The Effect of Changing Scores for Multi-way Tables with Open-ended Ordered Categories

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Abstract

Log-linear models are used to analyze the contingency tables. If the variables are ordinal or interval, because the score values affect both the model significance and parameter estimates, selection of score values has importance. Sometimes an interval variable contains open-ended categories as the first or last category. While the variable has open-ended classes, estimates of the lowermost and/or uppermost values of distribution must be handled carefully. In that case, the unknown values of first and last classes can be estimated firstly, and then the score values can be calculated. In the previous studies, the unknown boundaries were estimated by using interquartile range (IQR). In this study, we suggested interdecile range (IDR), interpercentile range (IPR), and the mid-distance range (MDR) as alternatives to IQR to detect the effects of score values on model parameters.

Keywords: Contingency tables, Log-linear models, Interval measurement, Open-ended categories, Scores.

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1. Introduction

Categorical variables, which have a measurement scale consisting of a set of categories, are of importance in many fields often in the medical, social, and behavioral sciences. The tables that represent these variables are called contingency tables. Log-linear model equations are applied to analyze these tables. Interaction, row effects, and association parameters are strictly important to interpret the tables.

In the presence of an ordinal variable, score values should be considered. As using row effects parameters for nominal–ordinal tables, association parameter is suggested for ordinal–ordinal tables. Score values are used to weight these parameters. In that case, selection of score values is important. For instance, taking the score values equal does not fit in many studies because these scores may not represent true intervals between categories. Choice of scores affects estimates of model parameters and results of Goodness-of-fit test statistics.

To use quantitative data in contingency tables, the data need to be converted to qualitative form. If one category (class) of a variable has either no lower or upper

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limit, this category is called open-ended. Age, income, serum cholesterol levels, systolic blood pressure are some examples of variable which can have open-ended categories. Ku and Kullback [12] used a contingency table which one of its variable is systolic blood pressure with the levels: (1) $' < 127'$, (2) 127-146, (3) 147-166, (4) $' \geq 167'$. Lower bound of the first and upper bound of the fourth categories are unknown. Agresti [3] applied linear-by-linear association model to the data and accepted that the distance between (1–2), (2–3), and (3–4) categories are equal. If it is not allowed to get raw data, it is not possible to find minimum and maximum values. Therefore, it is impossible to find the boundaries of open-ended categories. In this situation, the boundaries need to be estimated first. Then the score values can be calculated.

Determining these boundaries and fitted score values have been discussed by authorities. The author who studied on score values initially was Birch [6]. Simon [14], Goodman [9], Agresti [3], Graubard and Korn [10] discussed the equally spaced score values in their studies. Inequally spaced scores were discussed in the studies of Bross [7] and Agresti [3]. Iki *et al.* [11] used ridit scores to analyze square contingency tables by using cumulative probabilities. More recently, Bagheban and Zayeri [5] proposed exponential score values as an alternative to equal spaced scores. Initially, Frigge et al. [8] proposed the interquartile range to illustrate the outlier, then Tibshirani and Hastie [15], and Liu and Wu [13] focused on the interquartile range (IQR) to detect genes with over-expressed outlier disease samples as we used on estimate of the open ended boundaries. Aktas and Saracbasi [4] used median and quartile ranges to calculate standardized score values on open-ended categories. We suggested three different methods as alternative to IQR for ordinal categories that are grouped from quantitative data.

In this paper, through an application with one open-ended variable, we discussed the effects of score values on model parameters. The proposed new methods used to determine the boundaries of open-ended classes. In section 2, the log-linear models were introduced. Section 3 outlined the score methods and suggested the methods to estimate the boundaries of open-ended categories were represented in Section 4. The log-linear models and the estimation methods were illustrated in Section 5 by an application.

2. Log-Linear Models

2.1. Models for Two-way Tables. Consider an $R \times C$ contingency table that the first variable is represented by X and the second variable is represented by Y. In this two-way table, cross-classifies constitute multinominal sample of n subjects on two categorical responses. Let n_{ij} denote the frequency of $(i, j)^{th}$ cell and the cell probabilities are π_{ij} and the expected values m_{ij} where $i = 1, 2, \ldots, R$ and $j = 1, 2, \ldots, C$. The properties of independence [2], linear by linear association [9], and row effects [3] models for two-way contingency tables are given in Table 1.

Table 1. The properties of most used log-linear models for two-way contingency tables

Model	X	Y	Equation	*df*
Independence	N, O*	N,O	$log\ m_{ij} = \lambda + \lambda_i^X + \lambda_j^Y$	$(R-1)(C-1)$
Linear by Linear Association	O	O	$log\ m_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \beta * u_i v_j$	$(R-1)(C-1)-1$
Row Effects	N	O	$log\ m_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \mu_i * v_j$	$(R-1)(C-2)$

*:N: Nominal O: Ordinal

Here, in the equations λ is the overall effect parameter, λ_i^X is effect of variable X at i and λ_j^Y is effect of variable Y at j with constraints such as $\sum_{i=1}^{R} \lambda_i^X = \sum_{j=1}^{C} \lambda_j^Y = 0$. u_i and v_j in linear by linear association model are the the known scores where $u_1 \leq u_2 \leq \ldots \leq u_R$ are ordered row scores and $v_1 \leq v_2 \leq \ldots \leq v_C$ are column scores. β is the association parameter. Goodman [9] called the specifical case of model *uniform association model*, where $\{u_i = i\}$ and $\{v_j = j\}$. μ_i in row effect model is the row effect parameters where constraints are needed such as $\sum_{i=1}^{R} \mu_i = 0$.

The local log-odds ratios of linear by linear association, uniform association and row effects models are given in the Equations (2.1)-(2.3), respectively.

$$log\ \theta_{ij} = \beta(u_i - u_{i+1})(v_j - v_{j+1}), \tag{2.1}$$

$$log\ \theta_{ij} = \beta, \tag{2.2}$$

$$log\ \theta_{ij} = (\mu_{i+1} - \mu_i) * (v_{j+1} - v_j). \tag{2.3}$$

2.2. Models for Multi-way Tables for Nominal x Ordinal x Ordinal Categorical Data. Let X be a nominal variable, Y and Z be ordinal variables and, u_j are score values for variable Y and v_k are score values for variable Z. Then the full model is:

$$log\ m_{ijk} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \mu_i^{XY} u_j + \mu_i^{XZ} v_k + \beta^{YZ} * u_j v_k. \tag{2.4}$$

The constraints are $\sum_{i=1}^{R} \lambda_i^X = \sum_{j=1}^{C} \lambda_j^Y = \sum_{i=1}^{R} \mu_i^{XY} = \sum_{i=1}^{R} \mu_i^{XZ} = 0$. In this model, β^{YZ} represents the linear-by-linear association parameter, μ_i^{XY} and μ_i^{XZ} represent the row effects model parameters [2].

The $log\ \theta_{ij(k)}$ is the conditional log-odds ratio between X and Y for fixed levels of Z, the $log\ \theta_{i(j)k}$ is the conditional log-odds ratio between X and Z for fixed levels of Y and the $log\ \theta_{(i)jk}$ is the conditional log-odds ratio between Y and Z for fixed levels of X can be calculated from Equation (2.5).

$$\begin{aligned} log\ \theta_{ij(k)} &= (\mu_{i+1}^{XY} - \mu_i^{XY})(u_{j+1} - u_j) \\ log\ \theta_{i(j)k} &= (\mu_{i+1}^{XZ} - \mu_i^{XZ})(v_{k+1} - v_k) \\ log\ \theta_{(i)jk} &= \beta^{YZ}(u_{j+1} - u_j)(v_{k+1} - v_k). \end{aligned} \tag{2.5}$$

2.3. Scoring Methods. For log-linear model studies, assignment of score values is important. Assuming all distance between adjacent categories equal is not always fit the data. In this situation, the way to assign the scores causes a problem. The score equality of best fitting model is chosen as the distance between adjacent categories. As $\pi_{.j},\ j = 1, 2, \ldots, C$ are the marginal probabilities of the ordered variable Y, the properties of equal spaced, ridit [7, 11] and exponential [5] scores are summarized in Table 2.

Table 2. The recommended score equalities

Scores	**Variables**	u_i	v_j
Equal spaced	N, O	i	j
Ridit	O	-	$\sum_{k=1}^{j-1} \pi_{.k} + \frac{1}{2}\pi_{.j}$
Exponential	O	i^a	j^a

For application of equal spaced scores, all the intervals between adjacent categories are assumed as equal. The cumulative probabilities are used to calculate ridit scores. Sometimes, non-equality characteristic of scores are observed in the categories of variables. In this situation, the arithmetic progression between categories disappears. The exponential scores are used when the baseline characteristic of categories changing by a geometric progression. a in the exponential score equation is called the power parameter and the model gives the uniform association model with equal spaced score values for $a = 1$.

3. Suggested Methods to Estimate the Boundaries of Open-ended Categories

The most practical scoring method is the exponential scores because it permits different values of the power parameter. However, when working on the open-ended ordered categories, these methods are insufficient. Applying the same method both ordered and open-ended categories is only possible when ignoring the open-ended structure. It makes the minimum value (lower bound of the first category) and the maximum value (upper bound of the last category) unimportant. However, these unknown values are the proof of inequality of scores.

Instead of using equal or non-equal scoring method, the different methods need to be used. To avoid the outlier problem, the interquartile range was suggested as a measure of dispersion [13]. The first quartile of a raw data is defined as Q_1 and the third quartile is Q_3. Then the interquartile range is $IQR = Q_3 - Q_1$. For a frequency table with k categories, the values which are less and greater than the limits in the Equation (3.1) were defined as outliers by Frigge *et al.* [8] under the normality assumption.

$$\begin{aligned} &LowerBound(LB_1) = Q_1 - 1.5 * IQR \\ &UpperBound(UB_k) = Q_3 + 1.5 * IQR. \end{aligned} \tag{3.1}$$

The definition of quartiles can affect the number of observations which shown as outside. This estimation method is used with 25% trimmed range. Changes of trimmed range may have greater effects on the estimate of score values.

3.1. Interdecile and Interpercentile Ranges. In this study, the *Interdecile range (IDR)* and *Interpercentile range (IPR)* were suggested as the alternatives of IQR, having 10% and 5% trimmed ranges, respectively. The calculations of IDR ($IDR = P_{90} - P_{10}$) and IPR ($IPR = P_{95} - P_5$) are similar with IQR.

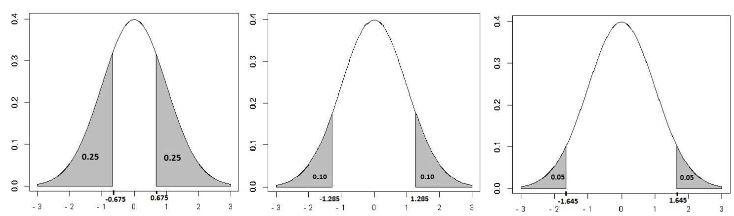
Under the normality assumption, the estimations of the boundaries with these methods can be limited as following equations, respectively.

$$LB_1 = P_{10} - 0.78 * IDR \quad and \quad UB_k = P_{90} + 0.78 * IDR, \tag{3.2}$$

$$LB_1 = P_5 - 0.61 * IPR \quad and \quad UB_k = P_{95} + 0.61 * IPR. \tag{3.3}$$

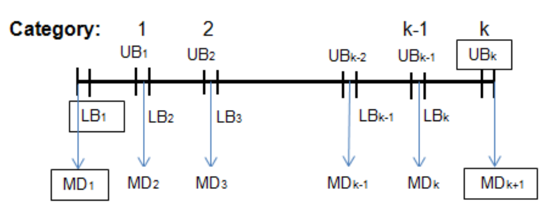
The standard normal distribution graphs and Z-values in order of IQR, IDR, and IPR are shown in Figure 1. Although the IPR seems to have wider range, this does not mean that it uses larger part of the distribution and it is better. The aim is to explain the data well and this depends on the distribution of frequencies.

Figure 1. The trimmed ranges for IQR, IDR, and IPR under the standard normal distribution



3.2. Mid-distance Range. Mid-distance range (MDR) was suggested to use as an alternative to IQR. The mid-distance ($MD_i = (LB_i + UB_{i-1})/2$) is the mid-point of i^{th} and $(i+1)^{th}$ categories where $i = 2, 3, \ldots, k$. The definition of MD can be shown in Figure 2. In this figure, first and last categories are open-ended and the values in the boxes are unknown. For a variable with k categories, the frequency table has $(k+1)$ MD. However, because of open-ended boundaries (LB_1 and UB_k) MD_1 and MD_{k+1}) are not calculated.

Figure 2. The mid-distances of a k-categories frequency table



Under the normality assumption, the percentage of the first category is $p_1 = P(x < MD_2)$ and the k^{th} category is $p_k = P(x > MD_k)$. Then the MDR is calculated from $MDR = MD_k - MD_2$. The distribution of frequencies is used to calculate MDR. Under the normality assumption, the boundaries are suggested,

$$LB_1 = MD_2 - [1/|Z_1|] * MDR \quad and \quad UB_k = MD_k + [1/|Z_k|] * MDR, \tag{3.4}$$

where $Z_1 = \Phi^{-1}(p_1)$ and $Z_k = \Phi^{-1}(p_k)$.

For Ku and Kullback [12] example, MD's of systolic blood pressure are calculated and shown in Table 3 [2].

Table 3. Mid-distances of systolic blood pressure

i	LB	UB	MD	
			–	MD_1
1	–	126		
			126.5	MD_2
2	127	146		
			146.5	MD_3
3	147	166		
			166.5	MD_4
4	167	–		
			–	MD_5

3.3. Standardized Score Values for Open-ended Categories. For an open-ended frequency table, because median is the appropriate measure of location and the quartile deviation is the appropriate measure of dispersion, Aktas and Saracbasi [4] suggested a score value that is calculated from quartile values. As s_i is the midpoint of i^{th} class, Q_2 is the median and Q_1, Q_3 are the first and third quartiles, respectively. The midpoint is,

$$s_i = \frac{LB_i + UB_i}{2} \qquad , i = 1, 2, ..., k. \tag{3.5}$$

Here, the estimated LB_1 and UB_k, which are defined in Equations (3.1)-(3.4), are used to calculate the midpoints. The standardized score values for row and column variables are

$$\begin{aligned} u_i &= \frac{s_i - Q_2}{(Q_3 - Q_1)/2} \qquad , i = 1, 2, ..., R \\ v_j &= \frac{s_j - Q_2}{(Q_3 - Q_1)/2} \qquad , j = 1, 2, ..., C. \end{aligned} \tag{3.6}$$

4. An Application

The $2 \times 4 \times 4$ contingency table, which is shown in Table 4, is taken from *General Social Survey, 1991, National Opinion Research Center*. It refers to the relationship between job satisfaction and income, stratified by gender, for 104 African–Americans [3].

The described models in Section 2 with equal spaced score values for ($nominal \times ordinal \times ordinal$) structure were applied to the data in Table 4. Because the data set contains sampling zeros, a correction factor for zero of 6 cells ($n_{ij} = 0 + 0.5$) was used. Table 5 shows the value of likelihood ratio statistics (G^2) for testing the Goodness-of-fit of each model. λ_i^G is the effect of gender at i,λ_j^I is the effect of income at j, and λ_k^S is the effect of job satisfaction at k. μ_i^{GI} and μ_i^{GS} are the

Table 4. Job Satisfaction and income, controlling for gender

		Job Satisfaction			
Gender	**Income**	**Very Dissatisfied**	**A Little Satisfied**	**Moderately Satisfied**	**Very Satisfied**
Female	< 5000	1	3	11	2
	5000–15,000	2	3	17	3
	15,000–25,000	0	1	8	5
	$> 25,000$	0	2	4	2
Male	< 5000	1	1	2	1
	5000–15,000	0	3	5	1
	15,000–25,000	0	0	7	3
	$> 25,000$	0	1	9	6

row effects parameters between gender–income and gender–job satisfaction, respectively. β^{IS} is the association parameter between income and job satisfaction. Then, the Akaike Information Criteria (AIC) was used to select the best fitting model [1]. Regarding the presented results, all models were fit the data. Because the 6^{th} model that contains both association parameter between income–job satisfaction and the row effects parameter between gender–income had the smallest value of AIC, this model was chosen as best fitting model.

Table 5. The results of Goodness-of-fit test for equal spaced score values

	Models	G^2	**df**	**P-Value**	**AIC**
1	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S$	25.326	24	0.388	–22.674
2	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \mu_i^{GI} u_j$	13.716	23	0.935	–32.284
3	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \mu_i^{GS} v_k$	24.983	23	0.351	–21.017
4	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \beta^{IS} * u_j v_k$	20.794	23	0.594	–25.206
5	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \mu_i^{GI} u_j + \mu_i^{GS} v_k$	13.373	22	0.922	–30.627
6	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \mu_i^{GI} u_j + \beta^{IS} * u_j v_k$	9.184	22	0.992	**–34.816**
7	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \mu_i^{GS} v_k + \beta^{IS} * u_j v_k$	20.451	22	0.555	–23.549
8	$log\ m_{ijk} = \lambda + \lambda_i^G + \lambda_j^I + \lambda_k^S + \mu_i^{GI} u_j + \mu_i^{GS} v_k + \beta^{IS} * u_j v_k$	9.174	21	0.988	–32.826

Thereafter, the recommended score values were tried on the 6^{th} model to choose the appropriate score values. Considering the open-ended structure, the standardized score values for income were calculated. Because gender is a nominal variable, score alternatives were not considered. For job satisfaction, equal spaced, exponential, and ridit scores were applied. The IQR, IDR, IPR, and MDR values for income were calculated as 17936.92, 25855.86, 30441.32, and 20000 respectively. To use mid-distance range, the percentages of 1^{st} and 4^{th} categories, $p_1 = 0.2056$ and $p_4 = 0.2337$, were used. Then the LB_1, UB_k from the methods, that were previously mentioned, were estimated. The estimated boundaries and range of income are shown in the Table 6. The estimated values of lower bound are negative. This can be logical when considering the people's loans. Between these methods, MDR has the largest value.

The score values in the first part of the Table 7 were calculated for job satisfaction. In the second part of the table, the standardized score values in Equatin (3.6) were calculated for income. After analyzing the model with different power parameter

Table 6. Estimated lower and upper boundaries of open-ended classes

Method	LB_1	UB_k	Range
IQR	–20,523	51,219	71,742
IDR	–15,304	50,887	67,191
IPR	–16,151	51,429	67,580
MDR	–19,330	52,510	71,840

values of exponential score, much appropriate a was found as 2. Because of the differences between estimated lowermost and uppermost values, the only alteration happens on the first and last classes.

Table 7. Estimated score values for income and job satisfaction

	Scores	v_1	v_2	v_3	v_4
Job Satisfaction	Equal Spaced	1	2	3	4
	Exponential	1	4	9	16
	Ridit	0.0304	0.1285	0.4906	0.8925
	Scores	u_1	u_2	u_3	u_4
Income	IQR	–2.457	–0.477	0.638	2.658
	IDR	–2.166	–0.477	0.638	2.639
	IPR	–2.213	–0.477	0.638	2.670
	MDR	–2.390	–0.477	0.638	2.730

The 6^{th} model was analyzed with the score values in Table 7. The results with different score values for income and job satisfaction were shown in Table 8.

Table 8. The results of parameter estimates for different score values in the model 6

	Scores			$\hat{\beta}^{IS}$		$\hat{\mu}^{GI}$	
	Income–Job Satisfaction	G^2	P-value	Estimate	P-value	Estimate	P-value
1	IQR–Equal Sapced	10.063	0.986	0.146	0.057	–0.202	0.001
2	IQR–Exponential	9.584	0.990	0.028	0.043	–0.202	0.001
3	IQR–Ridit	9.687	0.989	0.458	0.045	–0.202	0.001
4	IDR–Equal Spaced	9.750	0.988	0.157	0.055	–0.215	0.001
5	IDR–Exponential	**9.273**	0.992	0.030	0.041	–0.215	0.001
6	IDR–Ridit	9.377	0.991	0.488	0.043	–0.215	0.001
7	IPR–Equal Spaced	9.794	0.988	0.154	0.056	–0.211	0.001
8	IPR–Exponential	9.321	0.991	0.030	0.042	–0.211	0.001
9	IPR–Ridit	9.426	0.991	0.480	0.044	–0.211	0.001
10	MDR–Equal Spaced	9.974	0.987	0.146	0.057	–0.202	0.001
11	MDR–Exponential	9.501	0.990	0.028	0.043	–0.202	0.001
12	MDR–Ridit	9.605	0.990	0.456	0.045	–0.202	0.001

Despite all the models in Table 8 fitted the data based on $df = 22$, the Goodness-of-fit test statistics differed depending on the score alternatives. For these models, the best fitting one is the 5^{th} model which have standardized scores for income with the estimate method of IDR and exponential scores with $a = 2$ for job satisfaction. The 10% trimmed range was found as more appropriate. Besides the variation on Goodness-of-fit statistics, the estimated association parameter changed for different scores of income and job satisfaction. In general, the exponential score for job satisfaction had a decreasing effect on G^2 for all the combinations.

The association between adjacent categories where the gender effect is constant could be explained by odds ratio that $\theta_{(i)jk} = exp\{\beta^{IS} * (u_j - u_{j+1})(v_k - v_{k+1})\}$. The local odds ratios from the scores in Table 7 were estimated. The association between adjoint categories where job satisfaction effect was constant could be explained by odds ratio that $\theta_{ij(k)} = exp\{(\mu_{i+1}^{GI} - \mu_i^{GI})(u_{j+1} - u_j)\}$. Table 9 and Table 10 show the odds ratios for different score values.

Table 9. $\theta_{(1)11}$ for income × job satisfaction for the fixed levels of gender

	Job Satisfaction		
Income	Equal Spaced	Exponential	Ridit
IQR	1.335	1.181	1.093
IDR	1.304	**1.164**	1.084
IPR	1.307	1.169	1.085
MDR	1.322	1.174	1.089

Table 10. $\theta_{11(1)}$ for gender × income for the fixed levels of job satisfaction

Scores for Income			
IQR	IDR	IPR	MDR
2.225	**2.067**	2.080	2.166

Regarding the presented results in Table 9, using different methods to estimate lower and/or upper boundaries of open-ended categories was varying odds ratios. Using the estimation methods of IDR and IPR generated the odds ratios similar but different from the odds ratios estimated by using the IQR and MDR. Any category change on gender does not affect the odds ratio. The reason of this is the odds ratio depends on only changing scores of ordinal variable in row effects model. Regarding the presented results in Table 10, the odds ratios were varied between different scores of income.

By the 5^{th} case in Table 8, which explained the data well, the local odds ratios, which were calculated from parameter estimates, are shown in the following matrix.

$$\hat{\theta}_{(i)jk} = \begin{bmatrix} 1.164 & 1.288 & 1.426 \\ 1.105 & 1.182 & 1.264 \\ 1.197 & 1.350 & 1.522 \end{bmatrix}$$

$$\hat{\theta}_{ij(k)} = \begin{bmatrix} 2.067 & 1.615 & 2.364 \end{bmatrix}$$

The odds ratio that income was $'5000 - 15,000'$ rather than $'15,000 - 25,000'$ estimated to be 1.182 times higher than when the job satisfaction was 'A little satisfied' rather than 'Moderately satisfied'. The odds ratio that males rather than femalse estimated to be 2.067 times higher than when the income was $' < 5000'$ rather than $'5000 - 15,000'$.

5. Conclusions

In this study, we focused on determining the model which explains the data well for open-ended categories. This determination depends on the changing score values. When working on the contingency tables, which contain open-ended ordered categories, the open-ended boundaries of the distribution is suggested to be estimated. In the previous studies, utilizing the interquartile range, which is calculated from the first and the third quartiles, the unknown boundaries were estimated. In this study, we suggested alternative methods of interquartile range. We estimated the unknown boundaries of the table with these methods.

The used method is important because different methods cause differences on the estimated boundaries and accordingly midpoints. Differences in midpoints cause differences in score values. The changing score values also influenced the model significance and model fit. Parameter estimates and odds ratios varied between the methods which we utilized.

The difference between these four methods is that the estimation methods of IQR, IDR, and IPR use the trimmed range, which is a constant value, and trimmed ranges from the both side of the frequency distribution is equal. However, to estimate the MDR, we used the trimmed range where the information comes from the distribution of open-ended variable itself. Therefore, the trimmed ranges are different between the left and the right sides of the distribution. This difference comes from the percentages of the first and last categories.

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