100 customers, he would like more. In order to attract more clients, he advertises a special. For each "bonus customer" over 100, he will give a discount of 1,5TL to all customers in the development. Under this policy, how many customers will provide the greatest revenue for the contractor? Show this on a model (Swetz & Hartzler, 1991).

2.3.4. Getting the word out

A concert promoter is negotiating radio and television time for advertising an upcoming concert. She has up to 20000TL to spend on the promotion. Each 22 second radio commercial costs100TL, whereas thirty seconds of television time associated with major programming costs 800TL. She wants at least 30 radio spots distributed among various stations, but no more than 60 radio announcements in total. She would also like to have at least 15 television commercials. How much radio time and TV time can she schedule to maximize advertising exposure and yet remain within the allotted budget? Show this on a model (Swetz & Hartzler, 1991).

2.4. Data Analysis

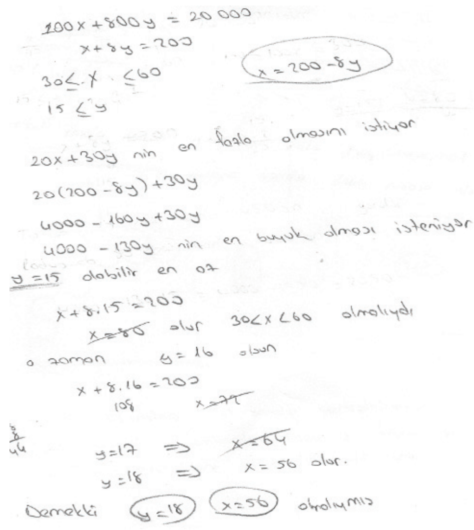
This part of the study involves two phases. In the first phase, modeling performances of the students were analyzed considering modeling stages by the researchers separately and then compared the results. Afterwards, according to modeling performances, five high and four low achiever students were chosen for the interviews. All the interviews were videotaped and transcribed by researchers. Transcribed interviews were analyzed using descriptive analyze.

3. Results (Findings)

Research findings were analyzed in two parts. In the first part, results of students' workings for modeling were presented and in the second part results gathered from dept interviews were given.

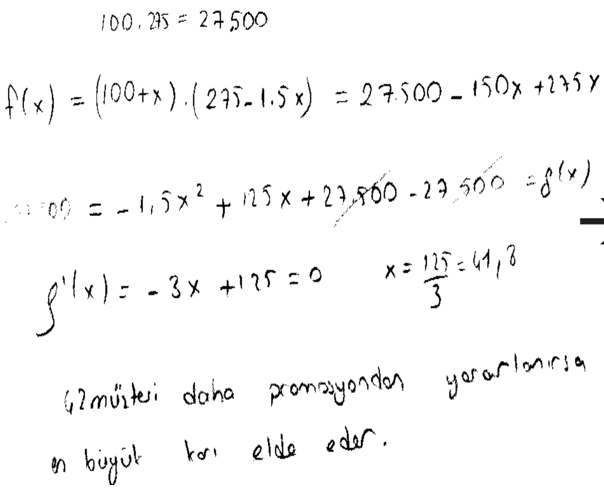
3.1. Activity Results

"Getting the Word Out" and "The Greening Forest Acres" are the two modeling activities that the students are most successful in. When the modeling activities are examined, it is seen that these activities are the ones that contain the most numerical data. With this characteristic, it resembles the problem type which the students are used to and which they encounter in almost all stages of their school life. In this respect, compared to the other modeling activities, it was easier for them to understand the problem, to determine the variables, and to form a mathematical model and to reach the solution. The students tended to form mathematical models mostly in these questions. In Figure-1 and Figure-2, some examples of mathematical models formed by the students for these questions are given.



100x+800y= 20000
x+8y=200 x=200-8y
30≤x≤60
It is asked to 20x+30y have the maximum value.
20(200-8y)+30y
4000-60y+30y
It is asked to 4000-130y have the maximum value.
y=15 is the minimum value of y
x+8*15=200 so ~~x=80~~. x have to be 30<x<80
then if y=16 is, x+8*16=200 so ~~x=72~~
y=17 ⇒ ~~x=64~~
y=18 ⇒ x=56, so y=18 and x=56

Figure 1. Student's solution for "Getting the word out" activity in Turkish and English.



f(x)= (100x)*(275-1,5x)=27500-150x+275-1,5

= -1,5x^2+125x+27500-27500=g(x)

g'(x)=-3x+125=0

x=125/3=41,8

If 42 more client profit from the promotion, maximum revenue wil be gained.

Figure 2. Student's solution for "Greening the forest acres" activity in Turkish and English

"Pack it them" is the one that presents itself as one of the most difficult questions for students. For most of the students, although they have formed different models for this question, the number of students that have reached the ideal result is significantly low. Among the solutions of the students that are not correct, there are results that are mathematically logical but impossible to apply to real life such as dividing the volume of the depot to the cumulative volume of the cylinders, or thinking of the base area of the cylinder as a square thus trying to find the appropriate cylinder number by dividing the base into squares (Figure-3) or ones that do not give the ideal solution. In addition to this, there are students who have reached the ideal solution (Figure-4).

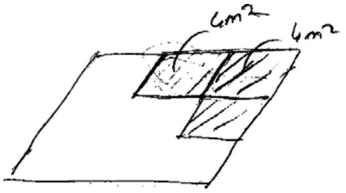


Figure 3. Student's diagram for "Pack it them " activity

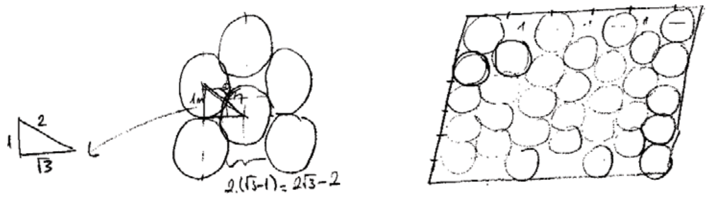


Figure 4. Student's diagrams for "Pack it them " activity

A similar result to the ones that are found in "Greening the Forest Acres" activity is realized in "Getting the Word out" activity. In this activity, there is a significant fall in the success of the students in passing from the forth modeling stage to the fifth modeling stage. The reason for this is that although the students have found the demanded ideal result, instead of applying the mathematical model they have formed into the given problem case, they have changed the ideal solution to the problem according to the mathematical model. In this activity, the majority of the students, although they have found the 60-second advertisement result in the solution process, because this solution does not fit the mathematical model correctly, they have ignored the "maximum how many seconds can the advertisement be seen?" expression and they have preferred the solution that fits their mathematical models. Apart from this, there are students who have associated the ideal solution with real life and have reached the correct solution.

Similarly, among the modeling activities, the activity that includes the least mathematical data is "Delivering the Mail." The students do not have a problem in the first stage of the modeling, which is understanding the problem and determining the variables, however, they face problems in forming the appropriate mathematical modeling and applying this mathematical model to the solution of the problem. In fact, many of the students, instead of forming a mathematical model for this question, tended to answer this question only by intuitive or verbal ways. The answers start with expressions such as "in my opinion…" and mostly, although they have made comments on the variables they determined, they have not used any mathematical model to support this expression. (Students' answers for "Delivering the Mail" activity, D)

> *D: In my opinion, 1st method is better since if the distance between the houses facing one another, the postman may cover more distance. However, if the width of the street is small, 2nd method is more advantageous.*

The answers that have mathematical quality among the answers given to this question are formed by expressing the verbal answers using variables (x, y, a, b …). Although the students are aware of the fact that the solution to this question is dependent on the values of the variables, they could not make a comparison taking into consideration the possible values of these variables.

In the light of the results achieved, the performances of preservice teachers determined by the modeling stages are summarized in Table-1. In Table-1, for the modeling activities, the number of students in each modeling stage is seen.

Table 1: Frequencies of students' performances

Stages	Delivering the mail	The greening of forest acres	Getting the word out	Pack it them in
1. Stage	55	43	60	41
2. Stage	29	31	48	19
3. Stage	16	19	46	5
4. Stage	2	15	41	5
5. Stage	1	10	14	3

As can be seen in Table-1, the stages that the students are most successful are the ones including understanding the problem and determining the variables (Stage 1, Stage 2). However, there is a significant fall in the number of successful students in noticing the association between the variables and forming an appropriate mathematical modeling. This fall is not observed only in "Getting the Word out" activity. Researchers associated this case with the students' aptitude to activities that include mathematical data. The success of the students in the last two stages is very low. The reason for this is that some students could not find an appropriate mathematical model in some questions (Pack it Them, Delivering the Mail), and that in some questions, they could not interpret the mathematical model in relation to the problem.

3.2 Interview Results

In the interviews thoroughly done in the second part of the study, the participants expressed that the problems given here did not look like the problems they had encountered before, and that they required more thinking and logical reasoning. The student answers are as follows:

R: Is there a difference between the problems here and the problems you have encountered before?

E: They required versatile thinking. For example, in the training we had, there are a lot of formule and you have to place them accordingly. We could immediately start the operation but here, first you have to think thoroughly. You have to interpret them. You have to think what to do and when to do that. These were on the foreground.

C: For example (in the problems we have solved before) everything was given, it was explained step by step. Here, it is more verbal, mathematics is given less way to.

The students who agree that mathematics facilitates in creating solutions to almost every problem that can be faced in daily life expressed that they have used mathematical modeling in many fields of their daily lives. In this respect, the student answers are as follows:

R: Can you mathematically express problems you face in daily life?

E: There is mathematics in every aspect of life. I am interested in music, for example, in note calculations, time calculations …

Another issue emphasized in the interviews was how the preservice teachers achieved modeling skills and what the success achieved in the modeling process was related to. For this, the participants think that the ability to form mathematical modeling can be achieved in time, and that it can be nurtured by the training they get. They are also of the opinion that the success in this process is dependent on the activities or applications which show the importance of mathematics in their lives. Thus mathematics won't be only a course for them but be a part of their lives.

R: What, in your opinion, determines the success of the modeling process?

F: Actually, this is something that develops in time. When we look at something, we see only what we know. We can look from different perspectives by using the mathematical modeling. We can have a wider perspective. For example, the cylinder question. At first, I have aligned the cylinders in this question like beads on an abacus. But I wanted to have a cigarette at the time, instead of being in the classroom. Then, I realized that I should align them like cigarettes in a pack. Then, I managed to put more. That's why I have named the model as cigarette stack. If there are more relations with daily life … for example, without the cigarette pack, I would have never come up with the idea.

In the answers given by the preservice teachers during the interviews, it is noticed that they have indicated the absence of a course among the undergraduate courses that can develop modeling skills. Although they think that the mathematics courses are helpful in developing reasoning skills, and their mathematical knowledge and perspective, they also express that they do not learn much about the daily-life use of such knowledge, but that such associations would both develop this skill and increase interest in mathematics.

R: Do you think there should be a course on mathematical modeling in undergraduate curriculum?

F: Actually, it would be really nice. You could think better. For integral, for example, we say you calculate the area, but for example Architect Sinan makes the dome of the mosque through eight-layered integral. In such things, the brain makes connections, and one thinks more.

According to the preservice teachers that participated in this study, mathematical modeling skills, although related to individual characteristics, can be developed through training, and students need more experience with such activities, and such experimental environments make students have more modeling skills.

C: Students may be given more applications in this subject. Such skills can be achieved through application, I think. How do the students understand the givens, how they interpret them? Some trials can be made; feedbacks from the students can be evaluated. New activities can be arranged afterwards.

4. Conclusion and Recommendation

The basic aim of education in general and of mathematics teaching in particular, is to prepare the students to life, and to raise them as individuals who can produce solutions for the problems they face, and who have a wide perspective. However, real-life problems, as has been stated by the preservice teachers that participated in this study, have a multi-variety, ill-defined structures instead of simple structures where what is given and wanted are clearly expressed. Although one expects to find individuals who have the training for mathematics to be more proficient in mathematically modeling the real-life related problems, it is seen, as can be seen in the results of this study, that they are not really that successful. In addition to that, the students have difficulty in applying the achieved mathematical models to real life problems. This result is consistent with the findings of Verschaffel, De Corte and Borghart (1997) and Christiansen (2001, cit. Barbosa, 2006).

Again, the fact that the students do not feel the need to turn the problem cases which do not have mathematical data into mathematical ones or that they do not recognize such necessity, and that they are more successful in questions with more mathematical data ratio can be given as a result of their education. As Lingefjard and Holmquist (2005) state in their study, although students take a modeling course, this attitude of theirs towards mathematics is a result of their education, which has a firm foundation due to the fact that it has been drilled into them throughout the years. The students who have stated that they did not take a course on mathematical modeling or that there has not been a consciousness-raising on this issue, all agree that modeling courses that they could use to turn daily-life problems into mathematics should be added to the curriculum.

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