# SELF-STARTING CONTROL CHARTS IN SOFTWARE DEVELOPMENT PROJECTS

# YAZILIM GELİŞTİRME PROJELERİNDE KENDİ KENDİNE BAŞLAYAN KONTROL GRAFİKLERİ

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# ABSTRACT

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To improve processes, generally some metrics are defined, data are collected and these are analyzed by appropriate methods. Statistical process control (SPC) is one of the methods used to improve processes. Maybe the most common tool of SPC is control charts, which are used to monitor changes in a process or characteristic over time. Control charts were originally developed for the mass production industry where production amounts are large. In such environments, a Phase I study is carried out with sufficient amount of data to estimate in-control process parameters and to determine control limits. Nevertheless, in some environments data is not abundant as in mass production and alternative approaches are needed to estimate process parameters and design control charts. For short production runs, where there is few data available to estimate parameters, self-starting control charts were developed.

Recently, the SPC has expanded beyond manufacturing and has started to be used for process improvement in software development processes. Just like in short production runs, there may not be enough data available to estimate parameters for designing control charts in software development processes. In this study, self-starting methods and change point models are considered for monitoring performance of software development processes over time.

Main performance metrics used in software development processes are generally counts, which have discrete probability distributions. Performance measures can be used to monitor software development processes as well as to monitor project scope. Examples of these metrics could be the number of defects calculated for different scenarios or the number of requests from customers. In this thesis, a simulation study was conducted based on the assumption that the data follows a Poisson distribution. Different project durations, change points, and shift magnitudes in the parameters were compared using self-starting methods Q and Exponentially Weighted Moving Average (EWMA) Q control charts, and a change point method Generalized Likelihood Ratio (GLR) control chart. Comparisons were made based on the probabilities of giving an out-of-control signal during the project duration, and after a change point in the case of a shift from the mean of a software quality metric.

The selected control charts were applied to monitor real software development process metrics and obtained results were compared. In addition to comparing the performance of Q, EWMA Q, and GLR control charts, robustness was evaluated by taking into account the responses to different change points.

The thesis demonstrates the applicability of self-starting and change point methods designed for Poisson distributed data in software development processes and presents comparisons of these methods under different scenarios.

**Keywords:** Statistical Process Control, Software Development Process, Short Run Processes, Self-Starting Control Charts, Change Point Models, Poisson Distribution

# ÖZET

## YAZILIM GELİŞTİRME PROJELERİNDE KENDİ KENDİNE BAŞLAYAN KONTROL GRAFİKLERİ

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Süreçleri iyileştirmek için genellikle bazı metrikler tanımlanır, veriler toplanır ve bunlar uygun yöntemlerle analiz edilir. İstatistiksel süreç kontrolü (İSK), süreçleri iyileştirmek için kullanılan yöntemlerden biridir. İSK'nin belki de en yaygın aracı, zaman içinde bir süreç veya özellikteki değişiklikleri izlemek için kullanılan kontrol grafikleridir. Kontrol grafikleri başlangıçta üretim miktarlarının büyük olduğu seri üretim endüstrisi için geliştirilmiştir. Bu tür ortamlarda, kontrol içi süreç parametrelerini tahmin etmek ve kontrol limitlerini belirlemek için yeterli miktarda veri ile bir Faz I çalışması gerçekleştirilir. Bununla birlikte, bazı ortamlarda, seri üretimdeki kadar veri bol değildir ve süreç parametrelerini tahmin etmek ve kontrol grafikleri tasarlamak için alternatif yaklaşımlara ihtiyaç vardır. Parametreleri tahmin etmek için çok az verinin mevcut olduğu durumlarda, kısa üretim çalışmaları için kendi kendine başlayan kontrol grafikleri geliştirilmiştir.

Son zamanlarda İSK, üretimin ötesine geçmiş ve yazılım geliştirme süreçlerinde süreç iyileştirme için kullanılmaya başlanmıştır. Kısa üretim süreçlerinde olduğu gibi, yazılım geliştirme süreçlerinde de kontrol grafikleri tasarlamak için parametreleri tahmin etmeye yeterli veri mevcut olmayabilir. Bu çalışmada, yazılım geliştirme süreçlerinin zaman içindeki performansını izlemek için kendi kendine başlayan yöntemler ve değişim noktası modelleri ele alınmıştır.

Yazılım geliştirme süreçlerinde kullanılan temel performans ölçütleri genellikle ayrık olasılık dağılımlarına sahip sayımlardır. Performans ölçütleri yazılım geliştirme süreçlerinin izlenmesinde kullanıldığı gibi proje kapsamının izlenmesinde de kullanılabilir. Bu metriklere örnek olarak, farklı senaryolar için hesaplanan hata sayısı veya müşterilerden gelen talep sayısı verilebilir. Bu tezde, verilerin bir Poisson dağılımını takip ettiği varsayımına dayalı olarak bir simülasyon çalışması yapılmıştır. Parametrelerdeki farklı proje süreleri, değişim noktaları ve kayma büyüklükleri, kendi kendine başlayan yöntemler Q ve Üstel Ağırlıklı Hareketli Ortalama (EWMA) Q kontrol grafikleri ve bir değişim noktası yöntemi Genelleştirilmiş Olabilirlik Oranı (GLR) kontrol grafiği kullanılarak karşılaştırıldı. Karşılaştırmalar, proje süresi boyunca ve bir yazılım kalite metriğinin ortalamasından kayma durumunda bir değişim noktasından sonra kontrol dışı bir sinyal verme olasılıklarına göre yapılmıştır.

Gerçek yazılım geliştirme süreci metriklerini izlemek için seçilen kontrol grafikleri uygulanmış ve elde edilen sonuçlar karşılaştırılmıştır. Q, EWMA Q ve GLR kontrol grafiklerinin performansının karşılaştırılmasına ek olarak, gürbüzlük, farklı değişim noktalarına verilen yanıtlar da dikkate alınarak değerlendirilmiştir.

Tez, Poisson dağılan veriler için tasarlanan kendi kendine başlama ve değişim noktası yöntemlerinin yazılım geliştirme süreçlerinde uygulanabilirliğini göstermekte ve bu yöntemlerin farklı senaryolar altında karşılaştırmalarını sunmaktadır.

**Anahtar Kelimeler:** İstatistiksel süreç kontrolü, Yazılım geliştirme süreci, Kısa koşumlu süreçler, Kendi kendine başlayan kontrol grafikleri, Değişim noktası modeli, Poisson dağılımı

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# SYMBOLS AND ABBREVIATIONS

**Symbols**

| | |
|---|---|
| $\alpha$ | EWMA Chart Smoothing Constant |
| $\lambda$ | Poisson Process Mean |
| $\tau$ | Change Point |
| $\Delta$ | Shift Magnitude |
| D | Project Duration |

**Abbreviations**

| | |
|---|---|
| CL | Centerline |
| CMMI | Capability Maturity Model Integration |
| CUSUM | Cumulative Sum |
| EWMA | Exponentially Weighted Moving Average |
| GLR | Generalized Likelihood Ratio |
| LCL | Lower Control Limit |
| SPC | Statistical Process Control |
| UCL | Upper Control Limit |

# 1. INTRODUCTION

In today's rapidly evolving technology environment, the demand for high-quality software is higher than ever before. To meet this demand, software development organizations strive to continuously improve their processes for increasing efficiency, reducing costs, and delivering better products. Process improvement has become a key focus area for many organizations seeking to achieve these goals. Hence, quantitative management is necessary to monitor and control process performance to reduce, eliminate, or prevent deficiencies in software quality and is widely used today. Software metrics are quantitative measures of software processes and are an input to process improvement. The most important factor in determining software metrics is organizational goals. It is important to identify measurements in alignment with the goals to provide input for process improvement.

In software projects, the most fundamental factor is the customer, and meeting or exceeding customer expectations can only be achieved through quantitatively managed stable software development processes. In this way, product quality, service quality, process performance, and business objectives are guaranteed by being controlled throughout the software life cycle, through a quantitatively managed process. There may always be some variation in software development processes, no matter how careful one is. Software metrics provide information about process performance, not about process stability. Therefore, measurement data are not directly supportive of a result or decision. Interpreting measurement outputs for decision-making is difficult without using systematic methods. Analysis and control of variation in software development processes are essential for software process improvement. Statistical process control (SPC) is one of the quantitative management techniques used to detect, measure, control, and reduce variation by identifying them.

Shewhart control charts (Shewhart, 1926) are one of the most commonly used SPC methods and have also gained considerable interest in the software industry. The benefits of using SPC methods to improve the quality of software processes and the successful outcomes of using control charts have been demonstrated (Florac & Carleton, 1999; Jalote & Saxena, 2002). At the same time there are some debates about the difficulty of implementing control charts in software development processes (Komuro, 2006; Manlove & Kan, 2007a; Sargut & Demirörs, 2006). Some of the reasons that have been debated are intensive human activities, diversity of measurements, multiple common causes, and a small amount of process data, which are actually difficult-to-change characteristics of the software industry. Setting valid control limits when creating control charts requires a large amount of data from a homogeneous source of variation. However, providing such a large dataset in software development processes is quite challenging. Therefore, modified SPC approaches are needed to deal with these types of problems instead of directly applying traditional control charts (i.e., the Shewhart control chart) to software development process data.

Insufficient number of data can result in poor process parameter estimates when a traditional control chart is applied, and control limits calculated using these estimated parameters may be inappropriate for efficient process monitoring. On the other hand, even if there is enough data to estimate process parameters, for monitoring performance of a software project, control charting is often required at the beginning or very close to the start of the projects. For such cases, Q charts, Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control charts based on a Q statistic have been developed. There are also change point methods that can be used alternatively. One of these change point methods is the Generalized Likelihood Ratio (GLR) control charts. These control charts can be designed with a small amount of data and may be more appropriate and useful than traditional control charts.

In this study, it will be assumed that the process data is discrete and follow a Poisson distribution. Q and EWMA Q control charts developed for cases where there is insufficient data, as well as GLR chart, which is a change point method, will be designed and compared for software development processes.

This work has four main goals: to demonstrate how each chosen Q, EWMA Q and GLR control chart performs under different scenarios, to provide a comparison with each other, to demonstrate the robustness of the charts when dealing with an unknown change point, and to illustrate the practical applicability of the self-starting charting techniques in monitoring a software process.

The remainder of the thesis is structured as follows. Section 2 provides an introduction to the process monitoring. An introduction to short run applications of control charts and quality control in software development life cycle (SDLC) are also provided. Section 3 presents the methodologies that includes charting with Q, EWMA Q and GLR, and explains how these can be utilized in order to monitor processes with defined parameters.

Results and discussion are presented in Section 4. Section 5 presents real life examples that compare the performance of the Q, EWMA Q and GLR charts and discusses the pros and cons of these charts as self-starting charting methods. Conclusions and directions for future work are then presented in Section 6.

# 2. BACKGROUND AND REVIEW OF LITERATURE FOR PROCESS MONITORING

This section provides overview of process monitoring. Section 2.1 introduces the Control Charts. In Sections 2.2 and 2.3 Statistical Process Control and Short Run Applications of Control Charts are examined. Finally, in Section 2.4 Quality Control in Software Development Life Cycle charts are discussed.

## 2.1. Control Charts

Control charts were first introduced by Shewhart in the 1920s and has been widely applied in various contexts. A common use is setting acceptable limits for process variation and monitoring process performance over time. Variations in process performance are mainly caused by two factors: common causes, resulting from regular interactions between people, machines, the environment, and techniques, and assignable causes, resulting from unusual interactions between these factors. Control charts are used to statistically identify existence of assignable causes of variation, thereby determining whether a process is in-control or out-of-control. For this purpose, a control statistic is plotted on a control chart over time, compared with the upper and lower control limits that indicate the allowable variations in the control statistic when the process in in-control. If there are assignable sources of variability, the probability of a control statistic plotting outside the control limits increases. A point plotted outside the control limits indicates that the process needs to be investigated and corrective actions should be taken to eliminate the causes of these anomalous fluctuations. Systematic use of control charts is a great way to reduce variability.

Control charts are typically used for quality control in product or process management. There are two main types of control charts: variables and attributes charts. Variables charts are used for data that is measured on a continuous scale, such as weight or length. The most commonly used variables charts are Shewhart type $\overline{\mathrm{X}}$ and Individuals (I) charts. Attributes charts are typically used for count type data or for categorizations such as pass or fail. They are less sensitive to variation in the process than variables charts but can be effective when used with real-time Pareto analysis (Anjard, 1995). Attributes charts include Shewhart type p-charts, c-charts, np-charts, and u-charts.

Shewhart and EWMA control charts are widely used in industry. While Shewhart type control charts are used for monitoring large shifts in processes, EWMA charts are more effective in detecting small to moderate sized shifts. In addition to these charts, GLR charts can also be used in monitoring different sized shifts in the processes. The GLR control chart offers an advantage in that it takes into account estimates of the process change point and shift size when calculating the GLR statistic (Huang et al., 2013). This consideration can be useful in the post-signal diagnosis of the process. These three types of control charts for attributes are considered and examined in detail in the thesis.

### 2.1.1 Shewhart Control Charts for Attributes

Sometimes it is difficult to express certain quality features using numbers. When this happens, we typically categorize each item examined as meeting or not meeting the standards for that particular quality characteristic. These two classifications of products are often referred to as defective or non-defective. p-, c-, np- and u- control charts are the commonly used Shewhart type charts when quality characteristics are attributes.

#### 2.1.1.1 p Charts

p control charts are used to monitor proportion of nonconforming units in a sample of n units. The probability of a unit not complying with the defined conditions is expressed with p. In cases where p is unknown, it is estimated from past observations.

Assuming that we already know the true value of the ratio of nonconforming items p in the process, or have a specified standard value for it, we can determine the centerline (CL), upper and lower control limits (UCL and LCL, respectively) for the p control chart as given in the Table 1, where L represents the distance of the control limits from the center line in terms of standard deviation unit.

Table 1. Control Limits for p Control Chart

| p Control Chart | |
|---|---|
| UCL | $p + L\sqrt{\frac{p(1-p)}{n}}$ |
| CL | p |

| | |
|---|---|
| LCL | p - L $\sqrt{\frac{p(1-p)}{n}}$ |

A common practice is to select L = 3.

### 2.1.1.2 np Charts

np charts are used for monitoring the number of nonconforming units in samples of n units. np charts can control single or multiple quality characteristics. For the np charts, centerline and control limit calculations are given in Table 2.

Table 2. Control Limits for np Control Chart

| np Control Chart | |
|---|---|
| UCL | np + 3$\sqrt{np(1-p)}$ |
| CL | np |
| LCL | np - 3 $\sqrt{np(1-p)}$ |

### 2.1.1.3 c Charts

A nonconforming item refers to a single product unit that fails to meet one or more of the requirements specified for that product. Whenever there is a violation of a particular requirement, it results in a defect or nonconformity. Therefore, any nonconforming item must have at least one nonconformity. However, some units may have multiple nonconformities of varying types and levels of severity but still not be classified as nonconforming. Control charts can be developed for either the total number of nonconformities or other specific aspects related to nonconformities. c chart is used for monitoring the number of nonconformities c in an inspection unit. Table 3 provides the L-sigma control limits for the c chart, assuming that there is an established standard value for c.

Table 3. Control Limits for c Control Chart

| c Control Chart | |
|---|---|
| UCL | $c + L\sqrt{c}$ |
| CL | c |
| LCL | $c - L\sqrt{c}$ |

### 2.1.1.4 u Charts

The purpose of the u control chart is to keep track of the average number of nonconformities per inspection unit. If a sample of n inspection units results in y total nonconformities, then the mean number of nonconformities per inspection unit can be calculated as follows,

$$\mathbf{u} = \frac{\mathbf{y}}{\mathbf{n}}$$

Let $\bar{\mathbf{u}}$ represent the observed average number of nonconformities per unit in a preliminary set of data, then control limits of the u chart are given in Table 4.

Table 4. Control Limits for u Control Chart

| u Control Chart | |
|---|---|
| UCL | $\bar{\mathbf{u}} + L\sqrt{\frac{\bar{\mathbf{u}}}{\mathbf{n}}}$ |
| CL | $\bar{\mathbf{u}}$ |
| LCL | $\bar{\mathbf{u}} - L\sqrt{\frac{\bar{\mathbf{u}}}{\mathbf{n}}}$ |

## 2.1.2 EWMA Chart

EWMA control chart was initially introduced to monitor the mean of a process. Based on performance evaluation studies, EWMA chart is found to be more sensitive to small changes in the process mean compared to the standard Shewhart chart (Roberts, 1959). However, when the changes in the process mean are large, the Shewhart chart is known to be more effective than the EWMA chart.

The EWMA control chart plots a statistic that combines current and past information, with the weights assigned to each observation determined by an exponential function. The EWMA statistic used to monitor a process mean over time is

$$z_r = \alpha y_r + (1 - \alpha)\, z_{r-1}$$

where $y_r$ is the observation at time r = 1, 2,… and α is a smoothing constant with $0 < \alpha \leq 1$. Starting value of the EWMA statistic is the in control process mean, i.e.

$$z_0 = \mu_0$$

In some cases, when using the EWMA chart, the initial value used is the average of some historical data. This means that the starting value of the EWMA is set to be equal to the average value of the data, so that $z_0 = \bar{y}$ (Montgomery, 2009).

The process of implementing an EWMA control chart involves plotting $z_r$ against either the sample number or time r. The EWMA chart's centerline and control limits are determined as in Table 5.

Table 5. Control Limits for EWMA Control Chart

| EWMA Control Chart | |
|---|---|
| UCL | $\mu_o + L\sigma\sqrt{\frac{\alpha}{(2-\alpha)}[1-(1-\alpha)^{2i}]}$ |
| CL | $\mu_o$ |
| LCL | $\mu_o - L\sigma\sqrt{\frac{\alpha}{(2-\alpha)}[1-(1-\alpha)^{2i}]}$ |

### 2.1.3 GLR Chart

The Shewhart chart is useful in detecting large shifts in the mean μ, but not as effective for detecting small shifts. In contrast, the EWMA charts are better suited for detecting small shifts, but may not be as effective for detecting large shifts. Since the magnitude of the shift in μ is often unknown in practice, it is important to be able to detect shifts of various sizes effectively. To achieve good performance across a broad range of shift sizes, one approach is to use multiple control charts together. For instance, combining a

Shewhart chart with a CUSUM chart can allow quick detection of both large and small shifts. This strategy has been explored by various researchers (Lucas & Crosier, 1982; Westgard et al., 1977). Adaptive CUSUM or EWMA control charts for detecting a range of shift sizes and tune the charts to react effectively to the size of the shift were also investigated (Capizzi & Masarotto, 2003; Jiang et al., 2008; Reynolds & Stoumbos, 2006). However, use of multiple charts results in a complex control chart design as many control chart parameters must be specified.

An alternative method for creating control charts that can identify various sizes of process changes is to use likelihood ratio tests as the foundation of the charts. These types of charts are commonly referred to as GLR charts (Lorden, 1971). When a special cause persists, it can result in a shift of the statistical distribution from the in-control distribution to a different one, and the distribution will continue to remain in this out-of-control state until corrective measures are implemented. The GLR method is designed to identify and detect such continuous changes.

Suppose that the change point is unknown, the value of an in-control parameter $\theta_0$, which represents the state before the change, is known but the value of the parameter after change $\theta_1$ is unknown. The formula for the GLR charts is as follows:

$$g_r = \max_{1 \le j \le r} \sup_{\theta_1 : |\theta_1 - \theta_0| \ge \upsilon_m > 0} S_j^r(\theta_1)$$

Where $S_j^r(\theta_1)$ is the log-likelihood ratio for the observations from point j up to point r and $\upsilon_m$ stands for a minimum magnitude of change in parameter $\theta$. The GLR statistic is found by calculating the log likelihood ratio first, then by using double maximization for two unknown independent parameters: change point and the value of the parameter after change (Riedel et al., 1994).

### 2.1.4 Two Phases in Control Chart Design

When the parameters that represent a particular quality characteristic of a process are unknown, control charts can be utilized in a two-step process with 2 different objectives. The first step, called Phase I, involves using control charts to examine a set of process data retrospectively. This step helps to establish an in-control state for the process and assess its stability to ensure a good representation of the process. Once an in-control reference sample is established, the parameters of the process are estimated, and control limits are determined for use in the second step, called Phase II.

In Phase II, the process is monitored prospectively by collecting samples and checking them for any deviations from the in-control process state. If the chart's control statistic remain within the control limits, the process is considered to be stable or in-control. However, if the control statistic falls outside of the control limits, it may indicate that the process is out of control, and corrective action may be necessary.

### 2.2 Short Run Applications of Control Charts

The literature has proposed self-starting control charts as a means of monitoring a process with limited available data. These charts are particularly useful in rapidly monitoring a process without the need to gather a substantial Phase I sample to estimate the in-control process parameters. In other words, self-starting charts for short runs can be employed to quickly initiate process monitoring without requiring a large amount of relevant data.

Self-starting cumulative sum (CUSUM) charts for tracking changes is introduced for both location and scale (Hawkins, 1987). This method involves continuously updating the sample mean and standard deviation with each new process sample that does not trigger an out-of-control signal. These updated statistics are then used as estimates for the unknown process parameters. A series of sequential Q statistics that can be utilized in Shewhart, EWMA, and CUSUM charts has also been introduced for detecting changes in the process mean or variance (Quesenberry, 1993, 1989, 1991a). Essentially, these statistics are used to monitor the process and detect any changes that may occur over time, even when historical data is not available. Effectiveness of EWMA and CUSUM of Q charts are examined and the characteristics of Q charts for variables are investigated in Quesenberry, 1989).

The design of the CUSUM Q chart was enhanced by taking into account the changes in the distribution of Q statistics that occur after a shift (Zantek & Smith, 2006). This improvement has increases the capability of the CUSUM Q chart to detect a wide range of shifts. On the other hand, effectiveness of ACUSCORE control charts for detecting sustained shifts in the mean of a process that follows a normal distribution was also investigated and it is demonstrated that they possess greater detection power than the CUSUM Q and EWMA Q charts (Capizzi & Masarotto, 2012).

As an alternative control chart, t chart, was suggested and evaluated in Zhang et al., (2009). The results indicated that t charts were more robust to variations in the standard deviation of the process compared to the traditional $\bar{X}$ chart. The t control chart was applied to short-run production processes with both perfect and imperfect setups (Celano et al., 2011). The study demonstrated that the EWMA t chart was most efficient in detecting shifts in the process mean when compared to the Shewhart t chart.

Additionally, a change-point model using the likelihood ratio for online monitoring is also developed, which could also be used as a self-starting approach (Hawkins et al., 2003; Hawkins & Zamba, 2005).

GLR charts have distinct advantages compared to other control charts. They are based on likelihood ratio tests and offer the benefit of not requiring additional control chart parameters aside from the control limit (Huang et al., 2012). GLR charts have been proven to be highly effective in detecting various types of parameter changes in a process. GLR charts used for monitoring mean and variance of a normally distributed process variables were shown to have good performance in detecting a wide range of shifts (Reynolds et al., 2013). GLR charts were also used for monitoring process proportions. Poisson count data has also been particularly studied, and it has been observed that GLR charts' performance is comparable to the CUSUM charts' performance. For some cases, GLR has been shown to be clearly superior to CUSUM (Lee & Park, 2014). GLR chart statistics have also been derived for count processes like Binomial, Bernoulli, Poisson, where(Lee & Woodall, 2018) has simplified the design of GLR charts without arbitrary bounds.

### 2.3 Quality Control in Software Development Life Cycle

The use of Statistical Process Control (SPC) as a process improvement tool in the software industry began in the 1980s (Burr & Owen, 1996; Carleton & Florac, 1999). One of the initial studies applying SPC to software processes is presented in (Lantzy, 1992). In this work, a comprehensive guideline consisting of seven steps to effectively implement SPC within a software organization is presented. The study emphasizes several important principles for SPC application in software processes, such as ensuring metrics align with customer-defined quality characteristics, choosing metrics for activities that yield tangible outcomes, restricting SPC implementation to critical processes, and verifying process capability to deliver desired software products. SPC is based on the use of control charts that monitor and evaluate performance once acceptable process variation limits have been established. Effective monitoring mechanisms are critical for the success of software projects, and a lack of monitoring can lead to project failures (Doraisamy et al., 2014). Different indicators can be derived from control charts, but some indicators are not frequently used in software processes because they differ significantly from those used in the production sector. Therefore, it is important to understand the nature of software processes to effectively apply SPC. Considering that it is possible to control and improve the quality of primary processes by measuring support processes, critical processes should be selected for SPC application in software processes (Manlove & Kan, 2007b).

In addition to the theoretical benefits of using SPC in software, potential problems in its practical implementation are also considered. One of these studies explores the application of SPC in software while addressing objections and potential implementation challenges (Card, 1994), where an example is provided for a control chart for testing effectiveness and it is concluded that SPC principles can bring advantages to a software organization, even without the need for formal statistical control techniques. The use of SPC charts and managerial training to derive benefits in software development is also emphasized in another study (Hirsch, 1999). The importance of embracing metrics is underscored, establishing defined processes, implementing a metric program, and fostering curiosity about metrics for effective SPC implementation. A case study showcases the utilization of SPC as part of achieving level 4 maturity (Meade, 1999), where the need for data comprehension is highlighted and it is noted that SPC may not be applicable to all software metrics. The study reveals that smaller programs tend to exhibit better performance when utilizing SPC techniques.

SPC is also widely used in organizations that follow the CMMI (Capability Maturity Model Integration) framework. CMMI is a process improvement model that helps organizations enhance their software development and management processes. Especially at higher maturity levels, such as CMMI Level 4 (Quantitatively Managed) and Level 5 (Optimizing), organizations focus on quantitative process management and continuous improvement (Software Engineering Institute, 2010). Hence, SPC plays a significant role in these levels by providing data-driven insights into process performance, identifying areas for optimization, and facilitating data analysis for decision-making.

Control charts are a valuable tool used in CMMI-certified organizations to monitor and control process performance, identify outliers, analyze trends, or establish performance goals. In (Humphrey, 1989), in which a framework of software process management is summarized, attention is drawn to the use of SPC as a data analysis method for organizations operating at the level 4. Note that SPC has emerged as a significant topic during the High Maturity Workshop held in 2001 and there are some recommendations about SPC usage for quantitative control (Paulk & Chrissis, 2002). In general, control charts enable CMMI-certified organizations to have a visual representation of process performance, make data-driven decisions, detect issues early on, and guide their process improvement efforts. Measurement and analysis, process control, and continuous improvement principles emphasized in the CMMI framework are supported by control charts. CMMI recognizes the direct relationship between improved processes and improved products and provides organizations with a framework to guide them in achieving these improvements.

Measurement of processes is a fundamental aspect of process improvement, as it helps understand current performance, identify areas for improvement, and monitor progress over time. To improve processes, it is necessary to define metrics. Careful selection of metrics is important for them to effectively represent process performance. One of the metric selection methods used in software projects is the Goal-Question-Metric (GQM) approach (Basili, 1993; Lazic & Mastorakis, 2008). The result of implementing the GQM approach is the establishment of a set of rules for interpreting a measurement system and measurement data targeting a specific subject area. The GQM approach is a methodology used to define and measure metrics in the context of process improvement. By aligning metrics with specific goals and questions, organizations can obtain meaningful and actionable data that guide their improvement efforts. Organizations can monitor the performance of their obtained metrics using this method and, accordingly, improve their processes to increase customer satisfaction, competitiveness, or achieve better business outcomes.

Careful selection of metrics is important as they provide enhanced comprehension of a project's advancement and aids in more effective project management (Downey & Sutherland, 2013) . Metrics offer transparency and understanding regarding our actions and the level of success achieved (Neto et al., 2006). Some of the important considerations for the selection of metrics are relevance to objectives, actionability, context, and level of detail. The metrics should be aligned with the project and business objectives, the determined metrics should provide insight for actions that will provide improvement, the data should be selected to reflect the metric to be measured, and the data should be collected and measured according to the level of detail determined for tracking.

Some of the metrics commonly used in software development projects are defect density, test coverage rate, defect removal efficiency, lead time and cycle time. These metrics can be used to monitor the performance of processes. Defect density is the number of defects per function points or defects per KLOC, where KLOC represents 1,000 lines of code (Malaiya & Denton, 1998). Test coverage rate is the rate at which codes are covered by tests and it can help find areas that are not covered by tests (Grinwald et al., 1998).Lead time measures the total time from the start of a task to its completion. Total time includes time spent developing, testing, reviewing, and other activities related to the delivery of the work item. Defect removal efficiency is the one of the important metric in software development projects which is the proportion of defects identified and resolved prior to software applications being delivered to customers, expressed as a percentage (Jones, 2008). Cycle time and lead time are important to identify bottlenecks and improve flow (Humble et al., 2014). Cycle time, unlike lead time, measures the time that the relevant work item is actively worked on, without including waiting or idle time. Lead and cycle time can be measured in different hours, days, or weeks depending on the level of detail required, and these metrics can be applied at different levels, such as individual tasks, user stories, or entire project releases. Time metrics can be tracked to gain insight into efficiency, identify bottlenecks, or make improvements to optimize delivery processes.

In software development projects, metrics that are input to process improvement are defined and monitored. However, the success of the project does not only depend on the development processes. Monitoring the scope in project management is also important to project success. If scope is not well-managed, more workload than planned may result, resulting in excess cost, poor quality, or inability to deliver on time. Metrics can also be defined for tracking scope. However, this is not a very common practice. Since metrics are need-based in nature, it may be a suggestion to identify and measure requests that differ from additional scope from the customer. With this measurement, it can be expected to support scope management by creating an alarm in case of more additional requests than expected.

Monitoring the determined metrics is as important as determining the metrics. The use of control charts is crucial for monitoring software performance. However, it may not always be possible to apply traditional methods. Control charts require a sufficient amount of data to perform Phase I analysis, which may not be feasible in software development projects. Therefore, alternative approaches, such as monitoring the code review rate metric for different review sequences using the developed Q-chart for short-term, have been utilized (Chang & Tong, 2013).

# 3. METHODOLOGY

Within the scope of this study, it is aimed to compare the performance of self-starting (Q and EWMA Q) and change point (GLR) control charts in detecting the shifts in Poisson process rate, considering limited time software development projects where there is not enough data to perform Phase I analysis. It is assumed that the process performance measurements can be modelled with a Poisson distribution having a known rate of process.

### 3.1 Q Chart for Poisson Distribution

Let the count of defects Y found in a standard size product unit that has been inspected, and the rate of defects occurring in that standard size unit is constant (known as a stable process), then Y can be modelled as a Poisson random variable. The Poisson probability function f (y;λ) and distribution function F (y;λ) are as follows:

$$f(y;\lambda) = \frac{\lambda^y e^{-\lambda}}{y!}, \; y = 0, 1, 2, \ldots$$

$$F(y;\lambda) = P(Y \leq y) = 0, \text{ for } y < 0$$

$$P(Y \leq y) = \sum_{y'=0}^{y} f(y';\ \lambda), \text{ for } y \geq 0$$

where λ is the rate of defects, with the known rate of defects in each unit being $\lambda = \lambda_0$ per standard inspection unit when the process is in control. Note that in cases when λ is not known, a target value can be used instead.

Let $y_r$ indicate the counts of defects found in a sample comprising $n_r$ standard inspection units at sample number or time r. The set of observed values $(n_1, y_1)$, $(n_2, y_2)$ … are converted into Q-statistics (Quesenberry, 1989);

$$u_r = F(y_r; n_r\lambda_0)$$

$$Q_r = \varphi^{-1}(u_r) \quad r = 1,2, \ldots$$

where $\varphi^{-1}$ is the inverse of the standard normal distribution function and $n_r$ =1 represents the case of individual observations.

The values $Q_1, Q_2, \ldots$ are distributed in a manner that is quite similar to independent standard normal statistics. It has been demonstrated that this normal approximation is far superior to the typical normal approximation for the Poisson distribution (Quesenberry, 1991b).

To detect a shift in $\lambda$ many tests were used and results were compared (Quesenberry, 1989). The 1-of-1 test, the 4-of-5 test are some examples of tests for detecting a shift in $\lambda$.

### 3.2 EWMA Q Chart for Poisson Distribution

EWMA is the one of the traditional control charting methods to detect small shifts in processes. EWMA charts can also be constructed by using Q statistics when the process metrics are distributed as Poisson. The EWMA statistic using Q is as follows:

$$Z_r = \alpha Q_r + (1 - \alpha) Z_{r-1} \quad r = 1,2,\ldots$$

with the control limits being:

$$\pm K \sqrt{\frac{\alpha}{2-\alpha}}$$

where $\alpha$ is smoothing constant and factor K is the width of control limits.

With parameters $(\alpha, K)$ = (0.25, 2.90) control limits calculated are $\pm 1.096$ when the in-control ARL value nearly equals 372 (Crowder, 1989).

In this thesis, upper control limits for the EWMA Q chart are calculated for each smoothing constant $\alpha$ value corresponding to different $\lambda_0$ values. When $\lambda_0$ is known and its value is 3, upper control limits calculated are; UCL = 0.45 for $\alpha$ = 0.05, UCL = 0.64 for $\alpha$ = 0.1, UCL = 1.02 for $\alpha$ = 0.25, and UCL = 1.33 for $\alpha$ = 0.4. In the case that $\lambda_0$ is known and its value equals 12, UCLs calculated are; 0.33, 0.51, 0.90 and 0.51 for $\alpha$ = 0.05, $\alpha$ = 0.1, $\alpha$ = 0.25 and $\alpha$ = 0.4, respectively. Details of $ARL_0$ and false alarm probabilities for each smoothing constant value are given in Table 6 and Table 10.

### 3.3 GLR Chart for Poisson Distribution

The GLR control chart utilizes a statistical test that uses maximum likelihood estimation to calculate values for unknown parameters, and is capable of delivering optimal detection outcomes since it employs the log-likelihood ratio to enhance the detection rate for a specified false alarm rate (Reynolds & Lou, 2010).

The Poisson GLR control chart was developed to monitor a Poisson process (Lee & Park, 2014). Let us assume that the count data y fits to a Poisson distribution with parameter $\lambda$. Here it is presumed that the Poisson process rate $\lambda_0$ is known or has been accurately estimated and the purpose of the control chart is to detect an increase in λ from the in-control parameter value $\lambda_0$.

Assume again that independent Poisson count data collected from a process is represented as $Y_r$ where r is the sample number or time. Then, the likelihood function at sample r is:

$$L\ (\tau,\ \lambda_1 \mid y_1, y_2, \dots, y_r\ ) = \frac{e^{-\tau\lambda_0}\lambda_0^{\sum_{i=1}^{\tau} y_i}}{\prod_{i=1}^{\tau} y_i!}\ \frac{e^{-(r-\tau)\lambda_1}\lambda_1^{\sum_{i=\tau+1}^{r} y_i}}{\prod_{i=\tau+1}^{r} y_i!}$$

That is, at observation r, we consider the hypothesis that a shift to a value of $\lambda_1 > \lambda_0$ has occurred after point τ, and the process data after point τ follows a Poisson distribution with parameter $\lambda_1$, where $\tau < r$.

If the shifted value $\lambda_1$ is unknown, then the maximum likelihood estimator of $\lambda_1$ is:

$$\hat{\lambda}_{1,\tau,r} = \frac{1}{r-\tau}\sum_{i=\tau+1}^{r} y_i$$

Then the log-likelihood ratio statistic utilized to determine if there has been a shift in $\lambda_0$ and is given by:

$$R_r = \ln \frac{\max_{0\le\tau<r,\ \ 0\le\lambda_1<\infty} L\ (\tau, \hat{\lambda}_{1,\tau,r}\ \mid y_1, y_2, \dots, y_r\ )}{L\ (\infty, \lambda_0 \mid y_1, y_2, \dots, y_r\ )}$$

$$= \max_{0\le\tau<r}\left[(r-\tau)\left(\lambda_0 - \hat{\lambda}_{1,\tau,r}\right) + \sum_{i=\tau+1}^{r} y_i\ (\ln\left(\hat{\lambda}_{1,\tau,r}\right) - \ln(\lambda_0))\ \right]$$

The difficulty of using $R_r$ directly has been mentioned in Reynolds & Lou (2010). This difficulty is computational as all historical data must be followed and the maximum in the equation must be found for each $0 \le \tau < r$ at each observation r. Some modifications by considering a moving window of the past m observations on $R_{kr}$ have also been introduced to reduce the computational burden (Willsky & Jones, 1976). However, in this study, the proposed moving-window approaches will not be used since the size of the historical data is not very large to have computational difficulty.

Only upper control limits are used in this study. For the known Poisson process rate $\lambda_0$ = 3 and $\lambda_0$ = 12 values for the GLR chart, the UCL was calculated as 3.01 and 3.30, respectively.

### 3.4 Simulation Model

In cases where the data is insufficient for a Phase I analysis, self-starting methods and change point models are useful for process monitoring. Assigning values to “unknown" change point $\tau$ and rate of performance metric $\lambda_0$, Q, EWMA Q and GLR charts were simulated to evaluate and compare the performance of the charts for different change point scenarios under different shift magnitudes for different project durations. The EWMA Q charts were also evaluated with different smoothing constants $\alpha$. The simulation details and assumptions made are explained in the following.

For the simulations, performance metric considered is the average run length (ARL), where $ARL_0$ indicates the average number of points plotted on a control chart until a control limit is exceeded by the control statistic when the process is actually in control (i.e. a false alarm) and $ARL_1$ indicates the average number of points plotted on a control chart until a control limit is exceeded by the control statistic when the process is out-of-control control (i.e. a true alarm). Note that a control chart having a large $ARL_0$ and a small $ARL_1$ is preferable.

In the study, for comparisons of the detection performance $ARL_1$, control limits of the control charts were determined to provide the same $ARL_0$ for different chart types. Control limits for different chart types and their parameters are shown in Table 6. Please note that classical charting methods assume 370 for $ARL_0$ where $3\sigma$ limits are utilized. However, in this study, considering that the duration of the software projects is not that long, the limits are set to yield $ARL_0 \approx 83$.

Table 6. Control limits for Q, EWMA Q and GLR charts when $\lambda_0 = 3$

| **Chart Type** | **Upper Control Limit** | **$ARL_0$** | **Probability of False Alarms** |
|---|---|---|---|
| Q | 2.66 | 83.78 | 0.013 |
| EWMA Q ($\alpha$ = 0.05, K = 2.82) | 0.45 | 83.87 | 0.151 |
| EWMA Q ($\alpha$ = 0.1, K = 2.78) | 0.64 | 83.51 | 0.061 |
| EWMA Q ($\alpha$ = 0.25, K = 2.70) | 1.02 | 83.74 | 0.023 |
| EWMA Q ($\alpha$ = 0.40, K = 2.65) | 1.33 | 83.95 | 0.016 |
| GLR | 3.01 | 83.88 | 0.106 |

Number of replications for the simulations is specified as 10,000. Software project's duration (D) is selected to be 48 and 64 with the length of every replication of the simulation are determined to be equal to the determined project duration. Shifts in the process is simulated through different "unknown" change points $\tau$. While the data up to a change point are produced from a Poisson distribution with rate of process $\lambda_0 = 3$, the data at the change point and after are produced from a Poisson distribution with process rate $\lambda_1$, where $\lambda_1 = \Delta\lambda_0$ with the shift magnitude $\Delta$ selected to be 1.1, 1.25, 1.5, 2, and 3. For example, when the project duration is 48 weeks and the change point is 6, the first 5 data are generated with performance metric rate $\lambda_0$ and the rest 43 data until the end of the project are produced from $\lambda_1$.

Q, EWMA Q and GLR control charting steps were applied to the generated data. Control statistics plotted on the charts are compared with the upper control limit UCL for an out of control signal. Out of control signals before the change point $\tau$ are omitted and the difference of the signal point from $\tau$ is considered to be the out of control run length (RL) of a replication. For example, assume that the change point is 16 and project duration is 48 months. Then the run length is 32. There may not be any out-of-control signals during a simulation replication period (project duration). In this case, we considered that the run length is greater than the project duration but did not specify its value.

The probability of a process shift being detected during the project duration is calculated with all replications where RL is specified and these are summed up and divided by the total number of replications to obtain the probability.

For a given control chart, while the performance metric rate $\lambda_0 = 3$ does not change to any $\lambda_1$ value (i.e. $\Delta = 1$), the probabilities of receiving false alarms (FA) during the project periods D = 48 and D = 64 are given in Table 7.

Table 7. False Alarm Probabilities

| **Chart Type** | **P(FA \| D = 48)** | **P(FA \| D = 64)** |
|---|---|---|
| Q | 0.44 | 0.54 |
| EWMA Q ($\alpha$ = 0.05) | 0.36 | 0.49 |
| EWMA Q ($\alpha$ = 0.1) | 0.39 | 0.50 |
| EWMA Q ($\alpha$ = 0.25) | 0.43 | 0.53 |
| EWMA Q ($\alpha$ = 0.4) | 0.42 | 0.53 |
| GLR | 0.44 | 0.53 |

# 4. RESULTS AND DISCUSSION

The probabilities of detecting out-of-control signals of the control charts were evaluated for various combinations of project durations, change points, shift magnitudes, and control charts when Poisson process rate $\lambda_0$ is assumed to have a value of 3. Comparisons of the control charts were made over their cumulative out-of-control signal detection probabilities. This is found by summing the out-of-control signal detection probability of each data point with the out-of-control signal detection probabilities calculated for all points preceding it. The results were evaluated under the headings of impact of project duration, effect of change point, effect of shift magnitude and effect of control chart types.

## 4.1 Evaluation of the Impact of Project Duration

When considering short-run processes with insufficient data, the amount of accumulated data is important. When evaluating software processes, the amount of accumulated data is dependent on the project duration. In this study, project data were evaluated on a weekly / monthly basis, and comparisons were made for D = 48 and D = 64.

In the following some representative figures are provided but further comparisons can be found in the tables in Appendices A-J. Consider Figure 1 and Figure 2, respectively for D = 48 and D = 64, when shift magnitude is $\Delta = 1.1$ and change point is $\tau = 0$. As expected, the cumulative probability of detecting an out of control signal increased for all control charts as the project duration increased. The cumulative probability in the charts is at the end point of the x-axis, which is equal to the defined project duration. In Figure 3 and Figure 4, comparisons are also made for different project durations when $\Delta$ is the same as before but $\tau = 12$. Again, the cumulative probability of detecting an out-of-control signal increased for all control charts as the project duration increased. The out-of-control signal detection capability also increases as the project duration increases for different values of $\tau$.

In the given representative figures, it is seen that for smaller $\alpha$ values, the performance of the EWMA Q is not very good immediately after the change point, but increases faster than the others as time progresses. As the project period comes to an end, these charts become comparable to others. Detailed examinations for different cases can be made through the tables in the Appendices A-J.

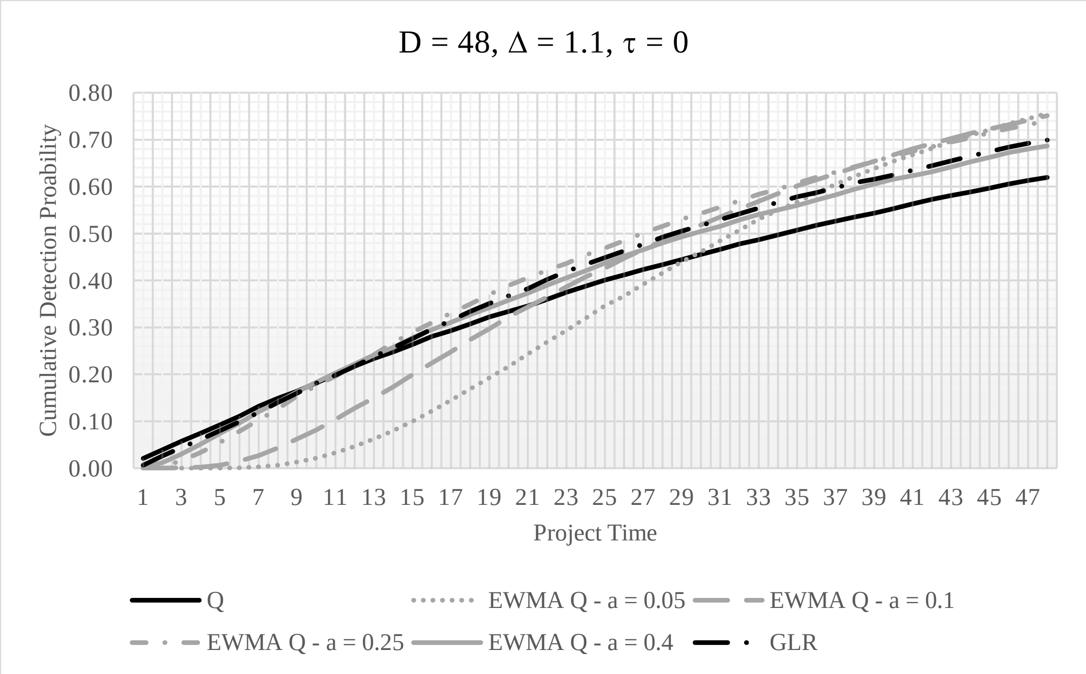


Figure 1. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 0

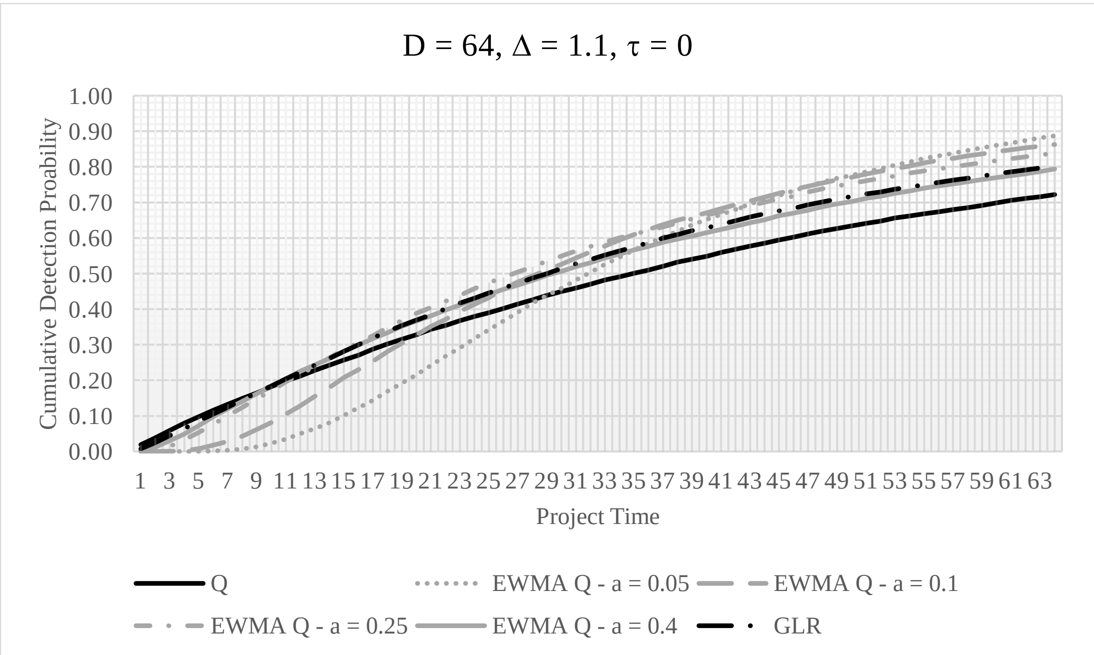


Figure 2. Comparison of Control Charts for D = 64, Δ = 1.1, τ = 0

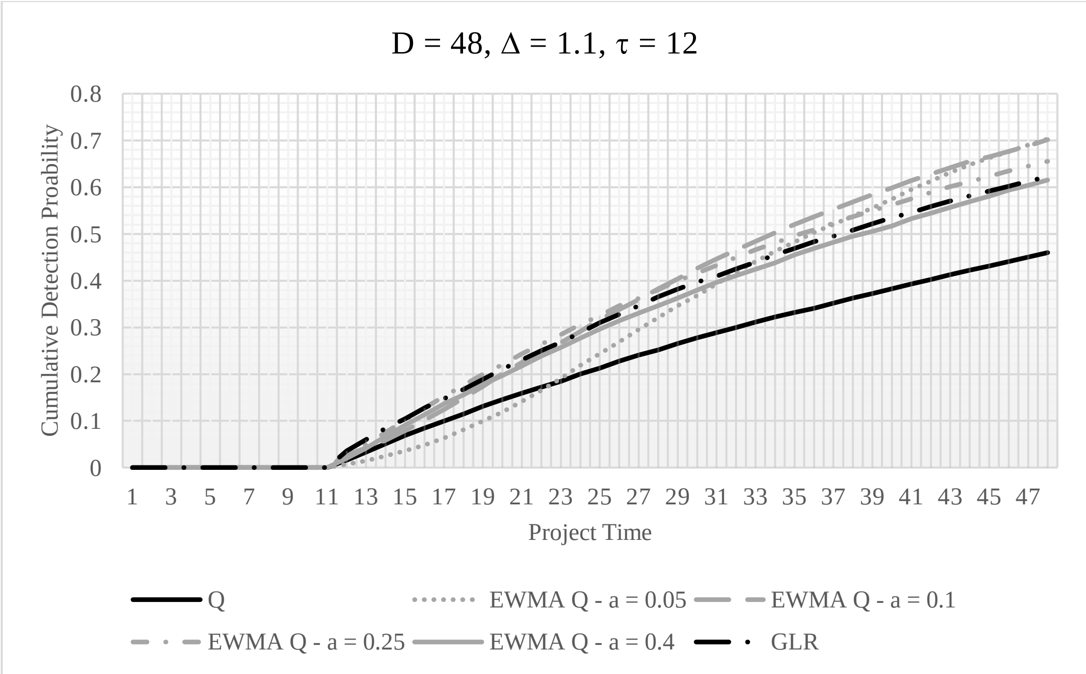


Figure 3. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 12

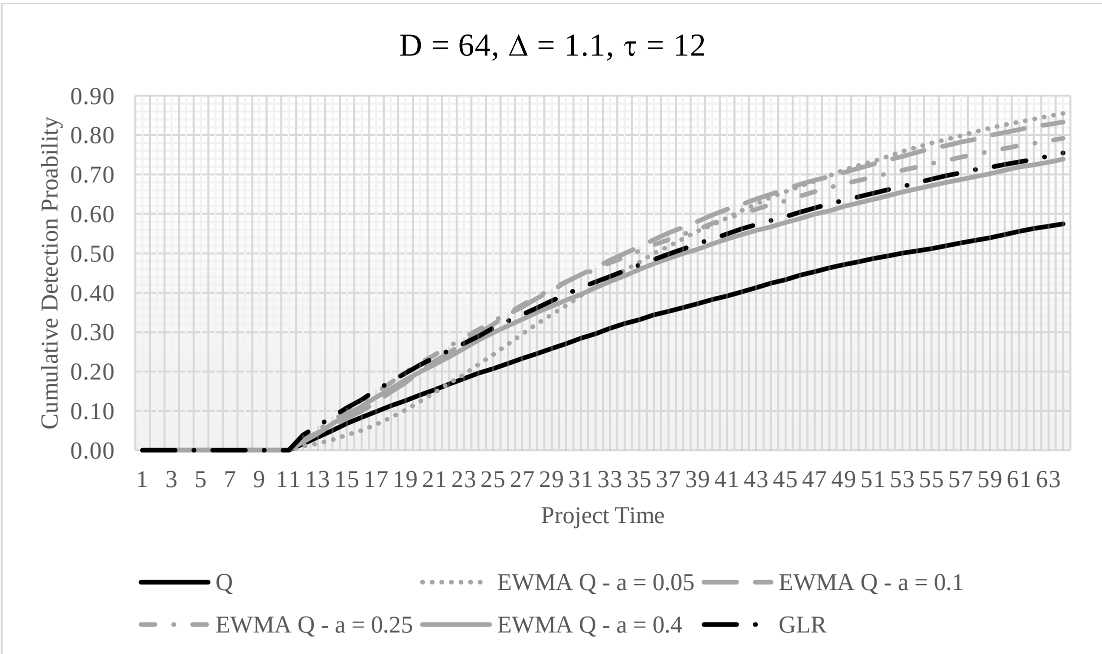


Figure 4. Comparison of Control Charts for D = 64, Δ = 1.1, τ = 12

### 4.2 Evaluation of the Effect of Change Point

A change from the in-control process mean can occur at any point during the project duration. Therefore, comparisons were made for $\tau = 0$ representing the shift at the beginning of the project and other values of $\tau = 3, 6, 9, 12$. Results are given in Figures 5-9, where the project duration is $D = 48$ and shift magnitude is $\Delta = 1.1$. The earlier the process shift occurs at the project, the higher the out-of-control signal detection capability will be, and this is true for all types of control charts. When the detection capabilities after the change point are compared, it is seen that Q and GLR charts stand out.

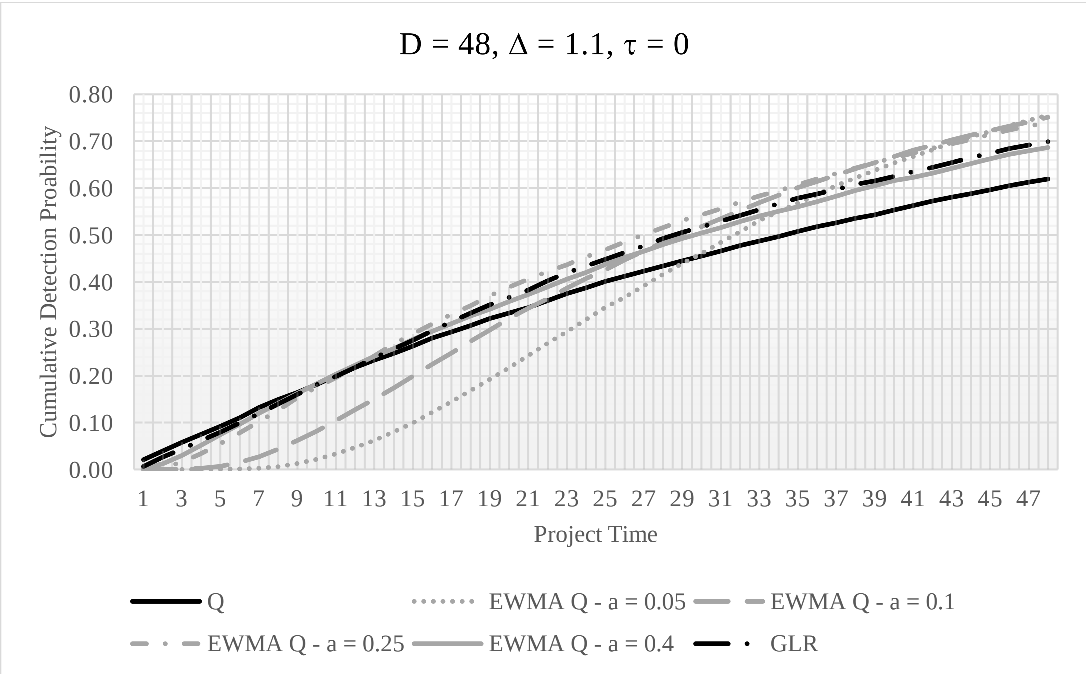


Figure 5. Comparison of Control Charts for $D = 48$, $\Delta = 1.1$, $\tau = 0$

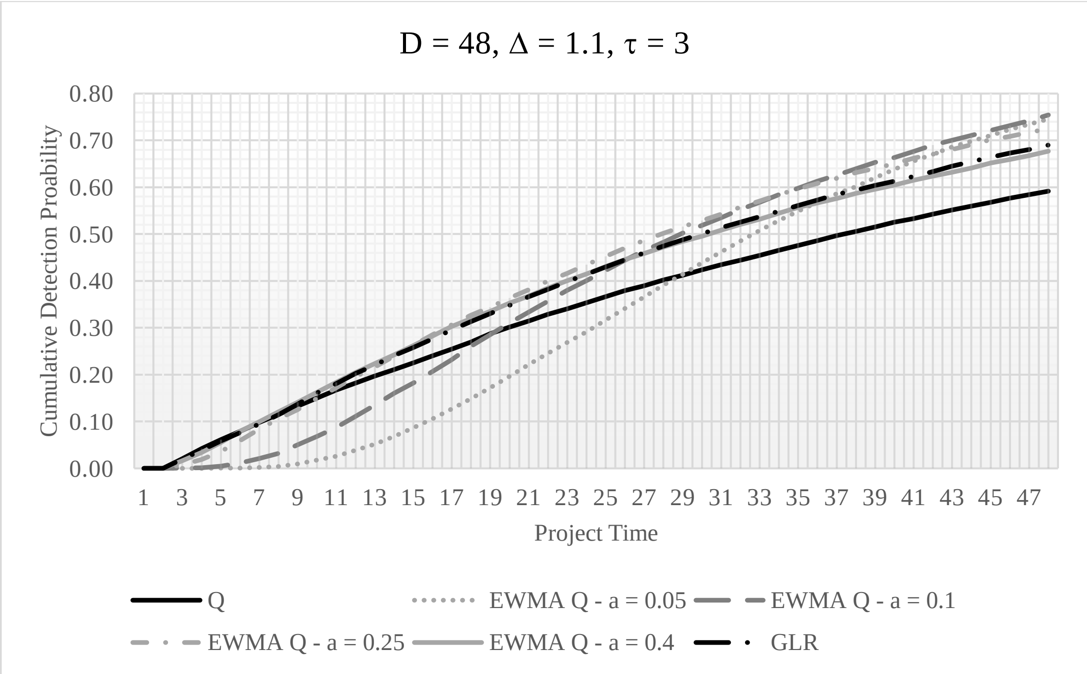


Figure 6. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 3

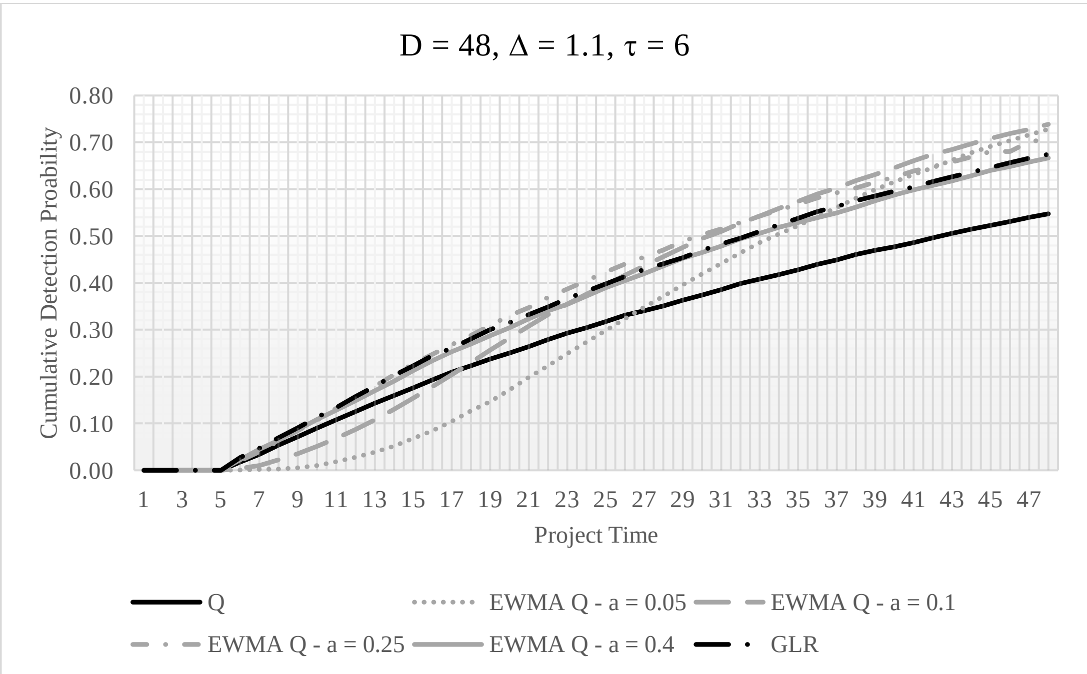


Figure 7. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 6

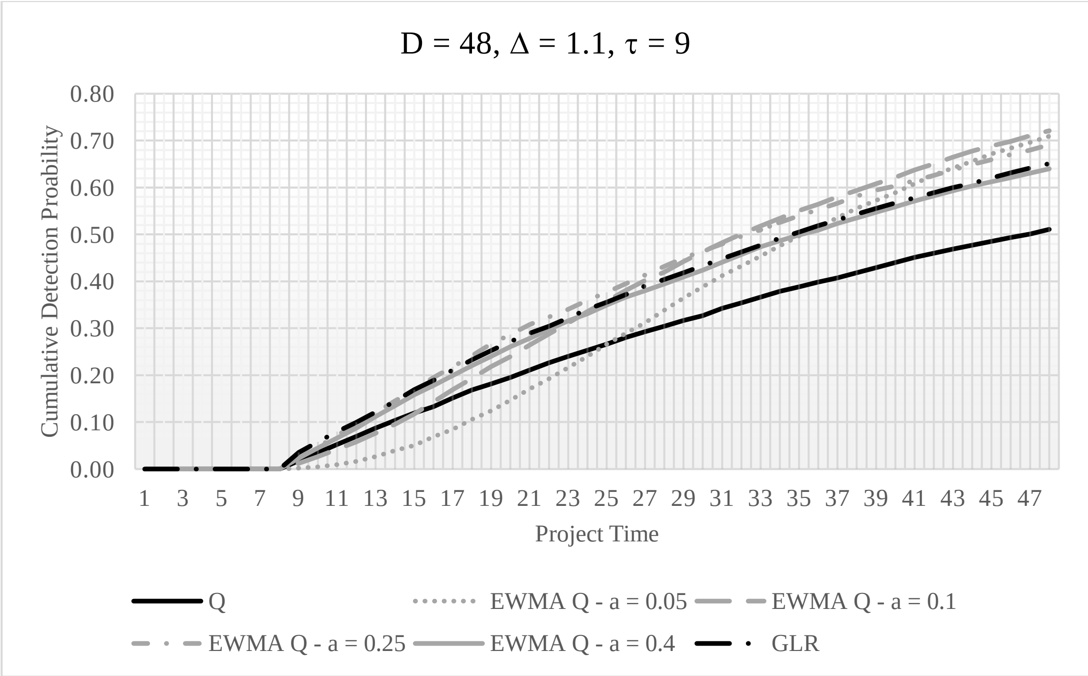


Figure 8. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 9

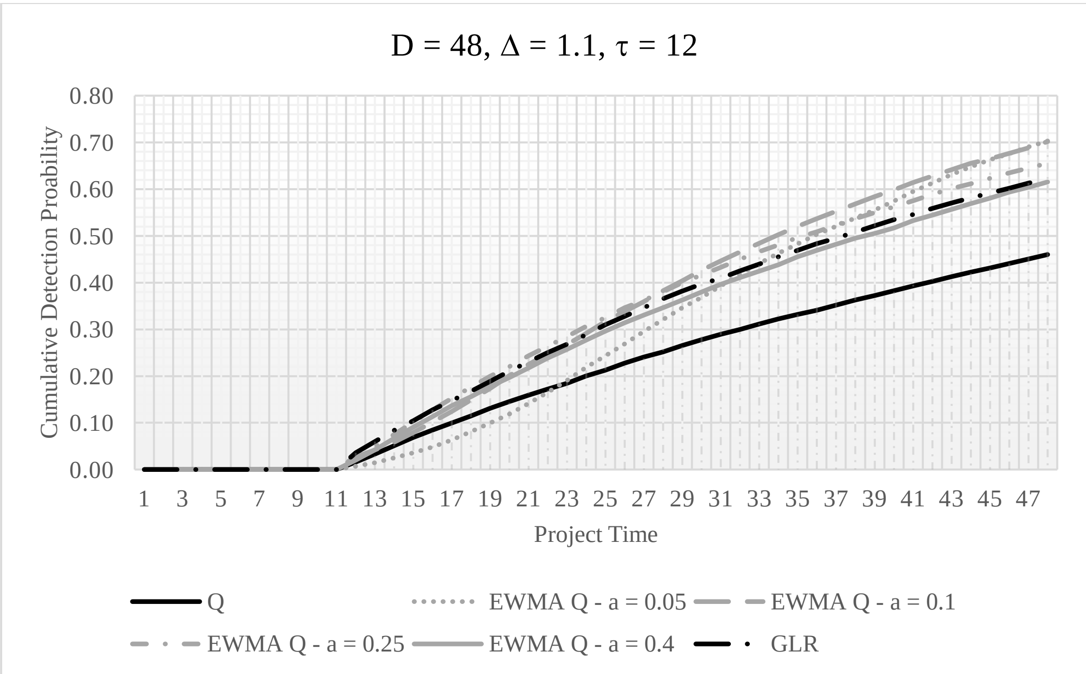


Figure 9. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 12

### 4.3 Evaluation of the Effect of Shift Magnitude

Shifts of different magnitudes may occur throughout the project. In order to simulate this situation, comparisons were made for different Δ values while τ is constant and equal to

0. Graphical comparisons are provided in Figures 10-14. As shift magnitude increases, overall detection capability also increases for all charts.

For the first observations after the change point, the detection capabilities of the EWMA Q charts are lower than those of the Q and GLR charts. At the end of the project, the cumulative detection probability for small values of the Δ is the lowest for the Q chart, while the detection capabilities of the GLR and EWMA Q – α= 0.4 charts are very close to each other. EWMA Q charts can detect out-of-control situations more slowly than Q and GLR charts after the change for large values of Δ.

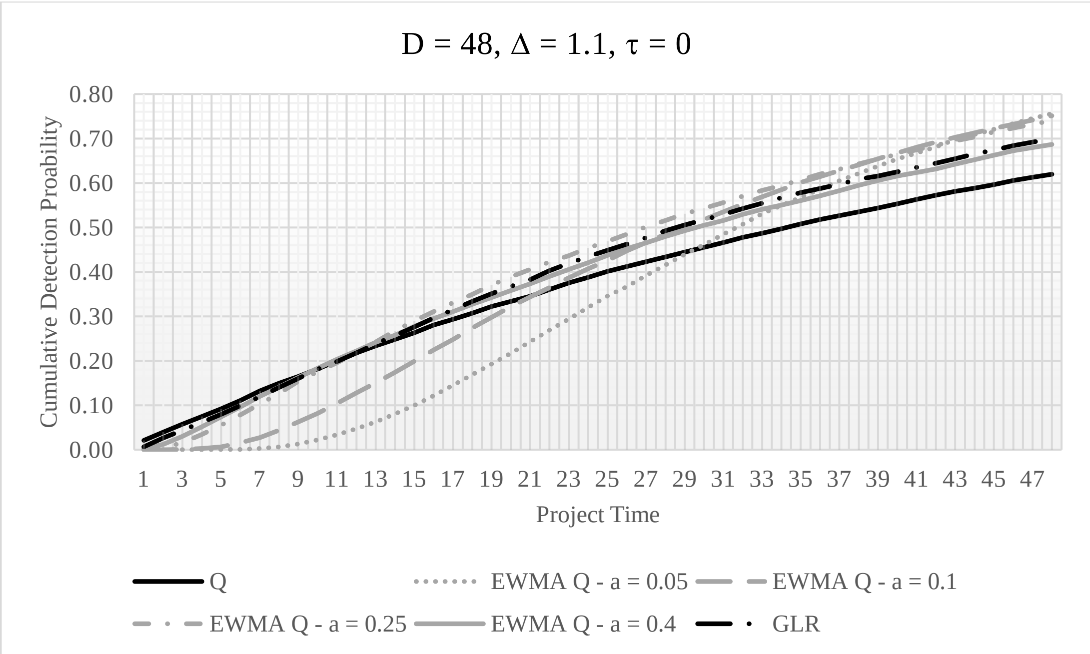


Figure 10. Comparison of Control Charts for D = 48, Δ = 1.1, τ = 0

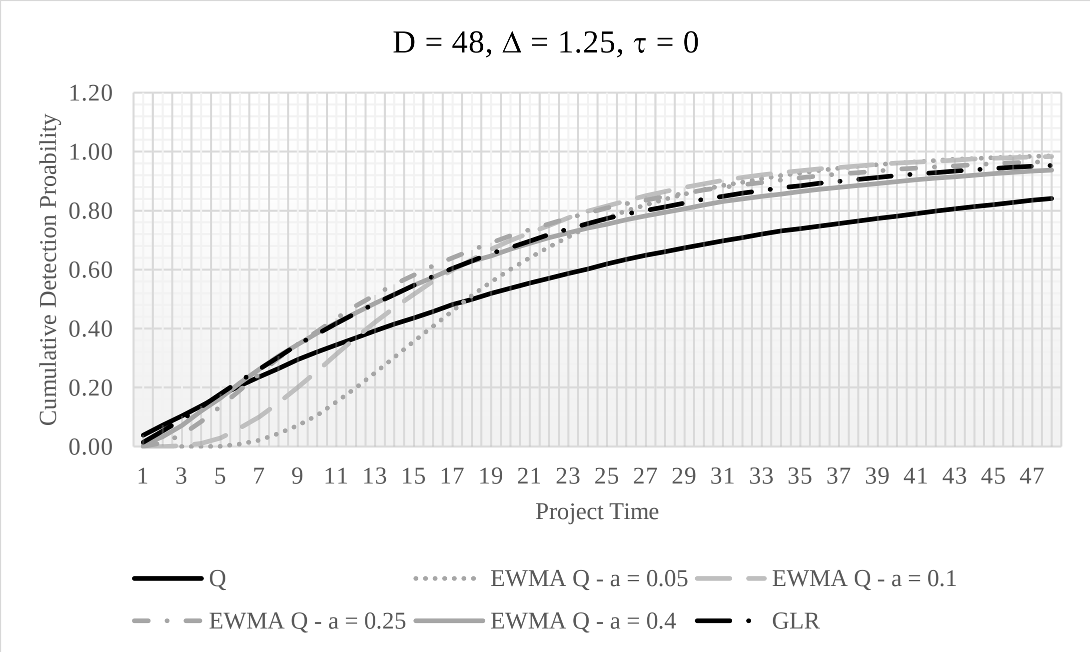


Figure 11. Comparison of Control Charts for D = 48, Δ = 1.25, τ = 0

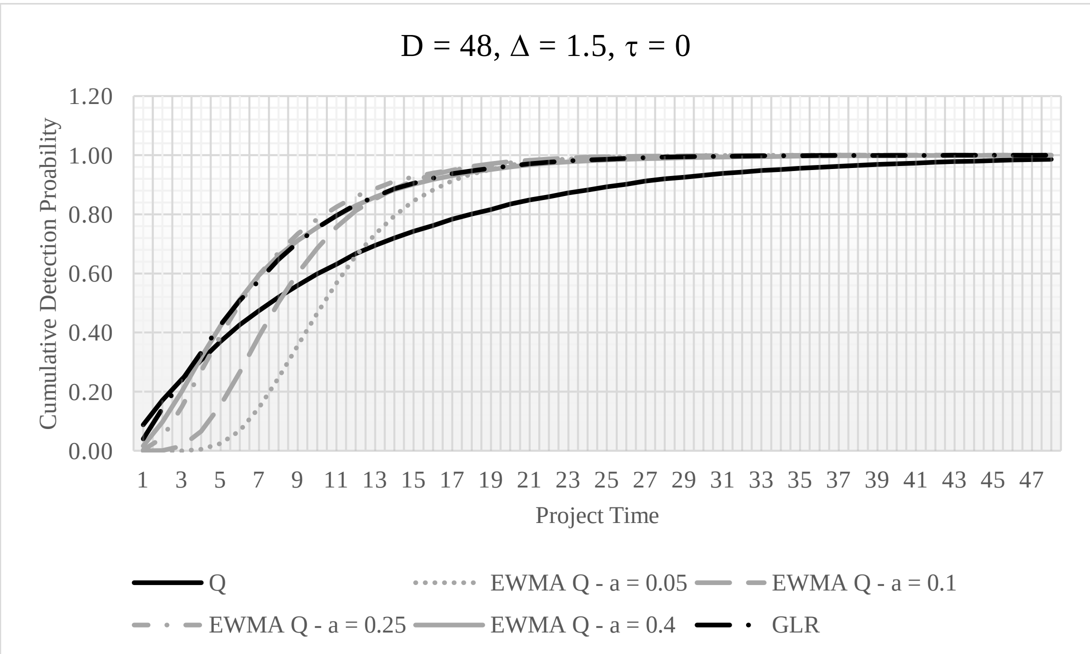


Figure 12. Comparison of Control Charts for D = 48, Δ = 1.5, τ = 0

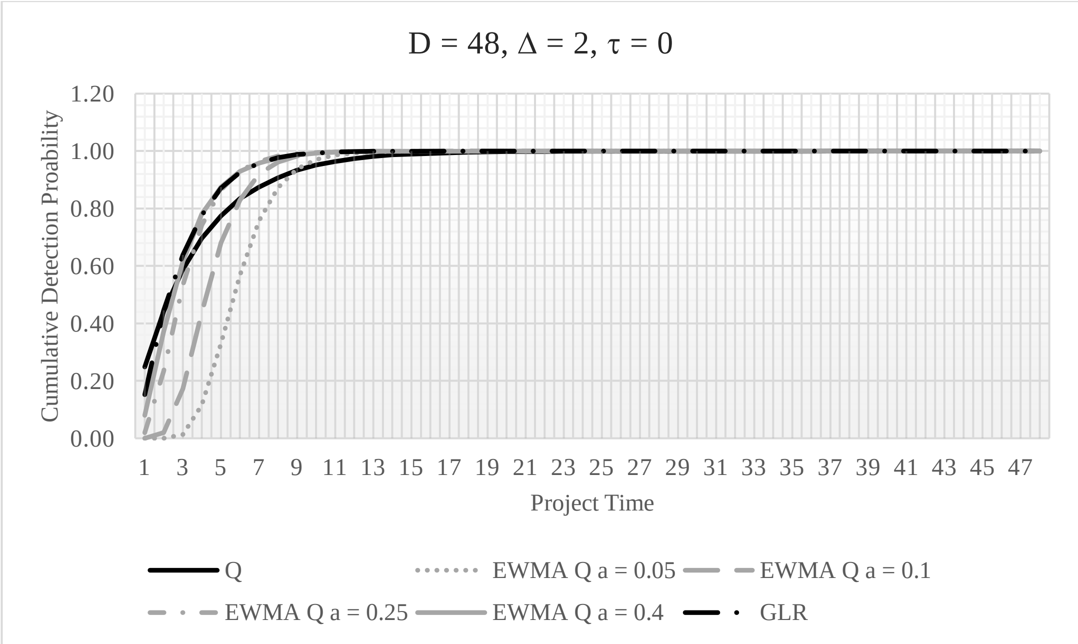


Figure 13. Comparison of Control Charts for D = 48, Δ = 2, τ = 0

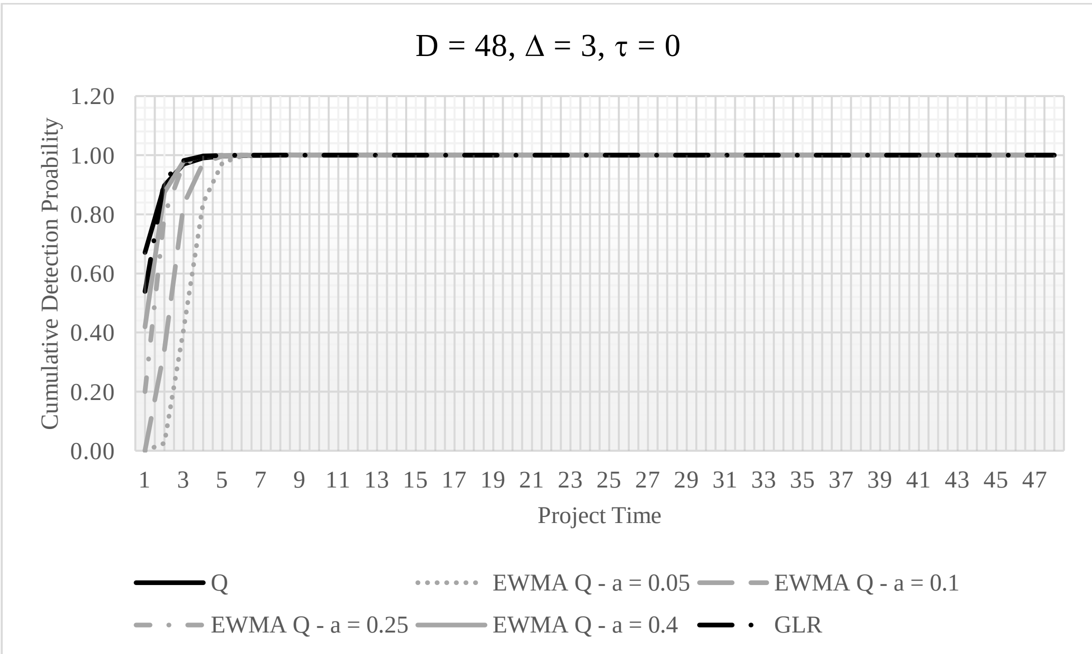


Figure 14. Comparison of Control Charts for D = 48, Δ = 3, τ = 0

### 4.4. Evaluation of the Effect of Control Chart Types

The Q, EWMA Q and GLR control charts were also compared according to their capability to detect out-of-control signals throughout the project period, how early they detected the shifts, and their capacity to detect the shift at the first observation after the change. When the cumulative probability of a process shift being detected during the project duration is evaluated, results for different project duration, shift magnitude and change point are given in Appendices A-J.

From these results, in general, the EWMA Q Chart has the best ability to detect out-of-control signals for both D = 48 and D = 64 but the best smoothing constant differs according to parameters $\Delta$ and $\tau$. As $\Delta$ increases, higher the value of $\alpha$ for EWMA Q charts are a better option. The EWMA Q and GLR charts' out-of-control signal detection performances converge as $\Delta$ increases. The Q chart's ability to detect out-of-control signals is the lowest in all comparisons. For both D = 48 and D = 64, the results are similar to each other but there are also some differences. The disparity between the top outcomes for identical $\Delta$ and $\tau$ values is insignificant enough that it does not impact decision-making. This is thought to be due to the stochastic nature of the simulation.

When we compare control charts according to their ability to detect out-of-control signals at the observation after the change, the Q and GLR charts stand out. These charts have a higher probability of detecting signals at the first observation after the change compared to the EWMA Q charts. Although the performance of the EWMA Q chart for $\alpha = 0.25$ and $\alpha = 0.4$ values is close to each other, for most $\tau$ and $\Delta$ values, the performance of $\alpha = 0.4$ is better. The out-of-control signal detection capability for $\alpha = 0.05$ and $\alpha = 0.1$ values remains lower compared to other control charts. The results are presented in Appendices A-J.

It should also be noted that there is a false alarm probability when the Q, EWMA Q and GLR control charts are evaluated for comparison. The decision should be made according to the structure of the processes to be used and the environment to be used, and the pros and cons of the control charts to be selected should also be considered.

# 5. CASE STUDY

In this section, control charts will be constructed using the known process rate and the upper control limit obtained from simulation results in a software development organization with CMMI Level 5 certification. CMMI Level 5 indicates that organization is performing at an optimizing level.

In this organization, which adopts agile methodologies, software development projects are managed in iterations called sprints. Sprints are short, iterative project increments, meaning that a sprint is a specific period of time during which a certain amount of work is performed. Sprints typically last for 1 or 2 weeks.

The total number of medium and major severity defects included in the sprints is defined as a metric. When the process rate of this metric is known to be 3, Q, EWMA Q, and GLR charts will be examined and compared.

Unlike sprints, metrics can also be set on a monthly basis. The number of monthly requests from the customer is also determined as a metric. When it is known that the process rate of this metric is 12, the Q, EWMA Q and GLR charts will be examined comparatively.

## 5.1 Number of Defects Included in Sprints

In software development, a defect refers to a flaw, fault, or error in the software that causes it to deviate from its intended behavior or produce incorrect or unexpected results. Defects can occur at various stages of the software development lifecycle, including design, coding, testing, and deployment.

Defects are reported on a project management tool and they are related with sprints. In the organization from which the dataset is obtained, defects are classified as minor, medium, and major. As the severity level of a defect increases, the need for resolution also becomes higher. Promptly addressing high-severity defects is critical as they directly impact customer satisfaction.

If there are a significant number of defects included in a sprint that would impact the planned development, it indicates potential issues related to the quality and stability of the software. In this study, the numbers of medium and major defects included in the sprints will be examined. Data table of minor and medium defects based on sprints of one of the teams in a project is given in Table 8.

Table 8. Dataset for Defect Count in Sprints

| **Sprint Number** | **Sprint Completion Date** | **Defect Count** |
|---|---|---|
| 1 | 24.01.2022 | 9 |
| 2 | 14.02.2022 | 3 |
| 3 | 1.03.2022 | 2 |
| 4 | 17.03.2022 | 8 |
| 5 | 7.04.2022 | 2 |
| 6 | 22.04.2022 | 1 |
| 7 | 20.05.2022 | 3 |
| 8 | 15.06.2022 | 0 |
| 9 | 18.07.2022 | 1 |
| 10 | 9.08.2022 | 3 |
| 11 | 26.08.2022 | 0 |
| 12 | 13.09.2022 | 1 |
| 13 | 23.09.2022 | 0 |
| 14 | 11.10.2022 | 2 |
| 15 | 24.10.2022 | 1 |
| 16 | 7.11.2022 | 0 |
| 17 | 21.11.2022 | 0 |
| 18 | 2.12.2022 | 0 |
| 19 | 19.12.2022 | 0 |
| 20 | 6.01.2023 | 0 |
| 21 | 20.01.2023 | 2 |
| 22 | 3.02.2023 | 4 |
| 23 | 17.02.2023 | 3 |
| 24 | 3.03.2023 | 3 |
| 25 | 17.03.2023 | 5 |
| 26 | 31.03.2023 | 4 |
| 27 | 14.04.2023 | 1 |
| 28 | 4.05.2023 | 3 |
| 29 | 22.05.2023 | 2 |

To test the distribution of the dataset, a Goodness-of-Fit Test for Poisson was used. The results, shown in Figure 15, indicate that the observed values for the defects are not very different from the expected values.

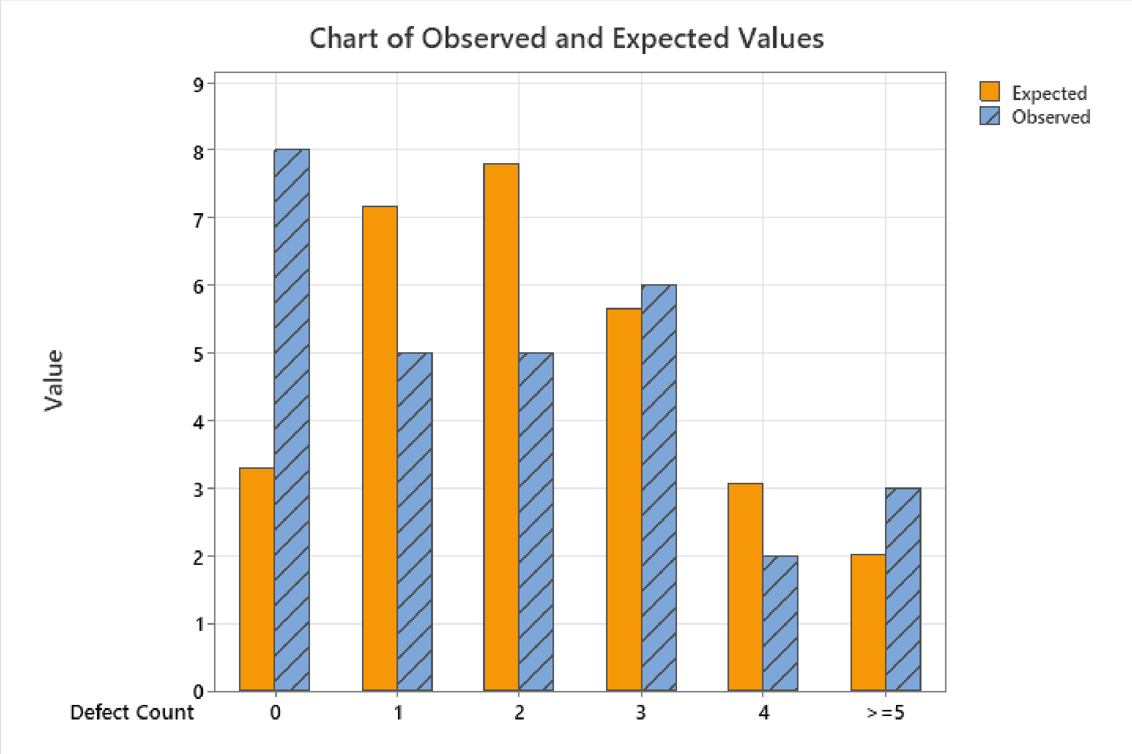


Figure 15. Observed and Expected Values of Defect Counts

As can be seen from the Chi-Square goodness of fit test results in Figure 16, the p-value is 0.056 and for the significance level of 0.01, we can consider the Poisson model as an approximation.

## Chi-Square Test

Null hypothesis — $H_0$: Data follow a Poisson distribution

Alternative hypothesis — $H_1$: Data do not follow a Poisson distribution

| DF | Chi-Square | P-Value |
|---|---|---|
| 4 | 9,21210 | 0,056 |

Figure 16. Chi-Square Test Results for Defect Counts

In Table 9, control statistics calculated for the Q, EWMA Q, and GLR charts for defect count data are given. In the design, $\lambda_0$ is taken as 3. The calculated control statistics are compared with the control limits. Out-of-control data points are shown in bold. For the EWMA Q and GLR charts, which take into account past data, in the event of an out-of-control data point the control charts are reset. Plots of control charts are given in Figures 17 to 22, where the upper control limits are as given in Table 6.

Table 9. Calculated Q, EWMA Q and GLR Statistics for Defect Counts

| **Sprint Number** | **Defect Count** | **Q** | **EWMA Q $\alpha$ = 0.05** | **EWMA Q $\alpha$ = 0.1** | **EWMA Q $\alpha$ = 0.25** | **EWMA Q $\alpha$ = 0.4** | **GLR** |
|---|---|---|---|---|---|---|---|
| 1 | 9 | **3.061** | 0.153 | 0.306 | 0.765 | 1.225 | **3.888** |
| 2 | 3 | 0.378 | 0.164 | 0.313 | 0.668 | 0.886 | 0.000 |
| 3 | 2 | -0.194 | 0.146 | 0.263 | 0.453 | 0.454 | 0.000 |
| 4 | 8 | **2.669** | 0.273 | 0.503 | 1.007 | **1.340** | 2.847 |
| 5 | 2 | -0.194 | 0.249 | 0.434 | 0.707 | -0.078 | 1.108 |
| 6 | 1 | -0.845 | 0.195 | 0.306 | 0.319 | -0.384 | 0.207 |
| 7 | 3 | 0.378 | 0.204 | 0.313 | 0.334 | -0.080 | 0.158 |
| 8 | 0 | -1.647 | 0.111 | 0.117 | -0.162 | -0.707 | 0.000 |
| 9 | 1 | -0.845 | 0.063 | 0.021 | -0.332 | -0.762 | 0.000 |
| 10 | 3 | 0.378 | 0.079 | 0.057 | -0.155 | -0.306 | 0.000 |
| 11 | 0 | -1.647 | -0.007 | -0.114 | -0.528 | -0.842 | 0.000 |
| 12 | 1 | -0.845 | -0.049 | -0.187 | -0.607 | -0.843 | 0.000 |
| 13 | 0 | -1.647 | -0.129 | -0.333 | -0.867 | -1.165 | 0.000 |
| 14 | 2 | -0.194 | -0.132 | -0.319 | -0.699 | -0.776 | 0.000 |
| 15 | 1 | -0.845 | -0.168 | -0.372 | -0.735 | -0.804 | 0.000 |
| 16 | 0 | -1.647 | -0.242 | -0.499 | -0.963 | -1.141 | 0.000 |
| 17 | 0 | -1.647 | -0.312 | -0.614 | -1.134 | -1.343 | 0.000 |
| 18 | 0 | -1.647 | -0.379 | -0.717 | -1.262 | -1.465 | 0.000 |
| 19 | 0 | -1.647 | -0.442 | -0.810 | -1.358 | -1.538 | 0.000 |
| 20 | 0 | -1.647 | -0.502 | -0.894 | -1.431 | -1.581 | 0.000 |
| 21 | 2 | -0.194 | -0.487 | -0.824 | -1.121 | -1.026 | 0.000 |
| 22 | 4 | 0.898 | -0.418 | -0.652 | -0.617 | -0.257 | 0.151 |
| 23 | 3 | 0.378 | -0.378 | -0.549 | -0.368 | -0.003 | 0.079 |
| 24 | 3 | 0.378 | -0.340 | -0.456 | -0.182 | 0.149 | 0.054 |
| 25 | 5 | 1.379 | -0.254 | -0.273 | 0.209 | 0.641 | 0.554 |
| 26 | 4 | 0.898 | -0.197 | -0.156 | 0.381 | 0.744 | 0.649 |
| 27 | 1 | -0.845 | -0.229 | -0.225 | 0.075 | 0.108 | 0.107 |
| 28 | 3 | 0.378 | -0.199 | -0.164 | 0.150 | 0.216 | 0.092 |
| 29 | 2 | -0.194 | -0.199 | -0.167 | 0.064 | 0.052 | 0.021 |

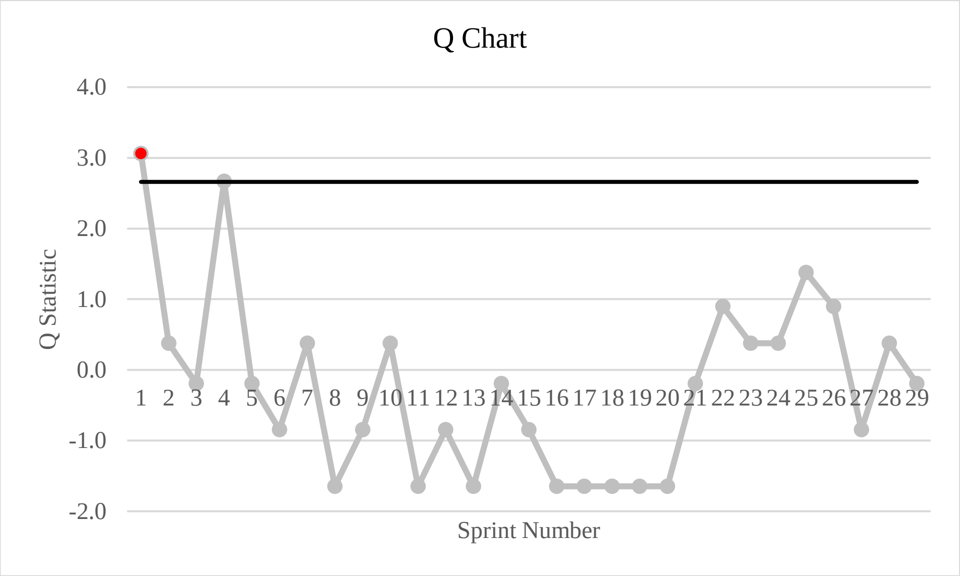


Figure 17. Q Chart for Defect Counts

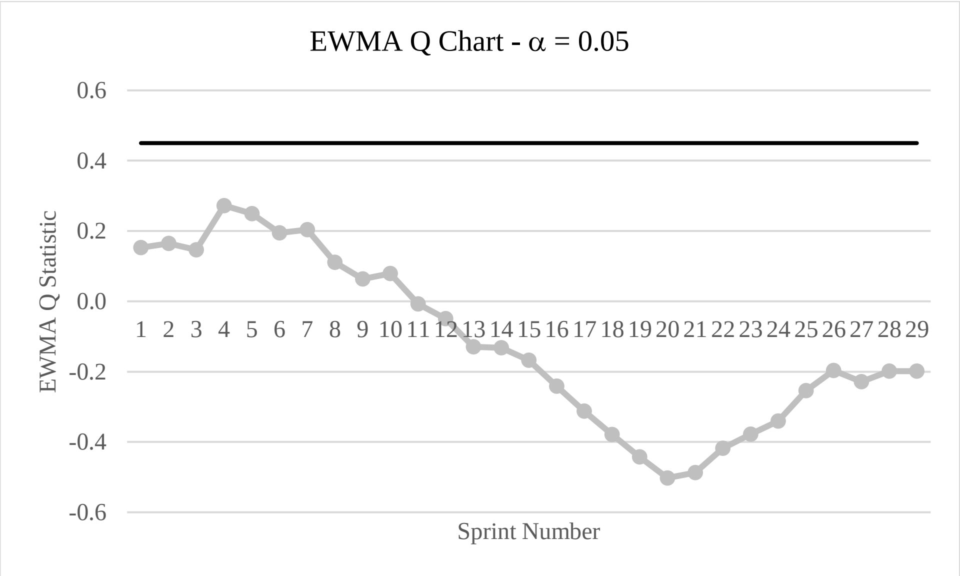


Figure 18. EWMA Q Chart - $\alpha = 0.05$ for Defect Counts

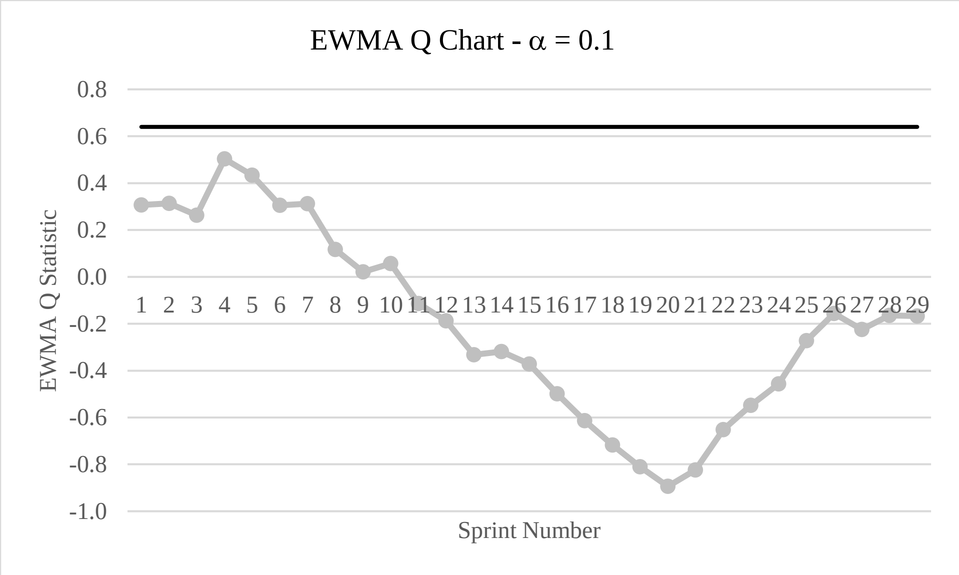


Figure 19. EWMA Q Chart - α = 0.1 for Defect Counts

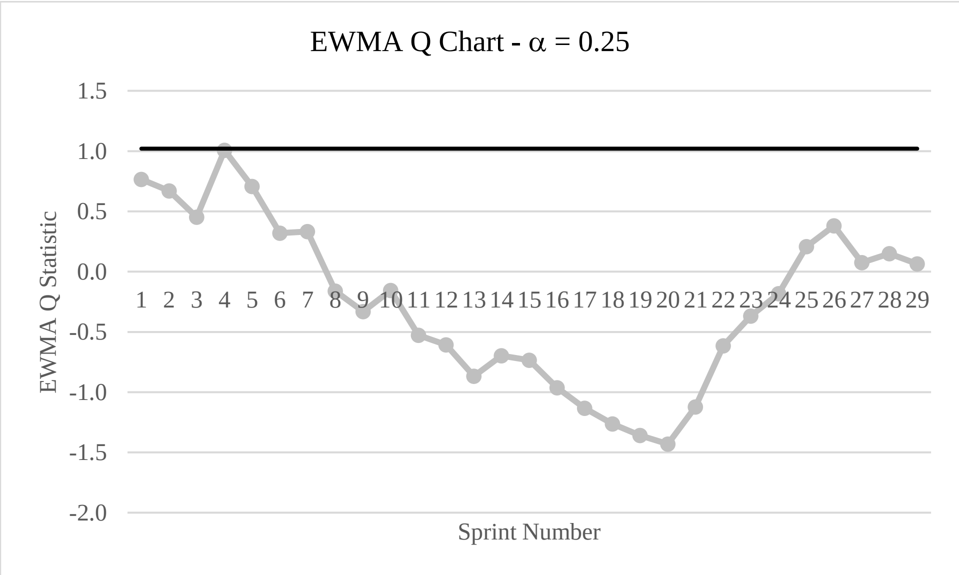


Figure 20. EWMA Q Chart - α = 0.25 for Defect Counts

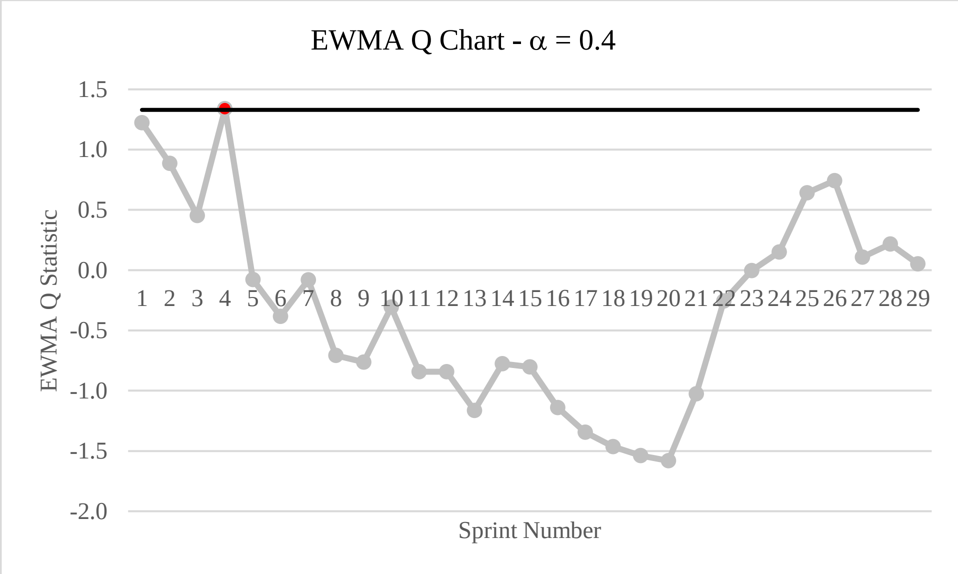


Figure 21. EWMA Q Chart - α = 0.4 for Defect Counts

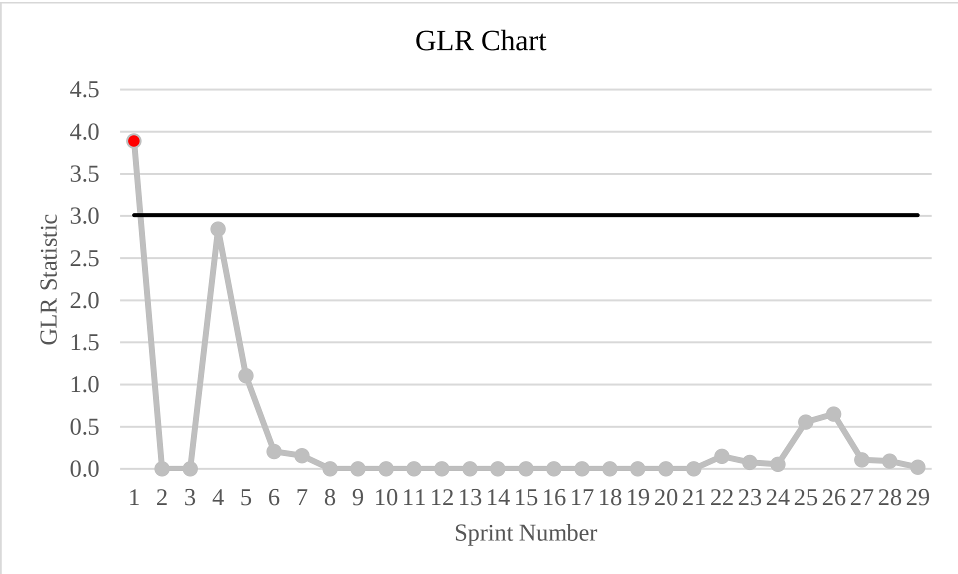


Figure 22. GLR Chart for Defect Counts

When examining the charts, it is observed that the Q and GLR control statistics signaled by exceeding the upper control limits at the same point. In addition, Q chart and EWMA Q Chart – $\alpha = 0.4$ signaled at fourth sprint. In contrast, the EWMA charts do not signal for any $\alpha$ value except $\alpha = 0.4$. Recall that for higher values of $\alpha$ the control chart converges to the classical Shewhart chart. This means that only instant out-of-control situations have occurred, without indicating any specific pattern of out-of-control behavior. This situation is consistent with the expectations for CMMI Level 5 organizations, which are expected to have continuously improving processes. The out-of-control point was determined to be due to the intense utilization of the project's end-of-period activities. Measures were taken to prevent its recurrence. In the following year, during the same period corresponding to the sprints, a high defect rate was not encountered. This indicates that the actions taken were effective.

### 5.2 Customer Requests

In software development projects, the scope of which is determined by a contract, demands from the customers should not expand the scope of the project. If the scope needs to be expanded, it would be appropriate to do so by expanding the scope of the contract, not with additional requests.

However, it is also not uncommon that the demands from the customers may be at a level expanding the scope of projects. In these cases, the contractor may need to tell the customer "these requests are now out of reach". In order to decide on this, the number of requests from a customer for a project is evaluated in the following. Known process rate $\lambda_0$ is taken as 12, considering the historical data. The Q, EWMA Q and GLR control charts were compared under this assumption. Control limits for the Q, EWMA Q and GLR charts were determined with same method mentioned in Methodologies section. The limits were set to yield an $ARL_0 \approx 86$. The limits determined are given in Table 10.

Table 10. Control limits for Q, EWMA Q and GLR charts when $\lambda_0 = 12$

| Chart Type | Upper Control Limit | ARL |
|---|---|---|
| Q | 2.28 | 86.11 |
| EWMA Q ($\alpha$ = 0.05, K = 2.04) | 0.33 | 85.54 |
| EWMA Q ($\alpha$ = 0.1, K = 2.22) | 0.51 | 86.08 |
| EWMA Q ($\alpha$ = 0.25, K = 2.37) | 0.90 | 85.71 |
| EWMA Q ($\alpha$ = 0.40, K = 2.21) | 0.51 | 85.12 |
| GLR | 3.30 | 85.29 |

The data set for customer requests for a certain project monitored for a change of the scope is given in Table 11.

Table 11. Periodic Project Dataset for Customer Requests

| Data Point | Period | Customer Requests |
|---|---|---|
| 1 | Oct-21 | 12 |

| | | |
|---|---|---|
| 2 | Nov-21 | 3 |
| 3 | Dec-21 | 11 |
| 4 | Jan-22 | 17 |
| 5 | Feb-22 | 11 |
| 6 | Mar-22 | 10 |
| 7 | Apr-22 | 13 |
| 8 | May-22 | 3 |
| 9 | Jun-22 | 13 |
| 10 | Jul-22 | 3 |
| 11 | Aug-22 | 21 |
| 12 | Sep-22 | 20 |
| 13 | Oct-22 | 13 |
| 14 | Nov-22 | 13 |
| 15 | Dec-22 | 10 |
| 16 | Jan-23 | 15 |
| 17 | Feb-23 | 18 |
| 18 | Mar-23 | 19 |

The data set is checked for appropriateness of Poisson distribution. The Chi-square goodness of fit test result is given in Figure 23 and considering a significance level of 0.01, the Poisson distribution is again found to be suitable.

## Chi-Square Test

Null hypothesis $H_0$: Data follow a Poisson distribution
Alternative hypothesis $H_1$: Data do not follow a Poisson distribution

| DF | Chi-Square | P-Value |
|---|---|---|
| 3 | 4,25454 | 0,235 |

Figure 23. Chi-Square Test Results for Customer Requests

Calculated Q, EWMA Q and GLR control statistics for customer requests are given in Table 12. Out of control points are showed in bold.

Table 12. Calculated Q, EWMA Q and GLR Statistics for Customer Requests

| **Data Point** | **Q** | **EWMA Q $\alpha$ = 0.05** | **EWMA Q $\alpha$ = 0.1** | **EWMA Q $\alpha$ = 0.25** | **EWMA Q $\alpha$ = 0.4** | **GLR** |
|---|---|---|---|---|---|---|
| 1 | 0.192 | 0.010 | 0.019 | 0.048 | 0.077 | 0.000 |
| 2 | -2.835 | -0.133 | -0.266 | -0.673 | -1.088 | 0.000 |
| 3 | -0.096 | -0.131 | -0.249 | -0.529 | -0.691 | 0.000 |
| 4 | 1.530 | -0.048 | -0.071 | -0.014 | 0.197 | 0.921 |
| 5 | -0.096 | -0.050 | -0.074 | -0.035 | 0.080 | 0.316 |
| 6 | -0.393 | -0.067 | -0.106 | -0.124 | -0.109 | 0.055 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 | 0.472 | -0.040 | -0.048 | 0.025 | 0.123 | 0.092 |
| 8 | -2.835 | -0.180 | -0.327 | -0.690 | -1.060 | 0.000 |
| 9 | 0.472 | -0.148 | -0.247 | -0.400 | -0.447 | 0.041 |
| 10 | -2.835 | -0.282 | -0.506 | -1.008 | -1.402 | 0.000 |
| 11 | **2.508** | -0.142 | -0.204 | -0.129 | 0.162 | 2.752 |
| 12 | 2.270 | -0.022 | 0.043 | 0.471 | **1.005** | **4.956** |
| 13 | 0.472 | 0.003 | 0.086 | 0.471 | 0.189 | **3.895** |
| 14 | 0.472 | 0.026 | 0.125 | 0.471 | 0.302 | **3.344** |
| 15 | -0.393 | 0.005 | 0.073 | 0.255 | 0.024 | 2.209 |
| 16 | 1.013 | 0.056 | 0.167 | 0.445 | 0.420 | 2.551 |
| 17 | 1.782 | 0.142 | 0.328 | 0.779 | **0.964** | **3.663** |
| 18 | 2.028 | 0.236 | 0.498 | **1.091** | **0.811** | **5.115** |

The Q chart created for customer demands signals out-of-control at the 11th point for 21 demands per month. With detailed investigations, it has been determined that the requests from the customers due to the module developed have increased for this month. The plot of the Q chart is given in Figure 24.

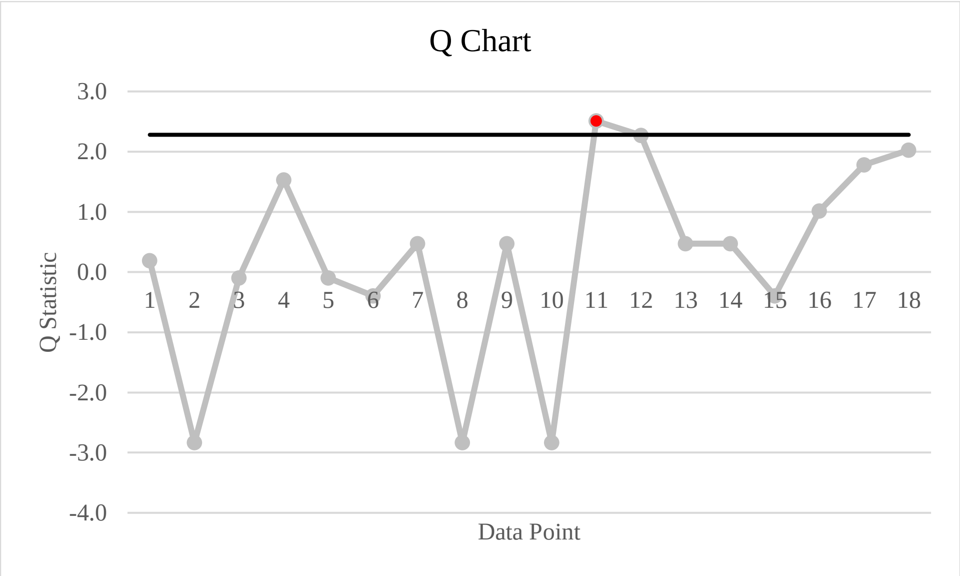


Figure 24. Q Chart for Customer Requests

On the other hand, the EWMA Q - $\alpha = 0.05$ chart created for customer requests did not give any out-of-control signal as shown in Figure 25.

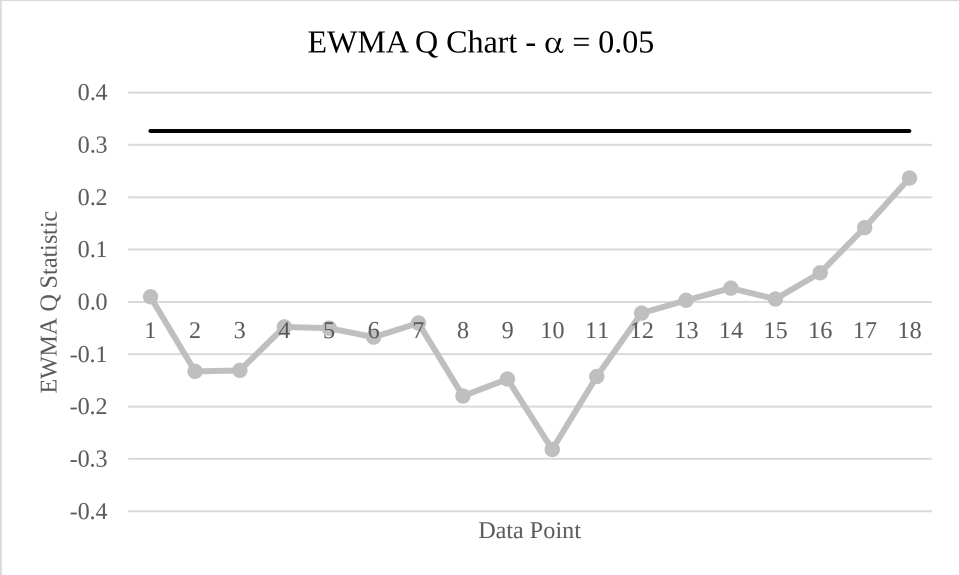


Figure 25. EWMA Q - α = 0.05 Chart for Customer Requests

Similarly, the EWMA Q - α = 0.1 chart created for customer requests did not give any out-of-control signal as shown in Figure 26.

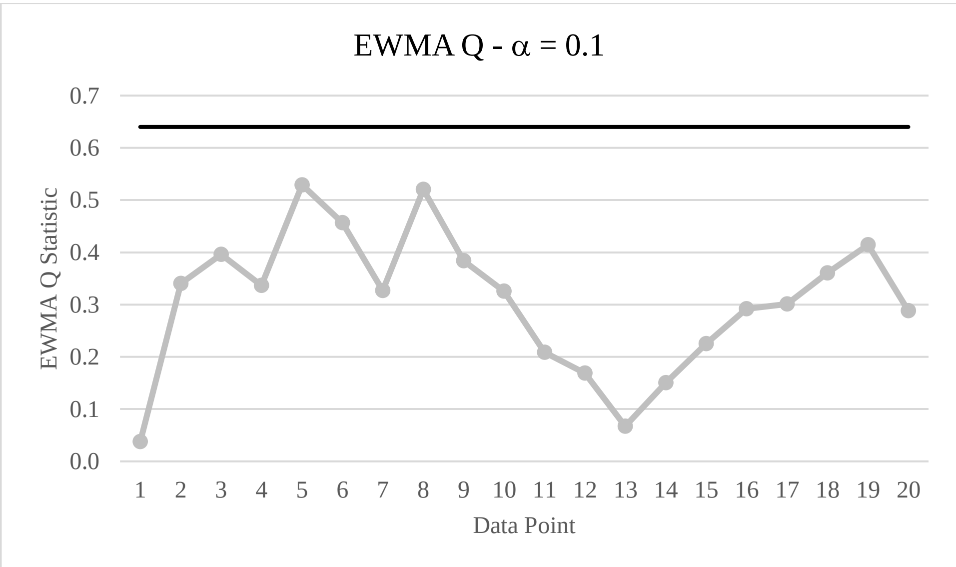


Figure 26. EWMA Q - α = 0.1 Chart for Customer Requests

However, the EWMA Q - α = 0.25 chart created for customer demands has signaled an out-of-control at the last point corresponding to 19 demands per month (see Figure 27).

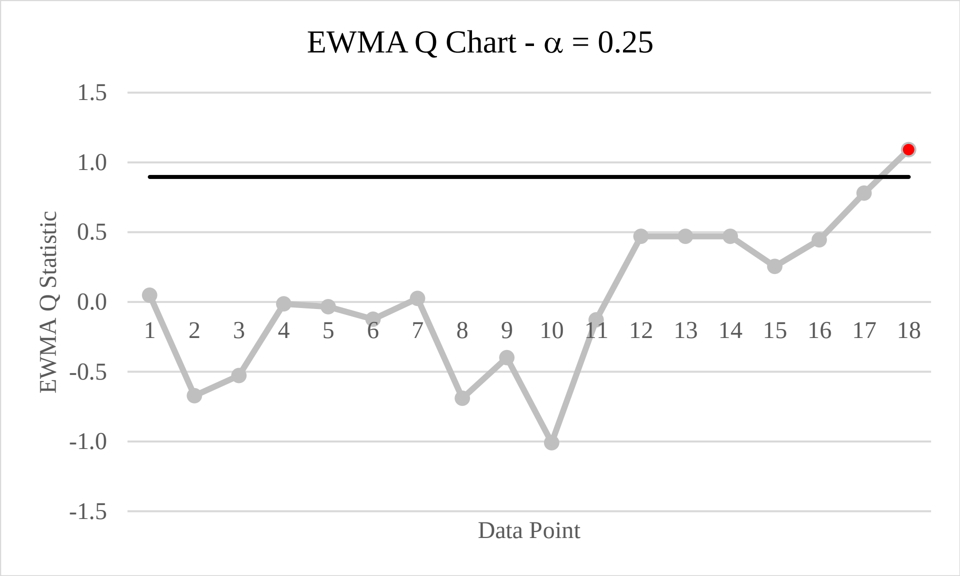


Figure 27. EWMA Q - α = 0.25 Chart for Customer Requests

As the smoothing constant is increased further, the EWMA Q - α = 0.4 chart created for customer requests (see Figure 28), has given out-of-control signals at the 12th, 17th, and 18th points for the 20, 18, and 19 monthly requests, respectively.

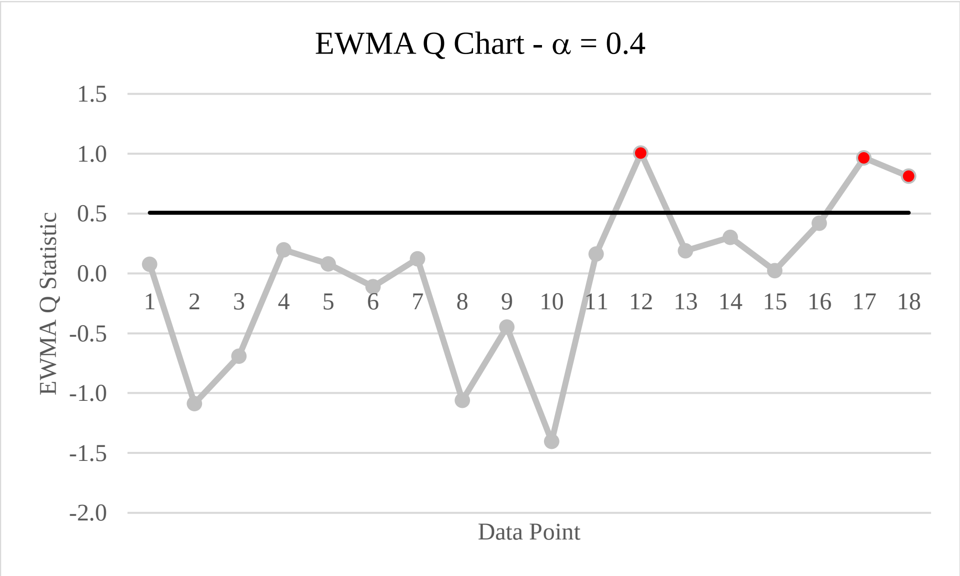


Figure 28. EWMA Q - α = 0.4 Chart for Customer Requests

Now consider Figure 29 for the GLR chart created for customer requests, which signals out-of-control for the 12th, 13th, 14$^{th}$, 17th, and 18th points corresponding to 20, 13, 13, 18, and 19 monthly requests, respectively.

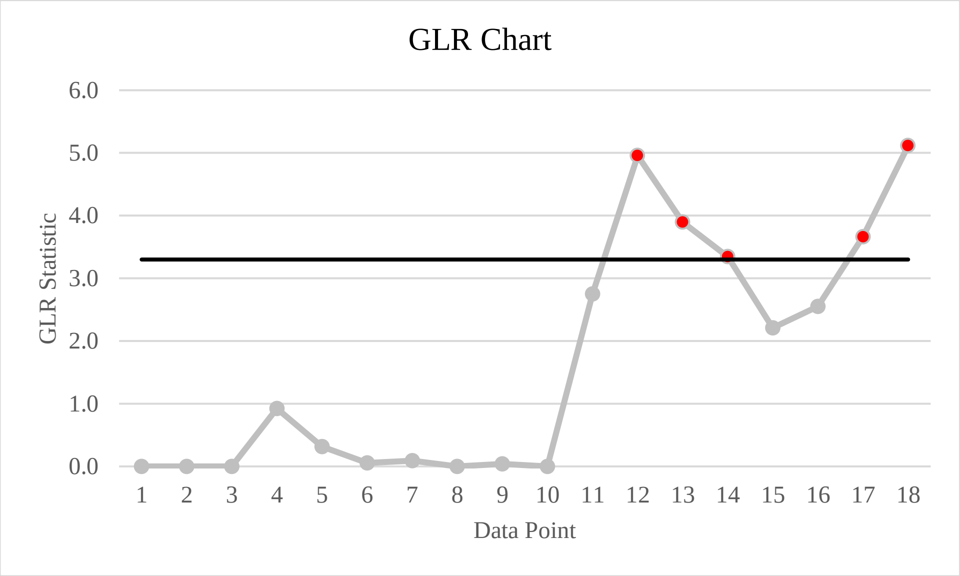


Figure 29. GLR Chart for Customer Requests

When the months with common out-of-control signals in more than one control chart types were examined, it was observed that out-of-control situations occurred due to new module developments. It was evaluated that there was no special case for the signals given only in the GLR chart and that these signals could be false.

It is more alarming that there is a pattern of increase in customer requests for this metric. Even if there are instant increases in customer demands, this should not cause a regular increase.

It is expected that there will not be a pattern in the number of defects. But instant increases are also important here. That is, the creation of different control charts according to the metrics taken will be the most effective way.

# 6. CONCLUSIONS

Control charts are used not only for monitoring production processes but also for monitoring software development processes. Unlike mass production, sufficient data may not always be available for Phase I analysis in software development processes. In such cases, self-starting methods developed for short production runs can be used.

In addition to the limited size of the data set, the data may be distributed differently from normal distribution. There are many methods developed for processes with data that is distributed differently from normal. In this study, situations where the data is distributed as Poisson have been evaluated.

In the study conducted within the scope of the thesis, a simulation study was carried out to monitor the Poisson rate parameter using self-starting methods, namely Q, EWMA Q, and GLR charts. The motivation is software development projects where the project duration is limited and, consequently, the data size is small, and the data follows a Poisson distribution. The simulation study evaluated the performance for different project durations, change points, and shift magnitudes, while the Poisson rate $\lambda_0$ was set to 3. As a result of the simulation study;

- Keeping all other parameters of $\tau$ and $\Delta$ constant, comparisons were made for project durations of D = 48 and D = 64, and as a result, it was observed that the cumulative out-of-control signal detection capability increases for all types of control charts as the project duration increases.
- During the project period, comparisons were made for different change points ($\tau$ = 0, 3, 6, 9 and 12) by considering that the deviation from the process mean could occur at any point. It was observed that the cumulative detection capability was higher for small values of $\tau$ in all chart types. It was also observed that the earlier the deviation occurred, the higher the detection capability. All charts were found to be robust against change point.
- Different $\Delta$ values were compared considering that different magnitudes of shifts can occur throughout the project, and $\Delta$ was determined as 1.1, 1.25, 1.5, 2, and 3. When the detection capability of the Q chart immediately after the change point was compared with other charts, it was found to be higher, but the cumulative detection capability remained lower than the other charts throughout the project.

For large values of $\Delta$, the GLR chart was observed to have a higher detection capability than other charts immediately after the change point.

One of the most important goals in software development projects is to ensure customer satisfaction. Customer satisfaction is ensured by the quality product delivered to the customer. One of the indicators of a quality product is that there are as few defects as possible. To ensure customer satisfaction, a case study was conducted on the number of medium and major severity defects included in the sprints. The Q, EWMA Q and GLR charts were created for known process rate $\lambda_0 = 3$, and the results were discussed.

Like in all projects, maintaining the scope is important in software development projects in many ways. To ensure that the scope is maintained, the counts of the number of requests received from a customer has been considered as a second case study. Control limits were calculated for a known Poisson rate $\lambda_0$ value of 12, and the Q, EWMA Q, and GLR charts were designed. Results indicated that these methods can be useful in practice.

To summarize, control charts including Q, EWMA Q, and GLR have been compared for processes such as software development projects where sufficient data is not available and process data follows a Poisson distribution. The EWMA Q charts have shown the best performance in terms of cumulative detection probabilities, while the Q and GLR charts have stood out in terms of detection capability immediately after the change point.

As a result, self-starting control charts provide valuable information on process performance and are essential tools for quality management. Throughout this thesis, we explored the concept of self-starting control charts and its application for datasets with different Poisson distribution parameters, while providing its application for monitoring Poisson distributed metrics in software development projects.

The implementation of self-starting control charts provides several benefits. First, it allows organizations to monitor new processes without a reference period, eliminating the need for historical data. This flexibility is particularly useful in dynamic environments, such as software development projects, where processes can change frequently. In addition, self-starting control charts offer enhanced ability to detect small shifts or deviations in process performance, increasing the accuracy and effectiveness of quality control efforts.

In software development projects, the use of self-starting control charts can improve the ability to identify and reduce quality issues, increasing product or service consistency and customer satisfaction. These charts allow organizations to make data-driven decisions, take proactive actions, and continuously improve processes.

It should be noted that while self-starting control charts offer significant advantages; they should not be viewed as a substitute for conventional control charts. Instead, they should be considered as complementary tools that provide additional insights and flexibility for practitioners. The distribution of data can affect the methods used. Practitioners should evaluate the methods suitable for them or the combinations of the methods used, taking into account the situation in their organization and the distribution of metrics. For future research, it would be valuable to explore the application of self-starting control charts in different process settings and to investigate the impact of different parameter values on the performance of the charts. Additionally, it may be useful to investigate the effectiveness of combining different control charts to further improve the quality control process.

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# APPENDICES

## Appendix A: Cumulative Shift Detection Probabilities for D = 48 and Δ = 1.1

| D | 48 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.1 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.021 | 0.000 | 0.000 | 0.000 | 0.003 | 0.006 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.039 | 0.000 | 0.000 | 0.004 | 0.012 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.058 | 0.000 | 0.001 | 0.016 | 0.030 | 0.044 | 0.020 | 0.000 | 0.000 | 0.008 | 0.016 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.075 | 0.000 | 0.002 | 0.034 | 0.051 | 0.062 | 0.043 | 0.000 | 0.001 | 0.019 | 0.034 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.092 | 0.000 | 0.007 | 0.055 | 0.074 | 0.080 | 0.062 | 0.000 | 0.005 | 0.036 | 0.055 | 0.059 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.111 | 0.001 | 0.015 | 0.079 | 0.097 | 0.099 | 0.079 | 0.001 | 0.012 | 0.060 | 0.079 | 0.076 | 0.018 | 0.000 | 0.004 | 0.021 | 0.022 | 0.028 |
| 7 | 0.132 | 0.003 | 0.027 | 0.103 | 0.120 | 0.119 | 0.098 | 0.002 | 0.021 | 0.084 | 0.100 | 0.095 | 0.034 | 0.002 | 0.011 | 0.039 | 0.045 | 0.047 |
| 8 | 0.149 | 0.006 | 0.043 | 0.126 | 0.141 | 0.140 | 0.115 | 0.004 | 0.032 | 0.106 | 0.121 | 0.115 | 0.054 | 0.003 | 0.022 | 0.063 | 0.064 | 0.070 |
| 9 | 0.164 | 0.013 | 0.062 | 0.154 | 0.162 | 0.160 | 0.133 | 0.010 | 0.051 | 0.126 | 0.141 | 0.137 | 0.072 | 0.005 | 0.035 | 0.085 | 0.087 | 0.091 |
| 10 | 0.182 | 0.022 | 0.082 | 0.175 | 0.183 | 0.181 | 0.150 | 0.017 | 0.068 | 0.149 | 0.163 | 0.161 | 0.090 | 0.010 | 0.051 | 0.107 | 0.108 | 0.113 |
| 11 | 0.200 | 0.033 | 0.104 | 0.196 | 0.203 | 0.198 | 0.168 | 0.026 | 0.087 | 0.172 | 0.184 | 0.181 | 0.108 | 0.018 | 0.068 | 0.133 | 0.128 | 0.134 |
| 12 | 0.218 | 0.047 | 0.128 | 0.218 | 0.222 | 0.217 | 0.182 | 0.039 | 0.111 | 0.196 | 0.205 | 0.203 | 0.125 | 0.028 | 0.087 | 0.157 | 0.149 | 0.158 |
| 13 | 0.233 | 0.062 | 0.150 | 0.242 | 0.241 | 0.238 | 0.197 | 0.052 | 0.135 | 0.217 | 0.224 | 0.221 | 0.143 | 0.039 | 0.108 | 0.179 | 0.170 | 0.180 |
| 14 | 0.248 | 0.080 | 0.174 | 0.268 | 0.258 | 0.257 | 0.211 | 0.068 | 0.161 | 0.240 | 0.242 | 0.241 | 0.160 | 0.052 | 0.130 | 0.204 | 0.191 | 0.203 |
| 15 | 0.263 | 0.100 | 0.199 | 0.290 | 0.278 | 0.276 | 0.226 | 0.087 | 0.182 | 0.263 | 0.262 | 0.258 | 0.176 | 0.068 | 0.154 | 0.227 | 0.213 | 0.223 |
| 16 | 0.281 | 0.122 | 0.224 | 0.310 | 0.295 | 0.296 | 0.241 | 0.106 | 0.207 | 0.286 | 0.283 | 0.276 | 0.193 | 0.085 | 0.179 | 0.248 | 0.234 | 0.245 |
| 17 | 0.293 | 0.145 | 0.248 | 0.331 | 0.311 | 0.315 | 0.254 | 0.127 | 0.232 | 0.306 | 0.303 | 0.295 | 0.209 | 0.105 | 0.204 | 0.268 | 0.253 | 0.261 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 0.307 | 0.169 | 0.274 | 0.350 | 0.327 | 0.334 | 0.270 | 0.149 | 0.261 | 0.327 | 0.319 | 0.313 | 0.223 | 0.127 | 0.230 | 0.288 | 0.270 | 0.280 |
| 19 | 0.322 | 0.193 | 0.297 | 0.370 | 0.342 | 0.351 | 0.287 | 0.172 | 0.286 | 0.344 | 0.335 | 0.331 | 0.238 | 0.147 | 0.257 | 0.309 | 0.287 | 0.300 |
| 20 | 0.334 | 0.217 | 0.322 | 0.389 | 0.358 | 0.367 | 0.301 | 0.196 | 0.311 | 0.364 | 0.353 | 0.348 | 0.251 | 0.172 | 0.282 | 0.330 | 0.304 | 0.315 |
| 21 | 0.345 | 0.243 | 0.344 | 0.406 | 0.373 | 0.382 | 0.315 | 0.222 | 0.334 | 0.382 | 0.368 | 0.366 | 0.264 | 0.199 | 0.308 | 0.348 | 0.323 | 0.333 |
| 22 | 0.360 | 0.269 | 0.364 | 0.422 | 0.390 | 0.403 | 0.329 | 0.246 | 0.357 | 0.400 | 0.385 | 0.382 | 0.279 | 0.224 | 0.333 | 0.369 | 0.340 | 0.349 |
| 23 | 0.375 | 0.294 | 0.387 | 0.436 | 0.406 | 0.420 | 0.341 | 0.269 | 0.381 | 0.416 | 0.401 | 0.399 | 0.293 | 0.249 | 0.355 | 0.387 | 0.354 | 0.367 |
| 24 | 0.388 | 0.320 | 0.407 | 0.454 | 0.420 | 0.434 | 0.354 | 0.292 | 0.401 | 0.435 | 0.415 | 0.414 | 0.305 | 0.274 | 0.376 | 0.405 | 0.372 | 0.383 |
| 25 | 0.401 | 0.346 | 0.426 | 0.469 | 0.437 | 0.449 | 0.367 | 0.317 | 0.422 | 0.452 | 0.431 | 0.430 | 0.317 | 0.299 | 0.396 | 0.423 | 0.390 | 0.398 |
| 26 | 0.412 | 0.367 | 0.447 | 0.485 | 0.452 | 0.463 | 0.379 | 0.341 | 0.444 | 0.471 | 0.445 | 0.445 | 0.331 | 0.323 | 0.417 | 0.440 | 0.405 | 0.414 |
| 27 | 0.423 | 0.392 | 0.466 | 0.501 | 0.466 | 0.477 | 0.389 | 0.366 | 0.465 | 0.487 | 0.458 | 0.460 | 0.341 | 0.348 | 0.436 | 0.455 | 0.420 | 0.429 |
| 28 | 0.434 | 0.416 | 0.484 | 0.516 | 0.480 | 0.492 | 0.402 | 0.391 | 0.483 | 0.501 | 0.473 | 0.475 | 0.351 | 0.372 | 0.456 | 0.471 | 0.436 | 0.441 |
| 29 | 0.445 | 0.440 | 0.500 | 0.531 | 0.493 | 0.506 | 0.412 | 0.415 | 0.502 | 0.516 | 0.484 | 0.488 | 0.363 | 0.396 | 0.476 | 0.489 | 0.453 | 0.454 |
| 30 | 0.455 | 0.461 | 0.518 | 0.543 | 0.505 | 0.517 | 0.424 | 0.439 | 0.519 | 0.529 | 0.496 | 0.501 | 0.374 | 0.419 | 0.495 | 0.503 | 0.465 | 0.470 |
| 31 | 0.466 | 0.485 | 0.535 | 0.556 | 0.516 | 0.530 | 0.435 | 0.462 | 0.535 | 0.542 | 0.508 | 0.514 | 0.386 | 0.442 | 0.510 | 0.516 | 0.479 | 0.483 |
| 32 | 0.478 | 0.507 | 0.552 | 0.570 | 0.529 | 0.542 | 0.444 | 0.486 | 0.553 | 0.557 | 0.521 | 0.526 | 0.399 | 0.464 | 0.528 | 0.529 | 0.494 | 0.495 |
| 33 | 0.487 | 0.531 | 0.569 | 0.584 | 0.541 | 0.554 | 0.455 | 0.508 | 0.568 | 0.570 | 0.532 | 0.537 | 0.408 | 0.486 | 0.543 | 0.543 | 0.506 | 0.510 |
| 34 | 0.497 | 0.550 | 0.585 | 0.594 | 0.550 | 0.567 | 0.466 | 0.529 | 0.584 | 0.584 | 0.545 | 0.549 | 0.418 | 0.504 | 0.559 | 0.555 | 0.519 | 0.524 |
| 35 | 0.507 | 0.567 | 0.602 | 0.607 | 0.560 | 0.578 | 0.476 | 0.549 | 0.598 | 0.597 | 0.557 | 0.561 | 0.428 | 0.522 | 0.574 | 0.568 | 0.529 | 0.538 |
| 36 | 0.518 | 0.586 | 0.614 | 0.620 | 0.571 | 0.587 | 0.486 | 0.568 | 0.613 | 0.608 | 0.566 | 0.573 | 0.440 | 0.541 | 0.590 | 0.582 | 0.539 | 0.552 |
| 37 | 0.526 | 0.605 | 0.628 | 0.631 | 0.583 | 0.597 | 0.497 | 0.587 | 0.626 | 0.619 | 0.576 | 0.585 | 0.449 | 0.561 | 0.603 | 0.593 | 0.550 | 0.563 |
| 38 | 0.535 | 0.622 | 0.641 | 0.643 | 0.595 | 0.608 | 0.506 | 0.602 | 0.639 | 0.631 | 0.587 | 0.594 | 0.460 | 0.581 | 0.618 | 0.603 | 0.562 | 0.576 |
| 39 | 0.543 | 0.638 | 0.654 | 0.654 | 0.606 | 0.616 | 0.515 | 0.621 | 0.653 | 0.641 | 0.596 | 0.604 | 0.469 | 0.600 | 0.632 | 0.615 | 0.575 | 0.586 |
| 40 | 0.553 | 0.654 | 0.668 | 0.665 | 0.616 | 0.625 | 0.526 | 0.638 | 0.663 | 0.651 | 0.605 | 0.613 | 0.477 | 0.616 | 0.646 | 0.626 | 0.588 | 0.596 |
| 41 | 0.563 | 0.668 | 0.681 | 0.675 | 0.623 | 0.635 | 0.533 | 0.656 | 0.677 | 0.662 | 0.615 | 0.623 | 0.486 | 0.631 | 0.661 | 0.639 | 0.599 | 0.605 |
| 42 | 0.573 | 0.681 | 0.692 | 0.685 | 0.632 | 0.644 | 0.543 | 0.670 | 0.690 | 0.671 | 0.624 | 0.633 | 0.496 | 0.646 | 0.674 | 0.648 | 0.609 | 0.616 |
| 43 | 0.581 | 0.697 | 0.703 | 0.695 | 0.642 | 0.655 | 0.552 | 0.686 | 0.701 | 0.681 | 0.632 | 0.645 | 0.506 | 0.662 | 0.685 | 0.658 | 0.618 | 0.627 |
| 44 | 0.588 | 0.710 | 0.713 | 0.704 | 0.652 | 0.665 | 0.560 | 0.698 | 0.711 | 0.691 | 0.641 | 0.654 | 0.515 | 0.678 | 0.697 | 0.669 | 0.628 | 0.636 |
| 45 | 0.597 | 0.721 | 0.723 | 0.715 | 0.662 | 0.674 | 0.568 | 0.710 | 0.722 | 0.702 | 0.652 | 0.664 | 0.523 | 0.691 | 0.708 | 0.681 | 0.640 | 0.646 |
| 46 | 0.605 | 0.734 | 0.732 | 0.724 | 0.672 | 0.684 | 0.577 | 0.724 | 0.731 | 0.709 | 0.659 | 0.673 | 0.531 | 0.704 | 0.719 | 0.681 | 0.649 | 0.657 |
| 47 | 0.613 | 0.745 | 0.741 | 0.732 | 0.680 | 0.692 | 0.585 | 0.735 | 0.742 | 0.717 | 0.668 | 0.681 | 0.540 | 0.716 | 0.728 | 0.701 | 0.658 | 0.666 |
| 48 | 0.620 | 0.756 | 0.751 | 0.740 | 0.687 | 0.699 | 0.592 | 0.746 | 0.754 | 0.725 | 0.677 | 0.690 | 0.548 | 0.728 | 0.739 | 0.708 | 0.667 | 0.675 |

| D | 48 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.1 | | | | | | | | | | | |
| τ | 9 | | | | | | 12 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha= 0.4 | GLR |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.018 | 0.002 | 0.012 | 0.026 | 0.024 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.035 | 0.005 | 0.027 | 0.047 | 0.045 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.053 | 0.010 | 0.042 | 0.072 | 0.066 | 0.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.070 | 0.016 | 0.058 | 0.097 | 0.088 | 0.100 | 0.016 | 0.007 | 0.027 | 0.029 | 0.020 | 0.036 |
| 13 | 0.087 | 0.027 | 0.076 | 0.120 | 0.112 | 0.122 | 0.033 | 0.015 | 0.042 | 0.050 | 0.042 | 0.060 |
| 14 | 0.104 | 0.040 | 0.096 | 0.146 | 0.135 | 0.145 | 0.051 | 0.025 | 0.058 | 0.075 | 0.067 | 0.084 |
| 15 | 0.119 | 0.051 | 0.117 | 0.169 | 0.158 | 0.169 | 0.069 | 0.036 | 0.079 | 0.102 | 0.091 | 0.105 |
| 16 | 0.133 | 0.069 | 0.144 | 0.195 | 0.178 | 0.189 | 0.085 | 0.048 | 0.102 | 0.128 | 0.113 | 0.128 |
| 17 | 0.151 | 0.085 | 0.169 | 0.218 | 0.199 | 0.211 | 0.100 | 0.064 | 0.124 | 0.152 | 0.137 | 0.147 |
| 18 | 0.168 | 0.106 | 0.193 | 0.242 | 0.221 | 0.233 | 0.115 | 0.081 | 0.149 | 0.177 | 0.156 | 0.167 |
| 19 | 0.182 | 0.124 | 0.218 | 0.267 | 0.241 | 0.252 | 0.131 | 0.100 | 0.174 | 0.200 | 0.177 | 0.188 |
| 20 | 0.195 | 0.147 | 0.240 | 0.287 | 0.261 | 0.271 | 0.146 | 0.119 | 0.201 | 0.221 | 0.197 | 0.210 |
| 21 | 0.211 | 0.171 | 0.264 | 0.308 | 0.278 | 0.289 | 0.159 | 0.141 | 0.225 | 0.244 | 0.218 | 0.231 |
| 22 | 0.226 | 0.193 | 0.288 | 0.324 | 0.298 | 0.305 | 0.172 | 0.165 | 0.247 | 0.264 | 0.239 | 0.251 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.240 | 0.216 | 0.312 | 0.340 | 0.316 | 0.323 | 0.185 | 0.191 | 0.268 | 0.285 | 0.257 | 0.268 |
| 24 | 0.253 | 0.240 | 0.336 | 0.360 | 0.331 | 0.341 | 0.200 | 0.219 | 0.291 | 0.306 | 0.277 | 0.289 |
| 25 | 0.266 | 0.266 | 0.358 | 0.377 | 0.350 | 0.356 | 0.212 | 0.243 | 0.317 | 0.325 | 0.296 | 0.310 |
| 26 | 0.281 | 0.290 | 0.381 | 0.395 | 0.367 | 0.372 | 0.228 | 0.269 | 0.338 | 0.346 | 0.314 | 0.328 |
| 27 | 0.293 | 0.312 | 0.402 | 0.414 | 0.381 | 0.390 | 0.241 | 0.296 | 0.360 | 0.362 | 0.331 | 0.346 |
| 28 | 0.305 | 0.339 | 0.420 | 0.432 | 0.395 | 0.404 | 0.252 | 0.321 | 0.383 | 0.381 | 0.346 | 0.366 |
| 29 | 0.317 | 0.365 | 0.443 | 0.450 | 0.410 | 0.419 | 0.266 | 0.346 | 0.404 | 0.400 | 0.363 | 0.382 |
| 30 | 0.327 | 0.388 | 0.463 | 0.465 | 0.424 | 0.434 | 0.278 | 0.368 | 0.426 | 0.416 | 0.380 | 0.397 |
| 31 | 0.342 | 0.412 | 0.483 | 0.480 | 0.441 | 0.448 | 0.289 | 0.393 | 0.446 | 0.433 | 0.397 | 0.410 |
| 32 | 0.354 | 0.433 | 0.501 | 0.496 | 0.457 | 0.463 | 0.300 | 0.418 | 0.466 | 0.450 | 0.411 | 0.425 |
| 33 | 0.366 | 0.454 | 0.518 | 0.510 | 0.474 | 0.477 | 0.311 | 0.441 | 0.484 | 0.466 | 0.425 | 0.439 |
| 34 | 0.379 | 0.476 | 0.535 | 0.526 | 0.487 | 0.492 | 0.322 | 0.463 | 0.503 | 0.480 | 0.438 | 0.455 |
| 35 | 0.388 | 0.497 | 0.551 | 0.540 | 0.499 | 0.505 | 0.332 | 0.482 | 0.520 | 0.496 | 0.455 | 0.469 |
| 36 | 0.399 | 0.516 | 0.565 | 0.553 | 0.509 | 0.519 | 0.341 | 0.504 | 0.537 | 0.509 | 0.469 | 0.483 |
| 37 | 0.408 | 0.536 | 0.580 | 0.567 | 0.523 | 0.531 | 0.352 | 0.520 | 0.553 | 0.523 | 0.482 | 0.495 |
| 38 | 0.418 | 0.556 | 0.594 | 0.582 | 0.535 | 0.543 | 0.363 | 0.538 | 0.569 | 0.537 | 0.495 | 0.508 |
| 39 | 0.430 | 0.573 | 0.608 | 0.594 | 0.547 | 0.556 | 0.372 | 0.555 | 0.584 | 0.550 | 0.505 | 0.522 |
| 40 | 0.441 | 0.589 | 0.622 | 0.604 | 0.559 | 0.567 | 0.383 | 0.574 | 0.599 | 0.562 | 0.517 | 0.535 |
| 41 | 0.451 | 0.608 | 0.637 | 0.615 | 0.572 | 0.578 | 0.393 | 0.595 | 0.615 | 0.575 | 0.532 | 0.546 |
| 42 | 0.460 | 0.627 | 0.651 | 0.626 | 0.582 | 0.589 | 0.403 | 0.613 | 0.628 | 0.589 | 0.545 | 0.559 |
| 43 | 0.469 | 0.642 | 0.665 | 0.638 | 0.593 | 0.600 | 0.413 | 0.631 | 0.642 | 0.601 | 0.557 | 0.570 |
| 44 | 0.477 | 0.656 | 0.678 | 0.649 | 0.603 | 0.609 | 0.422 | 0.648 | 0.655 | 0.612 | 0.569 | 0.582 |
| 45 | 0.486 | 0.672 | 0.689 | 0.660 | 0.612 | 0.621 | 0.431 | 0.663 | 0.665 | 0.623 | 0.581 | 0.592 |
| 46 | 0.494 | 0.684 | 0.699 | 0.671 | 0.621 | 0.631 | 0.441 | 0.676 | 0.677 | 0.635 | 0.594 | 0.602 |
| 47 | 0.501 | 0.696 | 0.711 | 0.680 | 0.631 | 0.641 | 0.450 | 0.690 | 0.688 | 0.645 | 0.604 | 0.613 |
| 48 | 0.511 | 0.709 | 0.721 | 0.691 | 0.640 | 0.651 | 0.460 | 0.703 | 0.702 | 0.655 | 0.615 | 0.622 |

## Appendix B: Cumulative Shift Detection Probabilities for D = 48 and Δ = 1.25

| **D** | **48** | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **1.25** | | | | | | | | | | | | | | | | | |
| **τ** | **0** | | | | | | **3** | | | | | | **6** | | | | | |
| **Run Length / Chart Type** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** |
| 1 | 0.038 | 0.000 | 0.000 | 0.001 | 0.006 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.072 | 0.000 | 0.000 | 0.012 | 0.033 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.104 | 0.000 | 0.003 | 0.040 | 0.072 | 0.092 | 0.035 | 0.000 | 0.000 | 0.011 | 0.025 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.138 | 0.000 | 0.011 | 0.084 | 0.120 | 0.134 | 0.070 | 0.000 | 0.004 | 0.039 | 0.063 | 0.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.173 | 0.001 | 0.029 | 0.136 | 0.165 | 0.179 | 0.104 | 0.000 | 0.014 | 0.080 | 0.111 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.206 | 0.008 | 0.062 | 0.191 | 0.215 | 0.222 | 0.136 | 0.003 | 0.031 | 0.130 | 0.161 | 0.158 | 0.032 | 0.001 | 0.005 | 0.032 | 0.036 | 0.046 |
| 7 | 0.236 | 0.022 | 0.100 | 0.246 | 0.262 | 0.262 | 0.168 | 0.009 | 0.062 | 0.185 | 0.208 | 0.203 | 0.063 | 0.002 | 0.018 | 0.073 | 0.075 | 0.090 |
| 8 | 0.265 | 0.044 | 0.148 | 0.300 | 0.307 | 0.303 | 0.198 | 0.020 | 0.106 | 0.241 | 0.253 | 0.246 | 0.095 | 0.007 | 0.041 | 0.122 | 0.126 | 0.133 |
| 9 | 0.295 | 0.071 | 0.200 | 0.347 | 0.346 | 0.346 | 0.227 | 0.043 | 0.155 | 0.295 | 0.298 | 0.289 | 0.130 | 0.017 | 0.073 | 0.171 | 0.175 | 0.179 |
| 10 | 0.321 | 0.106 | 0.256 | 0.391 | 0.385 | 0.382 | 0.254 | 0.067 | 0.209 | 0.342 | 0.339 | 0.331 | 0.162 | 0.033 | 0.114 | 0.221 | 0.222 | 0.225 |
| 11 | 0.345 | 0.151 | 0.313 | 0.435 | 0.420 | 0.417 | 0.281 | 0.101 | 0.263 | 0.388 | 0.380 | 0.368 | 0.189 | 0.054 | 0.158 | 0.272 | 0.265 | 0.267 |
| 12 | 0.369 | 0.199 | 0.370 | 0.477 | 0.454 | 0.452 | 0.308 | 0.144 | 0.314 | 0.429 | 0.420 | 0.405 | 0.219 | 0.085 | 0.206 | 0.325 | 0.305 | 0.304 |
| 13 | 0.392 | 0.250 | 0.421 | 0.514 | 0.486 | 0.485 | 0.336 | 0.192 | 0.367 | 0.473 | 0.455 | 0.439 | 0.247 | 0.119 | 0.262 | 0.376 | 0.348 | 0.343 |
| 14 | 0.415 | 0.302 | 0.471 | 0.550 | 0.517 | 0.515 | 0.362 | 0.246 | 0.414 | 0.511 | 0.487 | 0.472 | 0.271 | 0.163 | 0.319 | 0.417 | 0.391 | 0.386 |
| 15 | 0.436 | 0.356 | 0.517 | 0.582 | 0.547 | 0.546 | 0.386 | 0.298 | 0.461 | 0.545 | 0.517 | 0.503 | 0.297 | 0.211 | 0.371 | 0.460 | 0.426 | 0.423 |
| 16 | 0.457 | 0.407 | 0.560 | 0.612 | 0.574 | 0.578 | 0.408 | 0.352 | 0.507 | 0.577 | 0.545 | 0.534 | 0.322 | 0.258 | 0.419 | 0.499 | 0.460 | 0.460 |
| 17 | 0.481 | 0.460 | 0.599 | 0.640 | 0.603 | 0.604 | 0.426 | 0.403 | 0.546 | 0.610 | 0.574 | 0.565 | 0.347 | 0.312 | 0.464 | 0.534 | 0.494 | 0.495 |
| 18 | 0.499 | 0.511 | 0.634 | 0.666 | 0.628 | 0.629 | 0.445 | 0.455 | 0.588 | 0.639 | 0.600 | 0.592 | 0.370 | 0.364 | 0.514 | 0.566 | 0.524 | 0.530 |
| 19 | 0.519 | 0.558 | 0.669 | 0.691 | 0.647 | 0.655 | 0.465 | 0.506 | 0.627 | 0.666 | 0.626 | 0.617 | 0.390 | 0.416 | 0.554 | 0.595 | 0.553 | 0.558 |
| 20 | 0.537 | 0.600 | 0.699 | 0.714 | 0.669 | 0.675 | 0.483 | 0.552 | 0.664 | 0.692 | 0.648 | 0.639 | 0.411 | 0.461 | 0.591 | 0.624 | 0.579 | 0.586 |
| 21 | 0.554 | 0.641 | 0.725 | 0.736 | 0.689 | 0.696 | 0.501 | 0.593 | 0.693 | 0.713 | 0.670 | 0.661 | 0.431 | 0.505 | 0.628 | 0.653 | 0.606 | 0.610 |
| 22 | 0.570 | 0.676 | 0.750 | 0.755 | 0.708 | 0.719 | 0.519 | 0.633 | 0.720 | 0.736 | 0.688 | 0.685 | 0.449 | 0.554 | 0.663 | 0.677 | 0.630 | 0.635 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.586 | 0.711 | 0.776 | 0.775 | 0.724 | 0.738 | 0.535 | 0.669 | 0.749 | 0.754 | 0.707 | 0.706 | 0.468 | 0.597 | 0.696 | 0.699 | 0.652 | 0.660 |
| 24 | 0.602 | 0.745 | 0.799 | 0.791 | 0.741 | 0.756 | 0.552 | 0.703 | 0.772 | 0.772 | 0.724 | 0.725 | 0.486 | 0.633 | 0.725 | 0.721 | 0.674 | 0.685 |
| 25 | 0.619 | 0.775 | 0.816 | 0.809 | 0.754 | 0.774 | 0.568 | 0.734 | 0.792 | 0.791 | 0.742 | 0.742 | 0.505 | 0.668 | 0.751 | 0.741 | 0.693 | 0.705 |
| 26 | 0.635 | 0.799 | 0.833 | 0.824 | 0.769 | 0.789 | 0.585 | 0.763 | 0.812 | 0.807 | 0.759 | 0.760 | 0.522 | 0.704 | 0.773 | 0.764 | 0.710 | 0.724 |
| 27 | 0.648 | 0.819 | 0.850 | 0.836 | 0.782 | 0.801 | 0.599 | 0.789 | 0.829 | 0.822 | 0.775 | 0.776 | 0.539 | 0.735 | 0.793 | 0.782 | 0.728 | 0.743 |
| 28 | 0.661 | 0.839 | 0.865 | 0.848 | 0.794 | 0.813 | 0.615 | 0.812 | 0.844 | 0.836 | 0.788 | 0.791 | 0.553 | 0.765 | 0.811 | 0.797 | 0.743 | 0.760 |
| 29 | 0.674 | 0.856 | 0.879 | 0.858 | 0.806 | 0.826 | 0.628 | 0.832 | 0.860 | 0.848 | 0.802 | 0.802 | 0.568 | 0.792 | 0.831 | 0.812 | 0.757 | 0.776 |
| 30 | 0.686 | 0.871 | 0.891 | 0.870 | 0.819 | 0.839 | 0.642 | 0.850 | 0.874 | 0.861 | 0.814 | 0.814 | 0.582 | 0.816 | 0.849 | 0.826 | 0.771 | 0.790 |
| 31 | 0.698 | 0.886 | 0.903 | 0.879 | 0.831 | 0.849 | 0.654 | 0.867 | 0.886 | 0.870 | 0.825 | 0.826 | 0.596 | 0.836 | 0.866 | 0.839 | 0.785 | 0.804 |
| 32 | 0.709 | 0.896 | 0.913 | 0.888 | 0.839 | 0.859 | 0.665 | 0.882 | 0.896 | 0.881 | 0.835 | 0.837 | 0.609 | 0.853 | 0.879 | 0.850 | 0.798 | 0.816 |
| 33 | 0.720 | 0.908 | 0.921 | 0.895 | 0.848 | 0.868 | 0.676 | 0.895 | 0.906 | 0.889 | 0.846 | 0.847 | 0.621 | 0.868 | 0.891 | 0.861 | 0.811 | 0.829 |
| 34 | 0.731 | 0.919 | 0.929 | 0.903 | 0.856 | 0.878 | 0.688 | 0.907 | 0.913 | 0.898 | 0.854 | 0.857 | 0.632 | 0.883 | 0.902 | 0.872 | 0.821 | 0.839 |
| 35 | 0.739 | 0.928 | 0.935 | 0.912 | 0.865 | 0.885 | 0.699 | 0.916 | 0.921 | 0.905 | 0.864 | 0.867 | 0.643 | 0.895 | 0.910 | 0.880 | 0.833 | 0.850 |
| 36 | 0.748 | 0.936 | 0.941 | 0.918 | 0.873 | 0.893 | 0.709 | 0.926 | 0.928 | 0.912 | 0.872 | 0.876 | 0.654 | 0.907 | 0.919 | 0.891 | 0.842 | 0.858 |
| 37 | 0.756 | 0.944 | 0.946 | 0.924 | 0.879 | 0.900 | 0.720 | 0.936 | 0.935 | 0.920 | 0.881 | 0.885 | 0.664 | 0.917 | 0.925 | 0.899 | 0.852 | 0.869 |
| 38 | 0.765 | 0.950 | 0.952 | 0.929 | 0.885 | 0.906 | 0.729 | 0.944 | 0.941 | 0.928 | 0.888 | 0.893 | 0.676 | 0.926 | 0.933 | 0.907 | 0.862 | 0.876 |
| 39 | 0.773 | 0.956 | 0.957 | 0.934 | 0.892 | 0.913 | 0.740 | 0.950 | 0.947 | 0.934 | 0.895 | 0.901 | 0.687 | 0.935 | 0.941 | 0.913 | 0.870 | 0.885 |
| 40 | 0.782 | 0.961 | 0.961 | 0.940 | 0.899 | 0.919 | 0.747 | 0.955 | 0.953 | 0.938 | 0.902 | 0.908 | 0.698 | 0.941 | 0.946 | 0.920 | 0.877 | 0.891 |
| 41 | 0.790 | 0.966 | 0.965 | 0.944 | 0.904 | 0.924 | 0.757 | 0.960 | 0.957 | 0.942 | 0.908 | 0.913 | 0.709 | 0.949 | 0.952 | 0.926 | 0.885 | 0.898 |
| 42 | 0.798 | 0.970 | 0.968 | 0.948 | 0.910 | 0.929 | 0.766 | 0.964 | 0.962 | 0.946 | 0.913 | 0.920 | 0.716 | 0.954 | 0.956 | 0.931 | 0.892 | 0.905 |
| 43 | 0.806 | 0.974 | 0.971 | 0.951 | 0.915 | 0.934 | 0.773 | 0.969 | 0.965 | 0.950 | 0.919 | 0.927 | 0.725 | 0.959 | 0.961 | 0.937 | 0.899 | 0.912 |
| 44 | 0.814 | 0.977 | 0.975 | 0.956 | 0.920 | 0.938 | 0.781 | 0.972 | 0.968 | 0.954 | 0.924 | 0.933 | 0.731 | 0.963 | 0.965 | 0.942 | 0.904 | 0.918 |
| 45 | 0.821 | 0.980 | 0.977 | 0.959 | 0.925 | 0.943 | 0.788 | 0.974 | 0.971 | 0.957 | 0.928 | 0.937 | 0.739 | 0.967 | 0.968 | 0.946 | 0.909 | 0.924 |
| 46 | 0.828 | 0.982 | 0.980 | 0.962 | 0.930 | 0.947 | 0.796 | 0.978 | 0.975 | 0.960 | 0.931 | 0.942 | 0.747 | 0.970 | 0.971 | 0.951 | 0.914 | 0.930 |
| 47 | 0.835 | 0.984 | 0.982 | 0.964 | 0.934 | 0.950 | 0.802 | 0.980 | 0.977 | 0.963 | 0.936 | 0.946 | 0.755 | 0.973 | 0.974 | 0.955 | 0.919 | 0.935 |
| 48 | 0.841 | 0.986 | 0.983 | 0.966 | 0.937 | 0.953 | 0.810 | 0.982 | 0.979 | 0.965 | 0.940 | 0.949 | 0.762 | 0.976 | 0.977 | 0.958 | 0.924 | 0.940 |

| D | 48 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.25 | | | | | | | | | | | |
| τ | 9 | | | | | | 12 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.036 | 0.003 | 0.021 | 0.042 | 0.041 | 0.050 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.070 | 0.009 | 0.044 | 0.086 | 0.083 | 0.096 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.102 | 0.020 | 0.078 | 0.133 | 0.132 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.131 | 0.038 | 0.115 | 0.185 | 0.183 | 0.189 | 0.031 | 0.010 | 0.032 | 0.040 | 0.038 | 0.058 |
| 13 | 0.159 | 0.062 | 0.161 | 0.238 | 0.229 | 0.231 | 0.063 | 0.025 | 0.064 | 0.085 | 0.084 | 0.105 |
| 14 | 0.190 | 0.092 | 0.212 | 0.286 | 0.280 | 0.273 | 0.094 | 0.043 | 0.101 | 0.137 | 0.131 | 0.153 |
| 15 | 0.217 | 0.127 | 0.265 | 0.338 | 0.323 | 0.316 | 0.120 | 0.066 | 0.145 | 0.187 | 0.179 | 0.200 |
| 16 | 0.241 | 0.172 | 0.320 | 0.384 | 0.364 | 0.356 | 0.151 | 0.098 | 0.193 | 0.239 | 0.225 | 0.245 |
| 17 | 0.265 | 0.215 | 0.372 | 0.427 | 0.401 | 0.396 | 0.175 | 0.135 | 0.244 | 0.292 | 0.270 | 0.293 |
| 18 | 0.293 | 0.263 | 0.427 | 0.467 | 0.437 | 0.431 | 0.201 | 0.179 | 0.294 | 0.343 | 0.313 | 0.336 |
| 19 | 0.316 | 0.311 | 0.477 | 0.502 | 0.470 | 0.468 | 0.228 | 0.224 | 0.349 | 0.390 | 0.354 | 0.377 |
| 20 | 0.338 | 0.361 | 0.523 | 0.538 | 0.502 | 0.503 | 0.252 | 0.275 | 0.401 | 0.435 | 0.391 | 0.414 |
| 21 | 0.359 | 0.415 | 0.566 | 0.572 | 0.535 | 0.534 | 0.277 | 0.327 | 0.449 | 0.478 | 0.426 | 0.450 |
| 22 | 0.379 | 0.467 | 0.605 | 0.606 | 0.562 | 0.564 | 0.303 | 0.375 | 0.498 | 0.516 | 0.459 | 0.482 |
| 23 | 0.397 | 0.512 | 0.641 | 0.637 | 0.589 | 0.589 | 0.329 | 0.425 | 0.538 | 0.549 | 0.497 | 0.514 |
| 24 | 0.417 | 0.559 | 0.672 | 0.663 | 0.614 | 0.614 | 0.348 | 0.473 | 0.575 | 0.582 | 0.528 | 0.545 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 0.434 | 0.603 | 0.699 | 0.688 | 0.636 | 0.637 | 0.368 | 0.518 | 0.617 | 0.609 | 0.558 | 0.573 |
| 26 | 0.452 | 0.642 | 0.729 | 0.710 | 0.658 | 0.659 | 0.386 | 0.565 | 0.652 | 0.632 | 0.584 | 0.604 |
| 27 | 0.469 | 0.677 | 0.758 | 0.733 | 0.681 | 0.681 | 0.405 | 0.605 | 0.686 | 0.657 | 0.609 | 0.628 |
| 28 | 0.487 | 0.711 | 0.783 | 0.754 | 0.701 | 0.701 | 0.422 | 0.641 | 0.716 | 0.683 | 0.634 | 0.653 |
| 29 | 0.501 | 0.742 | 0.804 | 0.769 | 0.718 | 0.722 | 0.438 | 0.676 | 0.744 | 0.706 | 0.656 | 0.674 |
| 30 | 0.518 | 0.769 | 0.822 | 0.786 | 0.736 | 0.741 | 0.455 | 0.708 | 0.770 | 0.726 | 0.677 | 0.695 |
| 31 | 0.529 | 0.796 | 0.841 | 0.801 | 0.753 | 0.758 | 0.472 | 0.740 | 0.792 | 0.748 | 0.696 | 0.715 |
| 32 | 0.544 | 0.817 | 0.856 | 0.814 | 0.769 | 0.772 | 0.487 | 0.766 | 0.810 | 0.767 | 0.714 | 0.735 |
| 33 | 0.556 | 0.837 | 0.870 | 0.827 | 0.781 | 0.788 | 0.502 | 0.792 | 0.827 | 0.785 | 0.733 | 0.753 |
| 34 | 0.570 | 0.857 | 0.884 | 0.841 | 0.794 | 0.802 | 0.518 | 0.814 | 0.844 | 0.803 | 0.749 | 0.770 |
| 35 | 0.582 | 0.871 | 0.895 | 0.853 | 0.808 | 0.815 | 0.533 | 0.834 | 0.860 | 0.819 | 0.766 | 0.786 |
| 36 | 0.594 | 0.886 | 0.904 | 0.864 | 0.819 | 0.828 | 0.546 | 0.852 | 0.872 | 0.833 | 0.780 | 0.799 |
| 37 | 0.605 | 0.898 | 0.912 | 0.873 | 0.830 | 0.840 | 0.558 | 0.867 | 0.882 | 0.846 | 0.797 | 0.812 |
| 38 | 0.616 | 0.910 | 0.920 | 0.883 | 0.840 | 0.854 | 0.570 | 0.881 | 0.895 | 0.858 | 0.809 | 0.824 |
| 39 | 0.628 | 0.921 | 0.928 | 0.891 | 0.850 | 0.866 | 0.581 | 0.896 | 0.905 | 0.869 | 0.821 | 0.836 |
| 40 | 0.637 | 0.930 | 0.936 | 0.901 | 0.858 | 0.875 | 0.591 | 0.907 | 0.918 | 0.879 | 0.833 | 0.848 |
| 41 | 0.648 | 0.937 | 0.943 | 0.909 | 0.867 | 0.883 | 0.600 | 0.916 | 0.926 | 0.887 | 0.843 | 0.858 |
| 42 | 0.658 | 0.945 | 0.950 | 0.916 | 0.875 | 0.892 | 0.611 | 0.926 | 0.933 | 0.896 | 0.853 | 0.868 |
| 43 | 0.668 | 0.951 | 0.955 | 0.922 | 0.882 | 0.899 | 0.621 | 0.934 | 0.940 | 0.904 | 0.862 | 0.876 |
| 44 | 0.677 | 0.957 | 0.960 | 0.927 | 0.890 | 0.906 | 0.631 | 0.941 | 0.947 | 0.911 | 0.871 | 0.884 |
| 45 | 0.684 | 0.961 | 0.964 | 0.934 | 0.897 | 0.912 | 0.639 | 0.948 | 0.952 | 0.918 | 0.879 | 0.890 |
| 46 | 0.693 | 0.966 | 0.968 | 0.939 | 0.902 | 0.919 | 0.647 | 0.953 | 0.957 | 0.925 | 0.887 | 0.897 |
| 47 | 0.700 | 0.970 | 0.971 | 0.945 | 0.908 | 0.924 | 0.654 | 0.958 | 0.961 | 0.929 | 0.893 | 0.904 |
| 48 | 0.709 | 0.974 | 0.974 | 0.950 | 0.913 | 0.930 | 0.662 | 0.962 | 0.965 | 0.934 | 0.900 | 0.912 |

## Appendix C: Cumulative Shift Detection Probabilities for D = 48 and Δ = 1.5

| D | 48 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.5 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.088 | 0.000 | 0.000 | 0.003 | 0.016 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.171 | 0.000 | 0.001 | 0.044 | 0.097 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.242 | 0.000 | 0.016 | 0.149 | 0.202 | 0.239 | 0.081 | 0.000 | 0.001 | 0.025 | 0.051 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.306 | 0.005 | 0.066 | 0.269 | 0.316 | 0.332 | 0.162 | 0.000 | 0.013 | 0.104 | 0.146 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.370 | 0.024 | 0.155 | 0.391 | 0.423 | 0.427 | 0.235 | 0.004 | 0.055 | 0.213 | 0.262 | 0.269 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.426 | 0.068 | 0.266 | 0.498 | 0.512 | 0.509 | 0.297 | 0.021 | 0.125 | 0.332 | 0.372 | 0.366 | 0.084 | 0.001 | 0.017 | 0.056 | 0.076 | 0.079 |
| 7 | 0.474 | 0.146 | 0.388 | 0.595 | 0.595 | 0.576 | 0.355 | 0.060 | 0.225 | 0.443 | 0.470 | 0.455 | 0.155 | 0.006 | 0.051 | 0.150 | 0.176 | 0.182 |
| 8 | 0.519 | 0.247 | 0.502 | 0.670 | 0.660 | 0.648 | 0.409 | 0.121 | 0.340 | 0.542 | 0.555 | 0.535 | 0.220 | 0.021 | 0.110 | 0.262 | 0.283 | 0.282 |
| 9 | 0.559 | 0.356 | 0.601 | 0.734 | 0.711 | 0.704 | 0.460 | 0.211 | 0.450 | 0.631 | 0.626 | 0.605 | 0.284 | 0.056 | 0.199 | 0.382 | 0.389 | 0.379 |
| 10 | 0.598 | 0.465 | 0.686 | 0.784 | 0.756 | 0.754 | 0.507 | 0.310 | 0.556 | 0.703 | 0.688 | 0.667 | 0.337 | 0.113 | 0.300 | 0.487 | 0.482 | 0.470 |
| 11 | 0.631 | 0.568 | 0.757 | 0.824 | 0.795 | 0.796 | 0.549 | 0.417 | 0.650 | 0.760 | 0.737 | 0.720 | 0.388 | 0.196 | 0.410 | 0.579 | 0.560 | 0.549 |
| 12 | 0.666 | 0.660 | 0.812 | 0.861 | 0.829 | 0.833 | 0.584 | 0.518 | 0.721 | 0.809 | 0.778 | 0.765 | 0.435 | 0.290 | 0.512 | 0.656 | 0.629 | 0.623 |
| 13 | 0.695 | 0.732 | 0.852 | 0.887 | 0.858 | 0.859 | 0.616 | 0.614 | 0.781 | 0.845 | 0.812 | 0.804 | 0.481 | 0.397 | 0.607 | 0.724 | 0.687 | 0.684 |
| 14 | 0.720 | 0.795 | 0.887 | 0.911 | 0.883 | 0.886 | 0.647 | 0.698 | 0.830 | 0.873 | 0.843 | 0.838 | 0.521 | 0.500 | 0.687 | 0.780 | 0.736 | 0.737 |
| 15 | 0.743 | 0.845 | 0.913 | 0.927 | 0.902 | 0.905 | 0.671 | 0.771 | 0.872 | 0.896 | 0.869 | 0.865 | 0.555 | 0.594 | 0.753 | 0.824 | 0.774 | 0.778 |
| 16 | 0.762 | 0.882 | 0.934 | 0.940 | 0.918 | 0.922 | 0.700 | 0.824 | 0.902 | 0.918 | 0.888 | 0.888 | 0.588 | 0.675 | 0.809 | 0.856 | 0.810 | 0.816 |
| 17 | 0.783 | 0.914 | 0.949 | 0.951 | 0.931 | 0.937 | 0.726 | 0.868 | 0.924 | 0.935 | 0.903 | 0.908 | 0.619 | 0.745 | 0.858 | 0.886 | 0.843 | 0.844 |
| 18 | 0.802 | 0.936 | 0.962 | 0.958 | 0.942 | 0.947 | 0.749 | 0.902 | 0.941 | 0.949 | 0.918 | 0.927 | 0.648 | 0.803 | 0.892 | 0.908 | 0.869 | 0.874 |
| 19 | 0.817 | 0.952 | 0.971 | 0.966 | 0.951 | 0.956 | 0.770 | 0.929 | 0.954 | 0.959 | 0.933 | 0.942 | 0.674 | 0.855 | 0.917 | 0.926 | 0.889 | 0.899 |
| 20 | 0.835 | 0.966 | 0.978 | 0.974 | 0.960 | 0.964 | 0.788 | 0.948 | 0.966 | 0.966 | 0.943 | 0.952 | 0.696 | 0.890 | 0.936 | 0.942 | 0.908 | 0.917 |
| 21 | 0.848 | 0.974 | 0.983 | 0.979 | 0.967 | 0.971 | 0.804 | 0.963 | 0.974 | 0.972 | 0.951 | 0.961 | 0.717 | 0.917 | 0.953 | 0.955 | 0.922 | 0.930 |
| 22 | 0.860 | 0.981 | 0.988 | 0.983 | 0.973 | 0.977 | 0.822 | 0.972 | 0.981 | 0.978 | 0.959 | 0.969 | 0.736 | 0.940 | 0.966 | 0.964 | 0.934 | 0.942 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.872 | 0.987 | 0.990 | 0.987 | 0.978 | 0.980 | 0.835 | 0.980 | 0.987 | 0.983 | 0.965 | 0.974 | 0.754 | 0.955 | 0.973 | 0.971 | 0.943 | 0.953 |
| 24 | 0.882 | 0.990 | 0.993 | 0.989 | 0.981 | 0.984 | 0.847 | 0.986 | 0.990 | 0.986 | 0.971 | 0.980 | 0.769 | 0.967 | 0.980 | 0.977 | 0.953 | 0.962 |
| 25 | 0.893 | 0.993 | 0.995 | 0.992 | 0.984 | 0.986 | 0.856 | 0.990 | 0.993 | 0.989 | 0.976 | 0.983 | 0.784 | 0.977 | 0.985 | 0.982 | 0.961 | 0.971 |
| 26 | 0.902 | 0.995 | 0.996 | 0.994 | 0.986 | 0.990 | 0.865 | 0.993 | 0.995 | 0.991 | 0.980 | 0.986 | 0.799 | 0.983 | 0.989 | 0.986 | 0.968 | 0.977 |
| 27 | 0.913 | 0.996 | 0.997 | 0.995 | 0.988 | 0.992 | 0.875 | 0.996 | 0.996 | 0.993 | 0.983 | 0.989 | 0.811 | 0.989 | 0.992 | 0.988 | 0.973 | 0.981 |
| 28 | 0.920 | 0.997 | 0.997 | 0.996 | 0.990 | 0.993 | 0.884 | 0.997 | 0.997 | 0.994 | 0.986 | 0.990 | 0.823 | 0.992 | 0.994 | 0.990 | 0.977 | 0.984 |
| 29 | 0.926 | 0.998 | 0.998 | 0.997 | 0.991 | 0.995 | 0.893 | 0.997 | 0.998 | 0.995 | 0.988 | 0.993 | 0.833 | 0.994 | 0.995 | 0.993 | 0.981 | 0.987 |
| 30 | 0.932 | 0.999 | 0.998 | 0.998 | 0.992 | 0.996 | 0.899 | 0.998 | 0.998 | 0.996 | 0.990 | 0.994 | 0.843 | 0.996 | 0.996 | 0.993 | 0.984 | 0.990 |
| 31 | 0.938 | 0.999 | 0.999 | 0.998 | 0.993 | 0.996 | 0.905 | 0.999 | 0.999 | 0.997 | 0.991 | 0.995 | 0.849 | 0.997 | 0.997 | 0.995 | 0.986 | 0.991 |
| 32 | 0.943 | 1.000 | 0.999 | 0.999 | 0.995 | 0.997 | 0.912 | 0.999 | 0.999 | 0.998 | 0.992 | 0.996 | 0.858 | 0.999 | 0.997 | 0.996 | 0.988 | 0.993 |
| 33 | 0.948 | 1.000 | 0.999 | 0.999 | 0.995 | 0.998 | 0.917 | 0.999 | 0.999 | 0.998 | 0.994 | 0.997 | 0.866 | 0.999 | 0.998 | 0.997 | 0.990 | 0.994 |
| 34 | 0.952 | 1.000 | 0.999 | 0.999 | 0.996 | 0.998 | 0.921 | 1.000 | 1.000 | 0.999 | 0.995 | 0.998 | 0.873 | 0.999 | 0.998 | 0.998 | 0.991 | 0.996 |
| 35 | 0.956 | 1.000 | 1.000 | 0.999 | 0.997 | 0.998 | 0.926 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 | 0.877 | 0.999 | 0.999 | 0.998 | 0.993 | 0.996 |
| 36 | 0.959 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 | 0.930 | 1.000 | 1.000 | 1.000 | 0.996 | 0.999 | 0.882 | 1.000 | 0.999 | 0.998 | 0.994 | 0.997 |
| 37 | 0.963 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 | 0.934 | 1.000 | 1.000 | 1.000 | 0.997 | 0.999 | 0.887 | 1.000 | 1.000 | 0.999 | 0.995 | 0.998 |
| 38 | 0.966 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 | 0.938 | 1.000 | 1.000 | 1.000 | 0.997 | 0.999 | 0.892 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 |
| 39 | 0.969 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.941 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.895 | 1.000 | 1.000 | 0.999 | 0.997 | 0.998 |
| 40 | 0.971 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.944 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.899 | 1.000 | 1.000 | 1.000 | 0.997 | 0.999 |
| 41 | 0.973 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.947 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.903 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 |
| 42 | 0.976 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.949 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.906 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 |
| 43 | 0.978 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.951 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.909 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 |
| 44 | 0.980 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.953 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.913 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 45 | 0.982 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.955 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.916 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 46 | 0.984 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.956 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.917 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 47 | 0.985 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.958 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.919 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 48 | 0.986 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.959 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.921 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |

| D | 48 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.5 | | | | | | | | | | | |
| τ | 9 | | | | | | 12 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.080 | 0.006 | 0.035 | 0.070 | 0.071 | 0.090 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.157 | 0.022 | 0.083 | 0.165 | 0.175 | 0.204 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.220 | 0.055 | 0.159 | 0.280 | 0.290 | 0.305 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.278 | 0.106 | 0.267 | 0.398 | 0.394 | 0.401 | 0.078 | 0.018 | 0.051 | 0.074 | 0.077 | 0.096 |
| 13 | 0.328 | 0.179 | 0.372 | 0.501 | 0.486 | 0.486 | 0.146 | 0.047 | 0.114 | 0.169 | 0.181 | 0.208 |
| 14 | 0.376 | 0.263 | 0.481 | 0.598 | 0.565 | 0.566 | 0.213 | 0.093 | 0.199 | 0.290 | 0.295 | 0.310 |
| 15 | 0.424 | 0.362 | 0.579 | 0.673 | 0.638 | 0.637 | 0.270 | 0.160 | 0.302 | 0.399 | 0.398 | 0.409 |
| 16 | 0.471 | 0.459 | 0.662 | 0.734 | 0.696 | 0.697 | 0.321 | 0.242 | 0.410 | 0.501 | 0.491 | 0.494 |
| 17 | 0.507 | 0.550 | 0.734 | 0.784 | 0.746 | 0.749 | 0.369 | 0.334 | 0.512 | 0.597 | 0.571 | 0.570 |
| 18 | 0.541 | 0.634 | 0.793 | 0.824 | 0.784 | 0.792 | 0.412 | 0.434 | 0.603 | 0.672 | 0.638 | 0.637 |
| 19 | 0.572 | 0.713 | 0.837 | 0.858 | 0.819 | 0.826 | 0.450 | 0.524 | 0.679 | 0.733 | 0.698 | 0.698 |
| 20 | 0.599 | 0.775 | 0.874 | 0.882 | 0.847 | 0.859 | 0.486 | 0.609 | 0.747 | 0.784 | 0.748 | 0.746 |
| 21 | 0.626 | 0.832 | 0.902 | 0.906 | 0.872 | 0.883 | 0.518 | 0.685 | 0.804 | 0.827 | 0.787 | 0.788 |
| 22 | 0.652 | 0.877 | 0.925 | 0.925 | 0.892 | 0.905 | 0.548 | 0.752 | 0.846 | 0.863 | 0.822 | 0.821 |
| 23 | 0.673 | 0.906 | 0.944 | 0.940 | 0.907 | 0.923 | 0.578 | 0.806 | 0.880 | 0.890 | 0.851 | 0.854 |
| 24 | 0.692 | 0.931 | 0.955 | 0.953 | 0.921 | 0.937 | 0.605 | 0.849 | 0.908 | 0.912 | 0.877 | 0.880 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 0.710 | 0.948 | 0.966 | 0.961 | 0.934 | 0.949 | 0.628 | 0.884 | 0.930 | 0.931 | 0.896 | 0.904 |
| 26 | 0.730 | 0.961 | 0.974 | 0.970 | 0.944 | 0.959 | 0.649 | 0.909 | 0.946 | 0.945 | 0.912 | 0.922 |
| 27 | 0.744 | 0.970 | 0.981 | 0.976 | 0.954 | 0.969 | 0.671 | 0.931 | 0.959 | 0.957 | 0.925 | 0.935 |
| 28 | 0.759 | 0.979 | 0.985 | 0.981 | 0.962 | 0.977 | 0.689 | 0.949 | 0.969 | 0.966 | 0.936 | 0.947 |
| 29 | 0.773 | 0.985 | 0.989 | 0.985 | 0.969 | 0.981 | 0.705 | 0.962 | 0.976 | 0.972 | 0.947 | 0.955 |
| 30 | 0.785 | 0.989 | 0.992 | 0.988 | 0.973 | 0.984 | 0.722 | 0.974 | 0.982 | 0.978 | 0.954 | 0.964 |
| 31 | 0.796 | 0.991 | 0.994 | 0.990 | 0.978 | 0.987 | 0.738 | 0.980 | 0.987 | 0.983 | 0.961 | 0.971 |
| 32 | 0.805 | 0.994 | 0.995 | 0.992 | 0.982 | 0.990 | 0.750 | 0.985 | 0.990 | 0.986 | 0.967 | 0.977 |
| 33 | 0.814 | 0.996 | 0.997 | 0.994 | 0.985 | 0.992 | 0.761 | 0.989 | 0.993 | 0.988 | 0.972 | 0.982 |
| 34 | 0.822 | 0.997 | 0.998 | 0.995 | 0.987 | 0.994 | 0.772 | 0.993 | 0.994 | 0.991 | 0.977 | 0.986 |
| 35 | 0.830 | 0.998 | 0.999 | 0.996 | 0.990 | 0.995 | 0.782 | 0.995 | 0.995 | 0.992 | 0.982 | 0.989 |
| 36 | 0.836 | 0.999 | 0.999 | 0.997 | 0.991 | 0.996 | 0.791 | 0.996 | 0.996 | 0.995 | 0.984 | 0.991 |
| 37 | 0.842 | 1.000 | 0.999 | 0.998 | 0.992 | 0.996 | 0.798 | 0.997 | 0.998 | 0.996 | 0.986 | 0.993 |
| 38 | 0.848 | 1.000 | 1.000 | 0.998 | 0.993 | 0.997 | 0.805 | 0.998 | 0.998 | 0.997 | 0.988 | 0.993 |
| 39 | 0.853 | 1.000 | 1.000 | 0.998 | 0.994 | 0.997 | 0.811 | 0.998 | 0.999 | 0.998 | 0.990 | 0.995 |
| 40 | 0.858 | 1.000 | 1.000 | 0.999 | 0.995 | 0.998 | 0.817 | 0.999 | 0.999 | 0.998 | 0.992 | 0.996 |
| 41 | 0.862 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 | 0.822 | 0.999 | 0.999 | 0.999 | 0.993 | 0.996 |
| 42 | 0.866 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 | 0.827 | 0.999 | 1.000 | 0.999 | 0.994 | 0.997 |
| 43 | 0.869 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 | 0.832 | 0.999 | 1.000 | 0.999 | 0.995 | 0.998 |
| 44 | 0.871 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 | 0.836 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 |
| 45 | 0.873 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 0.840 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 |
| 46 | 0.876 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 0.843 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 |
| 47 | 0.879 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 0.846 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 |
| 48 | 0.881 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.849 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 |

# Appendix D: Cumulative Shift Detection Probabilities for D = 48 and Δ = 2

| D | 48 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 2 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.249 | 0.000 | 0.000 | 0.020 | 0.080 | 0.152 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.442 | 0.000 | 0.020 | 0.234 | 0.372 | 0.445 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.588 | 0.013 | 0.172 | 0.534 | 0.616 | 0.639 | 0.255 | 0.000 | 0.005 | 0.081 | 0.158 | 0.196 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.696 | 0.116 | 0.438 | 0.743 | 0.781 | 0.777 | 0.434 | 0.004 | 0.078 | 0.326 | 0.440 | 0.478 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.775 | 0.330 | 0.681 | 0.865 | 0.875 | 0.871 | 0.574 | 0.047 | 0.270 | 0.594 | 0.664 | 0.670 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.834 | 0.567 | 0.829 | 0.931 | 0.929 | 0.926 | 0.676 | 0.189 | 0.516 | 0.774 | 0.810 | 0.799 | 0.238 | 0.003 | 0.033 | 0.146 | 0.188 | 0.216 |
| 7 | 0.875 | 0.757 | 0.920 | 0.964 | 0.959 | 0.958 | 0.756 | 0.407 | 0.721 | 0.878 | 0.892 | 0.881 | 0.416 | 0.025 | 0.157 | 0.402 | 0.461 | 0.497 |
| 8 | 0.907 | 0.872 | 0.962 | 0.983 | 0.976 | 0.977 | 0.808 | 0.623 | 0.853 | 0.937 | 0.937 | 0.933 | 0.548 | 0.114 | 0.373 | 0.647 | 0.667 | 0.681 |
| 9 | 0.933 | 0.939 | 0.983 | 0.991 | 0.987 | 0.988 | 0.849 | 0.785 | 0.926 | 0.969 | 0.966 | 0.966 | 0.649 | 0.291 | 0.600 | 0.810 | 0.813 | 0.809 |
| 10 | 0.952 | 0.970 | 0.994 | 0.995 | 0.993 | 0.993 | 0.880 | 0.888 | 0.966 | 0.983 | 0.980 | 0.983 | 0.723 | 0.501 | 0.769 | 0.902 | 0.892 | 0.888 |
| 11 | 0.964 | 0.986 | 0.997 | 0.998 | 0.996 | 0.997 | 0.905 | 0.946 | 0.985 | 0.991 | 0.988 | 0.990 | 0.782 | 0.687 | 0.881 | 0.950 | 0.937 | 0.936 |
| 12 | 0.974 | 0.993 | 0.999 | 0.999 | 0.998 | 0.998 | 0.923 | 0.977 | 0.994 | 0.996 | 0.993 | 0.995 | 0.824 | 0.825 | 0.942 | 0.975 | 0.965 | 0.966 |
| 13 | 0.982 | 0.997 | 1.000 | 1.000 | 0.999 | 0.999 | 0.938 | 0.991 | 0.997 | 0.998 | 0.996 | 0.997 | 0.855 | 0.912 | 0.972 | 0.988 | 0.980 | 0.981 |
| 14 | 0.987 | 0.999 | 1.000 | 1.000 | 0.999 | 1.000 | 0.947 | 0.996 | 0.999 | 0.999 | 0.998 | 0.998 | 0.879 | 0.956 | 0.988 | 0.994 | 0.989 | 0.991 |
| 15 | 0.989 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.954 | 0.998 | 1.000 | 0.999 | 0.999 | 0.999 | 0.894 | 0.981 | 0.995 | 0.997 | 0.994 | 0.996 |
| 16 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.961 | 0.999 | 1.000 | 1.000 | 0.999 | 1.000 | 0.909 | 0.992 | 0.998 | 0.999 | 0.997 | 0.998 |
| 17 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.964 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.917 | 0.996 | 1.000 | 0.999 | 0.998 | 0.999 |
| 18 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.967 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.923 | 0.998 | 1.000 | 1.000 | 0.999 | 1.000 |
| 19 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.969 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.928 | 0.999 | 1.000 | 1.000 | 0.999 | 1.000 |
| 20 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.971 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.932 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 21 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.935 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 22 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.937 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.973 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.938 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 24 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.973 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.942 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.942 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 43 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.943 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| D | 48 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 2 | | | | | | | | | | | |
| τ | 9 | | | | | | 12 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.231 | 0.016 | 0.071 | 0.167 | 0.188 | 0.226 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.405 | 0.073 | 0.227 | 0.422 | 0.470 | 0.502 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.535 | 0.204 | 0.441 | 0.657 | 0.685 | 0.691 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.635 | 0.388 | 0.652 | 0.810 | 0.819 | 0.814 | 0.222 | 0.035 | 0.100 | 0.169 | 0.207 | 0.228 |
| 13 | 0.704 | 0.579 | 0.803 | 0.900 | 0.899 | 0.894 | 0.393 | 0.117 | 0.276 | 0.427 | 0.489 | 0.506 |
| 14 | 0.759 | 0.742 | 0.896 | 0.949 | 0.942 | 0.939 | 0.517 | 0.259 | 0.496 | 0.664 | 0.693 | 0.695 |
| 15 | 0.797 | 0.853 | 0.949 | 0.974 | 0.968 | 0.966 | 0.607 | 0.446 | 0.690 | 0.816 | 0.825 | 0.818 |
| 16 | 0.824 | 0.925 | 0.977 | 0.988 | 0.983 | 0.983 | 0.677 | 0.633 | 0.826 | 0.901 | 0.902 | 0.894 |
| 17 | 0.843 | 0.962 | 0.990 | 0.994 | 0.991 | 0.990 | 0.732 | 0.780 | 0.912 | 0.949 | 0.944 | 0.942 |
| 18 | 0.860 | 0.983 | 0.995 | 0.997 | 0.995 | 0.994 | 0.767 | 0.876 | 0.957 | 0.973 | 0.967 | 0.967 |
| 19 | 0.871 | 0.992 | 0.998 | 0.998 | 0.998 | 0.997 | 0.797 | 0.936 | 0.981 | 0.987 | 0.980 | 0.984 |
| 20 | 0.881 | 0.996 | 0.999 | 0.999 | 0.999 | 0.998 | 0.818 | 0.970 | 0.992 | 0.993 | 0.989 | 0.992 |
| 21 | 0.888 | 0.999 | 1.000 | 1.000 | 0.999 | 0.999 | 0.834 | 0.986 | 0.997 | 0.997 | 0.995 | 0.996 |
| 22 | 0.891 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 0.844 | 0.993 | 0.999 | 0.998 | 0.998 | 0.998 |
| 23 | 0.895 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.852 | 0.997 | 0.999 | 0.999 | 0.999 | 0.999 |
| 24 | 0.897 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.857 | 0.998 | 1.000 | 1.000 | 0.999 | 1.000 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 0.900 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.862 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 0.902 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.865 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 0.903 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.868 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 0.903 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.870 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 0.904 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.872 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.873 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 43 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

## Appendix E: Cumulative Shift Detection Probabilities for D = 48 and Δ = 3

| D | 48 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 3 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.671 | 0.000 | 0.001 | 0.200 | 0.419 | 0.539 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.897 | 0.026 | 0.342 | 0.798 | 0.875 | 0.916 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.970 | 0.411 | 0.834 | 0.971 | 0.980 | 0.982 | 0.675 | 0.001 | 0.039 | 0.314 | 0.483 | 0.588 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.990 | 0.839 | 0.975 | 0.996 | 0.998 | 0.997 | 0.877 | 0.095 | 0.436 | 0.824 | 0.885 | 0.921 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.997 | 0.974 | 0.997 | 1.000 | 0.999 | 1.000 | 0.942 | 0.508 | 0.850 | 0.971 | 0.982 | 0.987 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.999 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 0.967 | 0.865 | 0.976 | 0.996 | 0.997 | 0.998 | 0.630 | 0.015 | 0.128 | 0.397 | 0.520 | 0.611 |
| 7 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.973 | 0.975 | 0.997 | 0.999 | 1.000 | 1.000 | 0.842 | 0.203 | 0.562 | 0.850 | 0.901 | 0.923 |
| 8 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 0.910 | 0.603 | 0.881 | 0.976 | 0.985 | 0.986 |
| 9 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.930 | 0.883 | 0.979 | 0.997 | 0.998 | 0.998 |
| 10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.937 | 0.976 | 0.998 | 1.000 | 1.000 | 1.000 |
| 11 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 |
| 12 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 13 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 14 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 15 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 17 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 18 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 19 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 21 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 22 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 24 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 43 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| D | 48 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 3 | | | | | | | | | | | |
| τ | 9 | | | | | | 12 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.619 | 0.050 | 0.185 | 0.427 | 0.538 | 0.611 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.808 | 0.303 | 0.620 | 0.864 | 0.908 | 0.927 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.875 | 0.680 | 0.903 | 0.979 | 0.985 | 0.985 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.897 | 0.908 | 0.984 | 0.998 | 0.998 | 0.999 | 0.587 | 0.086 | 0.240 | 0.447 | 0.529 | 0.613 |
| 13 | 0.902 | 0.983 | 0.998 | 1.000 | 1.000 | 1.000 | 0.782 | 0.378 | 0.659 | 0.867 | 0.904 | 0.931 |
| 14 | 0.904 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 0.844 | 0.726 | 0.912 | 0.981 | 0.985 | 0.987 |
| 15 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.865 | 0.918 | 0.984 | 0.997 | 0.998 | 0.999 |
| 16 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.870 | 0.985 | 0.998 | 1.000 | 1.000 | 1.000 |
| 17 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.873 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| 18 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.873 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 19 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.873 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 20 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.873 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 21 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 22 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 23 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 24 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 43 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

## Appendix F: Cumulative Shift Detection Probabilities for D = 64 and Δ = 1.1

| D | 64 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.1 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.019 | 0.000 | 0.000 | 0.000 | 0.002 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.038 | 0.000 | 0.000 | 0.004 | 0.012 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.059 | 0.000 | 0.000 | 0.014 | 0.030 | 0.044 | 0.018 | 0.000 | 0.000 | 0.008 | 0.016 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.080 | 0.000 | 0.002 | 0.033 | 0.049 | 0.065 | 0.036 | 0.000 | 0.001 | 0.022 | 0.036 | 0.037 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.098 | 0.000 | 0.008 | 0.053 | 0.073 | 0.086 | 0.055 | 0.000 | 0.005 | 0.040 | 0.056 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.116 | 0.001 | 0.018 | 0.076 | 0.098 | 0.105 | 0.074 | 0.001 | 0.012 | 0.060 | 0.075 | 0.079 | 0.019 | 0.000 | 0.004 | 0.019 | 0.020 | 0.029 |
| 7 | 0.133 | 0.003 | 0.029 | 0.102 | 0.120 | 0.125 | 0.092 | 0.002 | 0.023 | 0.082 | 0.098 | 0.099 | 0.037 | 0.001 | 0.010 | 0.037 | 0.042 | 0.051 |
| 8 | 0.149 | 0.007 | 0.043 | 0.124 | 0.141 | 0.146 | 0.108 | 0.004 | 0.036 | 0.104 | 0.119 | 0.122 | 0.055 | 0.002 | 0.019 | 0.061 | 0.064 | 0.073 |
| 9 | 0.165 | 0.014 | 0.061 | 0.148 | 0.163 | 0.165 | 0.124 | 0.008 | 0.052 | 0.127 | 0.145 | 0.144 | 0.071 | 0.005 | 0.032 | 0.084 | 0.087 | 0.096 |
| 10 | 0.182 | 0.023 | 0.082 | 0.171 | 0.185 | 0.184 | 0.142 | 0.014 | 0.070 | 0.149 | 0.166 | 0.166 | 0.087 | 0.010 | 0.049 | 0.106 | 0.109 | 0.118 |
| 11 | 0.199 | 0.035 | 0.105 | 0.196 | 0.205 | 0.204 | 0.157 | 0.024 | 0.088 | 0.173 | 0.184 | 0.182 | 0.102 | 0.017 | 0.069 | 0.131 | 0.132 | 0.139 |
| 12 | 0.212 | 0.050 | 0.129 | 0.217 | 0.226 | 0.223 | 0.172 | 0.035 | 0.112 | 0.196 | 0.204 | 0.202 | 0.117 | 0.025 | 0.086 | 0.156 | 0.152 | 0.159 |
| 13 | 0.228 | 0.065 | 0.155 | 0.238 | 0.243 | 0.242 | 0.187 | 0.049 | 0.136 | 0.221 | 0.224 | 0.221 | 0.135 | 0.037 | 0.108 | 0.180 | 0.174 | 0.178 |
| 14 | 0.242 | 0.081 | 0.180 | 0.263 | 0.262 | 0.262 | 0.201 | 0.065 | 0.162 | 0.246 | 0.244 | 0.238 | 0.154 | 0.049 | 0.130 | 0.205 | 0.197 | 0.199 |
| 15 | 0.257 | 0.102 | 0.208 | 0.286 | 0.280 | 0.282 | 0.217 | 0.085 | 0.185 | 0.269 | 0.263 | 0.256 | 0.167 | 0.064 | 0.155 | 0.229 | 0.219 | 0.217 |
| 16 | 0.271 | 0.123 | 0.230 | 0.306 | 0.299 | 0.301 | 0.233 | 0.105 | 0.211 | 0.292 | 0.284 | 0.275 | 0.183 | 0.082 | 0.178 | 0.253 | 0.238 | 0.238 |
| 17 | 0.287 | 0.144 | 0.253 | 0.325 | 0.318 | 0.319 | 0.247 | 0.126 | 0.237 | 0.312 | 0.299 | 0.293 | 0.200 | 0.103 | 0.204 | 0.276 | 0.260 | 0.259 |
| 18 | 0.302 | 0.169 | 0.281 | 0.348 | 0.333 | 0.337 | 0.261 | 0.148 | 0.262 | 0.332 | 0.314 | 0.310 | 0.214 | 0.124 | 0.226 | 0.296 | 0.280 | 0.279 |
| 19 | 0.316 | 0.193 | 0.304 | 0.369 | 0.352 | 0.354 | 0.275 | 0.173 | 0.286 | 0.352 | 0.329 | 0.326 | 0.226 | 0.146 | 0.251 | 0.316 | 0.297 | 0.298 |
| 20 | 0.328 | 0.216 | 0.328 | 0.389 | 0.367 | 0.370 | 0.288 | 0.197 | 0.308 | 0.369 | 0.345 | 0.345 | 0.241 | 0.171 | 0.274 | 0.335 | 0.316 | 0.317 |
| 21 | 0.343 | 0.241 | 0.352 | 0.404 | 0.381 | 0.384 | 0.303 | 0.223 | 0.331 | 0.385 | 0.364 | 0.360 | 0.255 | 0.199 | 0.298 | 0.356 | 0.332 | 0.333 |
| 22 | 0.354 | 0.268 | 0.372 | 0.423 | 0.398 | 0.401 | 0.317 | 0.249 | 0.353 | 0.403 | 0.378 | 0.377 | 0.270 | 0.223 | 0.320 | 0.377 | 0.349 | 0.350 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.368 | 0.291 | 0.394 | 0.439 | 0.412 | 0.417 | 0.330 | 0.273 | 0.379 | 0.419 | 0.393 | 0.393 | 0.284 | 0.248 | 0.343 | 0.396 | 0.363 | 0.368 |
| 24 | 0.379 | 0.317 | 0.414 | 0.458 | 0.427 | 0.431 | 0.343 | 0.296 | 0.399 | 0.437 | 0.410 | 0.409 | 0.297 | 0.271 | 0.365 | 0.414 | 0.379 | 0.385 |
| 25 | 0.390 | 0.343 | 0.432 | 0.475 | 0.442 | 0.446 | 0.357 | 0.320 | 0.421 | 0.452 | 0.426 | 0.427 | 0.310 | 0.298 | 0.385 | 0.432 | 0.394 | 0.402 |
| 26 | 0.402 | 0.367 | 0.457 | 0.489 | 0.456 | 0.460 | 0.369 | 0.345 | 0.443 | 0.469 | 0.438 | 0.442 | 0.323 | 0.320 | 0.407 | 0.448 | 0.410 | 0.417 |
| 27 | 0.415 | 0.391 | 0.476 | 0.504 | 0.468 | 0.473 | 0.383 | 0.372 | 0.464 | 0.485 | 0.453 | 0.456 | 0.334 | 0.343 | 0.428 | 0.464 | 0.424 | 0.433 |
| 28 | 0.427 | 0.417 | 0.494 | 0.519 | 0.481 | 0.487 | 0.394 | 0.396 | 0.483 | 0.499 | 0.467 | 0.470 | 0.347 | 0.369 | 0.447 | 0.480 | 0.439 | 0.449 |
| 29 | 0.438 | 0.439 | 0.510 | 0.535 | 0.495 | 0.500 | 0.405 | 0.422 | 0.503 | 0.516 | 0.481 | 0.484 | 0.358 | 0.394 | 0.465 | 0.497 | 0.455 | 0.464 |
| 30 | 0.450 | 0.459 | 0.526 | 0.549 | 0.506 | 0.514 | 0.418 | 0.446 | 0.523 | 0.531 | 0.493 | 0.499 | 0.371 | 0.417 | 0.483 | 0.510 | 0.470 | 0.479 |
| 31 | 0.460 | 0.482 | 0.544 | 0.564 | 0.520 | 0.528 | 0.429 | 0.470 | 0.541 | 0.545 | 0.507 | 0.511 | 0.382 | 0.441 | 0.502 | 0.524 | 0.487 | 0.493 |
| 32 | 0.470 | 0.504 | 0.561 | 0.577 | 0.531 | 0.540 | 0.438 | 0.492 | 0.558 | 0.558 | 0.520 | 0.524 | 0.395 | 0.460 | 0.521 | 0.538 | 0.499 | 0.507 |
| 33 | 0.482 | 0.526 | 0.578 | 0.588 | 0.544 | 0.552 | 0.449 | 0.514 | 0.575 | 0.573 | 0.532 | 0.536 | 0.406 | 0.481 | 0.537 | 0.553 | 0.512 | 0.520 |
| 34 | 0.491 | 0.546 | 0.595 | 0.600 | 0.555 | 0.564 | 0.460 | 0.532 | 0.589 | 0.585 | 0.543 | 0.547 | 0.416 | 0.502 | 0.553 | 0.567 | 0.524 | 0.535 |
| 35 | 0.501 | 0.566 | 0.609 | 0.611 | 0.566 | 0.574 | 0.470 | 0.554 | 0.603 | 0.596 | 0.555 | 0.558 | 0.428 | 0.521 | 0.568 | 0.579 | 0.538 | 0.546 |
| 36 | 0.510 | 0.585 | 0.624 | 0.621 | 0.577 | 0.587 | 0.481 | 0.573 | 0.618 | 0.611 | 0.566 | 0.568 | 0.437 | 0.541 | 0.583 | 0.591 | 0.549 | 0.556 |
| 37 | 0.521 | 0.602 | 0.638 | 0.631 | 0.588 | 0.599 | 0.492 | 0.591 | 0.630 | 0.622 | 0.577 | 0.577 | 0.448 | 0.559 | 0.598 | 0.604 | 0.561 | 0.568 |
| 38 | 0.533 | 0.619 | 0.650 | 0.642 | 0.597 | 0.609 | 0.501 | 0.607 | 0.645 | 0.632 | 0.589 | 0.586 | 0.457 | 0.578 | 0.612 | 0.616 | 0.571 | 0.578 |
| 39 | 0.541 | 0.637 | 0.660 | 0.653 | 0.605 | 0.620 | 0.510 | 0.625 | 0.658 | 0.644 | 0.600 | 0.598 | 0.467 | 0.596 | 0.625 | 0.627 | 0.581 | 0.591 |
| 40 | 0.549 | 0.652 | 0.671 | 0.665 | 0.615 | 0.628 | 0.518 | 0.640 | 0.670 | 0.653 | 0.613 | 0.608 | 0.475 | 0.612 | 0.637 | 0.638 | 0.594 | 0.603 |
| 41 | 0.559 | 0.666 | 0.683 | 0.674 | 0.624 | 0.639 | 0.527 | 0.656 | 0.684 | 0.663 | 0.621 | 0.617 | 0.485 | 0.631 | 0.652 | 0.649 | 0.605 | 0.614 |
| 42 | 0.569 | 0.680 | 0.693 | 0.682 | 0.633 | 0.648 | 0.536 | 0.671 | 0.697 | 0.674 | 0.631 | 0.629 | 0.493 | 0.646 | 0.664 | 0.660 | 0.614 | 0.624 |
| 43 | 0.577 | 0.696 | 0.705 | 0.691 | 0.643 | 0.659 | 0.545 | 0.688 | 0.707 | 0.685 | 0.639 | 0.639 | 0.501 | 0.664 | 0.676 | 0.671 | 0.625 | 0.634 |
| 44 | 0.585 | 0.708 | 0.715 | 0.700 | 0.651 | 0.668 | 0.553 | 0.702 | 0.718 | 0.694 | 0.649 | 0.651 | 0.510 | 0.681 | 0.688 | 0.679 | 0.634 | 0.644 |
| 45 | 0.594 | 0.719 | 0.725 | 0.709 | 0.663 | 0.677 | 0.561 | 0.716 | 0.729 | 0.703 | 0.657 | 0.660 | 0.518 | 0.694 | 0.699 | 0.689 | 0.645 | 0.655 |
| 46 | 0.602 | 0.733 | 0.736 | 0.718 | 0.670 | 0.684 | 0.569 | 0.729 | 0.739 | 0.713 | 0.666 | 0.669 | 0.528 | 0.708 | 0.710 | 0.698 | 0.656 | 0.665 |
| 47 | 0.612 | 0.745 | 0.746 | 0.729 | 0.678 | 0.694 | 0.577 | 0.739 | 0.747 | 0.722 | 0.676 | 0.676 | 0.535 | 0.721 | 0.719 | 0.707 | 0.664 | 0.673 |
| 48 | 0.620 | 0.757 | 0.755 | 0.738 | 0.688 | 0.701 | 0.583 | 0.750 | 0.756 | 0.731 | 0.684 | 0.685 | 0.544 | 0.733 | 0.731 | 0.716 | 0.674 | 0.680 |
| 49 | 0.627 | 0.768 | 0.763 | 0.747 | 0.696 | 0.709 | 0.591 | 0.761 | 0.764 | 0.741 | 0.692 | 0.692 | 0.554 | 0.743 | 0.741 | 0.726 | 0.683 | 0.688 |
| 50 | 0.634 | 0.776 | 0.771 | 0.754 | 0.703 | 0.717 | 0.599 | 0.772 | 0.772 | 0.747 | 0.702 | 0.699 | 0.562 | 0.755 | 0.751 | 0.733 | 0.689 | 0.697 |
| 51 | 0.641 | 0.786 | 0.780 | 0.761 | 0.711 | 0.724 | 0.607 | 0.782 | 0.779 | 0.754 | 0.709 | 0.706 | 0.570 | 0.765 | 0.760 | 0.742 | 0.700 | 0.706 |
| 52 | 0.648 | 0.795 | 0.788 | 0.768 | 0.718 | 0.730 | 0.615 | 0.791 | 0.789 | 0.762 | 0.717 | 0.714 | 0.578 | 0.776 | 0.769 | 0.750 | 0.708 | 0.713 |
| 53 | 0.656 | 0.805 | 0.795 | 0.775 | 0.726 | 0.737 | 0.623 | 0.798 | 0.796 | 0.770 | 0.724 | 0.721 | 0.587 | 0.786 | 0.779 | 0.757 | 0.715 | 0.719 |
| 54 | 0.662 | 0.814 | 0.802 | 0.782 | 0.732 | 0.743 | 0.630 | 0.806 | 0.802 | 0.778 | 0.731 | 0.730 | 0.594 | 0.794 | 0.786 | 0.764 | 0.722 | 0.727 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 0.668 | 0.824 | 0.811 | 0.789 | 0.741 | 0.750 | 0.638 | 0.815 | 0.810 | 0.783 | 0.739 | 0.739 | 0.601 | 0.803 | 0.795 | 0.771 | 0.731 | 0.735 |
| 56 | 0.674 | 0.831 | 0.818 | 0.795 | 0.746 | 0.756 | 0.645 | 0.824 | 0.816 | 0.789 | 0.745 | 0.745 | 0.607 | 0.811 | 0.804 | 0.777 | 0.738 | 0.743 |
| 57 | 0.680 | 0.838 | 0.824 | 0.800 | 0.751 | 0.763 | 0.652 | 0.830 | 0.823 | 0.796 | 0.751 | 0.751 | 0.614 | 0.822 | 0.811 | 0.785 | 0.745 | 0.749 |
| 58 | 0.686 | 0.846 | 0.831 | 0.806 | 0.758 | 0.768 | 0.659 | 0.838 | 0.831 | 0.803 | 0.757 | 0.757 | 0.620 | 0.831 | 0.818 | 0.792 | 0.750 | 0.757 |
| 59 | 0.692 | 0.853 | 0.837 | 0.812 | 0.764 | 0.775 | 0.664 | 0.845 | 0.837 | 0.808 | 0.763 | 0.764 | 0.625 | 0.838 | 0.824 | 0.797 | 0.756 | 0.762 |
| 60 | 0.699 | 0.861 | 0.842 | 0.818 | 0.770 | 0.780 | 0.670 | 0.851 | 0.842 | 0.812 | 0.769 | 0.770 | 0.632 | 0.847 | 0.832 | 0.804 | 0.762 | 0.768 |
| 61 | 0.706 | 0.867 | 0.848 | 0.823 | 0.775 | 0.786 | 0.677 | 0.857 | 0.848 | 0.818 | 0.776 | 0.776 | 0.638 | 0.855 | 0.838 | 0.811 | 0.768 | 0.773 |
| 62 | 0.711 | 0.874 | 0.853 | 0.828 | 0.781 | 0.791 | 0.683 | 0.863 | 0.852 | 0.824 | 0.782 | 0.782 | 0.644 | 0.861 | 0.844 | 0.816 | 0.774 | 0.778 |
| 63 | 0.716 | 0.881 | 0.858 | 0.833 | 0.787 | 0.797 | 0.688 | 0.870 | 0.857 | 0.829 | 0.788 | 0.787 | 0.649 | 0.869 | 0.851 | 0.822 | 0.781 | 0.785 |
| 64 | 0.722 | 0.887 | 0.863 | 0.838 | 0.794 | 0.802 | 0.694 | 0.877 | 0.862 | 0.834 | 0.794 | 0.793 | 0.656 | 0.875 | 0.856 | 0.828 | 0.786 | 0.791 |

| **D** | **64** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **1.1** | | | | | | | | | | | |
| **τ** | **9** | | | | | | **12** | | | | | |
| **Run Length / Chart Type** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.018 | 0.003 | 0.017 | 0.027 | 0.022 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.035 | 0.006 | 0.029 | 0.048 | 0.043 | 0.059 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.051 | 0.011 | 0.043 | 0.071 | 0.066 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.068 | 0.018 | 0.059 | 0.093 | 0.088 | 0.103 | 0.017 | 0.010 | 0.027 | 0.026 | 0.021 | 0.039 |
| 13 | 0.084 | 0.029 | 0.078 | 0.117 | 0.111 | 0.125 | 0.033 | 0.017 | 0.045 | 0.051 | 0.042 | 0.062 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 0.102 | 0.042 | 0.098 | 0.141 | 0.134 | 0.148 | 0.050 | 0.027 | 0.061 | 0.076 | 0.067 | 0.086 |
| 15 | 0.118 | 0.053 | 0.122 | 0.167 | 0.156 | 0.172 | 0.068 | 0.039 | 0.081 | 0.098 | 0.090 | 0.108 |
| 16 | 0.133 | 0.068 | 0.146 | 0.192 | 0.178 | 0.195 | 0.084 | 0.051 | 0.100 | 0.122 | 0.112 | 0.128 |
| 17 | 0.150 | 0.085 | 0.171 | 0.216 | 0.200 | 0.218 | 0.098 | 0.066 | 0.122 | 0.147 | 0.135 | 0.154 |
| 18 | 0.165 | 0.102 | 0.197 | 0.240 | 0.218 | 0.236 | 0.113 | 0.083 | 0.148 | 0.171 | 0.156 | 0.174 |
| 19 | 0.180 | 0.121 | 0.223 | 0.265 | 0.236 | 0.255 | 0.126 | 0.102 | 0.171 | 0.198 | 0.179 | 0.196 |
| 20 | 0.195 | 0.143 | 0.248 | 0.287 | 0.255 | 0.276 | 0.140 | 0.124 | 0.201 | 0.221 | 0.199 | 0.217 |
| 21 | 0.211 | 0.165 | 0.275 | 0.307 | 0.274 | 0.296 | 0.154 | 0.146 | 0.226 | 0.242 | 0.218 | 0.236 |
| 22 | 0.224 | 0.189 | 0.295 | 0.324 | 0.295 | 0.313 | 0.169 | 0.171 | 0.248 | 0.264 | 0.238 | 0.254 |
| 23 | 0.235 | 0.213 | 0.318 | 0.344 | 0.310 | 0.331 | 0.182 | 0.193 | 0.274 | 0.286 | 0.259 | 0.272 |
| 24 | 0.247 | 0.241 | 0.340 | 0.364 | 0.328 | 0.347 | 0.196 | 0.218 | 0.296 | 0.307 | 0.280 | 0.289 |
| 25 | 0.259 | 0.266 | 0.365 | 0.385 | 0.347 | 0.364 | 0.207 | 0.243 | 0.319 | 0.327 | 0.298 | 0.311 |
| 26 | 0.272 | 0.289 | 0.386 | 0.401 | 0.364 | 0.382 | 0.220 | 0.269 | 0.344 | 0.348 | 0.316 | 0.328 |
| 27 | 0.284 | 0.314 | 0.406 | 0.418 | 0.378 | 0.400 | 0.233 | 0.295 | 0.364 | 0.369 | 0.332 | 0.344 |
| 28 | 0.297 | 0.342 | 0.426 | 0.434 | 0.392 | 0.414 | 0.246 | 0.321 | 0.386 | 0.389 | 0.350 | 0.362 |
| 29 | 0.309 | 0.368 | 0.445 | 0.450 | 0.407 | 0.429 | 0.259 | 0.344 | 0.405 | 0.410 | 0.366 | 0.379 |
| 30 | 0.321 | 0.390 | 0.464 | 0.466 | 0.421 | 0.445 | 0.270 | 0.367 | 0.427 | 0.428 | 0.381 | 0.395 |
| 31 | 0.332 | 0.413 | 0.484 | 0.482 | 0.435 | 0.459 | 0.284 | 0.392 | 0.446 | 0.444 | 0.396 | 0.412 |
| 32 | 0.343 | 0.436 | 0.503 | 0.499 | 0.448 | 0.473 | 0.296 | 0.415 | 0.464 | 0.459 | 0.413 | 0.428 |
| 33 | 0.356 | 0.460 | 0.520 | 0.514 | 0.461 | 0.489 | 0.309 | 0.434 | 0.482 | 0.476 | 0.428 | 0.441 |
| 34 | 0.366 | 0.481 | 0.537 | 0.528 | 0.476 | 0.501 | 0.322 | 0.457 | 0.500 | 0.490 | 0.443 | 0.456 |
| 35 | 0.377 | 0.502 | 0.553 | 0.544 | 0.490 | 0.515 | 0.332 | 0.478 | 0.517 | 0.506 | 0.459 | 0.471 |
| 36 | 0.388 | 0.522 | 0.568 | 0.556 | 0.501 | 0.528 | 0.343 | 0.500 | 0.535 | 0.522 | 0.473 | 0.484 |
| 37 | 0.399 | 0.541 | 0.584 | 0.569 | 0.515 | 0.540 | 0.353 | 0.518 | 0.552 | 0.534 | 0.487 | 0.498 |
| 38 | 0.410 | 0.563 | 0.598 | 0.582 | 0.528 | 0.553 | 0.363 | 0.537 | 0.566 | 0.548 | 0.500 | 0.511 |
| 39 | 0.418 | 0.580 | 0.613 | 0.593 | 0.540 | 0.564 | 0.372 | 0.557 | 0.581 | 0.561 | 0.510 | 0.525 |
| 40 | 0.426 | 0.597 | 0.626 | 0.604 | 0.552 | 0.578 | 0.383 | 0.572 | 0.597 | 0.576 | 0.524 | 0.536 |
| 41 | 0.436 | 0.614 | 0.639 | 0.615 | 0.564 | 0.589 | 0.392 | 0.590 | 0.611 | 0.588 | 0.537 | 0.549 |
| 42 | 0.444 | 0.633 | 0.651 | 0.628 | 0.576 | 0.600 | 0.402 | 0.609 | 0.624 | 0.600 | 0.548 | 0.562 |
| 43 | 0.454 | 0.647 | 0.663 | 0.637 | 0.587 | 0.611 | 0.413 | 0.625 | 0.638 | 0.612 | 0.558 | 0.573 |
| 44 | 0.462 | 0.660 | 0.676 | 0.647 | 0.600 | 0.622 | 0.424 | 0.642 | 0.650 | 0.624 | 0.567 | 0.583 |
| 45 | 0.471 | 0.675 | 0.688 | 0.657 | 0.612 | 0.631 | 0.433 | 0.656 | 0.662 | 0.633 | 0.578 | 0.593 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 46 | 0.480 | 0.686 | 0.700 | 0.666 | 0.622 | 0.641 | 0.445 | 0.670 | 0.675 | 0.645 | 0.588 | 0.605 |
| 47 | 0.488 | 0.700 | 0.712 | 0.677 | 0.630 | 0.652 | 0.453 | 0.683 | 0.686 | 0.656 | 0.599 | 0.615 |
| 48 | 0.496 | 0.714 | 0.722 | 0.686 | 0.639 | 0.661 | 0.463 | 0.697 | 0.695 | 0.667 | 0.607 | 0.625 |
| 49 | 0.504 | 0.727 | 0.730 | 0.696 | 0.650 | 0.670 | 0.471 | 0.711 | 0.705 | 0.677 | 0.619 | 0.634 |
| 50 | 0.513 | 0.740 | 0.739 | 0.704 | 0.660 | 0.679 | 0.478 | 0.723 | 0.715 | 0.685 | 0.628 | 0.643 |
| 51 | 0.521 | 0.749 | 0.748 | 0.713 | 0.669 | 0.688 | 0.486 | 0.734 | 0.726 | 0.693 | 0.637 | 0.653 |
| 52 | 0.528 | 0.760 | 0.758 | 0.720 | 0.677 | 0.697 | 0.493 | 0.746 | 0.737 | 0.703 | 0.646 | 0.662 |
| 53 | 0.536 | 0.772 | 0.767 | 0.727 | 0.684 | 0.705 | 0.501 | 0.759 | 0.746 | 0.711 | 0.655 | 0.670 |
| 54 | 0.545 | 0.784 | 0.776 | 0.736 | 0.693 | 0.714 | 0.506 | 0.769 | 0.756 | 0.719 | 0.664 | 0.679 |
| 55 | 0.550 | 0.793 | 0.784 | 0.743 | 0.702 | 0.722 | 0.512 | 0.780 | 0.766 | 0.728 | 0.672 | 0.688 |
| 56 | 0.558 | 0.802 | 0.792 | 0.751 | 0.708 | 0.730 | 0.519 | 0.789 | 0.773 | 0.735 | 0.680 | 0.697 |
| 57 | 0.565 | 0.811 | 0.797 | 0.759 | 0.716 | 0.736 | 0.527 | 0.799 | 0.782 | 0.744 | 0.687 | 0.705 |
| 58 | 0.571 | 0.819 | 0.805 | 0.766 | 0.724 | 0.745 | 0.533 | 0.809 | 0.790 | 0.751 | 0.695 | 0.712 |
| 59 | 0.578 | 0.827 | 0.813 | 0.774 | 0.730 | 0.751 | 0.540 | 0.818 | 0.799 | 0.759 | 0.702 | 0.718 |
| 60 | 0.584 | 0.835 | 0.819 | 0.782 | 0.737 | 0.759 | 0.548 | 0.826 | 0.807 | 0.766 | 0.711 | 0.726 |
| 61 | 0.591 | 0.842 | 0.825 | 0.790 | 0.743 | 0.767 | 0.555 | 0.834 | 0.814 | 0.773 | 0.719 | 0.732 |
| 62 | 0.597 | 0.849 | 0.831 | 0.795 | 0.750 | 0.772 | 0.563 | 0.841 | 0.822 | 0.779 | 0.725 | 0.739 |
| 63 | 0.604 | 0.855 | 0.837 | 0.801 | 0.757 | 0.778 | 0.569 | 0.848 | 0.827 | 0.786 | 0.732 | 0.747 |
| 64 | 0.611 | 0.860 | 0.843 | 0.806 | 0.764 | 0.784 | 0.575 | 0.855 | 0.833 | 0.792 | 0.739 | 0.755 |

## Appendix G: Cumulative Shift Detection Probabilities for D = 64 and Δ = 1.25

| D | 64 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.25 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.037 | 0.000 | 0.000 | 0.001 | 0.006 | 0.014 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.073 | 0.000 | 0.000 | 0.013 | 0.031 | 0.054 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.108 | 0.000 | 0.002 | 0.043 | 0.074 | 0.095 | 0.037 | 0.000 | 0.000 | 0.014 | 0.025 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.139 | 0.000 | 0.011 | 0.088 | 0.121 | 0.135 | 0.074 | 0.000 | 0.004 | 0.042 | 0.063 | 0.076 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.174 | 0.002 | 0.032 | 0.140 | 0.173 | 0.179 | 0.110 | 0.000 | 0.013 | 0.082 | 0.113 | 0.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.207 | 0.008 | 0.066 | 0.192 | 0.226 | 0.222 | 0.142 | 0.002 | 0.032 | 0.133 | 0.168 | 0.161 | 0.040 | 0.000 | 0.007 | 0.031 | 0.033 | 0.041 |
| 7 | 0.238 | 0.020 | 0.106 | 0.249 | 0.270 | 0.263 | 0.173 | 0.009 | 0.062 | 0.182 | 0.215 | 0.204 | 0.072 | 0.002 | 0.021 | 0.071 | 0.076 | 0.083 |
| 8 | 0.265 | 0.041 | 0.152 | 0.304 | 0.309 | 0.301 | 0.201 | 0.022 | 0.099 | 0.236 | 0.259 | 0.248 | 0.103 | 0.006 | 0.044 | 0.117 | 0.121 | 0.128 |
| 9 | 0.296 | 0.069 | 0.205 | 0.352 | 0.347 | 0.340 | 0.228 | 0.042 | 0.147 | 0.286 | 0.301 | 0.292 | 0.134 | 0.016 | 0.078 | 0.170 | 0.171 | 0.177 |
| 10 | 0.325 | 0.105 | 0.259 | 0.397 | 0.387 | 0.376 | 0.257 | 0.072 | 0.199 | 0.342 | 0.343 | 0.331 | 0.166 | 0.033 | 0.115 | 0.224 | 0.218 | 0.226 |
| 11 | 0.352 | 0.152 | 0.312 | 0.443 | 0.422 | 0.413 | 0.283 | 0.107 | 0.258 | 0.385 | 0.382 | 0.368 | 0.194 | 0.057 | 0.160 | 0.269 | 0.262 | 0.273 |
| 12 | 0.377 | 0.201 | 0.367 | 0.486 | 0.459 | 0.448 | 0.309 | 0.148 | 0.311 | 0.429 | 0.417 | 0.404 | 0.220 | 0.083 | 0.209 | 0.318 | 0.304 | 0.317 |
| 13 | 0.401 | 0.252 | 0.423 | 0.527 | 0.494 | 0.482 | 0.334 | 0.194 | 0.365 | 0.470 | 0.452 | 0.443 | 0.247 | 0.119 | 0.259 | 0.366 | 0.349 | 0.355 |
| 14 | 0.422 | 0.303 | 0.474 | 0.563 | 0.524 | 0.516 | 0.360 | 0.238 | 0.415 | 0.507 | 0.486 | 0.476 | 0.274 | 0.161 | 0.309 | 0.411 | 0.388 | 0.392 |
| 15 | 0.445 | 0.361 | 0.521 | 0.595 | 0.550 | 0.545 | 0.383 | 0.290 | 0.462 | 0.545 | 0.515 | 0.508 | 0.299 | 0.207 | 0.361 | 0.453 | 0.424 | 0.429 |
| 16 | 0.466 | 0.414 | 0.564 | 0.625 | 0.577 | 0.574 | 0.408 | 0.344 | 0.509 | 0.578 | 0.543 | 0.539 | 0.323 | 0.256 | 0.413 | 0.492 | 0.460 | 0.465 |
| 17 | 0.487 | 0.468 | 0.604 | 0.650 | 0.601 | 0.598 | 0.429 | 0.399 | 0.552 | 0.607 | 0.572 | 0.569 | 0.346 | 0.304 | 0.462 | 0.530 | 0.489 | 0.497 |
| 18 | 0.507 | 0.515 | 0.638 | 0.675 | 0.625 | 0.624 | 0.451 | 0.450 | 0.594 | 0.637 | 0.600 | 0.597 | 0.368 | 0.357 | 0.508 | 0.563 | 0.520 | 0.528 |
| 19 | 0.527 | 0.562 | 0.672 | 0.700 | 0.649 | 0.647 | 0.469 | 0.499 | 0.633 | 0.664 | 0.623 | 0.623 | 0.389 | 0.408 | 0.550 | 0.596 | 0.550 | 0.557 |
| 20 | 0.544 | 0.605 | 0.703 | 0.719 | 0.669 | 0.670 | 0.487 | 0.544 | 0.666 | 0.689 | 0.648 | 0.646 | 0.409 | 0.456 | 0.593 | 0.628 | 0.576 | 0.586 |
| 21 | 0.561 | 0.648 | 0.730 | 0.739 | 0.690 | 0.692 | 0.505 | 0.591 | 0.696 | 0.710 | 0.671 | 0.669 | 0.428 | 0.505 | 0.632 | 0.655 | 0.601 | 0.612 |
| 22 | 0.577 | 0.686 | 0.755 | 0.756 | 0.707 | 0.711 | 0.521 | 0.630 | 0.723 | 0.733 | 0.690 | 0.692 | 0.446 | 0.550 | 0.666 | 0.678 | 0.626 | 0.639 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.593 | 0.717 | 0.778 | 0.774 | 0.724 | 0.729 | 0.539 | 0.664 | 0.750 | 0.753 | 0.707 | 0.713 | 0.463 | 0.594 | 0.694 | 0.703 | 0.650 | 0.660 |
| 24 | 0.608 | 0.743 | 0.797 | 0.792 | 0.741 | 0.748 | 0.555 | 0.700 | 0.774 | 0.772 | 0.724 | 0.733 | 0.478 | 0.635 | 0.721 | 0.727 | 0.671 | 0.682 |
| 25 | 0.623 | 0.773 | 0.818 | 0.808 | 0.757 | 0.765 | 0.569 | 0.733 | 0.795 | 0.788 | 0.742 | 0.750 | 0.496 | 0.671 | 0.749 | 0.746 | 0.694 | 0.702 |
| 26 | 0.638 | 0.799 | 0.838 | 0.821 | 0.774 | 0.781 | 0.584 | 0.760 | 0.813 | 0.803 | 0.758 | 0.767 | 0.513 | 0.703 | 0.771 | 0.765 | 0.710 | 0.721 |
| 27 | 0.651 | 0.821 | 0.852 | 0.834 | 0.789 | 0.796 | 0.600 | 0.787 | 0.828 | 0.818 | 0.776 | 0.780 | 0.530 | 0.733 | 0.793 | 0.782 | 0.727 | 0.738 |
| 28 | 0.664 | 0.841 | 0.866 | 0.847 | 0.801 | 0.808 | 0.614 | 0.811 | 0.845 | 0.832 | 0.789 | 0.794 | 0.546 | 0.759 | 0.814 | 0.798 | 0.744 | 0.754 |
| 29 | 0.677 | 0.859 | 0.878 | 0.859 | 0.814 | 0.822 | 0.631 | 0.834 | 0.860 | 0.844 | 0.801 | 0.809 | 0.558 | 0.785 | 0.831 | 0.812 | 0.758 | 0.772 |
| 30 | 0.689 | 0.873 | 0.891 | 0.869 | 0.827 | 0.834 | 0.645 | 0.853 | 0.874 | 0.854 | 0.813 | 0.821 | 0.571 | 0.810 | 0.846 | 0.826 | 0.772 | 0.787 |
| 31 | 0.700 | 0.889 | 0.899 | 0.878 | 0.837 | 0.844 | 0.658 | 0.868 | 0.884 | 0.865 | 0.826 | 0.834 | 0.586 | 0.830 | 0.861 | 0.838 | 0.785 | 0.802 |
| 32 | 0.712 | 0.904 | 0.910 | 0.889 | 0.846 | 0.855 | 0.673 | 0.882 | 0.896 | 0.875 | 0.836 | 0.847 | 0.597 | 0.848 | 0.876 | 0.850 | 0.797 | 0.816 |
| 33 | 0.721 | 0.914 | 0.919 | 0.899 | 0.855 | 0.865 | 0.684 | 0.896 | 0.907 | 0.883 | 0.845 | 0.859 | 0.611 | 0.865 | 0.887 | 0.862 | 0.810 | 0.828 |
| 34 | 0.732 | 0.924 | 0.927 | 0.907 | 0.863 | 0.875 | 0.694 | 0.907 | 0.917 | 0.892 | 0.853 | 0.869 | 0.625 | 0.879 | 0.898 | 0.873 | 0.822 | 0.839 |
| 35 | 0.742 | 0.933 | 0.932 | 0.913 | 0.871 | 0.883 | 0.705 | 0.919 | 0.924 | 0.901 | 0.863 | 0.880 | 0.637 | 0.892 | 0.908 | 0.882 | 0.833 | 0.850 |
| 36 | 0.751 | 0.941 | 0.939 | 0.919 | 0.879 | 0.892 | 0.714 | 0.929 | 0.932 | 0.908 | 0.873 | 0.888 | 0.648 | 0.904 | 0.917 | 0.890 | 0.843 | 0.860 |
| 37 | 0.760 | 0.946 | 0.945 | 0.925 | 0.886 | 0.900 | 0.725 | 0.938 | 0.939 | 0.915 | 0.880 | 0.896 | 0.659 | 0.915 | 0.924 | 0.899 | 0.852 | 0.870 |
| 38 | 0.768 | 0.954 | 0.950 | 0.930 | 0.894 | 0.906 | 0.734 | 0.946 | 0.945 | 0.922 | 0.887 | 0.902 | 0.669 | 0.926 | 0.931 | 0.907 | 0.862 | 0.879 |
| 39 | 0.777 | 0.958 | 0.955 | 0.935 | 0.900 | 0.911 | 0.743 | 0.953 | 0.950 | 0.928 | 0.894 | 0.910 | 0.682 | 0.934 | 0.937 | 0.912 | 0.870 | 0.887 |
| 40 | 0.786 | 0.963 | 0.959 | 0.940 | 0.906 | 0.917 | 0.753 | 0.959 | 0.954 | 0.934 | 0.900 | 0.916 | 0.691 | 0.940 | 0.943 | 0.919 | 0.876 | 0.895 |
| 41 | 0.793 | 0.967 | 0.965 | 0.945 | 0.911 | 0.924 | 0.761 | 0.963 | 0.960 | 0.938 | 0.907 | 0.922 | 0.700 | 0.948 | 0.949 | 0.925 | 0.884 | 0.901 |
| 42 | 0.801 | 0.971 | 0.969 | 0.949 | 0.916 | 0.928 | 0.769 | 0.968 | 0.964 | 0.943 | 0.911 | 0.928 | 0.710 | 0.954 | 0.953 | 0.931 | 0.891 | 0.910 |
| 43 | 0.810 | 0.974 | 0.972 | 0.952 | 0.921 | 0.933 | 0.776 | 0.971 | 0.968 | 0.948 | 0.917 | 0.931 | 0.718 | 0.960 | 0.958 | 0.935 | 0.897 | 0.915 |
| 44 | 0.816 | 0.977 | 0.974 | 0.956 | 0.926 | 0.938 | 0.783 | 0.975 | 0.971 | 0.950 | 0.921 | 0.935 | 0.728 | 0.965 | 0.962 | 0.940 | 0.904 | 0.921 |
| 45 | 0.823 | 0.979 | 0.977 | 0.960 | 0.930 | 0.943 | 0.790 | 0.979 | 0.974 | 0.954 | 0.927 | 0.939 | 0.734 | 0.969 | 0.966 | 0.945 | 0.910 | 0.927 |
| 46 | 0.829 | 0.982 | 0.979 | 0.963 | 0.934 | 0.947 | 0.799 | 0.982 | 0.977 | 0.958 | 0.932 | 0.943 | 0.742 | 0.972 | 0.969 | 0.949 | 0.916 | 0.932 |
| 47 | 0.836 | 0.984 | 0.982 | 0.966 | 0.938 | 0.950 | 0.806 | 0.984 | 0.979 | 0.961 | 0.937 | 0.948 | 0.749 | 0.974 | 0.972 | 0.952 | 0.921 | 0.936 |
| 48 | 0.844 | 0.986 | 0.983 | 0.969 | 0.941 | 0.953 | 0.813 | 0.985 | 0.981 | 0.964 | 0.939 | 0.953 | 0.756 | 0.978 | 0.975 | 0.957 | 0.925 | 0.941 |
| 49 | 0.850 | 0.987 | 0.985 | 0.972 | 0.945 | 0.957 | 0.818 | 0.987 | 0.982 | 0.966 | 0.943 | 0.956 | 0.763 | 0.981 | 0.978 | 0.961 | 0.929 | 0.945 |
| 50 | 0.855 | 0.988 | 0.986 | 0.974 | 0.948 | 0.959 | 0.825 | 0.988 | 0.984 | 0.969 | 0.946 | 0.959 | 0.769 | 0.983 | 0.980 | 0.964 | 0.934 | 0.948 |
| 51 | 0.860 | 0.990 | 0.987 | 0.976 | 0.952 | 0.961 | 0.829 | 0.990 | 0.986 | 0.971 | 0.949 | 0.961 | 0.777 | 0.985 | 0.982 | 0.967 | 0.938 | 0.952 |
| 52 | 0.865 | 0.991 | 0.988 | 0.978 | 0.954 | 0.964 | 0.836 | 0.991 | 0.987 | 0.974 | 0.952 | 0.963 | 0.784 | 0.987 | 0.983 | 0.969 | 0.942 | 0.955 |
| 53 | 0.869 | 0.992 | 0.989 | 0.979 | 0.956 | 0.966 | 0.841 | 0.992 | 0.989 | 0.975 | 0.955 | 0.966 | 0.790 | 0.988 | 0.985 | 0.971 | 0.946 | 0.958 |
| 54 | 0.874 | 0.993 | 0.990 | 0.981 | 0.959 | 0.969 | 0.845 | 0.993 | 0.990 | 0.977 | 0.958 | 0.968 | 0.796 | 0.991 | 0.986 | 0.973 | 0.948 | 0.961 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 0.879 | 0.994 | 0.991 | 0.983 | 0.962 | 0.971 | 0.850 | 0.994 | 0.991 | 0.978 | 0.961 | 0.971 | 0.801 | 0.992 | 0.988 | 0.975 | 0.952 | 0.963 |
| 56 | 0.883 | 0.995 | 0.992 | 0.984 | 0.964 | 0.973 | 0.856 | 0.995 | 0.992 | 0.981 | 0.963 | 0.972 | 0.806 | 0.993 | 0.988 | 0.977 | 0.956 | 0.965 |
| 57 | 0.888 | 0.995 | 0.993 | 0.986 | 0.966 | 0.975 | 0.861 | 0.995 | 0.994 | 0.982 | 0.965 | 0.974 | 0.811 | 0.994 | 0.989 | 0.979 | 0.959 | 0.967 |
| 58 | 0.893 | 0.996 | 0.994 | 0.987 | 0.968 | 0.976 | 0.865 | 0.996 | 0.995 | 0.983 | 0.967 | 0.975 | 0.816 | 0.995 | 0.990 | 0.980 | 0.961 | 0.970 |
| 59 | 0.896 | 0.996 | 0.994 | 0.988 | 0.970 | 0.978 | 0.868 | 0.996 | 0.996 | 0.984 | 0.970 | 0.977 | 0.821 | 0.995 | 0.991 | 0.982 | 0.964 | 0.972 |
| 60 | 0.899 | 0.997 | 0.995 | 0.989 | 0.971 | 0.979 | 0.873 | 0.997 | 0.996 | 0.986 | 0.971 | 0.978 | 0.825 | 0.996 | 0.992 | 0.984 | 0.966 | 0.974 |
| 61 | 0.903 | 0.997 | 0.996 | 0.990 | 0.973 | 0.981 | 0.876 | 0.997 | 0.996 | 0.987 | 0.974 | 0.980 | 0.829 | 0.996 | 0.993 | 0.985 | 0.968 | 0.975 |
| 62 | 0.907 | 0.997 | 0.996 | 0.990 | 0.975 | 0.982 | 0.880 | 0.997 | 0.997 | 0.988 | 0.975 | 0.982 | 0.833 | 0.997 | 0.993 | 0.986 | 0.969 | 0.977 |
| 63 | 0.911 | 0.997 | 0.996 | 0.991 | 0.977 | 0.983 | 0.885 | 0.998 | 0.997 | 0.989 | 0.976 | 0.984 | 0.836 | 0.997 | 0.995 | 0.987 | 0.971 | 0.978 |
| 64 | 0.914 | 0.998 | 0.997 | 0.992 | 0.978 | 0.984 | 0.888 | 0.998 | 0.997 | 0.990 | 0.978 | 0.985 | 0.840 | 0.998 | 0.995 | 0.988 | 0.972 | 0.980 |

| **D** | **64** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **1.25** | | | | | | | | | | | |
| **τ** | **9** | | | | | | **12** | | | | | |
| **Run Length / Chart Type** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.032 | 0.004 | 0.021 | 0.041 | 0.036 | 0.050 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.065 | 0.009 | 0.045 | 0.085 | 0.081 | 0.098 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.098 | 0.020 | 0.079 | 0.133 | 0.125 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.129 | 0.036 | 0.116 | 0.185 | 0.178 | 0.189 | 0.031 | 0.011 | 0.034 | 0.045 | 0.037 | 0.059 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | 0.157 | 0.058 | 0.161 | 0.240 | 0.226 | 0.231 | 0.063 | 0.022 | 0.062 | 0.089 | 0.080 | 0.107 |
| 14 | 0.186 | 0.088 | 0.208 | 0.294 | 0.271 | 0.277 | 0.095 | 0.039 | 0.100 | 0.139 | 0.124 | 0.152 |
| 15 | 0.213 | 0.126 | 0.259 | 0.345 | 0.314 | 0.321 | 0.127 | 0.065 | 0.141 | 0.189 | 0.174 | 0.200 |
| 16 | 0.239 | 0.170 | 0.311 | 0.392 | 0.352 | 0.367 | 0.154 | 0.097 | 0.191 | 0.242 | 0.216 | 0.243 |
| 17 | 0.265 | 0.216 | 0.365 | 0.438 | 0.389 | 0.407 | 0.182 | 0.134 | 0.242 | 0.294 | 0.265 | 0.288 |
| 18 | 0.288 | 0.264 | 0.418 | 0.482 | 0.426 | 0.444 | 0.211 | 0.174 | 0.295 | 0.345 | 0.309 | 0.331 |
| 19 | 0.311 | 0.316 | 0.466 | 0.519 | 0.463 | 0.479 | 0.233 | 0.218 | 0.348 | 0.392 | 0.353 | 0.375 |
| 20 | 0.332 | 0.371 | 0.515 | 0.554 | 0.496 | 0.516 | 0.260 | 0.267 | 0.397 | 0.436 | 0.392 | 0.410 |
| 21 | 0.353 | 0.418 | 0.558 | 0.585 | 0.524 | 0.548 | 0.284 | 0.316 | 0.449 | 0.479 | 0.428 | 0.446 |
| 22 | 0.375 | 0.466 | 0.596 | 0.616 | 0.553 | 0.575 | 0.307 | 0.365 | 0.494 | 0.515 | 0.461 | 0.479 |
| 23 | 0.394 | 0.514 | 0.634 | 0.646 | 0.578 | 0.602 | 0.328 | 0.412 | 0.540 | 0.554 | 0.497 | 0.513 |
| 24 | 0.415 | 0.557 | 0.670 | 0.675 | 0.601 | 0.630 | 0.348 | 0.459 | 0.580 | 0.585 | 0.528 | 0.541 |
| 25 | 0.436 | 0.599 | 0.699 | 0.697 | 0.626 | 0.654 | 0.366 | 0.508 | 0.612 | 0.618 | 0.558 | 0.569 |
| 26 | 0.454 | 0.636 | 0.723 | 0.721 | 0.648 | 0.677 | 0.385 | 0.554 | 0.647 | 0.645 | 0.590 | 0.596 |
| 27 | 0.469 | 0.671 | 0.750 | 0.742 | 0.669 | 0.699 | 0.403 | 0.593 | 0.681 | 0.672 | 0.613 | 0.622 |
| 28 | 0.487 | 0.708 | 0.772 | 0.760 | 0.687 | 0.717 | 0.419 | 0.629 | 0.713 | 0.697 | 0.638 | 0.647 |
| 29 | 0.502 | 0.738 | 0.794 | 0.777 | 0.704 | 0.737 | 0.436 | 0.668 | 0.738 | 0.719 | 0.659 | 0.669 |
| 30 | 0.517 | 0.763 | 0.815 | 0.794 | 0.722 | 0.756 | 0.453 | 0.703 | 0.762 | 0.737 | 0.680 | 0.691 |
| 31 | 0.531 | 0.791 | 0.831 | 0.809 | 0.739 | 0.773 | 0.469 | 0.735 | 0.784 | 0.757 | 0.698 | 0.712 |
| 32 | 0.544 | 0.813 | 0.850 | 0.823 | 0.755 | 0.787 | 0.484 | 0.763 | 0.805 | 0.776 | 0.717 | 0.732 |
| 33 | 0.558 | 0.834 | 0.863 | 0.837 | 0.771 | 0.803 | 0.500 | 0.788 | 0.826 | 0.792 | 0.733 | 0.751 |
| 34 | 0.572 | 0.852 | 0.878 | 0.849 | 0.784 | 0.817 | 0.517 | 0.807 | 0.842 | 0.807 | 0.751 | 0.768 |
| 35 | 0.585 | 0.869 | 0.889 | 0.861 | 0.796 | 0.828 | 0.532 | 0.828 | 0.856 | 0.820 | 0.767 | 0.782 |
| 36 | 0.597 | 0.886 | 0.899 | 0.869 | 0.809 | 0.840 | 0.544 | 0.847 | 0.872 | 0.832 | 0.781 | 0.796 |
| 37 | 0.609 | 0.900 | 0.908 | 0.880 | 0.822 | 0.853 | 0.557 | 0.863 | 0.885 | 0.844 | 0.795 | 0.810 |
| 38 | 0.622 | 0.912 | 0.917 | 0.889 | 0.834 | 0.863 | 0.568 | 0.881 | 0.898 | 0.856 | 0.809 | 0.823 |
| 39 | 0.632 | 0.921 | 0.925 | 0.899 | 0.844 | 0.873 | 0.580 | 0.894 | 0.907 | 0.865 | 0.819 | 0.834 |
| 40 | 0.643 | 0.931 | 0.932 | 0.907 | 0.854 | 0.882 | 0.590 | 0.909 | 0.916 | 0.875 | 0.829 | 0.845 |
| 41 | 0.654 | 0.937 | 0.939 | 0.914 | 0.865 | 0.891 | 0.601 | 0.920 | 0.923 | 0.885 | 0.838 | 0.856 |
| 42 | 0.664 | 0.945 | 0.945 | 0.921 | 0.872 | 0.898 | 0.611 | 0.928 | 0.930 | 0.892 | 0.847 | 0.867 |
| 43 | 0.673 | 0.953 | 0.949 | 0.927 | 0.879 | 0.904 | 0.620 | 0.937 | 0.938 | 0.902 | 0.857 | 0.876 |
| 44 | 0.681 | 0.960 | 0.954 | 0.933 | 0.886 | 0.911 | 0.630 | 0.945 | 0.943 | 0.909 | 0.865 | 0.885 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | 0.690 | 0.964 | 0.958 | 0.937 | 0.893 | 0.918 | 0.638 | 0.953 | 0.949 | 0.916 | 0.874 | 0.893 |
| 46 | 0.698 | 0.968 | 0.961 | 0.942 | 0.901 | 0.924 | 0.646 | 0.958 | 0.955 | 0.923 | 0.883 | 0.900 |
| 47 | 0.706 | 0.971 | 0.964 | 0.947 | 0.907 | 0.928 | 0.655 | 0.963 | 0.959 | 0.929 | 0.892 | 0.906 |
| 48 | 0.713 | 0.974 | 0.968 | 0.950 | 0.912 | 0.933 | 0.662 | 0.967 | 0.963 | 0.934 | 0.898 | 0.911 |
| 49 | 0.721 | 0.977 | 0.971 | 0.954 | 0.918 | 0.937 | 0.671 | 0.970 | 0.967 | 0.939 | 0.903 | 0.916 |
| 50 | 0.729 | 0.980 | 0.973 | 0.957 | 0.924 | 0.941 | 0.679 | 0.973 | 0.970 | 0.946 | 0.909 | 0.923 |
| 51 | 0.736 | 0.983 | 0.975 | 0.960 | 0.928 | 0.946 | 0.687 | 0.976 | 0.975 | 0.950 | 0.914 | 0.927 |
| 52 | 0.744 | 0.984 | 0.977 | 0.962 | 0.933 | 0.949 | 0.695 | 0.979 | 0.978 | 0.953 | 0.920 | 0.932 |
| 53 | 0.752 | 0.985 | 0.980 | 0.964 | 0.937 | 0.953 | 0.703 | 0.982 | 0.981 | 0.957 | 0.924 | 0.937 |
| 54 | 0.758 | 0.987 | 0.982 | 0.966 | 0.940 | 0.955 | 0.710 | 0.984 | 0.983 | 0.960 | 0.929 | 0.943 |
| 55 | 0.764 | 0.988 | 0.984 | 0.968 | 0.944 | 0.958 | 0.715 | 0.986 | 0.985 | 0.963 | 0.932 | 0.946 |
| 56 | 0.770 | 0.990 | 0.986 | 0.970 | 0.948 | 0.962 | 0.720 | 0.987 | 0.986 | 0.965 | 0.937 | 0.949 |
| 57 | 0.774 | 0.992 | 0.987 | 0.972 | 0.952 | 0.965 | 0.726 | 0.988 | 0.988 | 0.968 | 0.941 | 0.954 |
| 58 | 0.779 | 0.993 | 0.988 | 0.973 | 0.955 | 0.968 | 0.732 | 0.990 | 0.989 | 0.971 | 0.944 | 0.957 |
| 59 | 0.783 | 0.994 | 0.990 | 0.975 | 0.957 | 0.971 | 0.738 | 0.991 | 0.990 | 0.973 | 0.948 | 0.959 |
| 60 | 0.789 | 0.995 | 0.991 | 0.977 | 0.960 | 0.973 | 0.746 | 0.993 | 0.991 | 0.975 | 0.951 | 0.962 |
| 61 | 0.794 | 0.995 | 0.991 | 0.979 | 0.962 | 0.974 | 0.752 | 0.994 | 0.992 | 0.977 | 0.954 | 0.966 |
| 62 | 0.798 | 0.996 | 0.992 | 0.980 | 0.965 | 0.976 | 0.755 | 0.994 | 0.993 | 0.978 | 0.957 | 0.968 |
| 63 | 0.802 | 0.996 | 0.993 | 0.983 | 0.967 | 0.978 | 0.760 | 0.995 | 0.994 | 0.980 | 0.960 | 0.970 |
| 64 | 0.805 | 0.997 | 0.993 | 0.984 | 0.970 | 0.980 | 0.764 | 0.995 | 0.994 | 0.982 | 0.962 | 0.972 |

## Appendix H: Cumulative Shift Detection Probabilities for D = 64 and Δ = 1.5

| D | 64 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 1.5 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.090 | 0.000 | 0.000 | 0.003 | 0.017 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.168 | 0.000 | 0.001 | 0.052 | 0.098 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.239 | 0.001 | 0.015 | 0.154 | 0.210 | 0.239 | 0.083 | 0.000 | 0.001 | 0.027 | 0.053 | 0.064 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.303 | 0.005 | 0.065 | 0.281 | 0.334 | 0.334 | 0.161 | 0.000 | 0.012 | 0.099 | 0.155 | 0.172 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.366 | 0.024 | 0.158 | 0.403 | 0.440 | 0.425 | 0.230 | 0.004 | 0.049 | 0.210 | 0.273 | 0.272 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.424 | 0.069 | 0.269 | 0.507 | 0.526 | 0.508 | 0.294 | 0.020 | 0.124 | 0.330 | 0.380 | 0.371 | 0.084 | 0.001 | 0.013 | 0.054 | 0.070 | 0.083 |
| 7 | 0.476 | 0.149 | 0.390 | 0.596 | 0.599 | 0.578 | 0.355 | 0.061 | 0.225 | 0.445 | 0.474 | 0.459 | 0.157 | 0.006 | 0.049 | 0.148 | 0.171 | 0.188 |
| 8 | 0.521 | 0.242 | 0.510 | 0.676 | 0.666 | 0.652 | 0.411 | 0.124 | 0.338 | 0.543 | 0.555 | 0.541 | 0.224 | 0.021 | 0.115 | 0.263 | 0.280 | 0.288 |
| 9 | 0.562 | 0.360 | 0.609 | 0.739 | 0.719 | 0.707 | 0.455 | 0.210 | 0.451 | 0.625 | 0.629 | 0.609 | 0.288 | 0.061 | 0.205 | 0.380 | 0.391 | 0.379 |
| 10 | 0.602 | 0.473 | 0.688 | 0.788 | 0.761 | 0.754 | 0.499 | 0.307 | 0.555 | 0.698 | 0.685 | 0.671 | 0.343 | 0.117 | 0.308 | 0.486 | 0.483 | 0.471 |
| 11 | 0.636 | 0.574 | 0.759 | 0.828 | 0.800 | 0.790 | 0.536 | 0.411 | 0.646 | 0.755 | 0.734 | 0.721 | 0.398 | 0.201 | 0.413 | 0.577 | 0.566 | 0.547 |
| 12 | 0.667 | 0.661 | 0.819 | 0.861 | 0.831 | 0.829 | 0.574 | 0.519 | 0.725 | 0.802 | 0.774 | 0.770 | 0.443 | 0.291 | 0.515 | 0.657 | 0.636 | 0.621 |
| 13 | 0.697 | 0.739 | 0.860 | 0.887 | 0.862 | 0.858 | 0.609 | 0.615 | 0.789 | 0.841 | 0.811 | 0.808 | 0.484 | 0.393 | 0.612 | 0.719 | 0.694 | 0.682 |
| 14 | 0.725 | 0.801 | 0.898 | 0.909 | 0.884 | 0.880 | 0.640 | 0.695 | 0.837 | 0.874 | 0.840 | 0.842 | 0.527 | 0.492 | 0.690 | 0.775 | 0.739 | 0.734 |
| 15 | 0.749 | 0.850 | 0.921 | 0.929 | 0.903 | 0.902 | 0.669 | 0.764 | 0.875 | 0.897 | 0.864 | 0.871 | 0.566 | 0.586 | 0.754 | 0.817 | 0.781 | 0.777 |
| 16 | 0.771 | 0.890 | 0.940 | 0.944 | 0.918 | 0.922 | 0.694 | 0.821 | 0.904 | 0.917 | 0.889 | 0.894 | 0.599 | 0.672 | 0.807 | 0.856 | 0.820 | 0.812 |
| 17 | 0.790 | 0.918 | 0.954 | 0.955 | 0.930 | 0.937 | 0.716 | 0.864 | 0.928 | 0.932 | 0.907 | 0.912 | 0.629 | 0.743 | 0.852 | 0.882 | 0.851 | 0.842 |
| 18 | 0.808 | 0.939 | 0.966 | 0.963 | 0.941 | 0.947 | 0.739 | 0.900 | 0.946 | 0.945 | 0.923 | 0.928 | 0.655 | 0.800 | 0.889 | 0.907 | 0.875 | 0.864 |
| 19 | 0.826 | 0.956 | 0.976 | 0.971 | 0.951 | 0.958 | 0.760 | 0.926 | 0.958 | 0.958 | 0.935 | 0.941 | 0.680 | 0.848 | 0.915 | 0.925 | 0.895 | 0.889 |
| 20 | 0.841 | 0.968 | 0.981 | 0.977 | 0.956 | 0.966 | 0.777 | 0.946 | 0.968 | 0.966 | 0.947 | 0.953 | 0.702 | 0.885 | 0.934 | 0.939 | 0.910 | 0.909 |
| 21 | 0.854 | 0.977 | 0.986 | 0.980 | 0.963 | 0.973 | 0.795 | 0.962 | 0.975 | 0.972 | 0.956 | 0.962 | 0.722 | 0.915 | 0.951 | 0.952 | 0.926 | 0.924 |
| 22 | 0.866 | 0.985 | 0.989 | 0.984 | 0.969 | 0.978 | 0.810 | 0.972 | 0.980 | 0.978 | 0.964 | 0.970 | 0.742 | 0.936 | 0.963 | 0.962 | 0.938 | 0.939 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.878 | 0.988 | 0.992 | 0.987 | 0.975 | 0.982 | 0.823 | 0.981 | 0.986 | 0.981 | 0.970 | 0.975 | 0.760 | 0.954 | 0.971 | 0.969 | 0.948 | 0.950 |
| 24 | 0.890 | 0.991 | 0.994 | 0.990 | 0.980 | 0.985 | 0.839 | 0.984 | 0.988 | 0.984 | 0.976 | 0.980 | 0.774 | 0.967 | 0.978 | 0.976 | 0.956 | 0.960 |
| 25 | 0.899 | 0.994 | 0.995 | 0.992 | 0.983 | 0.987 | 0.850 | 0.988 | 0.991 | 0.987 | 0.979 | 0.984 | 0.789 | 0.976 | 0.983 | 0.980 | 0.964 | 0.968 |
| 26 | 0.909 | 0.995 | 0.996 | 0.993 | 0.985 | 0.990 | 0.862 | 0.991 | 0.993 | 0.990 | 0.982 | 0.987 | 0.800 | 0.982 | 0.987 | 0.984 | 0.971 | 0.974 |
| 27 | 0.917 | 0.997 | 0.997 | 0.995 | 0.987 | 0.992 | 0.873 | 0.993 | 0.995 | 0.992 | 0.985 | 0.989 | 0.812 | 0.987 | 0.991 | 0.986 | 0.976 | 0.980 |
| 28 | 0.923 | 0.998 | 0.998 | 0.996 | 0.989 | 0.993 | 0.881 | 0.995 | 0.997 | 0.994 | 0.987 | 0.992 | 0.824 | 0.990 | 0.993 | 0.990 | 0.981 | 0.983 |
| 29 | 0.929 | 0.999 | 0.998 | 0.997 | 0.991 | 0.995 | 0.890 | 0.996 | 0.997 | 0.995 | 0.988 | 0.994 | 0.834 | 0.993 | 0.995 | 0.992 | 0.984 | 0.987 |
| 30 | 0.936 | 0.999 | 0.999 | 0.998 | 0.993 | 0.996 | 0.898 | 0.997 | 0.998 | 0.996 | 0.990 | 0.995 | 0.845 | 0.995 | 0.996 | 0.994 | 0.986 | 0.989 |
| 31 | 0.941 | 0.999 | 0.999 | 0.998 | 0.994 | 0.996 | 0.906 | 0.998 | 0.998 | 0.997 | 0.991 | 0.996 | 0.853 | 0.996 | 0.997 | 0.995 | 0.988 | 0.992 |
| 32 | 0.947 | 1.000 | 0.999 | 0.999 | 0.996 | 0.997 | 0.911 | 0.998 | 0.999 | 0.997 | 0.992 | 0.997 | 0.860 | 0.997 | 0.998 | 0.996 | 0.990 | 0.993 |
| 33 | 0.951 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 | 0.918 | 0.998 | 0.999 | 0.998 | 0.994 | 0.998 | 0.867 | 0.998 | 0.998 | 0.997 | 0.991 | 0.994 |
| 34 | 0.956 | 1.000 | 1.000 | 0.999 | 0.997 | 0.998 | 0.923 | 0.999 | 1.000 | 0.998 | 0.995 | 0.998 | 0.872 | 0.999 | 0.999 | 0.998 | 0.992 | 0.994 |
| 35 | 0.959 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.927 | 1.000 | 1.000 | 0.999 | 0.995 | 0.999 | 0.878 | 0.999 | 0.999 | 0.998 | 0.993 | 0.996 |
| 36 | 0.962 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.931 | 1.000 | 1.000 | 0.999 | 0.996 | 0.999 | 0.882 | 0.999 | 0.999 | 0.998 | 0.994 | 0.997 |
| 37 | 0.965 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.936 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 | 0.888 | 1.000 | 1.000 | 0.999 | 0.995 | 0.997 |
| 38 | 0.968 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.941 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 | 0.892 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 |
| 39 | 0.971 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.945 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 | 0.898 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 |
| 40 | 0.973 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.947 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.902 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 |
| 41 | 0.975 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.950 | 1.000 | 1.000 | 0.999 | 0.999 | 1.000 | 0.905 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 |
| 42 | 0.978 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.952 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.909 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 |
| 43 | 0.979 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.954 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.912 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 |
| 44 | 0.980 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.956 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.915 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 45 | 0.982 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.957 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.917 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 46 | 0.983 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.959 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.919 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 47 | 0.984 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.960 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.921 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 48 | 0.985 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.961 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.923 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 49 | 0.986 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.962 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.925 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 0.987 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.963 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.926 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 51 | 0.988 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.964 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.927 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 52 | 0.989 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.966 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.929 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 53 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.967 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.930 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 54 | 0.991 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.967 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.931 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.969 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.932 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 56 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.970 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.934 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 57 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.971 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.935 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 58 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.971 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.935 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 59 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.971 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.936 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 60 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.937 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 61 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.937 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 62 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.973 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.938 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 63 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.973 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.938 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 64 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.939 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| **D** | **64** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **1.5** | | | | | | | | | | | |
| **τ** | **9** | | | | | | **12** | | | | | |
| **Run Length / Chart Type** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.084 | 0.006 | 0.032 | 0.069 | 0.076 | 0.091 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.154 | 0.020 | 0.086 | 0.169 | 0.185 | 0.196 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 11 | 0.219 | 0.052 | 0.167 | 0.285 | 0.296 | 0.302 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 0.276 | 0.109 | 0.262 | 0.395 | 0.401 | 0.399 | 0.075 | 0.019 | 0.052 | 0.073 | 0.077 | 0.098 |
| 13 | 0.330 | 0.185 | 0.365 | 0.503 | 0.493 | 0.485 | 0.143 | 0.046 | 0.114 | 0.167 | 0.182 | 0.206 |
| 14 | 0.380 | 0.270 | 0.466 | 0.594 | 0.573 | 0.565 | 0.206 | 0.092 | 0.200 | 0.289 | 0.294 | 0.307 |
| 15 | 0.424 | 0.365 | 0.560 | 0.670 | 0.642 | 0.638 | 0.265 | 0.152 | 0.297 | 0.405 | 0.397 | 0.401 |
| 16 | 0.466 | 0.463 | 0.644 | 0.735 | 0.698 | 0.702 | 0.318 | 0.231 | 0.403 | 0.505 | 0.491 | 0.490 |
| 17 | 0.505 | 0.555 | 0.715 | 0.787 | 0.746 | 0.752 | 0.370 | 0.322 | 0.504 | 0.594 | 0.569 | 0.565 |
| 18 | 0.543 | 0.639 | 0.782 | 0.829 | 0.787 | 0.795 | 0.411 | 0.422 | 0.593 | 0.670 | 0.641 | 0.634 |
| 19 | 0.575 | 0.718 | 0.832 | 0.859 | 0.823 | 0.826 | 0.451 | 0.520 | 0.676 | 0.734 | 0.691 | 0.697 |
| 20 | 0.603 | 0.781 | 0.869 | 0.884 | 0.851 | 0.855 | 0.484 | 0.607 | 0.742 | 0.787 | 0.741 | 0.748 |
| 21 | 0.630 | 0.835 | 0.899 | 0.907 | 0.874 | 0.880 | 0.519 | 0.684 | 0.801 | 0.828 | 0.785 | 0.791 |
| 22 | 0.653 | 0.871 | 0.923 | 0.926 | 0.894 | 0.900 | 0.550 | 0.746 | 0.846 | 0.861 | 0.818 | 0.828 |
| 23 | 0.676 | 0.900 | 0.943 | 0.940 | 0.913 | 0.917 | 0.576 | 0.804 | 0.879 | 0.885 | 0.847 | 0.860 |
| 24 | 0.695 | 0.925 | 0.956 | 0.952 | 0.928 | 0.932 | 0.600 | 0.848 | 0.906 | 0.905 | 0.872 | 0.886 |
| 25 | 0.715 | 0.942 | 0.967 | 0.961 | 0.939 | 0.944 | 0.622 | 0.884 | 0.928 | 0.922 | 0.894 | 0.905 |
| 26 | 0.732 | 0.958 | 0.975 | 0.969 | 0.948 | 0.954 | 0.641 | 0.911 | 0.946 | 0.936 | 0.912 | 0.924 |
| 27 | 0.746 | 0.971 | 0.982 | 0.976 | 0.956 | 0.963 | 0.659 | 0.933 | 0.959 | 0.946 | 0.926 | 0.936 |
| 28 | 0.762 | 0.979 | 0.987 | 0.981 | 0.965 | 0.970 | 0.678 | 0.954 | 0.968 | 0.957 | 0.938 | 0.947 |
| 29 | 0.775 | 0.986 | 0.991 | 0.985 | 0.971 | 0.976 | 0.695 | 0.966 | 0.975 | 0.966 | 0.947 | 0.957 |
| 30 | 0.786 | 0.990 | 0.993 | 0.987 | 0.976 | 0.981 | 0.708 | 0.976 | 0.981 | 0.972 | 0.956 | 0.964 |
| 31 | 0.797 | 0.993 | 0.995 | 0.990 | 0.979 | 0.984 | 0.724 | 0.982 | 0.985 | 0.978 | 0.962 | 0.970 |
| 32 | 0.807 | 0.995 | 0.996 | 0.992 | 0.983 | 0.986 | 0.735 | 0.987 | 0.990 | 0.982 | 0.969 | 0.975 |
| 33 | 0.817 | 0.996 | 0.997 | 0.993 | 0.986 | 0.989 | 0.749 | 0.990 | 0.992 | 0.984 | 0.973 | 0.980 |
| 34 | 0.823 | 0.997 | 0.998 | 0.994 | 0.989 | 0.991 | 0.759 | 0.993 | 0.994 | 0.988 | 0.977 | 0.984 |
| 35 | 0.832 | 0.998 | 0.998 | 0.995 | 0.990 | 0.993 | 0.768 | 0.994 | 0.995 | 0.991 | 0.981 | 0.986 |
| 36 | 0.839 | 0.999 | 0.999 | 0.996 | 0.991 | 0.994 | 0.777 | 0.996 | 0.996 | 0.993 | 0.984 | 0.989 |
| 37 | 0.845 | 0.999 | 0.999 | 0.996 | 0.993 | 0.996 | 0.786 | 0.997 | 0.997 | 0.994 | 0.987 | 0.990 |
| 38 | 0.852 | 1.000 | 0.999 | 0.997 | 0.994 | 0.997 | 0.793 | 0.998 | 0.998 | 0.996 | 0.989 | 0.992 |
| 39 | 0.856 | 1.000 | 0.999 | 0.998 | 0.995 | 0.997 | 0.801 | 0.998 | 0.999 | 0.996 | 0.990 | 0.995 |
| 40 | 0.860 | 1.000 | 1.000 | 0.999 | 0.995 | 0.998 | 0.807 | 0.999 | 0.999 | 0.997 | 0.992 | 0.996 |
| 41 | 0.863 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 | 0.814 | 0.999 | 0.999 | 0.998 | 0.994 | 0.997 |
| 42 | 0.867 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 | 0.819 | 0.999 | 1.000 | 0.998 | 0.995 | 0.998 |
| 43 | 0.871 | 1.000 | 1.000 | 1.000 | 0.997 | 0.999 | 0.823 | 1.000 | 1.000 | 0.998 | 0.996 | 0.998 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 44 | 0.874 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.827 | 1.000 | 1.000 | 0.999 | 0.996 | 0.998 |
| 45 | 0.877 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.831 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 |
| 46 | 0.879 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 0.835 | 1.000 | 1.000 | 0.999 | 0.997 | 0.999 |
| 47 | 0.881 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 0.839 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 |
| 48 | 0.882 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.842 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 |
| 49 | 0.885 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.845 | 1.000 | 1.000 | 0.999 | 0.998 | 0.999 |
| 50 | 0.887 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.848 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 |
| 51 | 0.888 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.850 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 |
| 52 | 0.890 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.852 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 |
| 53 | 0.891 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 0.853 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 54 | 0.893 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.855 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 55 | 0.894 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.856 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 56 | 0.896 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.857 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| 57 | 0.896 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.858 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 58 | 0.898 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.860 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 59 | 0.899 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.861 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 60 | 0.899 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.861 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 61 | 0.900 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.862 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 62 | 0.901 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.863 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 63 | 0.901 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.864 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 64 | 0.901 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.865 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

## Appendix I: Cumulative Shift Detection Probabilities for D = 64 and Δ = 2

| D | 64 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 2 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.264 | 0.000 | 0.000 | 0.020 | 0.083 | 0.148 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.453 | 0.000 | 0.021 | 0.232 | 0.366 | 0.445 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.592 | 0.014 | 0.177 | 0.524 | 0.612 | 0.637 | 0.247 | 0.000 | 0.004 | 0.078 | 0.165 | 0.199 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.696 | 0.113 | 0.438 | 0.744 | 0.782 | 0.777 | 0.425 | 0.005 | 0.076 | 0.322 | 0.442 | 0.477 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.770 | 0.324 | 0.671 | 0.866 | 0.877 | 0.868 | 0.563 | 0.050 | 0.268 | 0.587 | 0.664 | 0.668 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.827 | 0.564 | 0.834 | 0.928 | 0.928 | 0.926 | 0.672 | 0.193 | 0.525 | 0.775 | 0.804 | 0.805 | 0.244 | 0.002 | 0.035 | 0.140 | 0.187 | 0.205 |
| 7 | 0.873 | 0.756 | 0.921 | 0.965 | 0.961 | 0.959 | 0.749 | 0.418 | 0.723 | 0.880 | 0.889 | 0.883 | 0.416 | 0.029 | 0.159 | 0.398 | 0.466 | 0.493 |
| 8 | 0.907 | 0.879 | 0.961 | 0.983 | 0.976 | 0.979 | 0.808 | 0.631 | 0.856 | 0.940 | 0.940 | 0.936 | 0.550 | 0.125 | 0.373 | 0.643 | 0.682 | 0.681 |
| 9 | 0.933 | 0.942 | 0.982 | 0.991 | 0.986 | 0.988 | 0.851 | 0.792 | 0.930 | 0.969 | 0.967 | 0.966 | 0.647 | 0.296 | 0.601 | 0.806 | 0.821 | 0.817 |
| 10 | 0.950 | 0.974 | 0.991 | 0.996 | 0.993 | 0.994 | 0.882 | 0.899 | 0.965 | 0.984 | 0.981 | 0.982 | 0.724 | 0.500 | 0.774 | 0.901 | 0.899 | 0.896 |
| 11 | 0.963 | 0.989 | 0.996 | 0.998 | 0.996 | 0.996 | 0.907 | 0.950 | 0.984 | 0.992 | 0.990 | 0.990 | 0.782 | 0.684 | 0.883 | 0.952 | 0.944 | 0.945 |
| 12 | 0.973 | 0.995 | 0.998 | 0.999 | 0.998 | 0.998 | 0.925 | 0.978 | 0.992 | 0.995 | 0.995 | 0.995 | 0.820 | 0.820 | 0.945 | 0.976 | 0.968 | 0.970 |
| 13 | 0.982 | 0.998 | 0.999 | 1.000 | 0.999 | 0.999 | 0.938 | 0.989 | 0.997 | 0.997 | 0.997 | 0.996 | 0.853 | 0.907 | 0.973 | 0.987 | 0.983 | 0.986 |
| 14 | 0.987 | 0.999 | 1.000 | 1.000 | 1.000 | 0.999 | 0.946 | 0.995 | 0.998 | 0.999 | 0.998 | 0.998 | 0.877 | 0.957 | 0.988 | 0.994 | 0.991 | 0.993 |
| 15 | 0.991 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.954 | 0.998 | 0.999 | 0.999 | 0.999 | 0.999 | 0.892 | 0.982 | 0.995 | 0.996 | 0.995 | 0.997 |
| 16 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.960 | 0.999 | 1.000 | 0.999 | 1.000 | 1.000 | 0.904 | 0.992 | 0.998 | 0.998 | 0.997 | 0.998 |
| 17 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.965 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.914 | 0.997 | 0.999 | 1.000 | 0.998 | 0.999 |
| 18 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.967 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.921 | 0.998 | 0.999 | 1.000 | 0.999 | 1.000 |
| 19 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.970 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.927 | 0.999 | 1.000 | 1.000 | 0.999 | 1.000 |
| 20 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.931 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 21 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.973 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.934 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 22 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.936 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.937 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 24 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.938 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.939 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.939 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 43 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 49 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 51 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 52 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 53 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 54 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 56 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 57 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 58 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 59 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 60 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 61 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 62 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 63 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 64 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| **D** | **64** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **2** | | | | | | | | | | | |
| **τ** | **9** | | | | | | **12** | | | | | |
| **Run Length / Chart Type** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.240 | 0.017 | 0.063 | 0.161 | 0.192 | 0.227 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.409 | 0.076 | 0.215 | 0.419 | 0.467 | 0.507 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 0.536 | 0.206 | 0.429 | 0.652 | 0.688 | 0.690 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.630 | 0.393 | 0.650 | 0.807 | 0.822 | 0.817 | 0.224 | 0.033 | 0.099 | 0.167 | 0.192 | 0.235 |
| 13 | 0.703 | 0.585 | 0.802 | 0.899 | 0.899 | 0.895 | 0.390 | 0.120 | 0.270 | 0.426 | 0.467 | 0.508 |
| 14 | 0.756 | 0.745 | 0.901 | 0.947 | 0.943 | 0.944 | 0.516 | 0.261 | 0.482 | 0.664 | 0.689 | 0.691 |
| 15 | 0.795 | 0.855 | 0.954 | 0.973 | 0.970 | 0.970 | 0.607 | 0.451 | 0.681 | 0.821 | 0.820 | 0.818 |
| 16 | 0.824 | 0.927 | 0.979 | 0.987 | 0.983 | 0.985 | 0.671 | 0.631 | 0.824 | 0.906 | 0.900 | 0.894 |
| 17 | 0.844 | 0.963 | 0.990 | 0.993 | 0.990 | 0.992 | 0.723 | 0.774 | 0.907 | 0.956 | 0.942 | 0.942 |
| 18 | 0.859 | 0.983 | 0.996 | 0.996 | 0.995 | 0.996 | 0.765 | 0.869 | 0.954 | 0.975 | 0.965 | 0.967 |
| 19 | 0.872 | 0.992 | 0.999 | 0.998 | 0.997 | 0.998 | 0.790 | 0.932 | 0.979 | 0.989 | 0.982 | 0.983 |
| 20 | 0.880 | 0.997 | 0.999 | 0.999 | 0.998 | 0.999 | 0.814 | 0.967 | 0.991 | 0.995 | 0.990 | 0.991 |
| 21 | 0.887 | 0.999 | 1.000 | 0.999 | 0.999 | 1.000 | 0.833 | 0.984 | 0.995 | 0.998 | 0.994 | 0.995 |
| 22 | 0.890 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 0.844 | 0.993 | 0.998 | 0.999 | 0.997 | 0.998 |
| 23 | 0.895 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.851 | 0.997 | 0.999 | 1.000 | 0.998 | 0.999 |
| 24 | 0.898 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.858 | 0.998 | 1.000 | 1.000 | 0.999 | 1.000 |
| 25 | 0.900 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.863 | 0.999 | 1.000 | 1.000 | 0.999 | 1.000 |
| 26 | 0.902 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.866 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 0.903 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.869 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 0.904 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.871 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 0.904 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.872 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.873 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.875 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 0.905 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.876 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 43 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 49 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 51 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 52 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 53 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 54 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 55 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 56 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 57 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 58 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 59 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 60 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 61 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 62 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 63 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 64 | 0.906 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.877 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

## Appendix J: Cumulative Shift Detection Probabilities for D = 64 and Δ = 3

| D | 64 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ | 3 | | | | | | | | | | | | | | | | | |
| τ | 0 | | | | | | 3 | | | | | | 6 | | | | | |
| Run Length / Chart Type | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR | Q | EWMA Q alpha=0.05 | EWMA Q alpha= 0.1 | EWMA Q alpha= 0.25 | EWMA Q alpha=0.4 | GLR |
| 1 | 0.675 | 0.000 | 0.001 | 0.202 | 0.415 | 0.552 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.892 | 0.026 | 0.339 | 0.798 | 0.878 | 0.915 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.966 | 0.413 | 0.827 | 0.969 | 0.980 | 0.982 | 0.666 | 0.001 | 0.043 | 0.313 | 0.486 | 0.591 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.989 | 0.839 | 0.971 | 0.996 | 0.997 | 0.997 | 0.878 | 0.091 | 0.453 | 0.823 | 0.896 | 0.923 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.996 | 0.975 | 0.998 | 1.000 | 0.999 | 1.000 | 0.945 | 0.506 | 0.856 | 0.972 | 0.985 | 0.985 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.999 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 0.963 | 0.859 | 0.975 | 0.997 | 0.997 | 0.997 | 0.636 | 0.015 | 0.128 | 0.397 | 0.522 | 0.609 |
| 7 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.970 | 0.974 | 0.997 | 1.000 | 1.000 | 1.000 | 0.842 | 0.202 | 0.552 | 0.852 | 0.901 | 0.922 |
| 8 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 | 0.910 | 0.604 | 0.881 | 0.977 | 0.984 | 0.986 |
| 9 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.931 | 0.883 | 0.979 | 0.997 | 0.997 | 0.999 |
| 10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.937 | 0.977 | 0.998 | 1.000 | 1.000 | 1.000 |
| 11 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.939 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| 12 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 13 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 14 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 15 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 17 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 18 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 19 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 21 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 22 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 24 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 43 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 49 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 51 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 52 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 53 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 54 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 56 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 57 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 58 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 59 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 60 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 61 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 62 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 63 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 64 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.974 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.941 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| **D** | **64** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Δ** | **3** | | | | | | | | | | | |
| **τ** | **9** | | | | | | **12** | | | | | |
| **Run Length / Chart Type** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** | **Q** | **EWMA Q alpha=0.05** | **EWMA Q alpha= 0.1** | **EWMA Q alpha= 0.25** | **EWMA Q alpha=0.4** | **GLR** |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 0.607 | 0.051 | 0.188 | 0.430 | 0.536 | 0.607 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 10 | 0.812 | 0.310 | 0.620 | 0.868 | 0.905 | 0.923 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 0.879 | 0.678 | 0.902 | 0.981 | 0.984 | 0.985 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 0.899 | 0.908 | 0.983 | 0.998 | 0.997 | 0.999 | 0.593 | 0.087 | 0.240 | 0.444 | 0.533 | 0.608 |
| 13 | 0.905 | 0.981 | 0.998 | 1.000 | 0.999 | 1.000 | 0.792 | 0.382 | 0.659 | 0.869 | 0.907 | 0.929 |
| 14 | 0.906 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 0.854 | 0.733 | 0.911 | 0.979 | 0.985 | 0.986 |
| 15 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.872 | 0.920 | 0.986 | 0.998 | 0.998 | 0.998 |
| 16 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.879 | 0.981 | 0.998 | 1.000 | 1.000 | 1.000 |
| 17 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.882 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| 18 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.882 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 19 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 20 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 21 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 22 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 23 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 24 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 25 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 26 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 27 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 28 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 29 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 30 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 31 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 36 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 37 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 38 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 39 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 40 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 41 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 42 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 43 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 44 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 45 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 46 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 47 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 48 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 49 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 51 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 52 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 53 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 54 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 55 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 56 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 57 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 58 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 59 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 60 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 61 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 62 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 63 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 64 | 0.907 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.883 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |