Applied Mathematics Letters 22 (2009) 1467–1470

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Forecasting nonlinear time series with a hybrid methodology

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ARTICLE INFO

Article history:
Received 4 February 2009
Accepted 4 February 2009

Keywords:
ARIMA
Canadian lynx data
Hybrid method
Recurrent neural networks
Time series forecasting

ABSTRACT

In recent years, artificial neural networks (ANNs) have been used for forecasting in time series in the literature. Although it is possible to model both linear and nonlinear structures in time series by using ANNs, they are not able to handle both structures equally well. Therefore, the hybrid methodology combining ARIMA and ANN models have been used in the literature. In this study, a new hybrid approach combining Elman's Recurrent Neural Networks (ERNN) and ARIMA models is proposed. The proposed hybrid approach is applied to Canadian Lynx data and it is found that the proposed approach has the best forecasting accuracy.

1. Introduction

In recent years, the artificial neural networks (ANN) have been applied to many areas of statistics. One of these areas is time series forecasting [1]. Since ANN can model both nonlinear and linear structures of time series, using neural networks in forecasting can give better results than the other methods. Zhang et al. [2] review the literature of forecasting time series using ANN.

Both theoretical and empirical findings in the literature show that combining different methods can be an affective and efficient way to improve forecasts. Therefore, hybrid ARIMA and ANNs methods have been used for modeling both linear and nonlinear patterns equally well. Pai and Lin [3] proposed hybrid ARIMA and support vector machines model. Tseng et al. [4] combined seasonal time series ARIMA model and feedforward neural network (FNN). Zhang [5] proposed a hybrid ARIMA and FNN model, composed of linear and nonlinear components as follows:

$$y_t = L_t + N_t, \tag{1}$$

where y_t denotes original time series, L_t denotes the linear component and N_t denotes the nonlinear component. Linear component is estimated by ARIMA model and residuals obtained from the ARIMA model

$$e_t = y_t - \hat{L}_t, \tag{2}$$

are estimated by FNN. Here $\hat{L}_t$ is the forecasting value for time t of the time series y_t by ARIMA. Zhang [5] claims that any ARIMA model can be selected for the data as this does not affect the final forecast accuracy.

With n input nodes, the ANN model for the residuals can be written as

$$e_t = f(e_{t-1}, e_{t-2}, \ldots, e_{t-n}) + \varepsilon_t, \tag{3}$$

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doi:10.1016/j.aml.2009.02.006

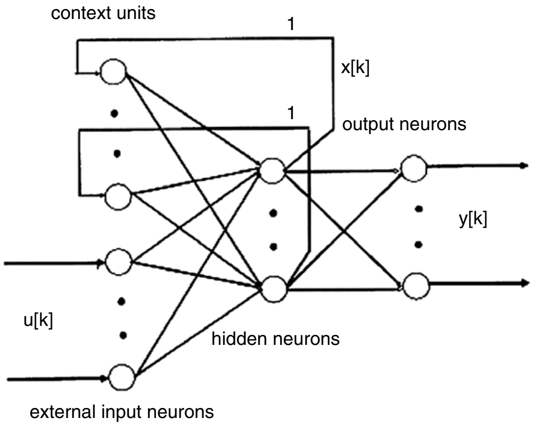

Fig. 1. Structure of an ERNN model [9].

where f is a nonlinear function determined by the FNN and ε_t is the random error. The estimation of e_t by (3) will yield the forecasting of nonlinear component of time series, N_t. By this way, forecasting values of the time series are obtained as follows:

$$\hat{y}_t = \hat{L}_t + \hat{N}_t. \tag{4}$$

In the next section, we modify Zhang's hybrid approach mentioned above. To obtain $\hat{N}_t$, we propose to use ERNN instead of FNN. In Section 3, the proposed hybrid method is applied to Canadian lynx data which is also used in Zhang [5] and Kajitani et al. [1]. By this way, we can compare the forecasting accuracy of the proposed method with the alternative methods. In the last section, we discuss the results of the application.

2. The proposed hybrid method

ARIMA and seasonal ARIMA (SARIMA) models were introduced by Box and Jenkins [6] and these models have recently been used successfully in forecasting linear time series. However, it is well known that the approximation of ARIMA models to complex nonlinear problems is not adequate [5]. Therefore, nonlinear time series have been forecasted by using nonlinear methods like ANNs. Although FNN has been used in many applications of ANNs, it is also possible to use recurrent neural networks. One type of recurrent neural networks is ERNN which was introduced by Elman [7]. According to the general principle of the recurrent networks, there is a feedback from the outputs of some neurons in the hidden layer to neurons in the context layer which seems to be an additional input layer. In the case of comparison with other type of multilayered network, the most important advantage of ERNN is a robust feature extraction ability, which provides feedback connections from the hidden layer to a context layer [8]. The structure of an ERNN is illustrated in Fig. 1.

Zhang [5]'s hybrid approach uses FNN to estimate N_t in (1). Since ERNN contains the context layer, it is certain that using ERNN, instead of FNN, can improve forecasting accuracy. Therefore, we propose a new hybrid approach as follows:

Step 1. Box–Jenkins models are used to analyze the linear part of the problem. That is, $\hat{L}_t$ is obtained by using Box–Jenkins method.

Step 2. ERNN model is developed to fit the residuals from the Box–Jenkins models. That is, $\hat{N}_t$ is obtained by using ERNN.

Step 3. Using (4), forecasts of the hybrid method are obtained by adding the estimates of linear and nonlinear components of the time series, found in Step 1 and Step 2, respectively.

3. Application

The proposed hybrid method is applied to Canadian lynx data consisting of the set of annual numbers of lynx trappings in the Mackenzie River District of North–West Canada for the period from 1821 to 1934. Canada lynx data, which is plotted in Fig. 2, was also examined by Zhang [5] and Kajitani et al. [1], beyond the other various studies in the time series literature. We would like to note that we use the logarithms (to the base 10) of the data in the analysis.

The proposed hybrid method is applied to the data as follows:

Firstly, Box–Jenkins method is used for estimating linear part of the problem. The Canadian lynx data shows a periodicity of approximately 10 years. Because of this, the data is fitted by *SARIMA* $(2, 0, 0) \times (0, 1, 1)_{10}$ model. We check that this model satisfies all statistical assumptions such as no autocorrelation, homoskedasticity, etc. using Box–Pierce and White Tests. Secondly, residuals obtained from *SARIMA* $(2, 0, 0) \times (0, 1, 1)_{10}$ model are estimated by the ERNN model. Note that the residuals are divided into training set (100 data points) and test set (last 14 data points). Number of input nodes is varied from 1 to 12, number of hidden layer nodes is also varied from 1 to 12 and by this way 114 architectures are examined totally. We find that the most appropriate ERNN architecture is $4 \times 4 \times 1$. Thirdly, forecasts of last 14 years were obtained using

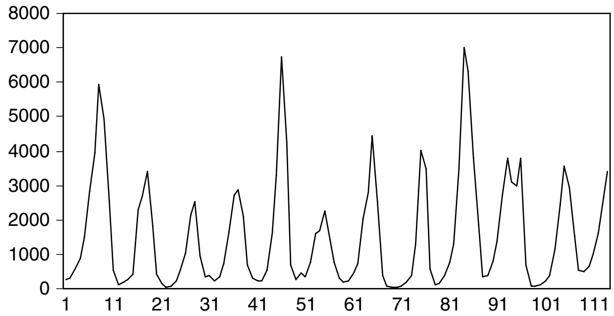

Fig. 2. Canadian lynx data series (1821–1934).

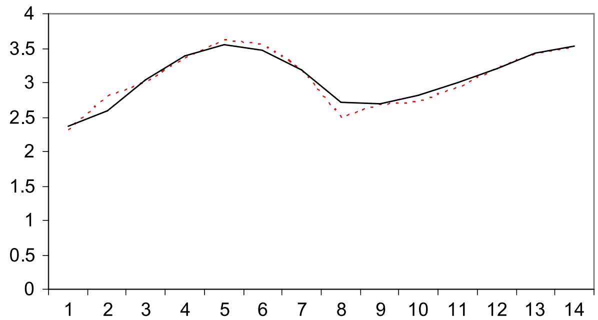

Fig. 3. Hybrid prediction of Canadian lynx data.

Table 1
Canadian lynx data forecasting results.

	Method	MSE
	FNN	0.020
Zhang [5]	Hybrid	0.017
Kajitani [1]	SETAR	0.014
Proposed	Hybrid	0.009

the proposed hybrid method. Finally, these forecasting values for last 14 years are shown in Fig. 3. Solid line represents the original time series data and dot line represents the forecasts.

The mean square error (MSE) values for the last 14 observations of the proposed approach, Zhang [5] and Kajitani et al. [1] are summarized in Table 1.

It is observed from Table 1 that the MSE of the proposed method is the smallest. Thus, it is concluded that the proposed approach has the best forecasting values for this widely used data.

4. Conclusions

Since artificial neural networks (ANN) can model both nonlinear and linear structures of time series, using ANN can give better results than other methods in forecasting. Therefore, in the literature, there have been many studies in which time series are solved by using ANN in recent years [10,2,11]. One type of ANN is recurrent neural network and one of the recurrent nets is ERNN.

Statisticians have studied to obtain better forecasts for long years and by these studies hybrid methods have been improved in the literature. In this paper, we consider that using ERNN instead of FNN in Zhang's hybrid method should improve the forecasting accuracy. Therefore, we propose a hybrid ARIMA and recurrent neural network model. It is observed that the proposed method yields better result than other methods for Canadian lynx data. It is well known that forecasting accuracy of ERNN is better than FNN, because of containing a context layer. Since ERNN is used in the proposed hybrid approach, as expected this approach is found better than Zhang [5]'s hybrid approach. In the future work we hope to increase the forecasting accuracy by changing the type of ANN used in hybrid methods such as Jordan recurrent neural networks [12].

References

[1] Y. Katijani, W.K. Hipel, A.I. Mcleod, Forecasting nonlinear time series with feedforward neural networks: A case study of Canadian lynx data, Journal of Forecasting 24 (2005) 105–117.
[2] G. Zhang, B.E. Patuwo, Y.M. Hu, Forecasting with artificial neural networks: The state of the art, International Journal of Forecasting 14 (1998) 35–62.

[3] P.F. Pai, C.S. Lin, A hybrid ARIMA and support vector machines model in stock price forecasting, The International journal of Management Science 33 (2005) 497–505.
[4] F.M. Tseng, H.C. Yu, G.H. Tzeng, Combining neural network model with seasonal time series ARIMA model, Technological Forecasting & Social Change 69 (2002) 71–87.
[5] G. Zhang, Time series forecasting using a hybrid ARIMA and neural network model, Neurocomputing 50 (2003) 159–175.
[6] G.E.P. Box, G.M. Jenkins, Time Series Analysis: Forecasting and Control, Holdan-Day, San Francisco, CA, 1976.
[7] J.L. Elman, Finding structure in time, Cognitive Science 14 (1990) 179–211.
[8] S. Seker, E. Ayaz, E. Turkcan, Elman's recurrent neural network applications to condition monitoring in nuclear power plant and rotating machinery, Engineering Applications of Artificial Intelligence 16 (2003) 647–656.
[9] D.T. Pham, D. Karaboga, Training Elman and Jordan networks for system identification using genetic algorithms, Artificial Intelligence in Engineering 13 (1999) 107–117.
[10] T.Y. Kim, K.J. Oh, C. Kim, J.D. Do, Artificial neural networks for non-stationary time series, Neurocomputing 61 (2004) 439–447.
[11] G. Zhang, B.E. Patuwo, Y.M. Hu, A simulation study of artificial neural network for nonlinear time series forecasting, Computers & Operations Research 28 (2001) 381–396.
[12] M.I. Jordan, Attractor dynamics and parallelism in a connectionist sequential machine, in: Conference of the Cognitive Science Society (1986).