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A new proposed model of restricted data envelopment analysis by correlation coefficients

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ABSTRACT

The concept of efficiency in data envelopment analysis (DEA) is defined as weighted sum of outputs/weighted sum of inputs. In order to calculate the maximum efficiency score, each decision making unit (DMU)'s inputs and outputs are assigned to different weights. Hence, the classical DEA allows the weight flexibility. Therefore, even if they are important, the inputs or outputs of some DMUs can be assigned zero (0) weights. Thus, these inputs or outputs are neglected in the evaluation. Also, some DMUs may be defined as efficient even if they are inefficient. This situation leads to unrealistic results. Also to eliminate the problem of weight flexibility, weight restrictions are made in DEA. In our study, we proposed a new model which has not been published in the literature. We describe it as the restricted Data Envelopment Analysis ((ARIII(COR))) model with correlation coefficients. The aim for developing this new model, is to take into account the relations between variables using correlation coefficients. Also, these relations were added as constraints to the CCR and BCC models. For this purpose, the correlation coefficients were used in the restrictions of input–output each one alone and their combination together. Inputs and outputs are related to the degree of correlation between each other in the production. Previous studies did not take into account the relationship between inputs/outputs variables. So, only with expert opinions or an objective method, weight restrictions have been made. In our study, the weights for input and output variables were determined, according to the correlations between input and output variables. The proposed new method is different from other methods in the literature, because the efficiency scores were calculated at the level of correlations between the input and/or output variables.

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1. Introduction

DEA was first developed in 1978 by Charnes, Cooper, Rhodes (CCR) [1]. DEA is a nonparametric method which is used for the evaluation of the efficiency of DMUs. The CCR model is developed depending on technical efficiency measurement for the single input-single output of Farrell [2] in the evaluation of the relative efficiency with multiple inputs and multiple outputs.

In order to obtain the maximum efficiency scores, the classical DEA allows weight flexibility. For this reason, some important inputs or outputs in DEA can be assigned weights of zero. In this situation, these inputs or outputs are neglected. So, unrealistic efficiency scores are obtained. Nowadays, weight restriction in DEA is one of the popular topics which is used to solve the existing problems. The first study about the weight restrictions was made by Thompson et al. [3]. Thompson

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et al. [4] defined the restriction of the input weights independent of the output weights as Assurance Region I (ARI). According to each one of the other units, the input and output weights were restricted together and they were described as Assurance Region II (ARII) by Thompson et al. [4]. The Cone-Ratio Approach is suggested by Charnes et al. [5], in order to solve the efficiency problem of DMUs which are inefficient in the real by the DEA model.

In our study, a new AR Aproach (the ARIII (COR)) which is restricted by correlation coefficients, is developed. The purpose of the proposed approach is to assign weights according to the correlations between the input and output variables in the weight restriction. In this method, each DMU takes a sequence in a set according to the weight results. In the study, Spearman rank test was used to check the ranks compatibility of models.

The goal of our study is given in the first section (input section). In the second section, earlier studies about the weight restrictions are placed. In the third section, the current methods of weight restriction are described. In the fourth section, the classical DEA methods (the CCR [1] and the BCC [6]) are mentioned. In the fifth section, ARIII, a new research approach for the restricted with correlation coefficient (ARIII (COR)), AR for the restricted with AHP [7] approach (the ARIII (AHP)) and Multiple Criteria Data Envelopment Analysis (MCDEA) [8] is described. In the sixth chapter, comparison of the similarities and differences with the models which are section three, four and five are examined. In the chapter seven, the new proposed correlation coefficient constrained DEA models (the CCRCOR and the BCCCOR), and the classical DEA models (the CCR, the BCC) and the AHP restricted DEA models (the AHPCCR, the AHPBCC) and the MCDEA model are applied on the data set. Then the obtained results were compared by application of Spearman's test on these methods. In the eighth chapter, the results are given.

2. Literature study

Mostly non-balanced weight distributions and unrealistic results are obtained in classical DEA models. To eliminate the problem which arises from the weight flexibility, different approaches in weight restrictions have been made in the literature. Most of the studies have been tried to incorporate the preference information of the decision makers into DMUs. The first study about weight restrictions was conducted by Thompson et al. [3] that was used in determination of the best location for establishment of a nuclear physic laboratory. Weight restrictions were developed for the preferred locations. So, this approach is called Assurance Region (AR) Approach. The reason for calling this region as AR is constraints limiting of the weight region with the specific region. The reduction of weight space is increasing the power of classification of DEA. The restriction of the input weights independently of output weights was considered as ARI and according to each one of other units, input and output weight restriction is defined as ARII [4]. Dyson and Thanassoulis [9] put direct restrictions on the weights in the evaluation of the departments proportionally. In the evaluation of the perinatal care to take the risk and other unknown factors in consideration, Thanassoulis et al. [10] carried out a weight restriction study. To make comparison and measurement for the performance of nations which were divided to four classes according to income level (low income, below the middle income, above middle income, high-income) in the Olympic games (6 different year), Li et al. [11] used the AR Approach in DEA. In order to give importance to inputs and outputs by AR Approach according to the number of medals won, a ratio of two variables is limited to a certain range (e.g. $1 < c_1/c_2 < 3$) [11]. AR approach was used to refine the results of the DEA by Li et al. [11]. Podinovski [12,13] argued it was estimated that DMUs had a low relative efficiency scores in adding weight restrictions in the CCR model. Thus, target values can be misleading for inefficient DMU and a wrong reference set of efficient peers can be determined. To avoid all these side effects, Podinovski and Athanassopoulos [12] proposed adding weight restrictions in the maximin DEA model. Cook et al. [14] give the first real-life example with the absolute weight restrictions in DEA. The pilot study has been done to measure the efficiency of 14 highway maintenance patrols. Two inputs and two outputs are available in the analysis. The AR limits for the separable inputs and outputs are placed on prices (multipliers) in the transition of overall efficiency from the technical efficiency in the model which is made by Thompson et al. [15]. Chilingerian and Sherman [16] used the weight restrictions to limit the factor weights in a cone which represents a doctor working model. This cone was established using the criteria which were determined by the head of primary health care unit. The application styles of the doctors which are compatible with the preference of the head of primary health care unit that they are defined in the preferred cone as efficient. The weight restrictions for the AR/cone ratio model which are used in this study, meet the criteria that are set by the head of health care system for efficient DMU's applications styles. These weight restrictions are defined as base on weight values assigned to these factors. Taylor et al. [17] assessed the efficiency of Mexican banks with DEA-AR productivity.

The VZA model presenting some banks efficient even though they faced financial difficulties and inefficient banks to be very good banks was seen as a problem by Charnes et al. [5] who suggested the Cone Ratio Approach in addressing these drawbacks.

Beasley [18] evaluated physics and chemistry departments of 52 universities by proposing to use proportionally weight restrictions to limit the weighted inputs and outputs. Wong and Beasley [19] developed the first virtual weight restriction method. Wong and Beasley [19] proposed the restriction of weighted input/output with virtual weight restriction, rather than directly restricting the weights. Sarrico and Dyson [20] discussed the use of virtual weights in the ARI and ARII.

From another perspective, there are approaches which are forced to use a common set of weights for all DMUs. Roll and Golany [21] suggested models which found a common set of weights to maximize the number of average efficiency of DMUs and to find the number of efficient DMUs. In order to obtain a common set of DMUs, a model was developed by Kao and Hung

[22]. An approach which includes the preference information in DEA and introduction of the term of value efficiency includes the values of decision makers as the concept of efficiency, is developed by Halme et al. [23]. This model has been useful to create a set of constraints through common weights to all DMUs. Korhonen et al. [24] had introduced a systematic approach to analyze the performance of research in R&D (research and development) institutes and universities, and also used value efficiency.

Furthermore, there are weight restriction approaches for ranking in the literature. Bal et al. [25], defining the coefficient of variation for the input and output weights, and adding them to the objective function of the CCR model (taking into account min CV = −max CV), for ranking that they obtained a new CVDEA model. This model serves the same purpose with the MCDEA which is developed by Li and Reeves [8]. In other words, to improve the separation power of DEA, the CVDEA model was used to obtain more reliable input and output weights. As opposed to Li and Reeves model [8], the CVDEA model [25] can be solved without any prior knowledge of decision-makers. Li and Reeves model [8] is a multi-goal linear programming technique. So, it is very hard to find a solution which always provides all the goals. Yet, the CVDEA model which is developed by Bal et al. [25], has a single objective function. It can be measured variability of weights related to average by the coefficient of variation which is used in the CVDEA model. The CVDEA model is obtained by adding the coefficient of variation to the CCR model, which reduces the number of efficient units and provides more stable (homogeneous) weight distribution. Wang et al. [26] suggested a linear programming method to produce the preferred weights from the pairwise comparison matrices. A model which provides the development of a balanced weight distribution using goal programming in DEA, is developed by Bal and Örkçü [27]. The only difference between the GPMCDEA model and the MCDEA model which is developed by Li and Reeves [8], is found to be the desired and undesired deviations for the input /output utilizing from the goal programming. Here, the aim is to minimize unwanted deviations. In the study of Bernroider and Stix [28], in order to sort of DMUs, multi-criteria analysis of decision making (MADM) is combined with the multiplier constrained DEA. The cross-efficiency method is used to measure the performance of nations in the Olympics by Wu et al. [29]. A major advantage of cross-efficiency can be used to rank all of the DMUs. Two input variables (the budget and population) and three output variables (the numbers of gold, silver and bronze medal winners) are available in the study. The weight restrictions which are made on the conditions show that the unit gets silver medal higher than unit gets bronze medal while the highest value of the units is gold. Soares De Mello et al. [30] used the cross-efficiency model in weight restricted DEA to each set for the ranking during Olympic games.

When the weight restrictions are used in DEA, sometimes infeasible solutions are obtained. In order to avoid infeasible solutions in the DEA model with weight restrictions, Estellita Lins et al. [31] have provided a theorem establishing the compability conditions for DEA multiplier programs with weight restrictions.

3. Weight restriction in DEA

In the weighting, subjective methods differ from objective methods during the assessment of efficiency. The value judgments are thought to reflect the preferences of decision makers. Even in DEA developed by Charnes et al. [1], there are the value judgments in the selection of input–output variables. For example, the variables with zero weights are removed from evaluation. The value judgments in evaluating the efficiency of a DMU affect the selection of optimal weights for inputs and outputs. The reasons for using of the value judgments in DEA were considered as follows [32]:

- To combine prior views on the values of inputs and outputs of DMUs.
- To establish a link between specific input and output values.
- To combine prior views on the efficient and inefficient DMUs,
- To estimate the marginal rate of substitution for inputs and outputs in the efficiency which are evaluated.
- To take into consideration of the situation of zero weights for inputs and outputs.
- To allow classification between the efficient units.

To install the weight restrictions in the literature, three different approaches are used as follows [32]:

- Direct restrictions (assurance region I (ARI), assurance region II (ARII), Absolute Weight Restrictions) [3,4,14].
- To adjust the observed input and output levels, in order to take into account the value judgments (Cone Ratio and Golany methods) [5,33].
- To restrict weighted inputs and outputs [32].

3.1. Direct restrictions on the weights

Many studies have been done to install directly on the weight restrictions in the literature. For example, the development of nuclear skills [3], to evaluate departments proportionally [9], military activities [34], efficiency in terms of the highway [14], the evaluation of perinatal protection [10], efficiency of physicians [16].

3.1.1. Assurance regions of type I (ARI)

ARI is used to include the preference information on input/ output values (prices) or to take into consideration the relative ranking of input/output values. Upper and lower limits in such restrictions are loaded using the price information on the ratios of factor weights [4,15,17,35]. AR model shows the transition of overall efficiency measurement from technical efficiency measurement. If there are no price information, the expert opinion on the relative importance of inputs/outputs is used to determine the boundaries [36].

ARI DEA model can be shown mathematically as follows:

$$\begin{aligned} &\max \sum_{r=1}^{s} u_r y_{ro} \\ &\text{s.t.} \sum_{i=1}^{m} v_i x_{io} = 1 \\ &\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad j = 1, \ldots, n, \\ &\quad A_i \leqslant \tfrac{v_i}{v_k} \leqslant B_i \quad i < k, i, k = 1, \ldots, m, \\ &\quad a_r \leqslant \tfrac{u_r}{u_t} \leqslant b_r \quad r < t, r, t = 1, \ldots, s, \\ &\quad -v_i \leqslant -\varepsilon \quad i = 1, \ldots, m, \\ &\quad -u_r \leqslant -\varepsilon \quad r = 1, \ldots, s. \end{aligned} \tag{1}$$

A_i and B_i are lower and upper limits on the ratios of the input weights. a_r and b_r are lower and upper limits on the ratios of the output weights.

ARI restrictions for x_1 and y_1:

$$\begin{aligned} &a_r \leqslant \tfrac{u_r}{u_1} \leqslant b_r \quad r = 2, \ldots, s, \\ &A_i \leqslant \tfrac{v_i}{v_1} \leqslant B_i \quad i = 2, \ldots, m. \end{aligned} \tag{2}$$

For ease of calculation in (2), AR restrictions can be written as follows:

$$\begin{aligned} &a_r u_1 \leqslant u_r \leqslant b_r u_1 \quad r = 2, \ldots, s, \\ &A_i v_1 \leqslant v_i \leqslant B_i v_1 \quad i = 2, \ldots, m. \end{aligned} \tag{3}$$

3.1.2. Assurance regions of type II (ARII)

Input and output weights are linked in ARII model. Therefore, such constraints are called as linked AR constraints. ARII models are used for the following purposes:

- To incorporate information on the relative importance according to an input of an output [10],
- To define the efficiency of DMUs [35].

The limits in ARII approach are based on the ratios of output weights to input weights.

ARII DEA model can be shown mathematically as follows:

$$\begin{aligned} &\max \sum_{r=1}^{s} u_r y_{ro} \\ &\text{s.t.} \sum_{i=1}^{m} v_i x_{io} = 1 \\ &\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad j = 1, \ldots, n, \\ &\quad \gamma_i v_i \geqslant u_r, \\ &\quad -v_i \leqslant -\varepsilon \quad i = 1, \ldots, m, \\ &\quad -u_r \leqslant -\varepsilon \quad r = 1, \ldots, s. \end{aligned} \tag{4}$$

Here, γ_i is the upper limit on the rate of the output weight, u_r to the input weight, v_i .

3.1.3. Absolute weight restrictions

Absolute weight restrictions load lower and upper limits on the input-output weights [21,37]. A basic difficulty of this approach, is to determine the limit values. The CCR model with the absolute weight restrictions can be shown in the following way:

$$\begin{aligned} &\max \ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &\text{s.t.} \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad j = 1, \ldots, n, \\ &\quad \delta_i \leqslant v_i \leqslant \tau_i, \\ &\quad \rho_r \leqslant u_r \leqslant \eta_r, \\ &\quad -v_i \leqslant -\varepsilon \quad i = 1, \ldots, m, \\ &\quad -u_r \leqslant -\varepsilon \quad r = 1, \ldots, s, \end{aligned} \tag{5}$$

where δ_i and τ_i are user-defined upper and lower limits on the input weights, respectively. However, ρ_r and η_r are upper and lower limits on the output weights, respectively.

3.2. To adjust the observed input-output levels for take into account of value judgments

Directly weight restrictions are installed by adding additional constraints to original DEA model. The current input-output data in this type of weight restrictions is multiplied by a vector. So, the input-output data is rearranged. Some studies in the literature for this purpose is as follows: The combined performance of banks [5], trade games [38], the selection of bills [39], etc. There are two approaches which are used to make the weight restrictions with the converted input–output.

3.2.1. The cone ratio model

The cone ratio model is proposed by Charnes et al. [5] who added the expert opinion to the analysis. This model includes creating a cone (smaller than the non-negative regions) within the range by the optimal virtual multipliers of efficient DMUs that provides the conditions which is determined by the decision maker.

3.2.2. The Golany method

To install the sequential relationships; $v_1 \geqslant v_2 \geqslant v_3 \geqslant \varepsilon$ among DEA weights have been proposed by Golany [32]. For this purpose, Golany [33] made equivalent transformations on the data without adding any other constraints. For instance, the equivalent of the constraint; $v_1 \geqslant v_2 \geqslant v_3 \geqslant \varepsilon$; x_{2j} can be replaced instead of $x_{2j} + x_{1j}$. Also, x_{3j} can be replaced instead of $x_{3j} + x_{2j} + x_{1j}, \ \forall_j$. Here, x_{ij} is the level of the ith input for the jth DMU.

3.3. The restriction of weighted inputs and outputs

There are two approaches which are proposed in the literature about this subject. The first, contingent weight restrictions are proposed by Pedraja et al. [40]. The second method includes the installation restrictions on the importance which is given of a DMU 's output (input) [19]. The importance is given to specific output by a DMU, which is the ratio of the output to total output. So, the importance is given to the rth output by the jth DMU which can be formulated as:

$$\frac{u_r y_{rj}}{\sum_{r=1}^{s} u_r y_{rj}} \tag{6}$$

Here, u_r is the weight on the rth output ($r = 1, \ldots, s$). Also, y_{rj} is the level of the rth output for the jth DMU.

Wong and Beasley [19] placed $[a_r, b_r]$ restrictions depending on the importance of the rth output for jth DMU. Using the limit values, the following constraints are added to the original DEA model:

$$a_r \leqslant \frac{u_r y_{rj}}{\sum_{r=1}^{s} u_r y_{rj}} \leqslant b_r. \tag{7}$$

$[a_r, b_r]$ is obtained by a common opinion which is reached from value judgments of experts on the relative importance of each output measurement in total output.

In order to contribute significantly to the total costs of inputs-outputs or to include the benefits of a DMU in the analysis, Pedraja et al. [40] argued that the weight restrictions should be added according to DMU 's input and output levels. The weight scheme is based on the input and output levels which are selected by a DMU. So, this approach is described as the contingent weight restriction approach by Pedraja et al. [40]. Because of this dependence, assessed DMU puts more weight on low-level inputs (for example, reducing its efficiency). In contrast to this situation, this DMU puts less weight on high-level input (such as increasing its inefficiency). Thus, the efficiency which is calculated by the contingent model tends to be more than the efficiency which is calculated to set limitations on input and output prices.

An input-space restriction in the following format is recommended by Pedraja et al. [40]:

$$c_i V_1 X_{1j} \leqslant V_i X_{ij} \leqslant d_i V_1 X_{1j}.$$

c_i and d_i are the values selected by the analyst.

4. The classical DEA methods

4.1. The CCR model

The CCR model was first introduced in 1978 by Charnes et al. [1]. In order to achieve maximum efficiency scores, this model evaluates the inputs and outputs for each DMUs that it is designed to assign different weights. The CCR model is based on the assumption of constant returns to scale (CRS). In the model, they are defined by:

θ_0: the efficiency score for the DMU_0,
u: the output weights,
v: the input weights,
x: inputs,
y: outputs.

4.1.1. Input oriented CCR model (CCR_i)

In the form of fractional programming, input oriented CCR model has been formulated in the following format:

$$\begin{aligned} &\max\ \theta_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \\ &\text{s.t.}\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad (j = 1, \ldots, n), \\ &\quad u_1, u_2, \ldots, u_s \geqslant 0, \\ &\quad v_1, v_2, \ldots, v_m \geqslant 0. \end{aligned} \tag{8}$$

The translated version to linear programming model of input oriented CCR is as follows:

$$\begin{aligned} &\max\ \theta_0 = \sum_{r=1}^{s} u_r y_{r0} \\ &\text{s.t.}\ \sum_{i=1}^{m} v_i x_{i0} = 1 \\ &\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad (j = 1, \ldots, n), \\ &\quad u_1, u_2, \ldots, u_s \geqslant 0, \\ &\quad v_1, v_2, \ldots, v_m \geqslant 0. \end{aligned} \tag{9}$$

4.1.2. Output oriented CCR model (CCR_o)

In the form of fractional programming, output oriented model is

$$\begin{aligned} &\min \theta_0 = \frac{\sum_{i=1}^{m} v_i x_{i0}}{\sum_{r=1}^{s} u_r y_{r0}} \\ &\text{s.t.}\ \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \geqslant 1 \quad (j = 1, \ldots, n), \\ &\quad u_1, u_2, \ldots, u_s \geqslant 0, \\ &\quad v_1, v_2, \ldots, v_m \geqslant 0. \end{aligned} \tag{10}$$

The translated version to linear programming model of output oriented CCR is

$$\begin{aligned} &\min\ \theta_0 = \sum_{i=1}^{m} v_i x_{i0} \\ &\text{s.t.}\ \sum_{r=1}^{s} u_r y_{r0} = 1 \\ &\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad (j = 1, \ldots, n), \\ &\quad u_1, u_2, \ldots, u_s \geqslant 0, \\ &\quad v_1, v_2, \ldots, v_m \geqslant 0. \end{aligned} \tag{11}$$

4.2. The BCC model

The BCC model was developed in 1984 as an alternative to the CCR model by Banker et al. [6]. The only difference between the BCC model and the CCR model is adding the variable of u_0 to the input oriented model and the variable of v_0 in the output oriented model to provide convexity. With the addition of the u_0 and v_0 variables (the signs of free), the BCC model is based on the variable returns to scale (VRS) assumption. In the model, they are shown by:

θ_0: the efficiency score for the DMU_0.
u: the output weights,
v: the input weights,
x: inputs,
y: outputs.

4.2.1. Input oriented BCC model (BCC_i)

$$\begin{aligned}
&\max\ \theta_0 = \sum_{r=1}^{s} u_r y_{r0} - u_0 \\
&\text{s.t.}\ \sum_{i=1}^{m} v_i x_{i0} = 1 \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \leqslant 0 \quad (j = 1, \ldots, n), \\
&\quad u_1, u_2, \ldots, u_s \geqslant 0,\ \ v_1, v_2, \ldots, v_m \geqslant 0, u_0\ \text{free}
\end{aligned} \tag{12}$$

4.2.2. Output oriented BCC model (BCC_o)

$$\begin{aligned}
&\min\ \theta_0 = \sum_{i=1}^{m} v_i x_{i0} - v_0 \\
&\text{s.t.}\ \sum_{r=1}^{s} u_r y_{r0} = 1 \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + v_0 \leqslant 0 \quad (j = 1, \ldots, n), \\
&\quad u_1, u_2, \ldots, u_s \geqslant 0,\ \ v_1, v_2, \ldots, v_m \geqslant 0, v_0\ \ \text{free.}
\end{aligned} \tag{13}$$

5. The weight restriction models

5.1. A new AR approach (ARIII)

We know that ARI approach will only restrict in the inputs or outputs constraints, while ARII approach restrict both inputs and outputs linked together. As an advantage in our study, ARIII approach can do by itself all the work which is done by ARI and ARII approaches simultaneously. Input and output variables (within their own group together and separate) can be restricted by this new approach. We suggest that ARIII can avoid the problem which is assigned of zero or very low weights to the important variables. Also, if they aren't actually efficient DMUs, they will be accurately found as inefficient. In this way, a more balanced distribution of the weights can be achieved.

ARIII is formulated as follows:

$$\begin{aligned}
&z_{i,i+1} v_{i+1} - v_i \leqslant 0 \quad (i = 1, \ldots, m-1), \\
&h_{i,r} u_r - v_i \leqslant 0 \quad (i = 1, \ldots, m), \quad (r = 1, \ldots, s), \\
&g_{r,r+1} u_{r+1} - u_r \leqslant 0 \quad (r = 1, \ldots, s-1), \\
&u_1, u_2, \ldots, u_s \geqslant 0,\ \ v_1, v_2, \ldots, v_m \geqslant 0.
\end{aligned} \tag{14}$$

In ARIII (14),

$z_{i,i+1}$ is a lower limit on the ratio of the weight of the ith input to the weight of the $(i+1)$th input.
$h_{i,r}$ is a lower limit on the ratio of the weight of the ith input to the weight of the rth output.
$g_{r,r+1}$ is a lower limit on the ratio of the weight of the rth output to the weight of the $(r+1)$th output.

5.1.1. A new AR model of restricted Data Envelopment Analysis by correlation coefficients (ARIII(COR))

In the production process, DMU's input and output variables are related to each other. The relationship between the variables was reflected to the model by the weights which are chosen in the correlation ratio in our study. Other known methods do not take into account the relationships to each other of the variables. The weights in the classical CCR and BCC models are determined to assign the highest efficiency scores. In the calculation of efficiency scores for some situations, a lot of important inputs and outputs are not taken into account or they are considered at very small amounts. In the literature, a lot of research has been done taking into account the problem of weighting. The studies to balance the weights may be considered in this context. Moreover, the weighted methods which are given in the literature, provide not to reach inconsistent results. For example, in the evaluation of the hospital efficiency, if a surgeon and general practitioner have been weighted at the same ratio, this situation will lead to incorrect results. The classical DEA method assesses without taking into account the levels of importance relative to each other for inputs and outputs. We think to use the correlation between each variable with the other variables to find more meaningful results. So, we proposed a new model in our study. The primary important input and output variables for the production must be given weights in that level. We believe that the binary combination (input and output) of weights to be taken into account can eliminate the above-mentioned drawbacks. Efficiency scores are calculated at the level of the correlation between input and/or output variables in our approach. To calculate the efficiency scores in our study, the weighted variables are taken into account as the existing relationship, in which case the weights are balanced according to us. The authors presented the method of balancing weight without taking into account such situations which considered only important weights that were different from zero and one. We propose a new concept of a balanced approach which is based on the principle of "if a variable is as important as what happened during production, it should be placed with a weight at the level". If the weights are given to this idea, they can be expressed as "balanced". Inputs or outputs are being compared among themselves by ARI, the ratio of inputs to outputs is based by ARII. Unlike them, we recommend ARIII (COR). While inputs/outputs are proportioned within their own group, inputs and outputs are proportioned in a way connected by ARIII (COR) at the same time. However, in order to take into account the relationships between variables, the weighting is done by the correlation matrix.

ARIII (COR) approach is superior to the subjective method which is as follows: first, ARIII (COR) does not require preference information. The results do not change according to analysts, because this method is objective. ARIII (COR) does not raise some well-known problems (the difference of alternative solutions in the cross-efficiency method, the difference of the preference information by experts, etc.). While the results are obtained, especially considering the relationship between variables, more realistic results are to be allowing. Thus, a more balanced distribution of weights is provided. So, in fact an inefficient DMU isn't found to be efficient.

Let m, be the number of inputs and s, be the number of outputs in ARIII (COR). The correlation matrix is symmetric. So, the total number of weight restrictions is obtained by the following formulation:

$$\frac{(m+s)^2-(m+s)}{2}=\frac{(m+s)(m+s-1)}{2}. \tag{15}$$

The weight restrictions with the correlation coefficients are defined in the following format:

$$\begin{aligned}
&c_{i,i+1}v_{i+1}-v_i \leqslant 0 \quad (i=1,\ldots,m-1),\\
&p_{i,r}u_r-v_i \leqslant 0 \quad (i=1,\ldots,m), \quad (r=1,\ldots,s),\\
&b_{r,r+1}u_{r+1}-u_r \leqslant 0 \quad (r=1,\ldots,s-1),\\
&u_1,u_2,\ldots,u_s \geqslant 0,\\
&v_1,v_2,\ldots,v_m \geqslant 0.
\end{aligned} \tag{16}$$

In ARIII (COR) (16),

$c_{i,i+1}$ is the correlation coefficient between the ith and $(i+1)$th input variables.
$p_{i,r}$ is the correlation coefficient between the ith input and the rth output variables.
$b_{r,r+1}$ is the correlation coefficient between the rth and $(r+1)$th output variables.

ARIII (COR) restrictions (16) were added in the classical CCR, BCC models (9), (11)–(13). So, the input and output oriented correlation weight restricted DEA models (the CCRCOR, the BCCCOR) were obtained.

The input oriented CCRCOR model can be shown mathematically as follows:

$$
\begin{aligned}
&\max\ \theta_0 = \sum_{r=1}^{s} u_r y_{r0} \\
&\text{s.t.}\ \sum_{i=1}^{m} v_i x_{i0} = 1 \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad (j = 1,\ldots,n), \\
&\quad c_{i,i+1} v_{i+1} - v_i \leqslant 0 \quad (i = 1,\ldots,m-1), \\
&\quad p_{i,r} u_r - v_i \leqslant 0 \quad (i = 1,\ldots,m), \quad (r = 1,\ldots,s), \\
&\quad b_{r,r+1} u_{r+1} - u_r \leqslant 0 \quad (r = 1,\ldots,s-1), \\
&\quad u_1, u_2, \ldots, u_s \geqslant 0, \quad v_1, v_2, \ldots, v_m \geqslant 0.
\end{aligned} \tag{17}
$$

5.1.2. An AR approach (ARIII (AHP)) of restricted with Analytical Hierarchy Process (AHP)

The AHP method was first developed by Saaty [7]. In this method, a preference matrix of the binary preferred coefficients is created by experts. Then, the consistency of the preference matrix is tested [7,41]. Often, necessary market information in DEA cannot be easily obtained [36]. In such cases, the limits of AR must be defined using expert opinions. First in the literature, in order to collect expert opinions to establish the limits of AR in DEA, the Analytical Hierarchy Process (AHP) was used by Zhu [36]. Later, to restrict the flexibility of the weight in DEA, Liu [42] has proposed to merge the objective knowledge and the subjective information obtained with the AHP. The AHP, DEA models and simulations are combined for the development of the railway system by Azadeh et al. [43]. First, the computer simulation model was performed to verify the model and to confirm the system which has been worked on. Second, the AHP method is used to define the weights of any qualitative criteria (inputs or outputs). Finally, DEA was applied to identify the best alternatives of model and to determine the current system mechanism. While earlier studies were based on quantitative variables, both qualitative and quantitative variables for evaluation of efficiency and integrated simulation of DEA and the AHP models were considered in this study. The restricted CCR and BCC models with the AHP were obtained by using the binary preferences for inputs and outputs in AHP method which was developed by Saaty [7,41]. Later, these preferences were transformed to the constraints which could be added into the CCR and BCC models. If AHP method is arranged for ARIII approach,

$$
\begin{aligned}
&a_{i,i+1} v_{i+1} - v_i \leqslant 0 \quad (i = 1,\ldots,m-1), \\
&k_{i,r} u_r - v_i \leqslant 0 \quad (i = 1,\ldots,m), (r = 1,\ldots,s), \\
&t_{r,r+1} u_{r+1} - u_r \leqslant 0 \quad (r = 1,\ldots,s-1), \\
&u_1, u_2, \ldots, u_s \geqslant 0, \quad v_1, v_2, \ldots, v_m \geqslant 0
\end{aligned} \tag{18}
$$

a set of constraints (18) will be obtained that we have called as ARIII (AHP).

In ARIII (AHP) (18),

$a_{i,i+1}$ is the AHP binary preference coefficient between the ith and $(i+1)$th input variables.
$k_{i,r}$ is the AHP binary preference coefficient between the ith input and the rth output variables.
$t_{r,r+1}$ is the AHP binary preference coefficient between the rth and $(r+1)$th output variables.

ARIII (AHP) was added in the classical CCR, BCC models (9), (11)–(13). So, the weights of the restricted input-output oriented AHP models (AHPCCR, AHPBCC) were obtained. The input oriented AHPCCR model can be shown mathematically as follows:

$$
\begin{aligned}
&\max\ \theta_0 = \sum_{r=1}^{s} u_r y_{r0} \\
&\text{s.t.}\ \sum_{i=1}^{m} v_i x_{i0} = 1 \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad (j = 1,\ldots,n), \\
&\quad a_{i,i+1} v_{i+1} - v_i \leqslant 0 \quad (i = 1,\ldots,m-1), \\
&\quad k_{i,r} u_r - v_i \leqslant 0 \quad (i = 1,\ldots,m), (r = 1,\ldots,s), \\
&\quad t_{r,r+1} u_{r+1} - u_r \leqslant 0 \quad (r = 1,\ldots,s-1), \\
&\quad u_1, u_2, \ldots, u_s \geqslant 0, \quad v_1, v_2, \ldots, v_m \geqslant 0.
\end{aligned} \tag{19}
$$

5.2. Multiple criteria data envelopment analysis (MCDEA)

Multiple criteria data envelopment analysis (MCDEA) was suggested by Li and Reeves [8]. This model was developed to solve the problem of ineligible weighting and the deficiency of separation. Three objective functions are introduced by

MCDEA. The first objective function is minimized of deviation (the value of inefficiency) of d_0. The efficiency value of DMU_0 is defined as $\theta_0 = 1 - d_0$ in the model. The second objective function is minimized of M which is the maximum deviation. The third target function is minimized the sum of deviations for all DMUs. MCDEA model is as follows:

$$
\begin{aligned}
&\min\ d_0\\
&\min\ M\\
&\min\ \sum_{j=1}^{n} d_j\\
&\sum_{i=1}^{m} v_i x_{i0} = 1,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad (j = 1,\ldots,n),\\
&M - d_j \geqslant 0 \quad (j = 1,\ldots,n),\\
&u_1, u_2, \ldots, u_s \geqslant 0,\\
&v_1, v_2, \ldots, v_m \geqslant 0,\\
&d_1, d_2, \ldots, d_n \geqslant 0.
\end{aligned}
\tag{20}
$$

6. Comparison of models based on similarities and differences

In this section, comparison of the similarities and differences with the models which are Sections 3–5 are listed in Table 1.

In Table 1, we examine the fundamentals of the models such as; restricted or unrestricted, weighting technique, taking into account the weight or not, the most prominent features of the method of weighting and cost of method.

As seen in Table 1, the CCR and BCC models of unrestricts methods, the MCDEA, CCRCOR and BCCCOR models of restricts methods are objective. The remaining methods are subjective. Subjective methods are based on expert opinions and value judgments. Objective methods don't include expert opinions or value judgments. Subjective methods affect the optimal solution region. If a weight restriction does not affect the solution region of the linear program, the weight restriction is objective [44].

Only the CCR and BCC models which are presented in Table 1, do not provide a balanced weight distribution. Because, the methods which are objective allow the flexibility of weight. Other methods provide a balanced weight distribution. The subjective methods are evaluated with expert opinions. To find an expert for each area is difficult. So, these methods costs are high. CCRCOR and BCCCOR are objective models. Therefore, their costs are low. The methods restrict weights with correlation coefficients. So, a balanced weight distribution is provided. These models don't require expert opinions. Thus, the new methods are advanced.

7. Application

The CCR, BCC, CCRCOR, BCCCOR, MCDEA, AHPCCR, AHPBCC models were applied on a data set (this data set has been used by Sun et al. [45]) which was given in Appendix 1. In our study on the evaluation of the efficiency of 27 robots, prices and repeatability were taken as the inputs. Also load capacity and speed were taken as the outputs.

By using the correlation coefficients matrix in Table 2, the weight restrictions with correlation coefficients will be added in the CCR and BCC models which are as follows:

$$
\begin{aligned}
&v_1/v_2 > 0.081 \Rightarrow 0.081\,v_2 - v_1 < 0,\\
&v_1/u_1 > 0.156 \Rightarrow 0.156u_1 - v_1 < 0,\\
&v_1/u_2 > 0.241 \Rightarrow 0.241u_2 - v_1 < 0,\\
&v_2/u_1 > 0.367 \Rightarrow 0.367u_1 - v_2 < 0,\\
&v_2/u_2 > 0.493 \Rightarrow 0.493u_2 - v_2 < 0,\\
&u_1/u_2 > 0.05 \Rightarrow 0.05u_2 - u_1 < 0.
\end{aligned}
\tag{21}
$$

By using AHP preference coefficients matrix in Table 3, the AHP weight restrictions will be added in the CCR and BCC models which are as follows:

Table 1
Comparison of the models.

The method	Restricted or unrestricted	Type of restriction	Weighting technique	The status of the balanced weight distribution	What does the current method takes into account when choosing the weights?	The most prominent features of the method of weighting	Cost of the method
ARIDEA	Restricted	Direct	Subjective	Yes	Assurance region determined by expert opinion on input or output variables	Input and output weights are not linked. Example: $A_i \leqslant \frac{v_i}{v_k} \leqslant B_i$, $i < k,\ i,k = 1,\ldots,m$, $a_r \leqslant \frac{u_r}{u_t} \leqslant b_r$, $r < t,\ r,t = 1,\ldots,s$	High
ARIIDEA	Restricted	Direct	Subjective	Yes	Assurance region determined by expert opinion on input and output variables	Input and output weights are linked. Example: $\gamma_i v_i \geqslant u_r$, $i = 1,\ldots,m,\ r = 1,\ldots,s$	High
Absolute weight restrictions	Restricted	Direct	Subjective	Yes	Lower and upper limits determined by expert opinion on input and output variables	Absolute weight restrictions load lower and upper limits on the input-output weights. A basic difficulty of this approach, is to determine the limit values. Example: $\delta_i \leqslant v_i \leqslant \tau_i$ $i = 1,\ldots,m$, $\rho_r \leqslant u_r \leqslant \eta_r$ $r = 1,\ldots,s$	High
Cone ratio	Restricted	To adjust the observed input-output levels	Subjective	Yes	Converted input and output data	A cone (smaller than the non-negative regions) is created by the optimal virtual multipliers of efficient DMUs with expert opinion	High
The Golany method	Restricted	To adjust the observed input-output levels	Subjective	Yes	Equivalent transformations on the data without adding any other constraints	The sequential relationships	High
Wong and Beasley method	Restricted	The restriction of weighted inputs and outputs	Subjective	Yes	$[a_r, b_r]$ restrictions depend on the importance of the rth output for jth DMU	$[a_r, b_r]$ is obtained by a common opinion which is reached from value judgments of experts on the relative importance of each output measurement in total output. Example: $a_r \leqslant \frac{u_r y_{rj}}{\sum_{r=1}^{s} u_r y_{rj}} \leqslant b_r$	High
Pedraja et al. method	Restricted	The restriction of weighted inputs and outputs	Subjective	Yes	Contingent weight restriction	For example; input-space restriction: $c_i V_1 x_{1j} \leqslant V_i x_{ij} \leqslant d_i V_1 x_{1j}$, c_i and d_i are the values selected by the analyst	High
CCR model	Unrestricted	–	Objective	No	Input and output weights must be positive	There is weight flexibility. The method doesn't require a priori assumption about the analytic form of the production function	Low
BCC model	Unrestricted	–	Objective	No	Input and output weights must be positive	There is weight flexibility. However, u_0 and v_0 are free variables for output, input variables, respectively. The method doesn't require a priori assumption about the analytic form of the production function	Low
AHPCCR	Restricted	The restricted CCR model with AHP coefficients	Subjective	Yes	The weights are selected according to the production function as determined by expert opinion	AHP coefficients are determined by expert opinion. Consistency of these coefficients will be tested. Then the restrictions are imposed with the coefficients in the CCR model	High
AHPBCC	Restricted	The restricted BCC model with AHP coefficients	Subjective	Yes	The weights are selected according to the production function as determined by expert opinion.	AHP coefficients are determined by expert opinion. Consistency of these coefficients will be tested. Then the restrictions are imposed with the coefficients in the BCC model	High
MCDEA	Restricted	Multi criteria	Objective	Yes	1 – Minimized of deviation (the value of inefficiency) of d_0 2 – Minimized of M which is the maximum deviation 3 – Minimized the sum of deviations for all DMUs	This model was developed to solve the problem of ineligible weighting and the deficiency of separation	Low

(*continued on next page*)

Table 1 (*continued*)

The method	Restricted or unrestricted	Type of restriction	Weighting technique	The status of the balanced weight distribution	What does the current method takes into account when choosing the weights?	The most prominent features of the method of weighting	Cost of the method
CCRCOR	Restricted	The restricted CCR model by correlation coefficients	Objective	Yes	In this method, the weights are determined by the correlations between input and output variables	In the production process, DMU ‘s input and output variables are related to each other. The relationship between the variables was reflected to the CCR model by the weights which are chosen in the correlation ratio in our study. Other known methods do not take into account the relationships among each of the other variables	Low
BCCCOR	Restricted	The restricted BCC model by correlation coefficients	Objective	Yes	In this method, the weights are determined by the correlations between input and output variables.	In the production process, DMUs input and output variables are related to each other The relationship between the variables was reflected to the BCC model by the weights which are chosen in the correlation ratio in our study. Other known methods do not take into account the relationships among each of the other variables	Low

Table 2
The correlation coefficients matrix.

	Prices (v_1)	Repeatability (v_2)	Load capacity (u_1)	Speed (u_2)
Prices (v_1)	1	0.081	0.156	0.241
Repeatability (v_2)	0.081	1	0.367	0.493
Load capacity (u_1)	0.156	0.367	1	0.05
Speed (u_2)	0.241	0.493	0.05	1

Table 3
The AHP preference coefficients matrix.

	Prices (v_1)	Repeatability (v_2)	Load capacity (u_1)	Speed (u_2)
Prices (v_1)	1	2	1/2	1/4
Repeatability (v_2)	1/2	1	1/5	1/3
Load capacity (u_1)	2	5	1	1/2
Speed (u_2)	4	3	2	1

Table 4
The efficiency scores obtained from the models.

DMUs	The efficiency of CCR	The efficiency of CCRCOR	The efficiency of AHPCCR	The efficiency of MCDEA	The efficiency of BCC	The efficiency of BCCCOR	The efficiency of AHPBCC
1	**1**	0.981420	0.4214601	0.48	**1**	**1**	0.466934
2	0.90376	0.610155	0.3463125	0.42	0.907407	0.628715	0.382893
3	0.52884	0.528588	0.4726979	0.47	0.666720	0.659670	0.522701
4	**1**	0.263219	0.1333884	0.16	1	0.305842	0.153262
5	0.59235	0.558651	0.125609	0.11	0.593779	0.567284	0.184290
6	0.48238	0.482050	0.3959284	0.47	0.864865	0.864865	0.864865
7	**1**	**1**	0.834411	**1**	**1**	**1**	0.899460
8	0.78254	0.782544	0.5240593	0.62	0.782948	0.782948	0.560730
9	0.37838	0.378383	0.2563228	0.31	0.383367	0.383367	0.282287
10	**1**	**1**	0.6307525	0.76	**1**	**1**	0.679364
11	0.67132	0.671317	0.591429	0.67	0.676624	0.676624	0.611468
12	0.10236	0.101919	0.06411219	0.09	0.141873	0.141851	0.141647
13	**1**	0.980572	0.5858928	0.7	**1**	**1**	0.658976
14	**1**	**1**	0.5652973	0.66	**1**	**1**	0.624570
15	0.6125	0.612294	0.5551437	0.56	0.623696	0.621425	0.584758
16	0.60351	0.602192	0.4006909	0.58	0.603854	0.602393	0.418108
17	0.40454	0.404024	0.3463358	0.27	**1**	0.680500	0.424708
18	0.36521	0.365215	0.2510529	0.3	0.366836	0.366836	0.269562
19	**1**	**1**	0.6394379	0.73	**1**	**1**	**1**
20	**1**	0.988171	**1**	**1**	**1**	**1**	**1**
21	0.85154	0.848660	0.7675496	0.15	**1**	0.973426	0.906979
22	0.82889	0.677921	0.379085	0.45	0.913151	0.677921	0.407843
23	0.69429	0.694161	0.5808784	0.69	0.923416	0.923416	0.802308
24	0.63613	0.636046	0.5317901	0.64	0.846536	0.846536	0.734507
25	0.55334	0.553343	0.4307609	0.99	0.556195	0.556195	0.462519
26	0.58102	0.581025	0.5252987	0.58	0.770664	0.744057	0.571405
27	**1**	**1**	**1**	**1**	**1**	**1**	**1**
The number of efficient DMUs	9	5	2	3	11	8	3

$$\begin{aligned}
&v_1/v_2 > 2 \Rightarrow 2v_2 - v_1 < 0,\\
&v_1/u_1 > 1/2 \Rightarrow (1/2)u_1 - v_1 < 0,\\
&v_1/u_2 > 1/4 \Rightarrow (1/4)u_2 - v_1 < 0,\\
&v_2/u_1 > 1/5 \Rightarrow (1/5)u_1 - v_2 < 0,\\
&v_2/u_2 > 1/3 \Rightarrow (1/3)u_2 - v_2 < 0,\\
&u_1/u_2 > 1/2 \Rightarrow (1/2)u_2 - u_1 < 0.
\end{aligned} \tag{22}$$

The results which are obtained from the evaluation of the input oriented models with two inputs and two outputs of 27 DMUs, were presented in Table 4. As shown that the DMUs are evaluated as efficient, which can obtain efficiency score of one from the models. From these results,

Table 5
The weights and efficiency scores obtained from CCR, CCRCOR, AHPCCR and MCDEA models.

DMUs	Models	Efficiency	u_1	u_2	v_1	v_2
1	CCR	**1**	0.014305	0.016345	0.10886	1.441366
	CCRCOR	0.981420	0.036919	0.015526	0.110475	1.363884
	AHPCCR	0.4214601	0.194341	0.002652	0.137457	0.068729
	MCDEA	0.48	0.24	0	0.14	0.09
2	CCR	0.90376	0.821596	0	0.008803	19.154930
	CCRCOR	0.610155	0.531814	0.004193	0.184594	2.278943
	AHPCCR	0.3463125	0.293022	0.003998	0.207254	0.103627
	MCDEA	0.42	0.36	0	0.21	0.13
3	CCR	0.52884	0.297845	0.003346	0.172609	0.107839
	CCRCOR	0.528588	0.297356	0.003354	0.172281	0.109130
	AHPCCR	0.4726979	0.250901	0.003423	0.177462	0.088731
	MCDEA	0.47	0.18	0.01	0.09	0.43
4	CCR	**1**	1.515152	0	0.016234	35.324677
	CCRCOR	0.263219	0.398816	0	0.13318	1.644196
	AHPCCR	0.1333884	0.196025	0.002675	0.138648	0.069324
	MCDEA	0.16	0.24	0	0.14	0.09
5	CCR	0.59235	0	0.011847	0.07696	1.044731
	CCRCOR	0.558651	0.000559	0.011172	0.078824	0.973141
	AHPCCR	0.125609	0.001255	0.002511	0.102828	0.051414
	MCDEA	0.11	0.18	0	0.1	0.06
6	CCR	0.48238	1.607929	0	0.881057	0.572687
	CCRCOR	0.482050	1.606832	0	0.879467	0.589707
	AHPCCR	0.3959284	1.262348	0.017224	0.892857	0.446429
	MCDEA	0.47	1.52	0.02	0.88	0.55
7	CCR	**1**	0.946815	0.010637	0.548704	0.342807
	CCRCOR	**1**	0.946609	0.010678	0.548443	0.347405
	AHPCCR	0.834411	0.781121	0.010658	0.552486	0.276243
	MCDEA	**1**	0.95	0.01	0.55	0.34
8	CCR	0.78254	0.536243	0.01642	0.265039	1.518738
	CCRCOR	0.782544	0.536243	0.01642	0.265039	1.518738
	AHPCCR	0.5240593	0.435024	0.005936	0.307692	0.153846
	MCDEA	0.62	0.53	0.01	0.31	0.19
9	CCR	0.37838	0.300156	0.004821	0.109837	1.309487
	CCRCOR	0.378383	0.300156	0.004821	0.109837	1.309487
	AHPCCR	0.2563228	0.207306	0.002829	0.146628	0.073314
	MCDEA	0.31	0.25	0	0.15	0.09
10	CCR	**1**	0.912098	0.01465	0.333767	3.979206
	CCRCOR	**1**	0.912098	0.01465	0.333767	3.979206
	AHPCCR	0.6307525	0.583023	0.007955	0.412371	0.206186
	MCDEA	0.76	0.71	0.01	0.41	0.26
11	CCR	0.67132	0.368092	0.011334	0.189835	0.906550
	CCRCOR	0.671317	0.368092	0.011334	0.189835	0.906550
	AHPCCR	0.591429	0.451703	0.006163	0.319489	0.159744
	MCDEA	0.67	0.37	0.01	0.19	0.91
12	CCR	0.10236	0.006524	0.007455	0.049651	0.657407
	CCRCOR	0.101919	0.017364	0.007303	0.05196	0.641478
	AHPCCR	0.06411219	0.191058	0.002607	0.135135	0.067568
	MCDEA	0.09	0.17	0.01	0.09	0.41
13	CCR	**1**	0.754717	0.009434	0.235849	4.905660
	CCRCOR	0.980572	0.721727	0.01145	0.261966	3.234153
	AHPCCR	0.5858928	0.438397	0.005982	0.310078	0.155039
	MCDEA	0.7	0.53	0.01	0.31	0.19
14	CCR	**1**	0.472005	0.014453	0.23329	1.336806
	CCRCOR	**1**	0.472005	0.014453	0.23329	1.336806
	AHPCCR	0.5652973	0.351262	0.004793	0.248447	0.124224
	MCDEA	0.66	0.43	0	0.25	0.15
15	CCR	0.6125	0.400846	0.004503	0.232301	0.145132
	CCRCOR	0.612294	0.400143	0.004514	0.231834	0.146853
	AHPCCR	0.5551437	0.338237	0.004615	0.239234	0.119617
	MCDEA	0.56	0.23	0.01	0.12	0.56
16	CCR	0.60351	0.006531	0.007462	0.049701	0.658060
	CCRCOR	0.602192	0.017382	0.00731	0.052014	0.642145
	AHPCCR	0.4006909	0.191576	0.002614	0.135501	0.067751
	MCDEA	0.58	0.17	0.01	0.09	0.41
17	CCR	0.40454	0.186555	0.002096	0.108114	0.067545
	CCRCOR	0.404024	0.186254	0.002101	0.107911	0.068355
	AHPCCR	0.3463358	0.157092	0.002143	0.111111	0.055556
	MCDEA	0.27	0.11	0	0.06	0.27

Table 5 (*continued*)

DMUs	Models	Efficiency	u_1	u_2	v_1	v_2
18	CCR	0.36521	0.314672	0.005054	0.115149	1.372817
	CCRCOR	0.365215	0.314672	0.005054	0.115149	1.372817
	AHPCCR	0.2510529	0.220911	0.003014	0.15625	0.078125
	MCDEA	0.3	0.27	0	0.16	0.1
19	CCR	**1**	1.64494	0.050652	0.84834	4.051214
	CCRCOR	**1**	1.64494	0.050652	0.84834	4.051214
	AHPCCR	0.6394379	1.465108	0.019991	1.036269	0.518135
	MCDEA	0.73	1.78	0.02	1.03	0.64
20	CCR	**1**	1.25	0	6.25	0
	CCRCOR	0.988171	1.209628	0.013645	0.70083	0.443934
	AHPCCR	**1**	1.25	0	0.862069	0.431034
	MCDEA	**1**	1.22	0.01	0.71	0.44
21	CCR	0.85154	0.425062	0.004775	0.246335	0.153900
	CCRCOR	0.848660	0.423361	0.004776	0.245286	0.155374
	AHPCCR	0.7675496	0.371084	0.005063	0.262467	0.131234
	MCDEA	0.15	0	0.01	0.03	0.45
22	CCR	0.82889	0.828893	0	0.008881	19.325045
	CCRCOR	0.677921	0.677921	0	0.226383	2.794858
	AHPCCR	0.379085	0.379085	0	0.261438	0.130719
	MCDEA	0.45	0.45	0.01	0.26	0.16
23	CCR	0.69429	1.314726	0.01477	0.761919	0.476015
	CCRCOR	0.694161	1.314197	0.014825	0.761415	0.482310
	AHPCCR	0.5808784	1.087561	0.014839	0.769231	0.384615
	MCDEA	0.69	1.31	0.01	0.76	0.48
24	CCR	0.63613	1.20459	0.013533	0.698092	0.436139
	CCRCOR	0.636046	1.204172	0.013584	0.697669	0.441931
	AHPCCR	0.5317901	0.995655	0.013585	0.704225	0.352113
	MCDEA	0.64	1.2	0.01	0.7	0.44
25	CCR	0.55334	0.423626	0.012972	0.209378	1.199786
	CCRCOR	0.553343	0.423626	0.012972	0.209378	1.199786
	AHPCCR	0.4307609	0.379043	0.005172	0.268097	0.134048
	MCDEA	0.99	0.0048	0.0001	0.0028	0.0017
26	CCR	0.58102	0.170615	0.005254	0.087991	0.420196
	CCRCOR	0.581025	0.170615	0.005254	0.087991	0.420196
	AHPCCR	0.5252987	0.238219	0.00325	0.168492	0.084246
	MCDEA	0.58	0.17	0.01	0.09	0.42
27	CCR	**1**	0	0.004878	0.25	0
	CCRCOR	**1**	0.006521	0.004854	0.248785	0.002393
	AHPCCR	**1**	0.002435	0.004869	0.249176	0.001623
	MCDEA	**1**	0	0	0.03	0.43

Table 6
The comparison of CCR. CCRCOR. AHPCCR and MCDEA models.

The general situation	Models	The number of efficient DMUs	The number of zero weights			
			u_1	u_2	v_1	v_2
	CCR	9	2	5	-	2
	CCRCOR	5	-	3	-	-
	AHPCCR	2	-	2	-	-
	MCDEA	3	2	9	-	-

the CCR model, 1, 4, 7, 10, 13, 14, 19, 20, 27 DMUs
the BCC model, 1, 4, 7, 10, 13, 14, 17, 19, 20, 21, 27 DMUs
the CCRCOR model, 7, 10, 14, 19, 27 DMUs
the BCCCOR model, 1, 7, 10, 13, 14, 19, 20, 27 DMUs
the MCDEA model, 7, 20, 27 DMUs
the AHPCCR model, 20, 27 DMUs
the AHPBCC model, 19, 20, 27 DMUs that
they were considered as efficient.

The efficiency scores and weights were given in Table 5 for the CCR, CCRCOR, AHPCCR, MCDEA models and in Table 7 for the BCC, BCCCOR, AHPBCC models. The results were given in Tables 6 and 8, respectively, which were compared in terms of the efficiency of scores and the zero weights for the CCR, CCRCOR, AHPCCR, MCDEA models and for the BCC, BCCCOR, AHPBCC models. Spearman's test results are included in Table 9 for all models with the help of SPSS program.

Table 7
The weights and efficiency scores obtained from BCC. BCCCOR and AHPBCC models.

DMUs	Models	Efficiency	u_0	u_1	u_2	v_1	v_2
1	BCC	**1**	2.120425	2.086898	0.005052	0.126297	0.604403
	BCCCOR	**1**	0.759108	0.773012	0.011926	0.120590	0.878355
	AHPBCC	0.466934	0.064488	0.274914	0.002671	0.137457	0.068729
2	BCC	0.907407	0.111111	0.925926	0	0	20.000000
	BCCCOR	0.628715	0.206563	0.759344	0	0.184594	2.278943
	AHPBCC	0.382893	0.097234	0.414508	0.004028	0.207254	0.103627
3	BCC	0.666720	0.584244	0.801125	0.005190	0.129742	0.276608
	BCCCOR	0.659670	0.513556	0.745446	0.005034	0.130511	0.273579
	AHPBCC	0.522701	0.083257	0.354925	0.003449	0.177462	0.088731
4	BCC	**1**	−0.891686	0.164111	0	0.078684	17.339148
	BCCCOR	0.305842	−0.125363	0.273453	0	0.133180	1.644196
	AHPBCC	0.153262	−0.117574	0.054073	0	0.138648	0.069324
5	BCC	0.593779	−0.007016	0	0.011735	0.077175	1.036476
	BCCCOR	0.567284	−0.011417	0.000556	0.011117	0.078824	0.973141
	AHPBCC	0.184290	−0.077577	0.001067	0.002133	0.102828	0.051414
6	BCC	0.864865	−0.864865	0	0	0.900901	0.360360
	BCCCOR	0.864865	−0.864865	0	0	0.900901	0.360360
	AHPBCC	0.864865	−0.864865	0	0	0.900901	0.360360
7	BCC	**1**	0.623879	1.623879	0	0.551506	0.293492
	BCCCOR	**1**	−0.010002	0.936829	0.010634	0.548647	0.343816
	AHPBCC	0.899460	0.259200	1.104972	0.010738	0.552486	0.276243
8	BCC	0.782948	0.009105	0.536175	0.017059	0.269192	1.385842
	BCCCOR	0.782948	0.009105	0.536175	0.017059	0.269192	1.385842
	AHPBCC	0.560730	0.144355	0.615385	0.005980	0.307692	0.153846
9	BCC	0.383367	0.537031	0.786015	0.005578	0.139453	0.314376
	BCCCOR	0.383367	0.537031	0.786015	0.005578	0.139453	0.314376
	AHPBCC	0.282287	0.068791	0.293255	0.002850	0.146628	0.073314
10	BCC	**1**	0.146667	1.066667	0.013333	0.333333	4.000000
	BCCCOR	**1**	0.146667	1.066667	0.013333	0.333333	4.000000
	AHPBCC	0.679364	0.193465	0.824742	0.008014	0.412371	0.206186
11	BCC	0.676624	−0.116184	0.422403	0.006009	0.311345	0.206652
	BCCCOR	0.676624	−0.116184	0.422403	0.006009	0.311345	0.206652
	AHPBCC	0.611468	0.149889	0.638978	0.006209	0.319489	0.159744
12	BCC	0.141873	−0.106229	0	0.002621	0.138551	0.043996
	BCCCOR	0.141851	−0.106187	0.000131	0.002621	0.138547	0.044028
	AHPBCC	0.141647	−0.105813	0.001310	0.002620	0.138505	0.044316
13	BCC	**1**	1.446231	2.038526	0	0.303385	0.583376
	BCCCOR	**1**	1.190678	1.728312	0.011670	0.302589	0.634291
	AHPBCC	0.658976	0.145474	0.620155	0.006026	0.310078	0.155039
14	BCC	**1**	0.936358	1.370484	0.009726	0.243148	0.548141
	BCCCOR	**1**	0.936358	1.370484	0.009726	0.243148	0.548141
	AHPBCC	0.624570	0.116560	0.496894	0.004829	0.248447	0.124224
15	BCC	0.623696	0.700744	1.012416	0.006639	0.175910	0.352653
	BCCCOR	0.621425	0.681236	0.988839	0.006677	0.173124	0.362904
	AHPBCC	0.584758	0.112237	0.478469	0.004649	0.239234	0.119617
16	BCC	0.603854	−0.004120	0.002396	0.007467	0.049507	0.659393
	BCCCOR	0.602393	−0.003603	0.014561	0.007303	0.052014	0.642145
	AHPBCC	0.418108	0.063571	0.271003	0.002633	0.135501	0.067751
17	BCC	**1**	1.003266	1.001633	0	0.057898	0.268409
	BCCCOR	0.680500	0.375807	0.504541	0.003148	0.078708	0.185166
	AHPBCC	0.424708	0.052128	0.222222	0.002159	0.111111	0.055556
18	BCC	0.366836	−0.041068	0.271517	0.005425	0.115316	1.367544
	BCCCOR	0.366836	−0.041068	0.271517	0.005425	0.115316	1.367544
	AHPBCC	0.269562	0.073305	0.312500	0.003037	0.156250	0.078125
19	BCC	**1**	−0.802102	0	0.019790	1.046160	0.332201
	BCCCOR	**1**	−0.801805	0.000989	0.019790	1.046146	0.332449
	AHPBCC	**1**	−0.799131	0.009895	0.019790	1.046027	0.334684
20	BCC	**1**	−1.000000	0	0	6.250000	0
	BCCCOR	**1**	−0.383747	0.770316	0	0.630173	0.449586
	AHPBCC	**1**	−0.731034	0.336207	0	0,862069	0.431034
21	BCC	**1**	0.781336	1.047845	0	0.355872	0
	BCCCOR	0.973426	0.561943	0.815681	0.005508	0.142808	0.299355
	AHPBCC	0.906979	0.123137	0.524934	0.005101	0.262467	0.131234
22	BCC	0.913151	−0.665012	0.248139	0	0.062035	15.285359
	BCCCOR	0.677921	0.253326	0.931247	0	0.226383	2.794858
	AHPBCC	0.407843	0.115033	0.522876	0	0.261438	0.130719

Table 7 (*continued*)

DMUs	Models	Efficiency	u_0	u_1	u_2	v_1	v_2
23	BCC	0.923416	−0.460861	0.925110	0	0.756806	0.539930
	BCCCOR	0.923416	−0.460861	0.925110	0	0.756806	0.539930
	AHPBCC	0.802308	−0.652308	0.300000	0	0.769231	0.384615
24	BCC	0.846536	−0.422491	0.848089	0	0.693797	0.494978
	BCCCOR	0.846536	−0.422491	0.848089	0	0.693797	0.494978
	AHPBCC	0.734507	−0.597183	0.274648	0	0.704225	0.352113
25	BCC	0.556195	0.007259	0.427457	0.013600	0.214609	1.104841
	BCCCOR	0.556195	0.007259	0.427457	0.013600	0.214609	1.104841
	AHPBCC	0.462519	0.125778	0.536193	0.005210	0.268097	0.134048
26	BCC	0.770664	1.120841	1.131918	0.006809	0.106124	0.344523
	BCCCOR	0.744057	0.523944	0.736925	0.004955	0.123873	0.270452
	AHPBCC	0.571405	0.079048	0.336984	0.003275	0.168492	0.084246
27	BCC	**1**	−0.032924	0	0.004717	0.250000	0
	BCCCOR	**1**	−0.037211	0.000235	0.004696	0.248825	0.002315
	AHPBCC	**1**	−0.034072	0.002352	0.004703	0.249204	0.001568

Table 8
The comparison of BCC, BCCCOR and AHPBCC models.

The general situation	Models	The number of efficient DMUs	The number of zero weights				
			u_0	u_1	u_2	v_1	v_2
	BCC	11	–	6	11	1	3
	BCCCOR	8	–	1	7	–	–
	AHPBCC	3	–	1	6	–	–

Table 9
Spearman's test results.

			CCR	BCC	CCRCOR	BCCCOR	MCDEA	AHPCCR	AHPBCC
Spearman rho	CCR	Correlation Coefficient	1.000	0.826**	0.827**	0.720**	0.543**	0.623**	0.518**
		Sig.(2 tailed)	,	0.000	0.000	0.000	0.003	0.001	0.006
		N	27	27	27	27	27	27	27
	BCC	Correlation Coefficient	0.826**	1.000	0.687**	0.800**	0.381	0.578**	0.592**
		Sig.(2-tailed)	0.000	,	0.000	0000	0.050	0.002	0.001
		N	27	27	27	27	27	27	27
	CCRCOR	Correlation Coefficient	0.827**	0.687**	1.000	0.890**	0.720**	0.838**	0.751**
		Sig.(2 tailed)	0.000	0.000	,	0.000	0.000	0.000	0.000
		N	27	27	27	27	27	27	27
	BCCCOR	Correlation Coefficient	0.720**	0.800**	0.890**	1.000	0.656**	0.802**	0.842**
		Sig.(2 tailed)	0.000	0.000	0.000	,	0.000	0.000	0.000
		N	27	27	27	27	27	27	27
	MCDEA	Correlation Coefficient	0.543**	0.381	0.720**	0.656**	1.000	0.797**	0.714**
		Sig.(2 tailed)	0.003	0.050	0.000	0.000	,	0.000	0.000
		N	27	27	27	27	27	27	27
	AHPCCR	Correlation Coefficient	0.623**	0.578**	0.838**	0.802**	0.797**	1.000	0.920**
		Sig.(2 tailed)	0.001	0.002	0.000	0.000	0.000	,	0.000
		N	27	27	27	27	27	27	27
	AHPBCC	Correlation Coefficient	0.518**	0.592**	0.751**	0.842**	0.714**	0.920**	1.000
		Sig.(2 tailed)	0.006	0.001	0.000	0.000	0.000	0.000	,
		N	27	27	27	27	27	27	27

** Correlation is significant at the 0,01 level (2-tailed).

When the CCR model and BCC models were examined separately in accordance with Spearman's test results in Table 9, the maximum relationship which was among the models derived from CCR model, was seen between CCRCOR and AHPCCR models with 83.8%. The maximum relationship among of models derived from BCC models was found between BCCCOR and AHPBCC with 84.2%. The highest relationship in terms of ranking with the classical CCR model was seen in CCRCOR model with correlation coefficient of 82.7%. The classical BCC model was similar to the BCCCOR model which had got the highest relationship in the terms of ranking with correlation coefficient of 80%. As shown the CCRCOR and BCCCOR models have led to greatly reduced the number of zero weights by assigning weights to the input and output variables according to the correlations between variables. Thus, when the CCRCOR and BCCCOR models were determined efficiency scores in the level of the relationship between input and output variables, the models also yielded very good results about the balanced weight distribution.

8. Conclusion

The classical DEA models which assign weights as the free, often give not well-balanced weight distribution and unrealistic results. Therefore, the weight restrictions are needed. A lot of work related to weight restrictions has been done in the literature. In some of these studies, without taking into account the relations between the variables (input/output), the inputs or outputs are multiplied by a vector and this approach is called as cone ratio weight restriction. Furthermore, if only the inputs or outputs are restricted among themselves, these weight restrictions are called as ARI approach. Moreover, if the inputs and outputs are taken as a ratio in conjunction with each other, the weight restrictions are called as ARII approach. In general, the restrictions of this approaches are made by expert opinions or a variety of ways with weighting. We recommend ARIII (COR) that also makes the task of ARI and ARII, taking into account the relationships between variables through the correlation matrix at the same time. The restricted DEA methods with correlation coefficients were called as CCRCOR, BCCCOR by us. In these methods (CCRCOR and BCCCOR), while the relationships between input and output variables were taken into account, the weight restrictions were considered according to the correlation coefficients. Weight restrictions were realized in earlier studies only by using expert opinion or by the idea to provide not zero of weights of the input/output variables. So, the relationships in previous studies are not taken into account. We recommend this approach which is not previously available in the literature and does not require the preferences of experts. The results do not change according to analysts, because the methods are objective. The new approach prevents actual inefficient DMUs from being found as efficient. In our proposed approach, the relationships between variables were especially considered, therefore more realistic results can be obtained. Thus, according to the correlations between input and output variables, a more balanced distribution of the weights is provided.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apm.2012.07.010.

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