ACTUATOR FAULT TOLERANT TERMINAL SLIDING MODE GUIDANCE LAW WITH IMPACT ANGLE AND ACCELERATION CONSTRAINTS

AKTÜATÖR ARIZASI TOLERANSLI ÇARPMA AÇISI VE İVME KISITLARI İLE TERMİNAL KAYAN KİPLİ GÜDÜM KANUNU

FATİH KIRIMLIOĞLU

ASST. PROF. DR. EMİR KUTLUAY

Supervisor

Submitted to

Graduate School of Science and Engineering of Hacettepe University

as a Partial Fulfillment to the Requirements

for the Award of the Degree of Master of Science

in Mechanical Engineering

2021

ABSTRACT

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Fatih KIRIMLIOĞLU

Master of Science, Department of Mechanical Engineering

Supervisor: Asst. Prof. Dr. Emir KUTLUAY

July 2021, 73 pages

In the guidance literature, various requirements on missile guidance performance have arisen and a significant number of research has been presented on these requirements. Ensuring a successful hit on the target is one of the principal properties that is studied on missile guidance problems. To guarantee high hit probability, the guidance law must be robust against disturbances. Undesired forces created on the missile in case of actuator failure and unknown target acceleration acts as a disturbance on engagement geometry. Thus, a robust guidance law can tolerate the actuator failure and unknown target acceleration effects. Another requirement of guidance problem is achieving the desired impact angle which is mainly important for deactivating the heavily armored ground targets effectively.

In this thesis, an actuator fault tolerant terminal sliding mode guidance law is proposed by considering impact angle and acceleration constraints. The sliding mode control method is known to be robust against unknown disturbances and provides an adequate solution for controlling non-linear systems. In this study, the sliding mode control method is adopted for guidance design, since actuator failure and unknown target acceleration behave as a disturbance on the non-linear missile-target engagement kinematics. A first order sliding mode guidance law is designed with equivalent control method. The selected sliding surface ensures achieving a successful hit on the target with the desired impact angle. Bounded target acceleration and actuator failure effects are considered in switching function architecture. Additionally, a guidance law needs necessary data to generate proper commands for the missile. Line-of-sight (LOS) angular rate, one of the commonly used parameters in guidance laws, may not be directly measured on missiles with strapdown seekers. In this study, LOS angular rate is estimated from LOS angle with a second order sliding mode differentiator, if this rate information is not accessible by the missile. The proposed guidance law is used in the terminal flight phase. Slowly moving heavily armored vehicles are considered as a target. The performance of the proposed guidance law is analyzed with a numerical simulation model. The results of the simulation studies prove that the guidance law is robust against actuator failures and unknown target acceleration. Estimation performance of LOS angular rate is interpreted as suitable for the guidance process.

Keywords: Guidance law, sliding mode control, sliding mode differentiator, impact angle, target acceleration, disturbances, LOS rate estimation

ÖZET

AKTÜATÖR ARIZASI TOLERANSLI ÇARPMA AÇISI VE İVME KISITLARI İLE TERMİNAL KAYAN KİPLİ GÜDÜM KANUNU

Fatih Kırımlıoğlu

Yüksek Lisans, Makina Mühendisliği Bölümü

Tez Danışmanı: Asst. Prof. Dr. Emir KUTLUAY

Temmuz 2021, 73 sayfa

Güdüm literatüründe, füze güdüm performansı üzerine çeşitli gereksinimler ortaya çıkmış ve bu gereksinimler üzerine önemli bir sayıda araştırma yapılmıştır. Hedefin başarılı bir şekilde vurulmasının sağlanması, füze güdümü problemleri üzerinde çalışılan başlıca niteliklerden biridir. Yüksek vuruş olasılığının garanti edilebilmesi için güdüm kanununun bozucu etkilere karşı gürbüz olması gerekmektedir. Aktüatör arızaları nedeniyle füze üzerinde oluşan istenmeyen kuvvetler ve bilinmeyen hedef ivmeleri, angajman geometrisi üzerinde bozucu etki yaratmaktadır. Bu nedenle, gürbüz bir güdüm kanunu aktüatör arızası ve bilinmeyen hedef ivmesi etkilerini tolere edebilmelidir. Güdüm probleminin bir başka gereksinimi ise, ağır zırhlı yer hedeflerinin etkisiz hale getirilmesinde önemli olan arzu edilen vuruş açısının elde edilmesidir.

Bu tezde, vuruş açısı ve ivme kısıtları dikkate alınarak aktüator arızası toleranslı kayan kipli bir terminal güdüm kanunu tasarlanmıştır. Kayan kipli kontrol metodu bilinmeyen bozuculara karşı gürbüz olmaktadır ve lineer olmayan sistemlerin kontrolünde elverişli bir çözüm sunmaktadır. Aktüatör arızası ve bilinmeyen hedef ivmesinin lineer olmayan füze-hedef angajman kinematiği üzerinde bozucu olarak davranması nedeniyle, bu çalışmada kayan kipli kontrol metodu kullanılmıştır. Eşdeğer kontrol metodu ile birinci dereceden kayan kipli bir güdüm kanunu tasarlanmıştır. Seçilen kayan yüzey, hedefin arzu edilen bir vuruş açısı ile başarılı bir şekilde vurulmasını sağlamaktadır. Anahtarlama fonksiyonu tasarımında sınırlandırılmış hedef ivmesi ve aktüatör arızası etkileri dikkate alınmıştır. Ayrıca, güdüm kanununun uygun komutları üretebilmesi için gerekli verilere ihtiyaç duymaktadır. Güdüm kanunlarında genellikle kullanılan görüş hattı açısal hızı, sabit arayıcı kullanılan füzelerde doğrudan ölçülememektedir. Bu çalışma içerisinde, görüş hattı açısal hızının füze tarafından ulaşılabilir olmadığı durumlarda bu açısal hız, görüş hattı açısı üzerinden ikinci dereceden kayan kipli türev alıcı ile kestirilmiştir. Tasarlanan güdüm kanunu uçuşun terminal safhasında kullanılabilmektedir. Yavaş ilerleyem ağız zırhlı vasıtalar hedef olarak belirlenmiştir. Tasarlanan güdüm kanununun performansı nümerik simülasyon modeli ile analiz edilmiştir. Simülasyon çalışmaları sonuçları, güdüm kanununun aktüatör arızasına ve bilinmeyen hedef ivmesine karşı gürbüz olduğunu ortaya koymuştur. Bakış hattı açısal hızının kestirim performansının güdüm işlemi için uygun olduğu değerlendirilmiştir.

Anahtar Kelimeler: Güdüm kanunu, kayan kipli kontrol, kayan kipli türev alıcı, vuruş açısı, hedef ivmesi, disturbances, görüş hattı açısal hız kestirimi

ACKNOWLEDGMENTS

First of all, I would like to express my sincere appreciation to my supervisor Asst. Prof. Dr. Emir KUTLUAY for his guidance. His advice and supports are priceless for me. I thank him for all the effort he put into this thesis.

I would like to express my gratitude to my bellowed family for their support.

Finally, I appreciate the precious support of my dear friends.

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SYMBOLS AND ABBREVIATIONS

Symbols

a_{com}	Guidance acceleration command
a_{ec}	Equivalent control term
a_{F}	Additive acceleration term
a_M	Normal (actual) missile acceleration
a_{rl}	Reaching law term
a_T	Normal target acceleration
d	Sigmoid function parameter
e	Error threshold
k_{1-3}	Guidance gains
L	Lipschitz constant
M_{0-2}	Sliding mode differentiator coefficients
R	Range
S	Laplace domain parameter
s	Sliding variable
s_f	Filtered sliding variable
T	Settling time
V	Lyapunov function
V_M	Missile velocity
V_T	Target velocity
x_{1-2}	Sliding variable states
z_{0-2}	Estimates of LOS angle derivatives
γ	Sliding variable parameter

ε	Sliding variable gain
θ_{imp}	Desired impact angle
θ_M	Missile flight path angle
θ_T	Target orientation angle
λ	Line-of-sight angle
λ_{des}	Desired LOS angle
μ	Multiplicative acceleration term

Abbreviations

ANFIS	Adaptive neuro-fuzzy interference system
CAS	Control actuator system
CF	Cost Function
FOC	Field of view
IMU	Inertial measurement unit
LOS	Line-of-sight
NLESO	Nonlinear extended state observer
SMC	Sliding mode control
SMD	Sliding mode differentiator
SMGL	Sliding mode guidance law

1. INTRODUCTION

1.1. Motivation

In the modern era, missiles are becoming one of the popular and indispensable concepts of the defense industry. Thanks to the relatively cheaper and simpler design of the missiles, these systems offer cost-effective solutions for eliminating targets. Missiles are able to deactivate a wide range of expensive and valuable targets from heavily armored vehicles like main battle tanks to fighter aircrafts, and even other missiles. Furthermore, the user can operate the missile without getting closer to the dangerous targets, since the self-propelled autonomous systems can take out targets from long distances.

The success of intercepting the target is heavily dependent on the control structure if the missile is guided. The position data of the targeted object is transferred to the missile computer from radar systems or acquired from onboard missile sensors in order to guide the missile to the target. A guidance law generates necessary guidance commands by using target and inertial measurement unit (IMU) data. After autopilot calculates proper control surface positions from guidance command, the control actuator system (CAS) realizes the calculated control surface positions. By changing the control surface positions, adequate aerodynamic moments and forces are generated on the missile in order to intercept the target. Although the missile's control system includes more than one sub-structure, autopilot and CAS serve the purpose of realizing the guidance commands generated by the guidance algorithm with desired control performance. Therefore, the guidance algorithm is the main element of the control structure to achieve a successful missile-target interception. Thus the performance properties are vital for the missile guidance.

Through the flight, engagement kinematics may include several unknown disturbances. Target normal acceleration is one of the main terms in missile-target engagement kinematic equations and acts as a disturbance if the parameter is unknown. If the term is not handled properly in the guidance algorithm, missile suffers from poor hit probability against maneuvering targets. It is possible to estimate the term from radar position data or onboard missile sensor data. However, radar target position data is not accessible for all the missiles. Apart from these, adding other sub-systems into the missile may not be possible in terms of mechanical design limitations. Hence robustness of the missile guidance against unknown target acceleration is significant.

Another concept that affects the guidance performance is actuator failures. CAS is an electromechanical subsystem of the missile as illustrated in Figure 1.1. It is possible to experience mechanical damages and malfunctions on CAS through the flight. CAS may not be able to ensure desired control surface locations because of the malfunctions of the mechanical parts of the system. Structural damages and defects on control surfaces may hinder missile to actuate desired guidance commands. Similarly, mechanical imperfections and uncertainties originated in manufacturing and montage process of control surfaces may result in discrepancies between guidance command and the response. A high probability of successful interception is achieved, only if the guidance law is robust against actuator failures.

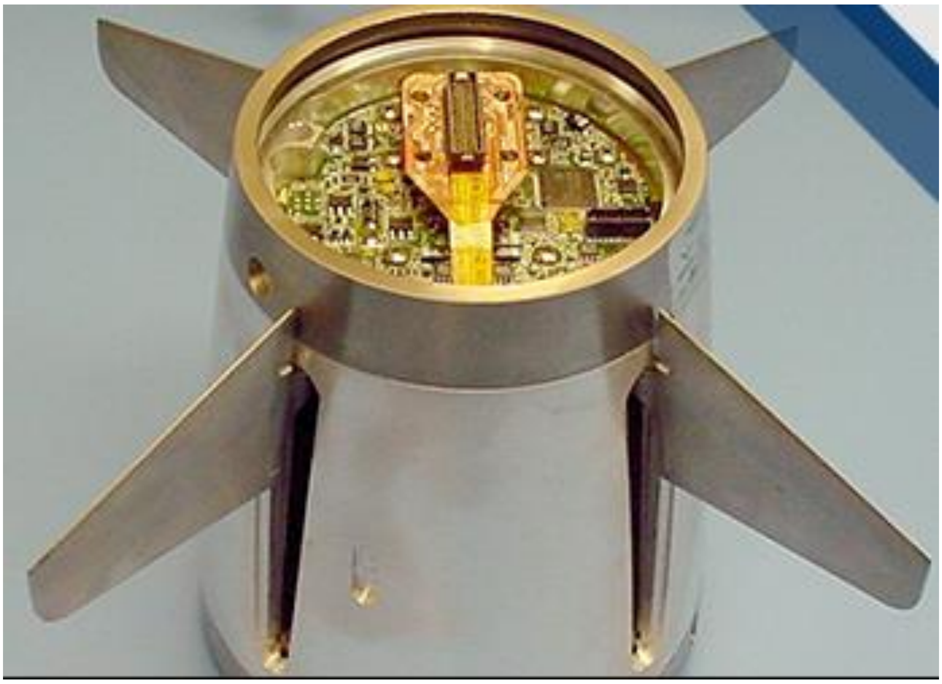

Figure 1.1. Missile control actuator system (CAS) [1]

Along with hitting a targeted object accurately, achieving the desired impact angle is another crucial goal or performance property of the guidance. It is important to achieve a high hit angle against heavily armored targets, in order to increase the missile warhead’s penetration performance. Figure 1.2. demonstrates firing of FGM-148 Javelin, which attacks with impact angles in order to disable heavily armored tanks. Using different impact angles on a salvo missile attack makes it possible to hit the target from diverse directions. Also, an exclusive desired impact angle makes it possible to overcome counter measures against the specific targets. Thus achieving the desired impact angle is an important property of a guidance law.

Figure 1.2. FGM-148 Javelin [2]

Strapdown seekers are widely used missile sub-systems to acquire target's angular position relative to the seeker and determine the line-of-sight (LOS) angle. Presented in Figure 1.3 the seeker is aligned with the body axis of missile. Depending on the position of the seeker on the missile body, the target's angular position is measured as seeker look angle with respect to the missile body. Thus, it is possible to acquire LOS angle from seeker look angle by using missile body angles. However, LOS angular rate is not provided by strapdown seekers, which is an important term used in guidance laws. Gimballed seekers are able to access this rate information from sensors on gimbal mechanisms, but this solution increases the cost of the product, and gimbal mechanisms occupy a considerable amount of space in the mechanical package. It is possible to derive LOS rate from measured LOS angle data alone. Herewith a guidance algorithm which can estimate LOS rate is capable of performing guidance without knowing the relevant term. But the guidance laws performance against the effect of measurement noise on LOS angle must be convenient.

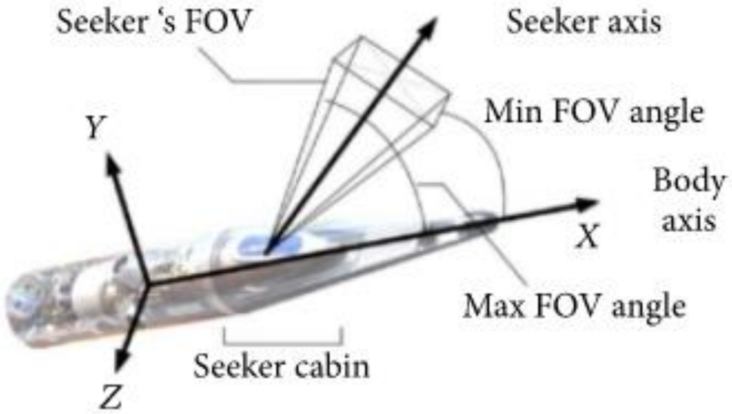

Figure 1.3. Illustration of strapdown seeker [3]

The sliding mode control (SMC) is stated as a successful robust control methods against parasitic dynamics and bounded disturbances [4]. As a variable structure control technique, SMC applies a switching control mechanism and manipulates the states of the plant with desired control performance. From the initial condition, the states of the plant are driven on a pre-defined trajectory. The trajectory of the states is defined as the sliding surface. The switching mechanism forces the plant states to slide along the sliding surface. Thus, the desired state values are achieved as a result of this sliding process. Because of the robustness property, this control method received large interest in the literature and was used in various missile guidance applications. SMC is a proper control method against disturbance and uncertainties in missile-target engagement kinematics. Actuator failures and target accelerations are one of the unknown parameters and act as disturbances or uncertainty on the engagement kinematics. Additionally, sliding mode control is appropriate for non-linear engagement kinematic; since the control method is proven to be suitable for complex non-linear systems [5]. Another advantage is that; the method is adequate for implementing design constraints like the impact angle. Sliding mode guidance literature contains several studies about the estimation of LOS angular rate with sliding mode differentiator. Sliding mode differentiator offers differentiation exactness and robustness on measurement and input noises [6]. Since the LOS angle is directly dependent on non-linear kinematics of missile-target engagement geometry, it is hard to predict measured LOS angle behavior on the flight. Sliding mode differentiator exhibits high performance on estimation errors without extensive knowledge about the base signal.

1.2. Contribution

In this thesis, actuator fault tolerant terminal sliding mode guidance law is discussed by considering impact angle with target acceleration. The literature contains several studies on guidance laws designed with sliding mode control and sliding mode differentiator structures. However, the aforementioned robustness properties in respect to unknown target acceleration, actuator failure and impact angle considerations are not taken into account in any single sliding mode guidance law in the past literature. The study in the thesis offers a single solution for this specific guidance problem. The proposed guidance law study is accepted and presented at Ankara International Aerospace Conference 2019 [7]. Moreover, the thesis contains studies on the use of the suggested guidance algorithm in absence of LOS rate information. Sliding mode differentiator is included into the proposed guidance law. Unknown data is estimated from measured LOS angle information with sliding mode differentiator. The resultant guidance law is capable of operating under measurement noise. Estimation of LOS rate subject on the SMGL will be incorporated in a journal article.

In this study, a first order SMC is used to derive the guidance algorithm. SMC may suffer from producing discontinuous high frequency control commands. This phenomenon is called chattering. Tracking high frequency commands is difficult for dynamic systems. In guidance law design, equivalent control method is followed for chattering attenuation. For the purpose of estimating LOS angular rate, a second order sliding mode differentiator structure is designed. Chattering behavior originated from the presence of measurement noise on LOS angle is attenuated with an estimated signum function. Proposed guidance law offers high accuracy on heavily armored slow targets. Guidance law is suitable for missiles with strapdown seekers since LOS angular rates can be estimated.

1.3. Organization

Subsequent to the introduction chapter, a literature survey is presented in the second chapter. In this thesis, the literature survey includes studies on guidance laws designed by sliding mode control method. The main concepts focused on sliding mode guidance law literature are fault tolerance, unknown target accelerations, impact angles and LOS angle and LOS rate estimations.

Second chapter covers the literature survey on sliding mode guidance laws.

In the third chapter, preliminary subjects are covered before discussing the SMGL design. The guidance concept in missiles is presented in the first place. Missile operating principle, sub-systems of the missile are introduced along with the guidance process. Secondly, missile-target engagement geometry is described. From the engagement geometry, essential kinematic relations for guidance law design and modeling are demonstrated. Subsequently, correlations of impact angle and actuator failure are introduced. Before entering into guidance law design, basic knowledge about sliding mode control technique are mentioned.

The fourth chapter includes the design of the SMGL. The sliding surface structure is introduced at the beginning of the chapter. Considering the design goals of achieving desired impact angle, performance against actuator failure and unknown target acceleration, a SMGL is designed with equivalent control method. After the guidance law is stated, stability of the controller is considered with the Lyapunov method. For the purpose of investigating the guidance law performance in absence of LOS angular rate information, a sliding mode differentiator (SMD) is presented. SMD is able to estimate LOS angle and its derivative from noisy LOS angle measurements. In order to improve chattering attenuation against LOS angle measurement noise, an estimated signum function is introduced, at the end of the chapter.

Simulation studies are covered in the fifth chapter of the thesis. First of all, the numerical simulation model used in the study is introduced. Afterward, the performance of the SMGL is analyzed, if LOS rate information is known by the missile. Actuator failure and maneuvering target scenarios are considered separately for different impact angles. Thereafter, both actuator failure and target maneuver are evaluated in a single scenario. Furthermore, the robustness of the SMGL is validated for various target motion directions and target ranges. Apart from this, the guidance law is tested if LOS angular rate data is not accessible by the missile. Actuator failures and target maneuver are considered along with LOS rate estimations. Additionally, the SMGL performance is inspected in presence of measurement noise on LOS angle. Effects of signum, sigmoid and proposed estimated signum function on guidance law performance are analyzed in another scenario. In the final part of the simulation studies, a Monte Carlo analysis is presented for various initial conditions.

Discussion on results is demonstrated in the sixth chapter. Simulation results on the previous chapter are evaluated and scenario outputs are compared. Possible future works are noted at the end of the chapter.

2. LITERATURE SURVEY

In order to give a decent insight into SMGL subject, related studies in the literature are introduced. The survey is mainly focused on SMGL studies on desired impact angle, actuator failure and LOS angular rate estimation along with target acceleration considerations. Hence, sliding mode guidance subjects are grouped and discussed under relevant sub-chapters.

2.1. Impact Angle

In absence the information of target acceleration, acceleration upper bound terms are included in sliding mode guidance law as a switching term in the studies of Cho et al. [8]. The study introduces two separate sliding surfaces for impact angle and LOS angular rate. Robustness of the SMGL is proved against non-maneuvering and maneuvering targets with several distinct desired impact angle constraints. The effect of target flight path angle noise on the guidance law is evaluated. Xiong et al. [9] use extended state observer to estimate unknown target acceleration. The approximated acceleration term is included in the derivation of the SMGL. Thus the performance of tracking desired impact angle is improved against maneuvering targets. An adaptive second order SMC method is presented by Wang [10] against unknown target acceleration. The guidance law provides smooth guidance commands and attenuates chattering by using an integrated continuous function. A modified sliding mode control algorithm is designed in order to intercept high maneuvering targets [11]. The proposed guidance uses equivalent control method for reducing chattering behavior. The results of the study offer high accuracy over intercepting maneuvering aerial target, however, the guidance law needs to access the location of the target relative to the missile. Adaptive fuzzy-sliding mode guidance law is presented by Wang et al. [12] by considering unknown target acceleration and autopilot dynamics. The guidance law takes into account autopilot time lag and proposes improved performance against the delayed response of the autopilot. Fuzzy logic provides an adaptive solution for guidance gains. Proper guidance gains are selected with respect to LOS rate and range parameters. Similarly, Li et al. [13] proposed fuzzy SMGL by including impact angle and missile acceleration considerations along with autopilot dynamics. The sliding surface is constructed with fuzzy logic based gains. Simulation results show that high precision is achieved in respect to miss distance and desired impact angle errors. But maneuvering target scenarios are not considered in this study. In another study by Li et al. [14], neuro-fuzzy logic is adapted in order to reduce chattering. The guidance structure uses an adaptive neuro-fuzzy interference system (ANFIS) for the ability of fast learning and it is able to succeed desired impact angles in absence of target acceleration data. Kumar et al. [15] proposes a SMGL and includes a sliding mode autopilot design in their study. SMGL is able to hit the target with a chosen impact angle. In the design process, equivalent control method is adopted for chattering attenuation. Altough autopilot dynamics is included in the model, the suggested algorithm ensures proper performance against maneuvering targets. A second order SMGL with impact angle consideration is introduced by Zhang et al. [16]. Different from the previous studies, autopilot is regarded as a second order system, in order to implement the dynamics accurately. Zhou et al. [17] integrated a nonhomogeneous disturbance observer on a second order SMGL by considering time delay of the autopilot and impact angle. The nonhomogeneous disturbance observer based on SMD is able to estimate target maneuver. Thus disturbance term created by target maneuver is accessible for the guidance law. Chen et al. [18] introduced an adaptive SMGL scheme for aircraft pursuit-evasion problem. The proposed SMGL is able to estimate target acceleration boundary. Estimated parameters allows guidance law to generate proper commands with respect to disturbance effect of the target maneuver. Wang et al. [19], included seeker field of view (FOV) limitations along with impact angle and target acceleration considerations. To keep the target in seeker FOV, a control integral barrier Lyapunov function is adopted in the guidance design. The influence of target acceleration over the engagement geometry is calculated with a nonlinear extended state observer (NLESO). In order to hit the target with desired impact angle in limited FOV, He et al [20] offer a different approach. Upon exceeding a predefined FOV limit, switching logic introduces an additional term on the guidance law. The term forces missile to keep the target in its FOV range. Although the guidance law offers a robust solution, the study does not introduce interest in unknown target acceleration. In addition to achieving a chosen impact angle, the impact time subject is contemplated in another study by Kumar et al. [21]. The proposed solution uses separate guidance laws for impact angle and impact time goal. Impact time is controlled over a sliding surface which contains time to go, elapsed time and desired impact time parameters. Equivalent control method is followed in the SMGL design. During the flight, the use of the proper guidance is decided with respect to the estimated time to go error. However, the study does not cover the impact of target maneuvers on guidance performance. Kim et al. [22] offer a SMGL for considering both impact time goal and unknown target acceleration. Apart from these, there is a considerable amount of research available on SMGL with terminal impact angle considerations [23]-[25].

2.2. Actuator Failure

Zhu et al. [26] proposed a SMGL to achieve the interception in presence of target acceleration and actuator failure. The actuator failure terms are considered as bias and scale errors on the guidance command. These errors result in discrepancies between the guidance command and actuated command. Unknown failure terms and target acceleration are acknowledged as bounded in finite interval. By selecting proper guidance gains according to bounded unknown terms, fault tolerant guidance law offers suitable miss distances against maneuvering targets in presence of actuator failure. By using backstepping and sliding mode control methods, an adaptive fault tolerant guidance law is derived by Jegarkandi et al. [27]. The failure model is designed based on the deflection angle of the control surfaces and the difference between actual and desired deflection is expressed as a failure. Although the guidance law is robust against bounded unknown actuator failure effects and target acceleration, the missile needs to access angle of attack information for guidance command calculations. Ashrafifar et al. [28], offer adaptive sliding guidance law for the failure problem. In this study, failure is related to the change of area on the control surfaces. The guidance law is adapted since guidance gains are calculated with respect to the area of control surfaces. Control surface area is estimated by using unscented Kalman filter. Similar to the previous study [27], angle of attack information must be accessible by the missile, in order to operate the proposed guidance law. The literature contains several solutions on sliding mode control and actuator failures. For instance, Corradini et al. [29] implemented state feedback and output feedback methods based on sliding mode control to compensate actuator failures. Nonetheless, SMGL with actuator failure concept is not studied extensively in the literature.

2.3. LOS Angular Rate Estimation

Lee et al. [30] estimated LOS rate from LOS angle information with a sliding mode differentiator algorithm in the proposed SMGL for strapdown seekers. The proposed algorithm uses a hybrid guidance law without considering target maneuver or impact angle. In Figure 2.1 simulation results prove that; the guidance law presents low estimation errors under measurement LOS angle measurement noise. By considering input saturation and autopilot lag together, an adaptive SMGL is designed by Guan et al. [31] and LOS rate information is acquired from a sliding mode differentiator. Adaptive structure determines the upper bound of target acceleration. Control commands are saturated, in order to keep the produced commands below the acceleration limit. Simulation results of the study demonstrate the robustness of the SMGL against maneuvering target and in absence of measurement noise. The impact of measurement noise on SMGL performance is tested with Monte Carlo analysis by investigating resultant miss distances. However, the effect of noise on guidance and flight parameters is not discussed in detail. Without the need for LOS rate data, He et al. [32] proposed an observer-based SMGL against maneuvering targets. Target acceleration disturbance on the engagement geometry and LOS angular rate are estimated with sliding mode observers. In another study of the author [33], LOS angular rate estimations are discussed and autopilot dynamics are integrated into guidance law. Detailed simulation studies are presented in both studies to prove guidance law effectiveness, however, measurement noise on LOS angle is not included in the simulations.

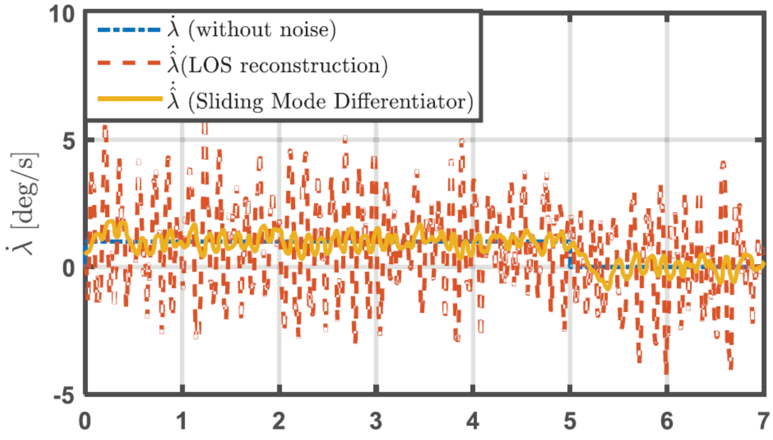

Figure 2.1. LOS angular rate estimation [30]

2.4. Conclusion

In sliding mode guidance literature, target acceleration is investigated in various studies. However, impact angle, actuator failure and LOS angular rate estimation subjects are discussed in separate articles. Related subjects are not considered together in the past. Also, the literature on sliding mode guidance law with actuator failure and LOS angular rate estimation concepts is limited. The guidance law proposed in this thesis offers a single solution for the entire aforementioned design goals.

3. PRELIMINARIES

3.1. Missile Guidance Concept

Guided missiles are autonomous systems, which once activated are capable of tracking and deactivating targets without user inputs. A missile is composed of four main subsystems: guidance section, warhead section, propulsion section and control section. The subsystems of a missile are visualized in Figure 3.1.

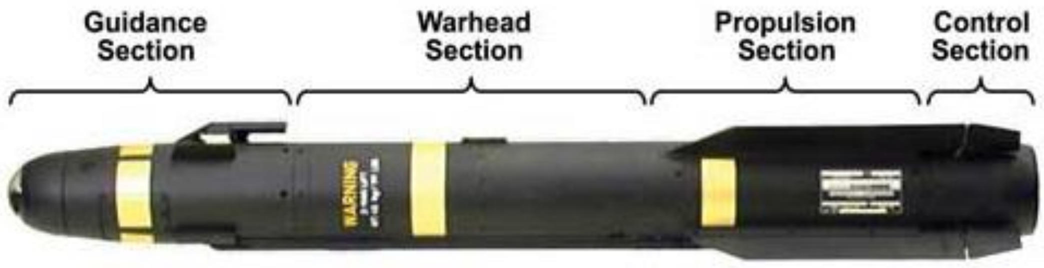

Figure 3.1. Missile subsystems [34]

Guidance section contains the sensors, required in order to acquire target data. Target information can be obtained from onboard seekers and radars. IR seekers are capable of tracking infrared light emissions of the target. Laser seekers can track reflected laser energy from the target if the target is designated with a laser source. Thus relative angular position of the target. Similarly, onboard radars can measure angle between missile and target. Besides onboard trackers, target data can be acquired from a ground radar with an onboard receiver. Depending on the missile mechanical design, missile computer and IMU are also located on the guidance section. IMU measures required information of missile dynamics. The information may contain missile accelerations and angular rates. Missile computer is responsible of calculating control actuator commands, in order to intercept the target. Warhead section carries the payload that deactivates the target upon the collision. Propulsion section ensures the necessary missile velocity to pursue the target. As the calculated control surface deflections are applied by CAS, aerodynamics or thrust forces on the missile are manipulated so that the desired maneuver is achieved.

In Figure 3.2 relation between concepts involving missile guidance is visualized.

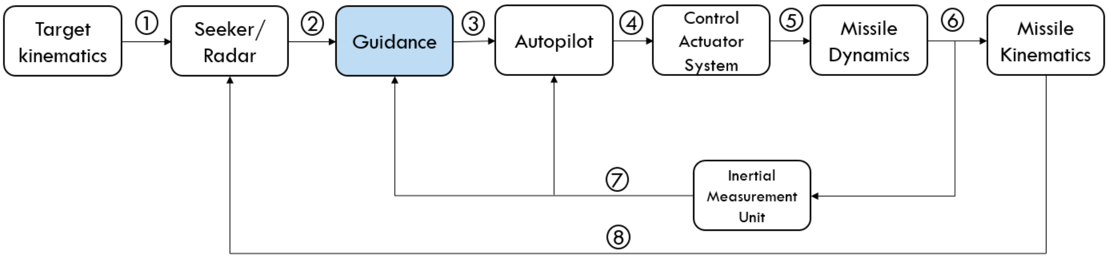

Figure 3.2. Simplified engagement model

Numbered data flow between sub-models are described below:

1. Target position, orientation
2. Missile-target relative position, velocity, orientation
3. Missile acceleration, orientation commands
4. Actuator commands
5. Aerodynamic/propulsive state of the missile
6. Aerodynamic/thrust forces on the missile
7. Missile acceleration and angular rates
8. Missile position and orientation

3.2. Missile-Target Engagement Geometry

In real world, missile and target are free to move in three-dimensional space. Therefore, guidance algorithms are represented in three-dimensional space or two separate orthogonal planes. In this study, a planar engagement geometry model is used with point mass approach for simplicity. Nevertheless, the guidance law can be used in two orthogonal planes separately. Besides this, it is still possible to derive the guidance law for three-dimensional space if the missile-target engagement geometry is also modeled in three dimensions.

The missile-target engagement model is represented on vertical plane, since the impact angle is related with top surface of the heavily armored targets. A fixed inertial reference frame is used in order to define vectorial terms. Kinematic engagement model is presented in Figure 3.1:

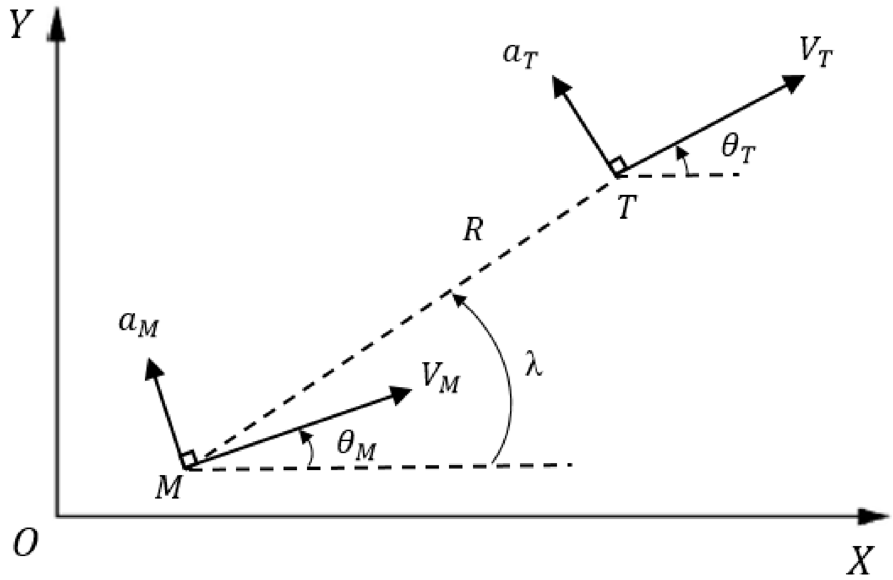

Figure 3.3. Missile-target engagement geometry

Missile and target properties are indicated with M and T respectively. Y and X axes represent position notion of altitude and longitudinal distance with respect to the reference frame. a_M and a_T are acceleration vectors normal to velocity components of missile V_M and target V_T. Missile flight path angle θ_M and orientation of the target θ_T declares direction of the velocity vectors. Orientation of relative position vector of the target with respect to the missile is denoted by LOS angle λ. Lastly, R represents the distance between target and missile.

Theoretically, it is possible for missile computer to acquire all the information described on the engagement geometry. To give an insight about the topic, accelerometer on the onboard IMU can measure the acceleration of the missile. A simple navigation algorithm is able to calculate position and velocity of the missile from integration of acceleration information. Missile flight path angle can be obtained from integration of rate gyro data. Depending on the guidance method; LOS angle, range and target properties can be measured, estimated or calculated from seeker or radar data.

In this study it is assumed that; a_M, θ_M, θ_T, V_M, V_T, λ, R and its time derivative $\dot{R}$ data are accessible for missile computer. LOS angular rate $\dot{\lambda}$ is not known, since the study is focused on missiles with strapdown seekers. Furthermore, a_T is an unknown parameter and behaves as a disturbance term on missile-target engagement geometry. Drag force is supposed as a negligible effect on the missile, since terminal guidance is considered to be taking place after the end of the boost phase. Besides that, target is accepted as a slowly moving heavily armored object. Thus V_M and V_T are assumed as constant in this problem. Effect of the gravity is not taken into account for simplicity, because it is possible to eliminate the effect with a gravity term correction on guidance command. Time lag of seeker, autopilot and CAS dynamics are neglected, since the concern of the study is the derivation proposed guidance law and its interaction with seeker noise.

The velocity components of missile and target on reference frame are obtained as:

$$V_{Mx} = V_M cos(\theta_M) \tag{1}$$

$$V_{My} = V_M sin(\theta_M) \tag{2}$$

$$V_{Tx} = V_T cos(\theta_T) \tag{3}$$

$$V_{Ty} = V_T sin(\theta_T) \tag{4}$$

Time derivative of missile and target position components can be expressed as:

$$\dot{X}_{Mx} = V_{Mx} \tag{5}$$

$$\dot{X}_{My} = V_{Mx} \tag{6}$$

$$\dot{X}_{Tx} = V_{Tx} \tag{7}$$

$$\dot{X}_{Ty} = V_{Ty} \tag{8}$$

Range component on the reference frame can be calculated from:

$$R_x = X_{Tx} - X_{Mx} \tag{9}$$

$$R_y = X_{Ty} - X_{My} \tag{10}$$

Magnitude of the range is given by:

$$R = \sqrt{{R_x}^2 + {R_y}^2} \tag{11}$$

Line of sight angle can be determined as angular orientation of range vector on the reference frame:

$$\lambda = \text{atan2}(R_y, R_x) \tag{12}$$

Geometric relation in the engagement kinematics shows the correlation between $\dot{R}$ and velocity components in the following equation:

$$\dot{R} = V_T cos(\lambda - \theta_T) - V_M cos(\lambda - \theta_M) \quad (13)$$

Trigonometric relation of λ in the engagement geometry states the following equation for $\dot{\lambda}$ term:

$$\dot{\lambda}R = V_M sin(\lambda - \theta_M) - V_T sin(\lambda - \theta_T) \quad (14)$$

Elementary mechanics relation for constant velocity gives the following equations for normal acceleration terms of missile and target:

$$\dot{\theta}_M = a_M / V_M \quad (15)$$

$$\dot{\theta}_T = a_T / V_T \quad (16)$$

3.3. Impact Angle

Impact angle is one of the concerns of the study. The parameter represents the angle of the missile's body with respect to the target's upper surface at the time of the interception. Angle of attack is another concept that describes the angle of missile velocity vector relative to the missile body. In real world, velocity vector of the missile is not always aligned with missile body. However, missiles are likely to be designed to operate with small angle of attacks, since higher angle of attacks increase non-linear characteristics of missile aerodynamics and bring difficulties in autopilot design. Therefore, it is assumed that effects of the angle of attack are negligible and velocity vector of the missile is aligned with the missile body. Similarly, target's upper surface is bounded to the orientation of the target and target velocity vector is perpendicular to the surface normal.

The illustration of velocity vector and orientations of missile and target at the interception time is shown at Figure 3.4:

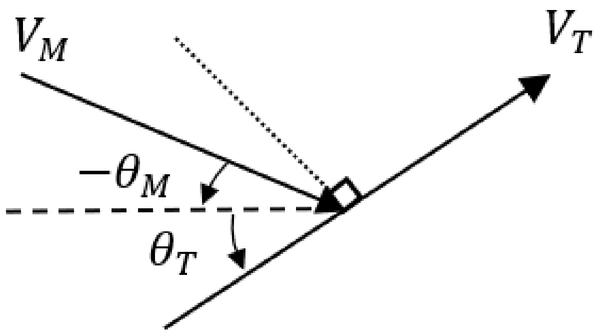

Figure 3.4. Representation of impact angle

From previous considerations, the impact angle θ_{imp} is defined in the following equation, where t_f is interception instant [13]:

$$\theta_{imp} = \theta_T(t_f) - \theta_M(t_f) \quad (17)$$

3.4. Actuator Failure

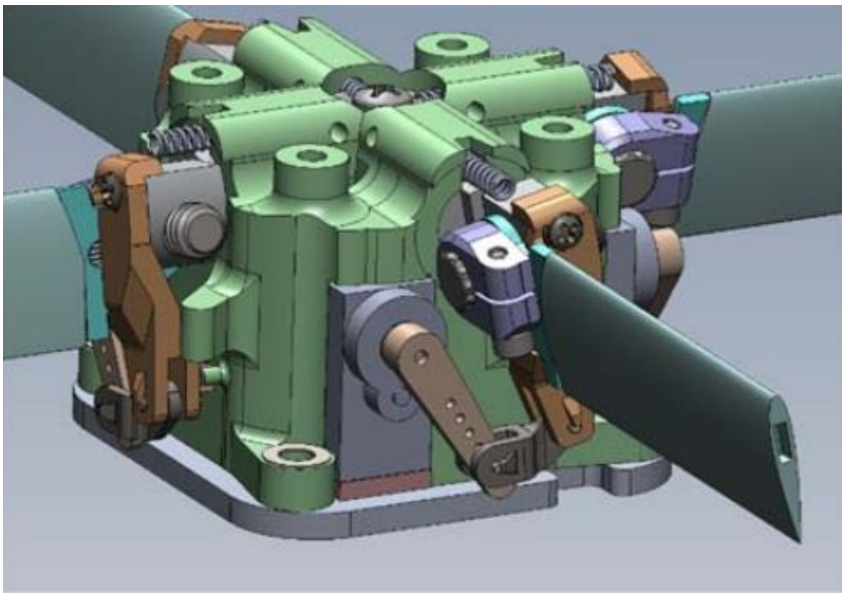

Figure 3.5. Control actuator system inner mechanisms [35]

During the flight it is desired that, the guidance command a_{com} is actuated perfectly by the missile, so that a_{com} is equal to a_M. Unfortunately, this is not possible if the CAS is not responding to the guidance commands as required. Various conditions the missile is exposed may cause malfunctions on the inner mechanisms of CAS or deformations on the control surfaces. Extreme temperature conditions, thermal shocks, strong vibrations, high and impulsive shocks or manufacturing uncertainties on the dynamic actuator system can alter the performance of the system, as well as projectiles fired by the active defense systems that are nowadays available for military land vehicles may harm the control surfaces. Besides, it is hard to detect these effect on the missile before the flight. Through the flight, sufficient lift forces may not be created if a small rupture on the control surface exists. On the contrary, a deformation of the aerodynamic design may increase the lift forces generated on the surface. As a result, performance of the control surfaces may change and produced normal accelerations on the missile may be more or less than the guidance command. Malfunctions on the inner actuator mechanisms or damages on the connection mechanisms may result in fluctuations or biases on the control surface positions. Consequently, unwanted normal accelerations occur on the missile. Herewith, the actuator failures are modeled with a multiplicative term of the guidance command μ and additive term a_{F} [26]. Multiplicative term μ represents the performance of the control surfaces and takes the value of 1 in the ideal condition. Additive term a_{F} denotes the unwanted accelerations induced one the missile. Thus, the actual missile normal acceleration a_M is calculated with respect to a_{com}, a_M and μ:

$$a_{\mathrm{M}} = a_{\mathrm{F}} + a_{\mathrm{com}}\mu \tag{18}$$

3.5. Sliding Mode Control

Architecture of controller is heavily dependent on plant dynamics. Although the plant dynamics are taken into consideration in controller design as much as possible, the discrepancies between the actual plant and the corresponding mathematical model always exist. Unknown disturbances and uncertainties aggravate achieving the desired controller performance. Furthermore, as the nonlinear properties and the order of the plant increases, controller design process becomes more complicated and troublesome. These circumstances in the controller design create need for application of robust controller methods. The SMC approach is stated as an effective method for high-order nonlinear structures and the method exhibits robust attributes against unknown disturbances and uncertainties [5].

As a variable structure control method, the SMC manipulates the states of the system on a desired trajectory with a switching control mechanism. In SMC structure, a sliding variable "s" is selected by designer which inherits proper characteristics of the system. Sliding variable represents the relation between the states of the system. The desired trajectory of the system states is characterized by sliding surface. The condition of sliding variable equal to zero denotes the sliding surface. From an initial condition, system states are attracted to the sliding surface in the reaching phase. After reaching the sliding surface the sliding phase initiates and switching control structure forces system states to slide along the surface. Behavior of a sliding mode controlled system consisting of two states is demonstrated in Figure 3.6.

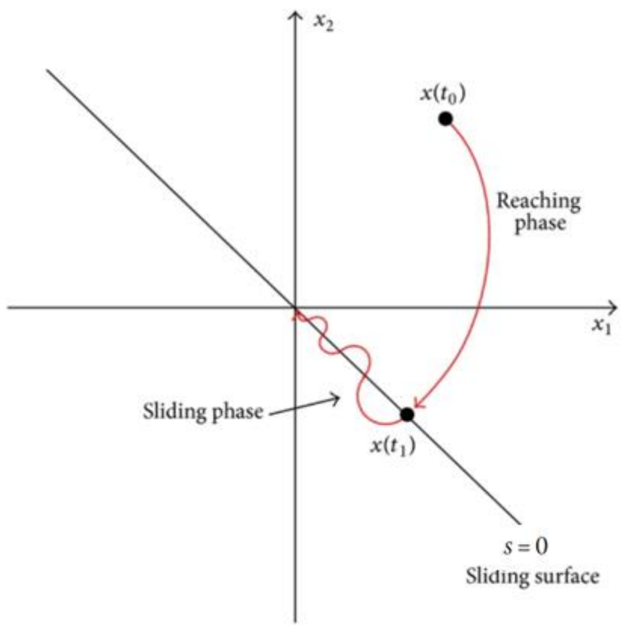

Figure 3.6. Demonstration of state trajectories in SMC [36]

The order of the SMC defines the constraints on the sliding variable derivatives. Higher order sliding modes offers smoother control commands. However, system suffers from slower responses. Suppose that, r is the order of the sliding mode. Up to $(r-1)$ order derivatives of the sliding variable correspond to zero, in higher order sliding modes. The relation is presented in Equation (18).

$$s = \dot{s} = \ddot{s} = \cdots = s^{(r-1)} = 0 \tag{18}$$

Chattering is another important concept in sliding mode control method. In sliding phase, high frequency discontinuous commands are produced to keep the system states on the sliding surface. Figure 3.7 demonstrates high frequency sliding mode control command. States slide along the sliding surface with zig-zag like motions. The produced discontinuous control commands may not be tracked perfectly by the dynamic systems, since dynamic systems actuates the control commands with continuous responses. Besides, high frequency motions may result in malfunctions on the actuator. Thus, smoother control commands are desired for dynamic systems. Chattering phenomenon is attenuated by applying higher sliding mode control, equivalent control method or replacing discontinuous switching function with a continuous one. In Figure 3.7 and 3.8, chattering on a control signal is illustrated.

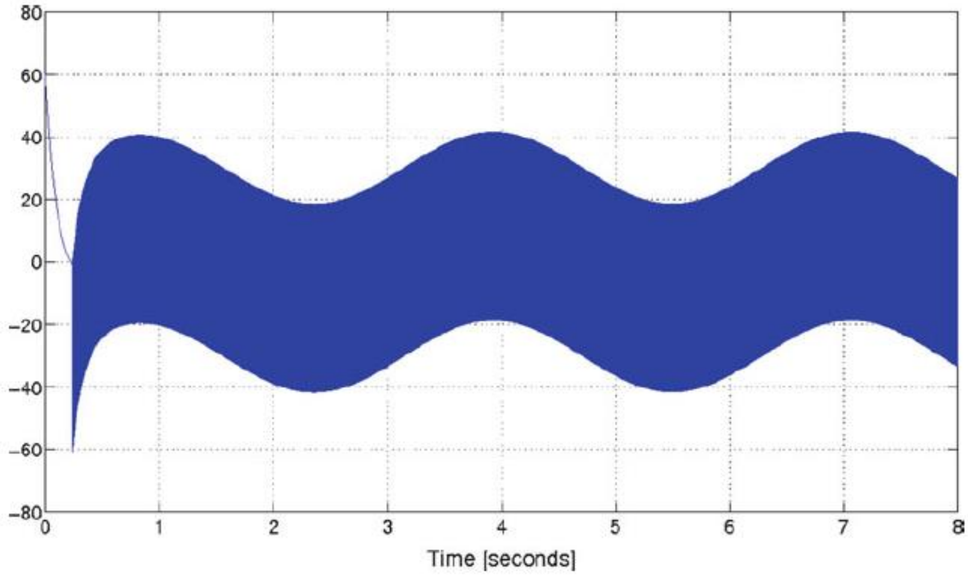

Figure 3.7. Sliding mode control command in chattering [37]

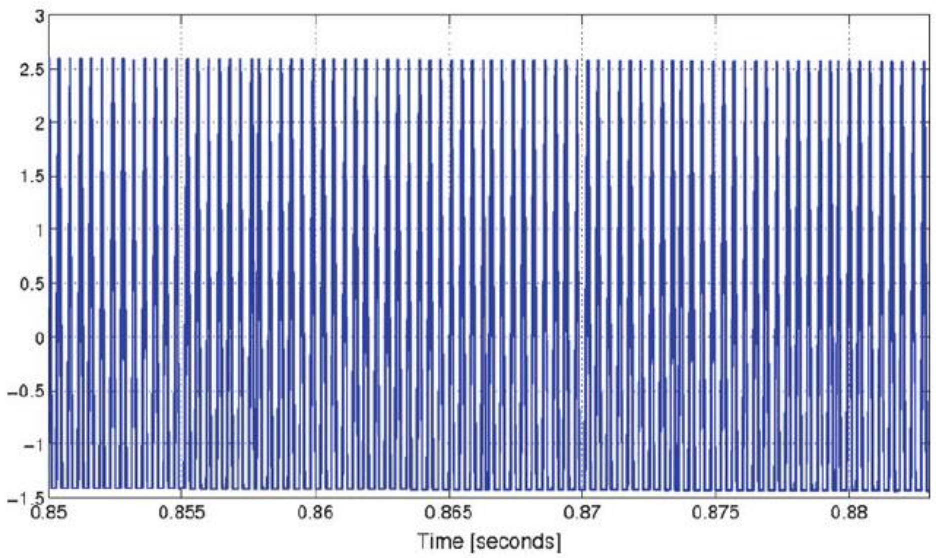

Figure 3.8. Sliding mode control command in chattering (zoomed) [37]

4. GUIDANCE LAW DESIGN

4.1. SLIDING-MODE GUIDANCE LAW DESIGN

4.1.1. Sliding Surface Design

The guidance law must be able to produce necessary commands to guide the missile. The main principle of the parallel navigation rule is that, if the missile and the target are on the collision course a successful interception is inevitable [38]. In other words, LOS angle is kept constant or LOS rate is zero until the interceptions occurs. This can be succeeded by controlling the missile with normal acceleration commands.

Impact angle is another parameter that should considered before starting design of the guidance law. It is known that, there exists a relation between impact angle and the LOS angle [13]. As discussed before, LOS rate must converge to zero before the end of the engagement. With this knowledge, following relation is acquired by using the equations (15) and (17), where λ_{des} is the desired impact angle that is related to the LOS angle:

$$\lambda_{des} = \theta_T - arctan(\frac{sin\theta_{imp}}{cos\theta_{imp} - V_T/V_M}) \tag{19}$$

In this thesis, the main objective of the SMGL is ensuring successful interception against the target and achieving the desired impact angle. With the aim of procure the goals, LOS rate $\dot{\lambda}$ and desired LOS angle λ_{des} are implemented on the sliding surface. The sliding surface states are comprise of LOS rate $\dot{\lambda}$ and error between LOS angle λ and desired LOS angle λ_{des}.

$$x_1 = \lambda - \lambda_{des} \tag{20}$$

$$x_2 = \dot{\lambda} \tag{21}$$

The target is considered as a slowly maneuvering object in this study. Thus, it is assumed that target orientation θ_T does not vary significantly in time. From this point of view, time derivative of x_1 can be equated to x_2.

$$x_2 = \dot{x}_1 \tag{22}$$

One can define the sliding variable as a homogenous linear time-invariant differential function for the purpose of including asymptotical convergence property on system states.

$$s = x_1 c + \dot{x}_1, \quad c > 0 \tag{23}$$

In Equation (23), the convergence rate of the states to zero is only tunable with parameter c. Increasing value of c results in higher convergence rate for desired impact angle or vice versa. However, achieving desired impact angle is only meaningful at the end of the engagement. It is redundant to spend control effort when the magnitude of the range is large. Therefore, range parameter R is included into the sliding variable and the sliding surface.

$$s = x_1 \frac{\varepsilon}{R^\gamma} + x_2 \tag{24}$$

$$0 = x_1 \frac{\varepsilon}{R^\gamma} + x_2 \tag{25}$$

Sliding variable parameters ε and γ are tunable variables where $\varepsilon > 0$ and $1 \geq \gamma \geq 0$. As the range decreases, the gain of x_1 increases and control effort on achieving desired impact angle becomes prominent. Tunable parameters offer extensive option to adjust guidance law. If it is desired to use the sliding variable independent from the range term, it is possible to select γ as zero.

4.1.2. Equivalent Control Method

In sliding phase, sliding mode controlled systems suffer from chattering phenomenon. One of the methods to solve this problem is the equivalent control method. In equivalent control method, control command comprises equivalent control and reaching law terms [39].

Equivalent control term corresponds to the continuous control command, that make the system states hang on the sliding surface. Thus, system states slide along the sliding surface with the calculated continuous command. Since, control effort on the sliding phase is undertaken by equivalent control term, the magnitude of the discontinuous control in the reaching law is reduced.

In sliding phase, since the system states are stuck on the sliding surface sliding variable’s time derivative is zero.

$$\dot{s} = 0 \tag{26}$$

Before solving the above equation time derivative of $\dot{\lambda}$ must be derived, owing to the fact that $\dot{\lambda}$ is the second state x_2. Considering equations (14), (15), (16), (18) and recalling that $\dot{V}_M = 0$ and $\dot{V}_T = 0$, the equality of $\ddot{\lambda}$ is found as:

$$\ddot{\lambda} = \frac{-(a_F + a_{com}\mu)\,cos(\lambda - \theta_M) + a_T\,cos(\lambda - \theta_T) - 2\dot{R}\dot{\lambda}}{R} \tag{27}$$

Solving the Equation (26) for guidance command a_{com} with using equations (14), (15), (16), (17) by ignoring the unknown disturbance and uncertainty terms a_T, μ and a_F reveals the equivalent control term:

$$a_{ec} = \frac{\varepsilon R^{1-\gamma} - 2\dot{R}}{cos(\lambda - \theta_M)}\dot{\lambda} - \frac{\varepsilon\gamma R^{-\gamma}\dot{R}}{cos(\lambda - \theta_M)}\lambda \tag{28}$$

Reaching law carries the states to the sliding surface expeditiously. With respect to the sliding surface, reaching law’s switching function designates the direction of states motion. Even though it is expected that the equivalent control term must keep the sliding variable on the sliding phase, unknown disturbance and uncertainty terms make the states deviate from the sliding surface in time. The reaching law also, assists system states to track the sliding surface in presence of disturbances. Thus, a proper reaching law is produced by taking in to account of stability conditions of the sliding surface.

$$a_{rl} = (k_1|a_{ec}| + \frac{k_2}{cos(\lambda - \theta_M)} + \frac{k_3}{\sqrt{2}cos(\lambda - \theta_M)})sgn[s]sgn[cos(\lambda - \theta_M)] \tag{29}$$

After equivalent control and reaching law terms are determined, terminal sliding mode guidance law is designated as sum of a_{ec} and a_{rl}**:**

$$a_{com} = a_{ec} + (k_1|a_{ec}| + \frac{k_2}{cos(\lambda - \theta_M)} + \frac{k_3}{\sqrt{2}cos(\lambda - \theta_M)})sgn[s]sgn[cos(\lambda - \theta_M)] \tag{30}$$

4.1.3. Stability Considerations

Derived sliding mode guidance law must be able to make system states converge to zero in finite-time. In order to determine stability condition of the guidance law, Lyapunov stability criteria is applied. Besides that, following lemma should be checked, for the purpose of proving finite time stability property.

Suppose that, $V(S)$ is a smooth positive definite function and it satisfies following condition:

$$\dot{V}(S) + cV(S)^{\alpha} \le 0 \tag{31}$$

Where real numbers c and α satisfies $c > 0$, $0 > \alpha > 1$. Thus, $V(S)$ is able to converge to zero in finite-time. $T(S)$ and $V(S_0)$ are settling time and initial state of $V(S)$ respectively [40].

$$T(S) \leq \frac{1}{c(1-\alpha)} V(S_0)^{1-\alpha} \tag{32}$$

Considering the finite time stability properties, a Lypunov function is nominated to evaluate stability property of the sliding mode guidance law.

$$V(s) = \frac{1}{2} s^2 \tag{33}$$

$\dot{V}(s)$ is calculated by taking time derivative of the sliding variable $\dot{s}$:

$$\dot{V}(s) = s(a_T\, cos(\lambda - \theta_T) + (1-\mu)a_{ec} cos(\lambda - \theta_T) - \mu a_{rl} cos(\lambda - \theta_T) - a_F\, cos(\lambda - \theta_T)) \leq\ -cV(s)^{\alpha} \leq 0 \tag{34}$$

Disturbance and uncertainty terms a_T, μ and a_F appears in the inequality. In this thesis it is assumed that, disturbance and uncertainty terms are unknown. On the other hand, unknown terms are accepted as they are bounded within a finite space.

The target has a limited maneuvering ability; hence it is able to move with a bounded acceleration a_T:

$$a_{T_{max}} > |a_T| \tag{35}$$

Additive term of actuator failure a_F represents real life reactions of the failures to the missile system. Thus, additive acceleration term should not exceed a physical limit.

$$a_{F_{max}} > |a_F| \tag{36}$$

Multiplicative actuator failure term μ is defined as a percentage unit. Together with the guidance command a_{com}, the term μ indicates the quantity of the control signal actuated by the missile. Hence, the term is bounded with an upper and lower limit.

$$1 > \mu > \mu_{min} > 0 \tag{37}$$

When the inequality (34) is solved with equations (29) and (30), resultant inequality reveals that, the system is stable with following conditions:

$$k_1 > \frac{1-\mu_{min}}{\mu_{min}} \quad (38)$$

$$k_2 > \frac{a_{F_{max}}+a_{T_{max}}}{\mu_{min}} \quad (39)$$

The proper choice of guidance gains suppresses the effect of bounded unknown parameters on the sliding variable. If k_1 and k_2 satisfies the given relations, Equation (34) is reduced to:

$$\dot{V}(S) \leq -V(S)^{0.5}\mu_{min}k_3 \leq -cV(S)^{\alpha} \leq 0 \quad (40)$$

From the inequality (40), $\boldsymbol{V}(\boldsymbol{S})$ converges to zero from any initial condition in finite time, if $\boldsymbol{k_3} > \frac{\boldsymbol{c}}{\boldsymbol{\mu_{min}}}$. Thus the settling time satisfies:

$$T(S) \leq \frac{2}{k_3\mu_{min}} V(S_0)^{0.5} \quad (41)$$

In order to compensate the effect of μ by its own, k_1 must be chosen high enough as a design consideration. Both a_F and a_T effects the sliding mode dynamics together with μ, therefore a proper k_2 must be used to eliminate the effects of these bounded unknown parameters. Larger selections of k_3 speeds up the reaching phase and decreases the settling time.

4.2. LOS ANGLE AND LOS RATE ESTIMATION

The terms LOS angle λ and LOS rate $\dot{\lambda}$ have significant role in calculating missile acceleration command. However, the knowledge of these terms may not be directly accessible by the missile or the measured data may contain noise. λ is mostly calculated by radar data or measured by onboard missile sensors and this information is transmitted to the missile computer. Even though λ is accessible by the missile computer, the data contains an amount of noise related to measurement quality. Depends on the use, noisy data reduces the performance of the control systems. As discussed before, it is assumed that missile can acquire the LOS angle data from the measurements of strapdown seeker but LOS rate is not provided for the guidance algorithm. Thus, LOS rate must be estimated, in order to provide this significant data into the sliding mode guidance law algorithm.

In this study, relatively different and simple method is used for LOS rate $\dot{\lambda}$ estimations. A second order SMD is formed to estimate the LOS rate:

$$\dot{z}_0 = -M_2 L^{1/3} |z_0 - \lambda|^{2/3} sign|z_0 - \lambda| + z_1 \quad (42)$$

$$\dot{z}_1 = -M_1 L^{2/3} |z_0 - \lambda|^{1/3} sign|z_0 - \lambda| + z_2 \quad (43)$$

$$\dot{z}_2 = -M_0 L sign|z_0 - \lambda| \quad (44)$$

Terms z_0, z_1 and z_2 are the estimated values of LOS angle and its derivatives $\hat{\lambda}$, $\dot{\hat{\lambda}}$ and $\ddot{\hat{\lambda}}$ respectively. M_0, M_1 and M_2 are defined as differentiator gains. The Lipschitz constant L and it is chosen bigger than (k+1)th derivative of the measurement signal. By feeding noisy λ data as an input to the SMD algorithm; $\hat{\lambda}$, $\dot{\hat{\lambda}}$ and $\ddot{\hat{\lambda}}$ parameters are acquired.

It is known that kth order differentiator has better accuracy than lth order differentiator (l<k) when estimating the lth derivative [41]. By considering this fact and approved performance of the method in guidance literature [33], a second order differentiator is adopted in order to present preferable performance.

The proposed SMGL includes a number of signum functions (sgn), that makes the guidance law discontinuous. Sign changes on the sliding surface may result in high frequency discontinuous guidance commands. These guidance commands are hard to be tracked by the system, since system dynamics of the actuator and missile aerodynamics are continuous and have limited response speed in real life.

Most of the time, a sigmoid function [42] is effective to attenuate chattering in the presence of actuator and aerodynamic disturbances and uncertainties [7]. However, including noisy data into the sliding surface, increases the chattering significantly.

Designed guidance law's sliding variable is consist of the estimated states $\hat{\lambda}$, $\dot{\hat{\lambda}}$. Since it is impossible to calculate exact values of the sliding variable states in the presence of the noise, high frequency fluctuations on the sliding variable is inevitable. In the sliding phase of the control, the sliding variable fluctuates around zero and sign of the variable changes rapidly. In order to avoid this situation, a feasible substitute method for signum function is proposed as:

$$s_f = s\frac{1}{TS+1} \quad (45)$$

$$\widetilde{sgn}(s) = \begin{cases} sgn(s), & |s_f| > e \\ \frac{s_f}{|s_f|+d}, & |s_f| \leq e \end{cases} \tag{46}$$

In this method of approximating signum function, sliding variable is fed into a first older filter to mitigate the amount of noisy fluctuations. If magnitude of the filtered variable s_f is greater than e, in other words if sliding variable is far away from the sliding surface; well-known signum function is used on s_f to maintain fast convergence speed of sliding variable. When s_f is smaller than e; instead of the signum function, a sigmoid function is operated by using s_f and chattering is attenuated when the sliding variable is around zero. Greater values of e provides a safe region against larger amplitudes of the sliding variable fluctuations around zero. On the other hand, convergence speed of sliding variable is decreased. Parameter d must be chosen small enough to maintain a viable convergence speed and high enough to attenuate chattering phenomena. Intended use of the approximate signum is presented in the analysis described in the next part of the paper.

5. SIMULATION STUDIES

5.1. SIMULATION MODEL

Detailed analyses are made in an effort to observe performance of the SMGL. A numerical simulation model is created through Matlab\Simulink environment. Time step of 0.001 s and solver method of ode4 (Runge-Kutta) is chosen for the numerical runs. The 3-DOF simulation model consists of two main parts which are a kinematic missile-target engagement geometry model and a guidance algorithm model.

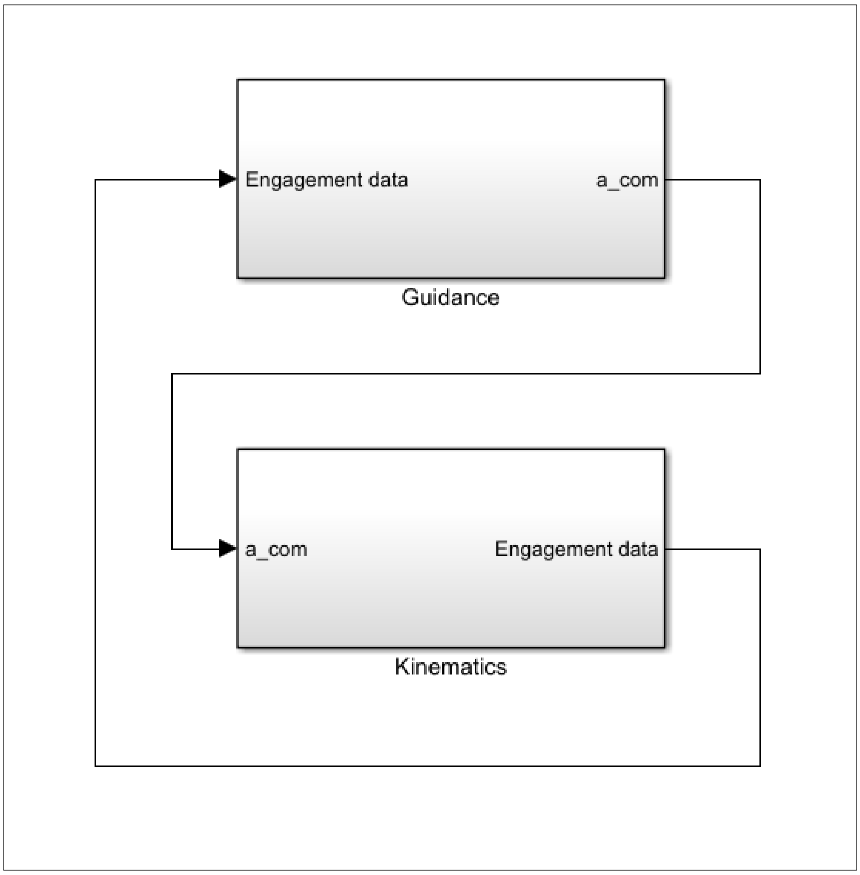

Figure 5.1. Simulation model

Guidance block consists of calculations of proposed guidance law. Engagement parameters acquired by the missile V_M, V_M, θ_M, θ_M, λ, $\dot{\lambda}$, R and $\dot{R}$ are fed from the Kinematics block. Desired impact LOS angle λ_{des} is calculated online through the flight. Power spectral density of the measurement noise on the LOS angle is considered constant. Thus in the simulation, the noise is modeled as white noise. The measurement LOS angle is obtained by adding Gaussian distributed random numbers on the exact value of the LOS angle. From the measurement LOS angle λ, estimated LOS angle $\hat{\lambda}$ and LOS angular rate $\hat{\dot{\lambda}}$ are calculated with sliding mode differentiator. λ_{des} and $\hat{\lambda}$ are used in sliding variable calculations. Estimated parameters and other known engagement parameters ensure generation of guidance command a_{com} in the guidance process.

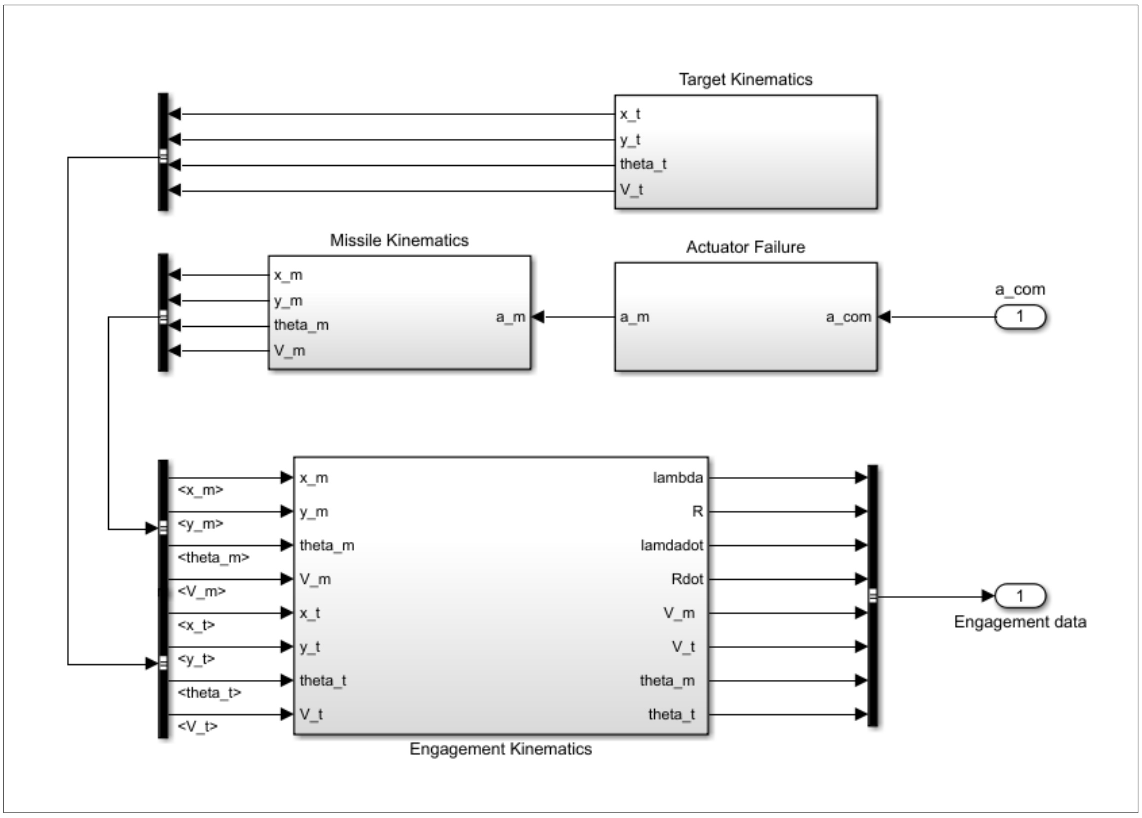

Figure 5.2. Simulation model (Kinematics block)

Kinematics block comprises actuator failure, missile kinematics, target kinematics and engagement kinematic sub-blocks. Guidance command a_{com} calculated in the Guidance block is manipulated with actuator failure model in the related sub-block and actual missile normal acceleration a_{M} is calculated. Missile and target position, velocity and angle information expressed in Local Cartesian Frame are computed in Missile and Target Kinematics sub-blocks. Individual data of missile and target are used in calculations of relative terms λ, $\dot{\lambda}$, R and $\dot{R}$ within the Engagement Kinematics block. Thus, the engagement kinematics parameters are produced and delivered to the Guidance block.

Only the terminal guidance phase is considered along the simulations. Therefore, guidance process starts immediately after the simulation starts. In real world, applicable normal accelerations by the missile are limited due to aerodynamic performance or structural strength of the missile. Thus in addition to the LOS rate estimator integrated in the guidance algorithm model, 100 m/s^2 saturation limit on guidance command is used in the simulations. In order to prohibit divergence of the guidance commands at final time of the flight, the latest guidance command is used if the range is no more than 5 m. Also it is accepted that, initial LOS and its time derivative are already estimated at the beginning of terminal guidance, since the discussed SMD structure is able start the estimation before the terminal phase of the flight.

5.2. SLIDING-MODE GUIDANCE LAW WITHOUT LOS RATE ESTIMATION

In this simulation study, LOS angular rate and LOS angle are assumed to be directly accessible by the missile. Effect of actuator failures and target motion are discussed in three different scenarios. To prove the robustness of the SMGL on achieving desired impact angle, separate impact angle goals are considered in the first three scenarios. On the last scenario, different target ranges and motions are studied.

Proper guidance gains k_1, k_2 and k_3 are chosen, in order to suppress the impact of bounded disturbances. Success of a scenario depends on miss distance and impact angle values at interception time. Therefore, sliding surface states are significant parameters at the final time interval of the engagement. By considering the state parameters, a cost function is used for choosing optimal sliding surface parameters ε and γ. Sum of the states between the last two seconds of the engagement is decided as the cost function. Thus, minimizing the cost function gives the optimum sliding surface gains for a specific scenario. The scenario is determined for the scenario when both actuator failure and target motion presents. Desired impact angle of $60°$ is chosen. The cost function is stated at Equation (47), where t_f is the final time of the engagement.

$$CF = \int_{t_f-2}^{t_f} s\ dt \qquad (47)$$

Selected sliding surface and guidance gains are presented in Table 5.1:

Table 5.1. Sliding surface and guidance gains

ε	γ	k_1	k_2	k_3
200	0.9	0.5	35	30

5.2.1. Actuator Failure

Performance of the SMGL is analyzed under actuator failures. Additive term a_F and multiplicative term μ are modeled as sinusoidal signals, in order to observe the guidance law robustness against time variant disturbance. Disturbance terms are bounded in between sinusoidal signal amplitude. Target is designated as a stationary target. Input parameters of the scenario are given at Table 5.2:

Table 5.2. Scenario inputs

Missile's initial position [x y] (m)	[0 600]
Missile's initial flight path angle (deg)	0
Missile velocity (m/s)	250
Target's initial position [x y] (m)	[1500 10]
Target's initial orientation angle (deg)	0
Target velocity (m/s)	0
Target normal acceleration (m/s^2)	0
Multiplicative uncertainty and disturbance term	1 - 0.15sin(t)
Additive uncertainty and disturbance term	20sin(t)

Table 5.3 demonstrates the desired impact angles of the cases:

Table 5.3. Desired impact angles of the cases

Case - 1	Desired impact angle = $\mathbf{20}$°
Case - 2	Desired impact angle = $\mathbf{40}$°
Case - 3	Desired impact angle = $\mathbf{60}$°

In Figures 5.3-5.8 results are presented:

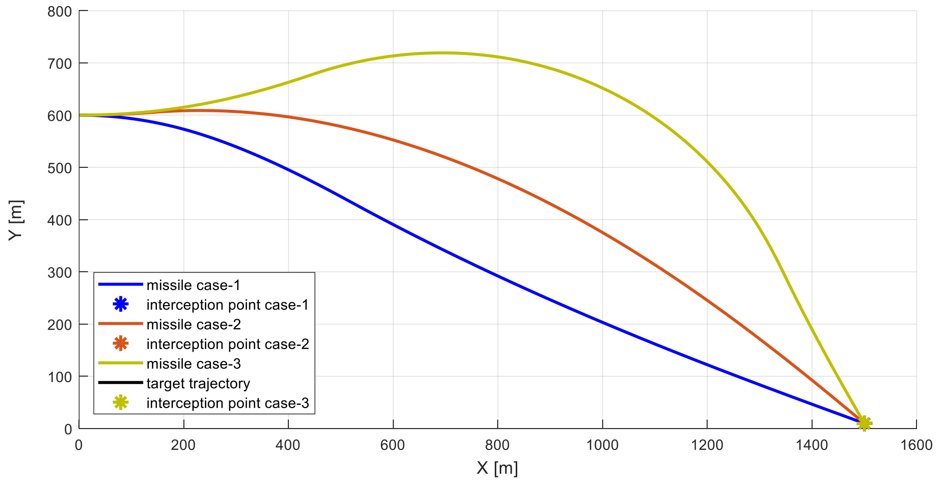

Figure 5.3. Missile and target trajectories

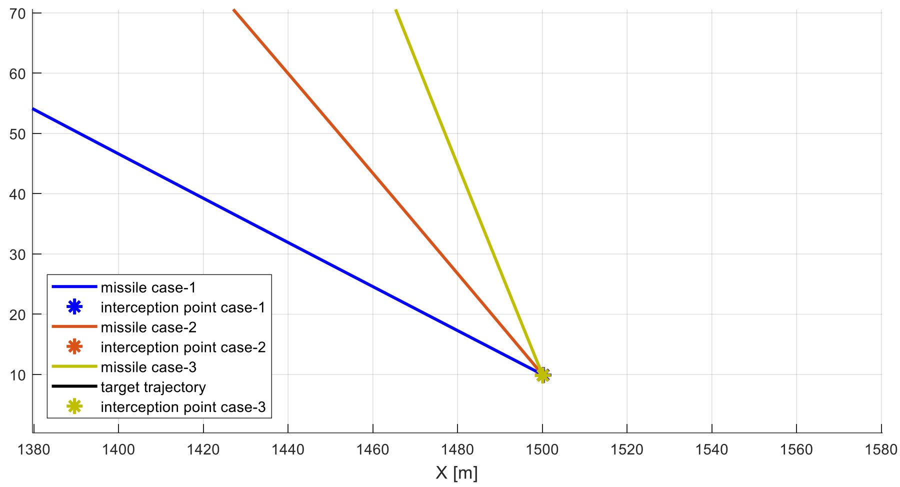

Figure 5.4. Missile and target trajectories (zoomed)

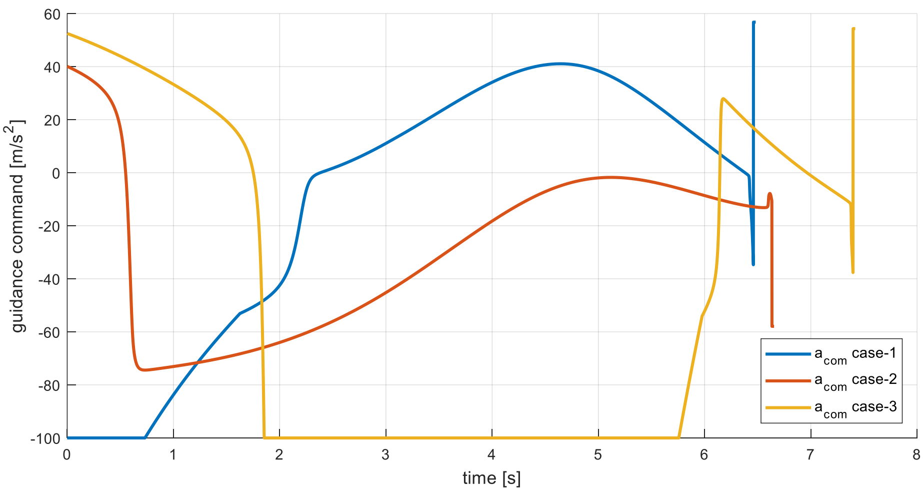

Figure 5.5. Guidance commands

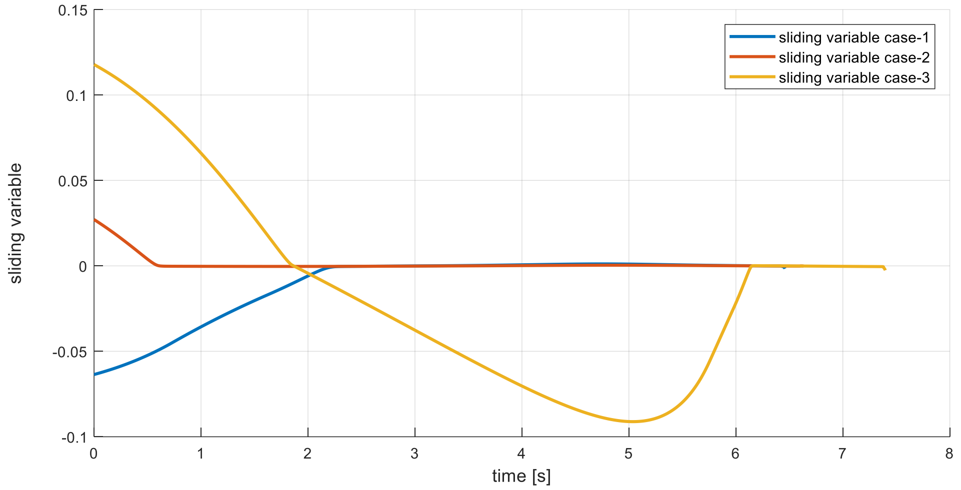

Figure 5.6. Sliding variable

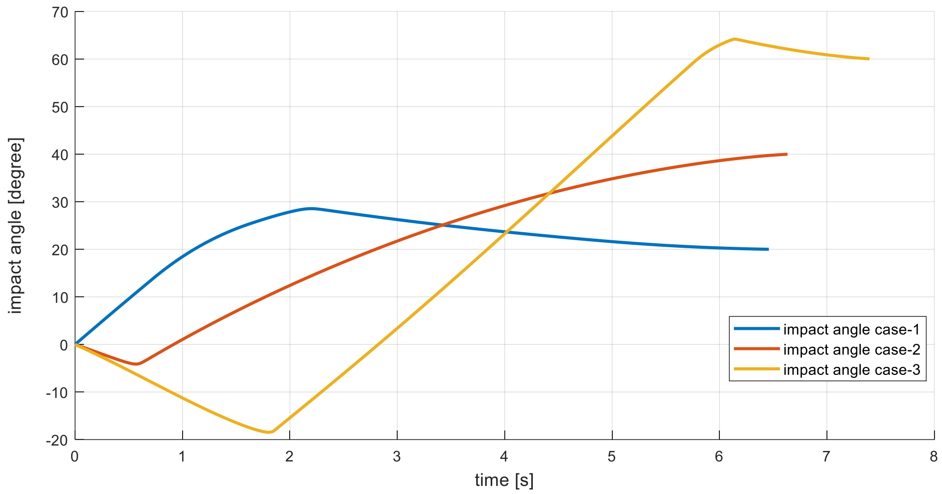

Figure 5.7. Impact angles

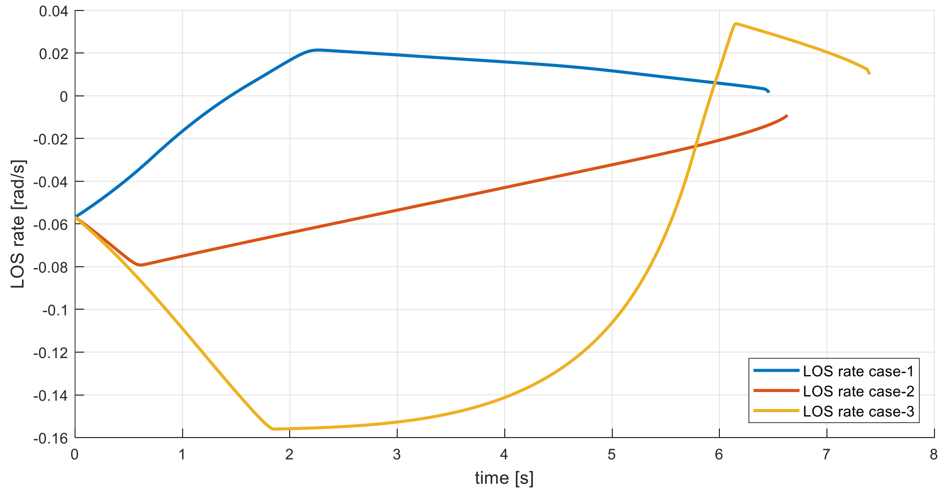

Figure 5.8. LOS angular rates

Resultant impact angle errors and miss distances are tabulated at Table 5.4:

Table 5.4. Impact angle errors and miss distances

	Miss distances [m]	Impact angle errors [deg]
Case - 1	0.160	0.106
Case - 2	0.084	0.360
Case - 3	0.057	0.186

As seen from the Table 5.4, targets are successfully hit with considerably low impact angle error values and miss distance. From Figure 5.3, variations between smooth missile trajectories are observed. Missile gains more altitude if the desired impact angle is higher. As desired impact angle value increases, negative signed acceleration command magnitude becomes larger. Difference between the initial conditions of the sliding variables are originated from desired impact angles, since other scenario inputs are identical in three cases. Before the final time of the interception, sliding variables converges to zero in Figure 5.6. Sliding variable in case-3 converges to zero much later than other cases. By looking guidance command and sliding variable data in Figures 5.5 and 5.4, it can be said that case-3 is more compelling scenario. Sliding variable does not directly converge to zero and higher guidance commands are observed for case-3. Despite the fact that, generated acceleration command is limited in great section of the flight in Figure 5.5, missile achieves high impact angle with low miss distance. Impact angle curves in Figure 5.7 shows that; desired impact angle is achieved at the final time of the flight. In fig. LOS angular rates in Figure 5.8 approaches to zero before the final interception time.

In this scenario, there are discrepancies between the calculated guidance commands and actuated acceleration commands, because actuator failure is included through the flight. Guidance commands and actual missile accelerations are presented in Figures 5.9-5.11:

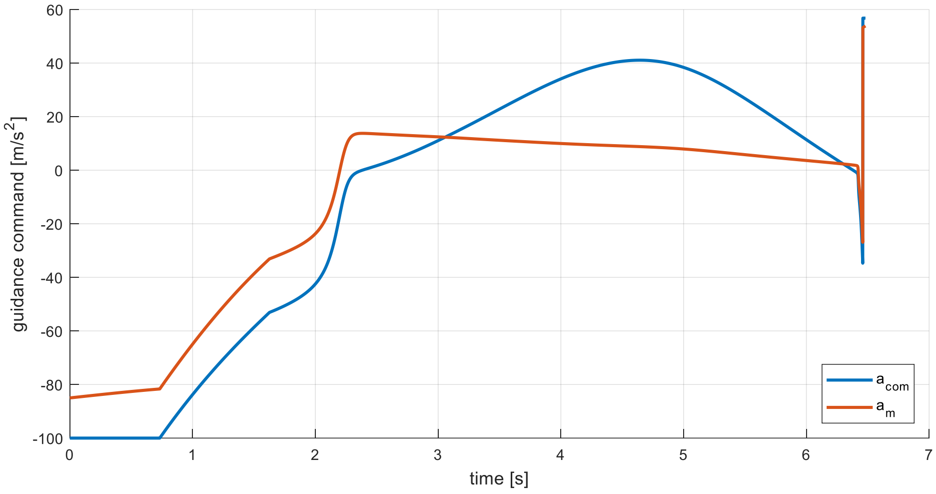

Figure 5.9. Actual missile acceleration and guidance command (case-1)

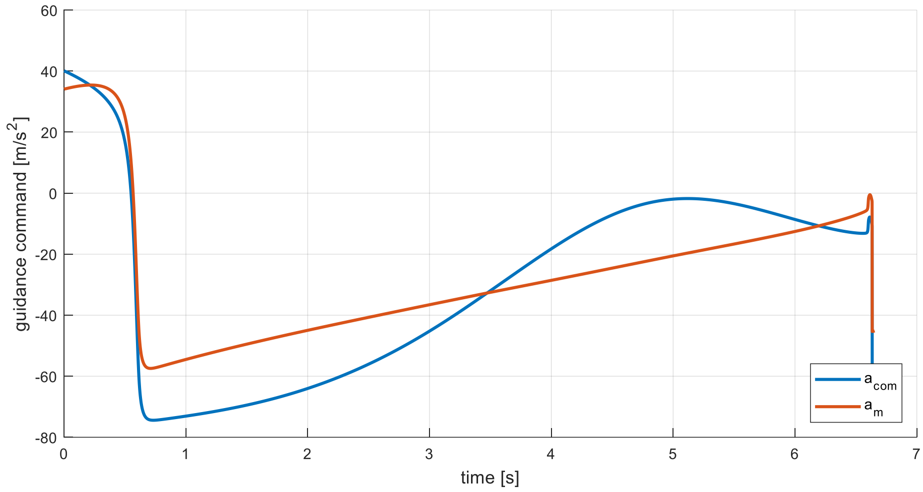

Figure 5.10. Actual missile acceleration and guidance command (case-2)

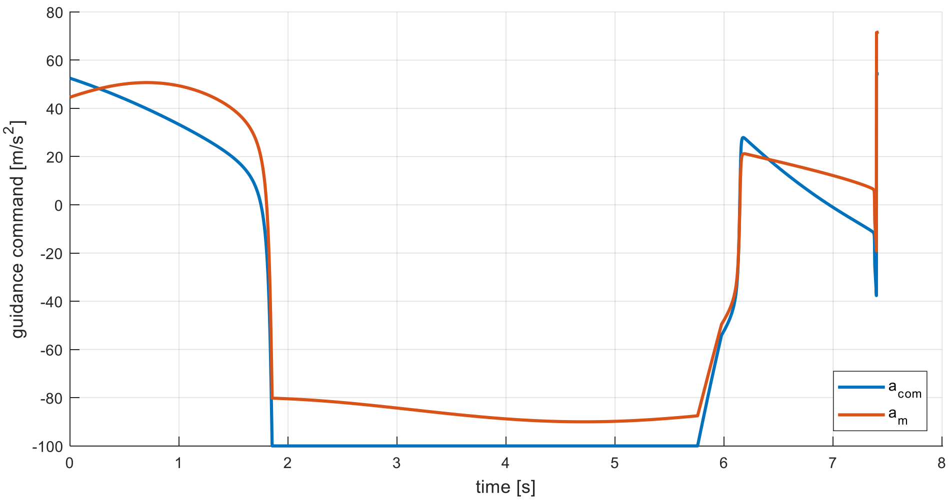

Figure 5.11. Actual missile acceleration and guidance command (case-3)

In Figures 5.9-5.11 it can be seen that, guidance command curves have fluctuating profile because of the sinusoidal actuator failure terms. However, sliding mode guidance law ensures more stable profile on the actual missile acceleration. Fluctuating missile acceleration commands diminishes the effect of the actuator failure.

According to results of this scenario, SMGL is robust under actuator failures. More than one desired impact angles are achievable with high accuracy. Although there exist discrepancies between guidance commands and the actual missile acceleration, the guidance law allows missile to hit the target with high precision.

5.2.2. Maneuvering Target

In this scenario engagement geometry includes maneuvering target. Target has time variant bounded normal acceleration. Acceleration term is denoted with a sinusoidal signal. In this way, targets motion on a bumpy environment is modeled. Cases are described as same in Table 5.4. Table 5.5 demonstrates the scenario inputs:

Table 5.5. Scenario input parameters

Missile’s initial position [x y] (m)	[0 600]
Missile’s initial flight path angle (deg)	0
Missile velocity (m/s)	250
Target’s initial position [x y] (m)	[1500 10]
Target’s initial orientation angle (deg)	0
Target velocity (m/s)	25
Target normal acceleration (m/s^2)	2sin(t)
Multiplicative uncertainty and disturbance term	0
Additive uncertainty and disturbance term	0

In Figures 5.12-5.17 simulation results are shown:

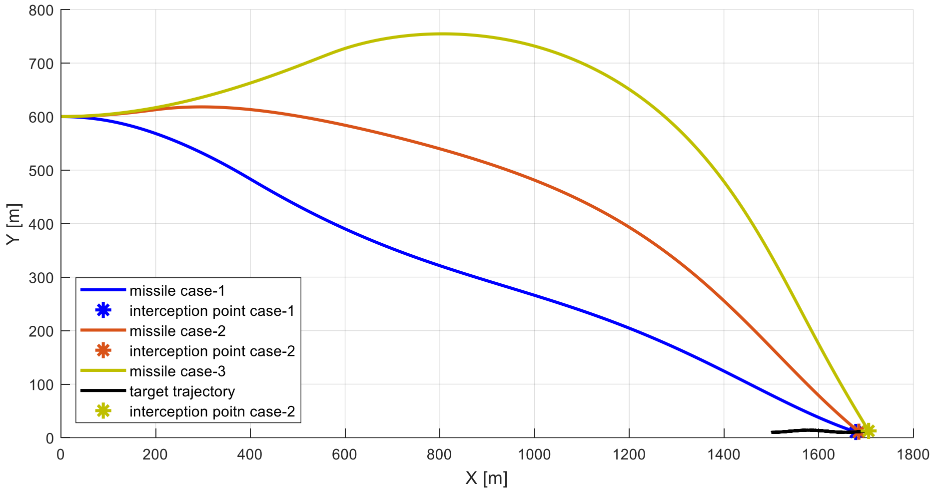

Figure 5.12. Missile and target trajectories

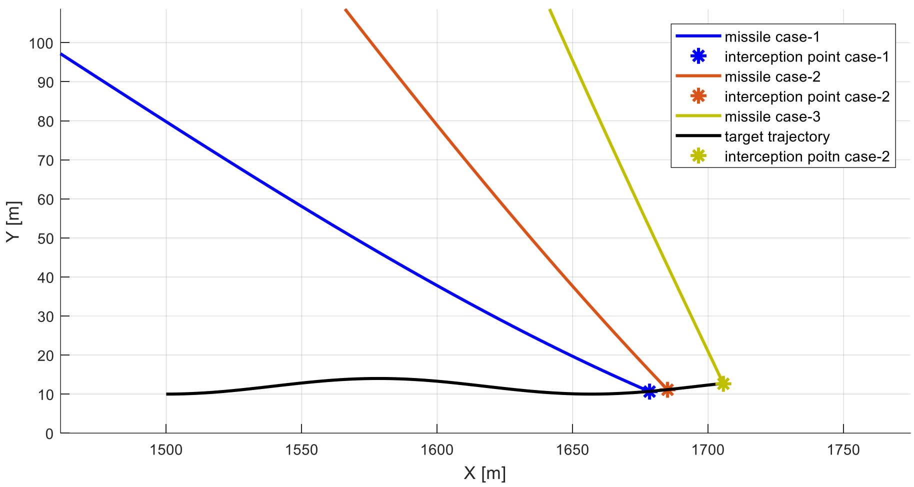

Figure 5.13. Missile and target trajectories (zoomed)

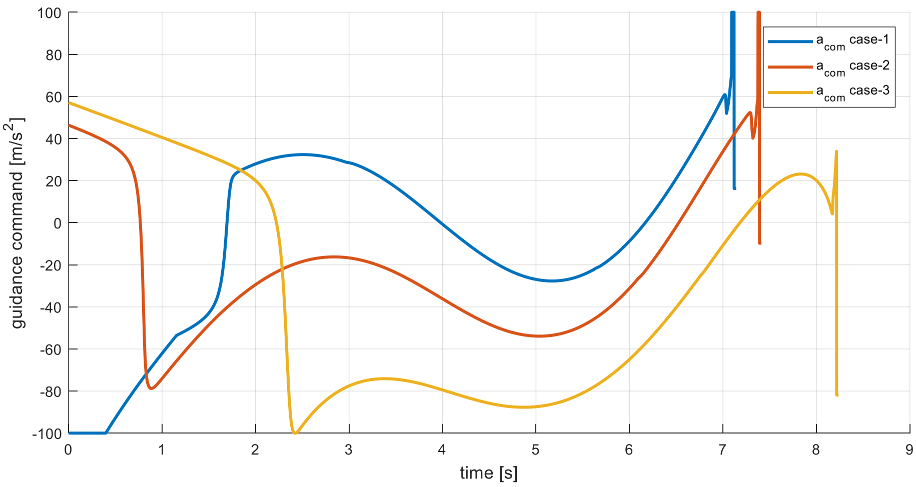

Figure 5.14. Guidance commands

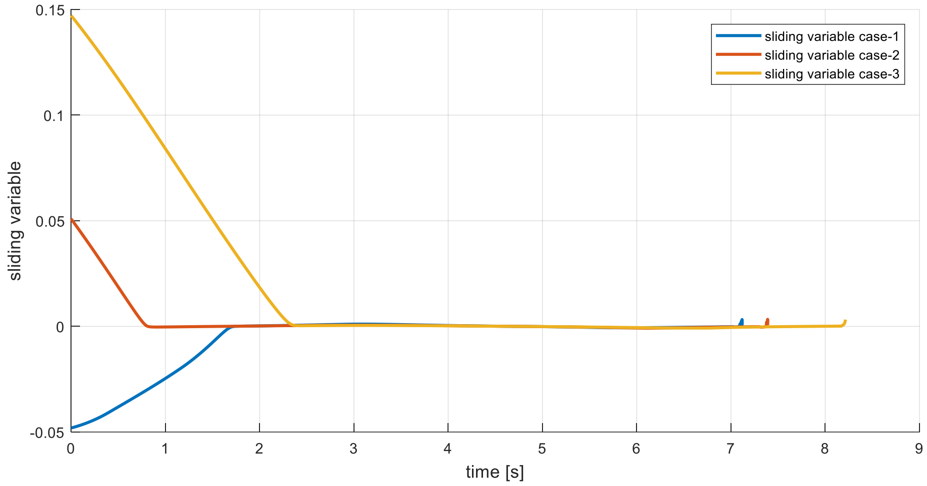

Figure 5.15. Sliding variables

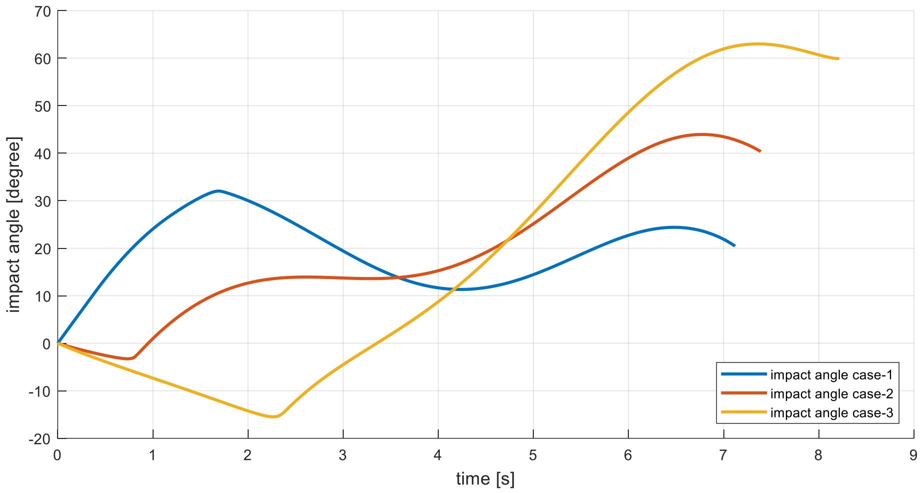

Figure 5.16. Impact angles

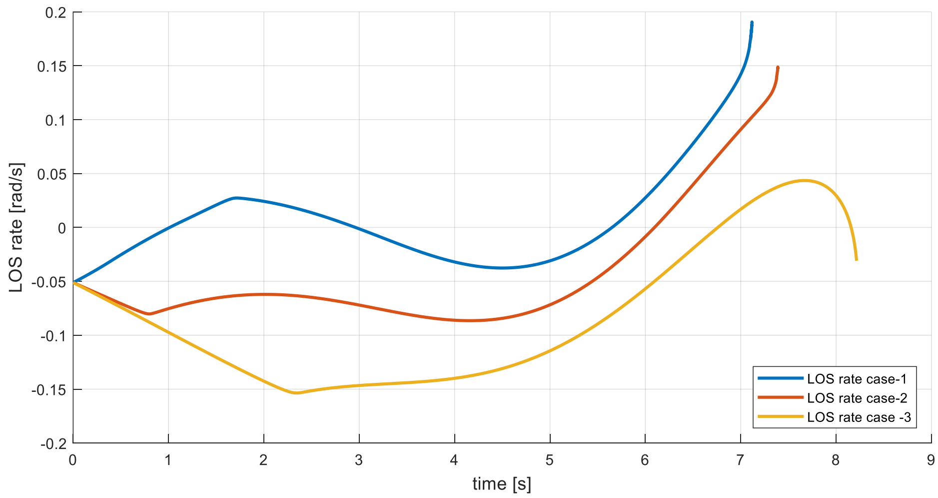

Figure 5.17. LOS angular rates

Impact angle errors and miss distance and are tabulated at Table 5.6:

Table 5.6. Impact angle errors and miss distance and errors

	Miss distances [m]	Impact angle errors [deg]
Case - 1	0.109	0.244
Case - 2	0.014	0.251
Case - 3	0.053	0.210

From Figure 5.12 it can be seen that, trajectory of missile is shaped with respect to the desired impact angle. Maximum altitude gained and time of flight increases with the value of the desired impact angle. In Figure 5.14, larger acceleration commands are produced, in order to achieve higher impact angles. If the sliding variable behavior in Figure 5.15 is compared with Figure 5.6 in the previous scenario, one can say that the sliding variable diminishes quicker in this scenario. Hereby actuator failure produces more dominant effect on the sliding variable than target maneuver for this scenario. From Table 5.6 and Figure 5.16 it can be observed that, desired impact angles are achieved with low errors. Motion of the target tends to diverge the LOS angular rates at Figure 5.17. From the kinematic relations if the range decreases, relative angular position changes rapidly. Even though LOS rate begins to diverge before the hit time, missile hits the target accurately. To conclude, SMGL is robust against unknown target acceleration and it is possible to hit target with various impact angles.

5.2.3. Maneuvering Target with Actuator Failures

After investigating performance of the SMGL on actuator failures and target maneuver separately, evaluating the performance in much harder scenario solidifies the robustness of the guidance law. In this scenario, both target acceleration and actuator failure disturbances are considered simultaneously. Three cases are considered, which are stated at Table 5.3. Scenario input parameters are shown at table:

Table 5.7. Scenario input parameters

Missile's initial position [x y] (m)	[0 600]
Missile's initial flight path angle (deg)	0
Missile velocity (m/s)	250
Target's initial position [x y] (m)	[1500 10]
Target's initial orientation angle (deg)	0
Target velocity (m/s)	25
Target normal acceleration (m/s^2)	2sin(t)
Multiplicative uncertainty and disturbance term	1 - 0.15sin(t)
Additive uncertainty and disturbance term	20sin(t)

Resultant simulation outputs are presented in Figures 5.18-5.23:

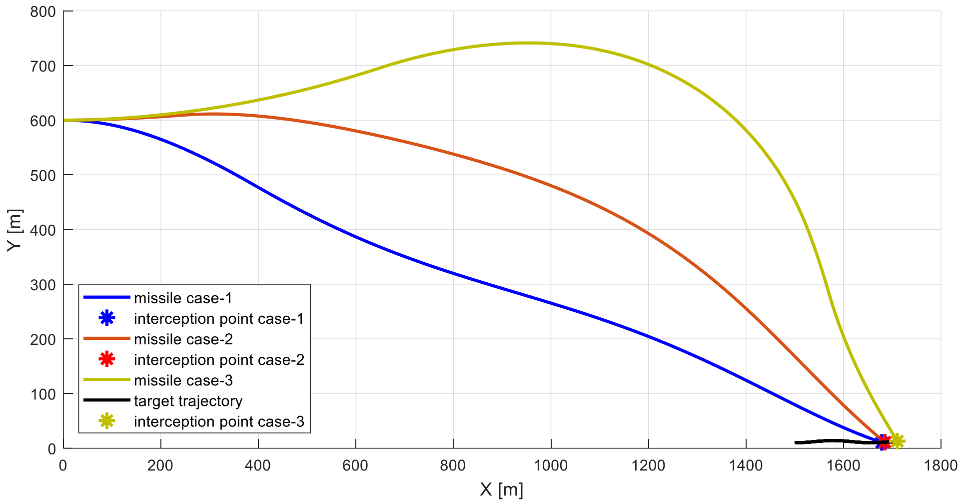

Figure 5.18. Missile and target trajectories

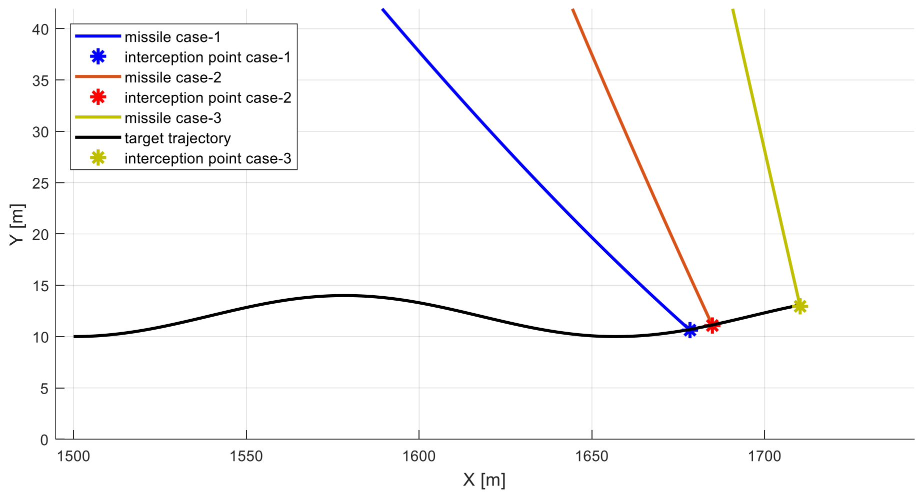

Figure 5.19. Missile and target trajectories (zoomed)

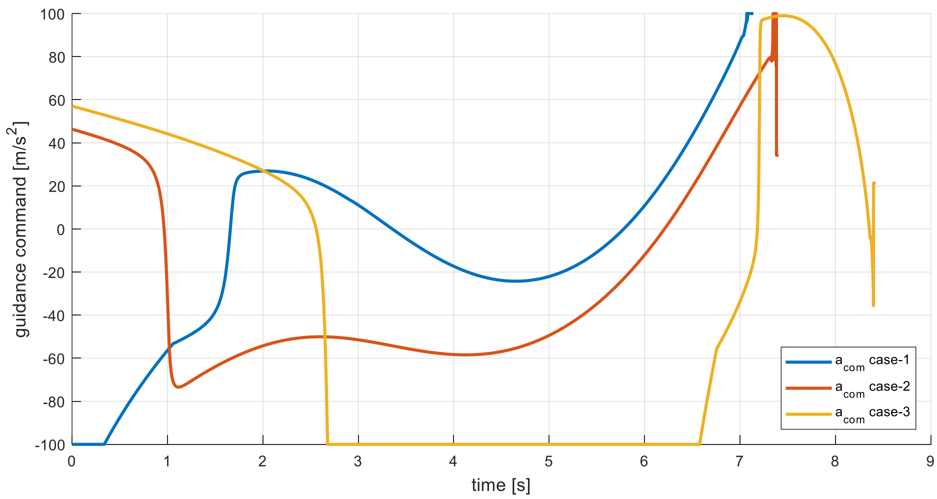

Figure 5.20. Guidance commands

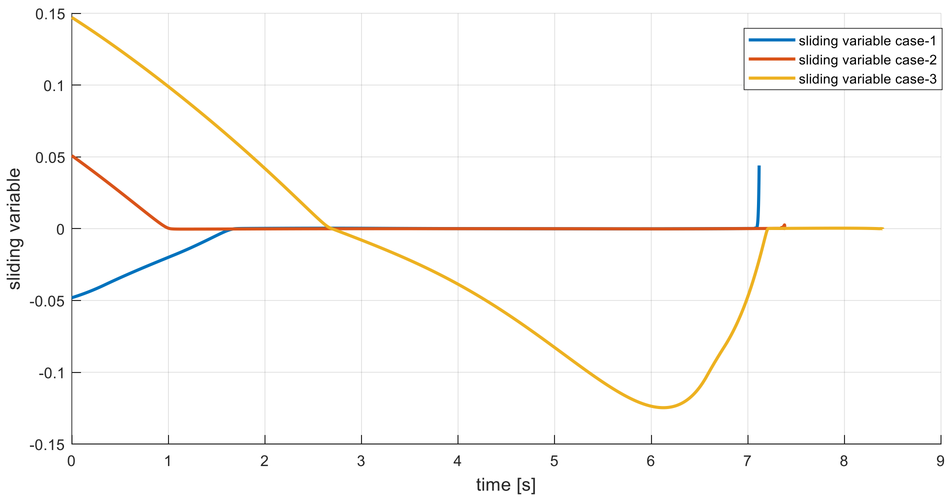

Figure 5.21. Sliding variables

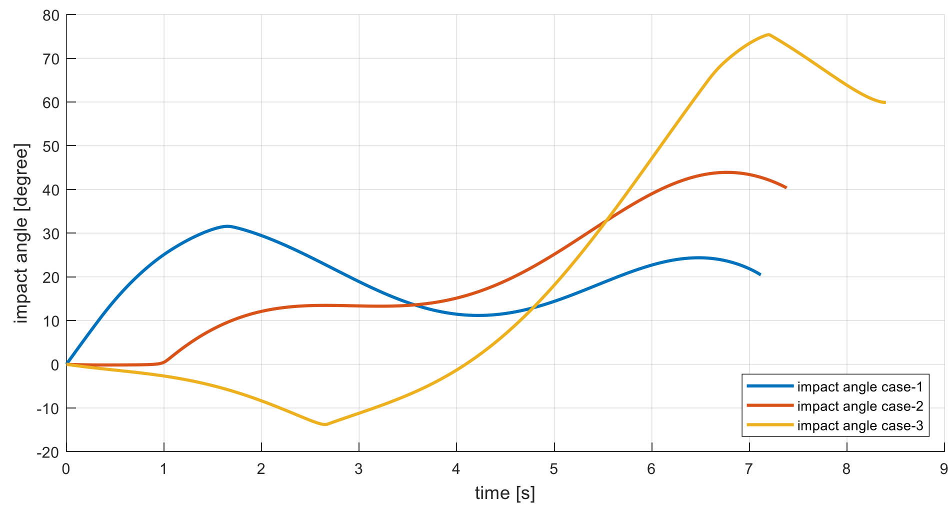

Figure 5.22. Impact angles

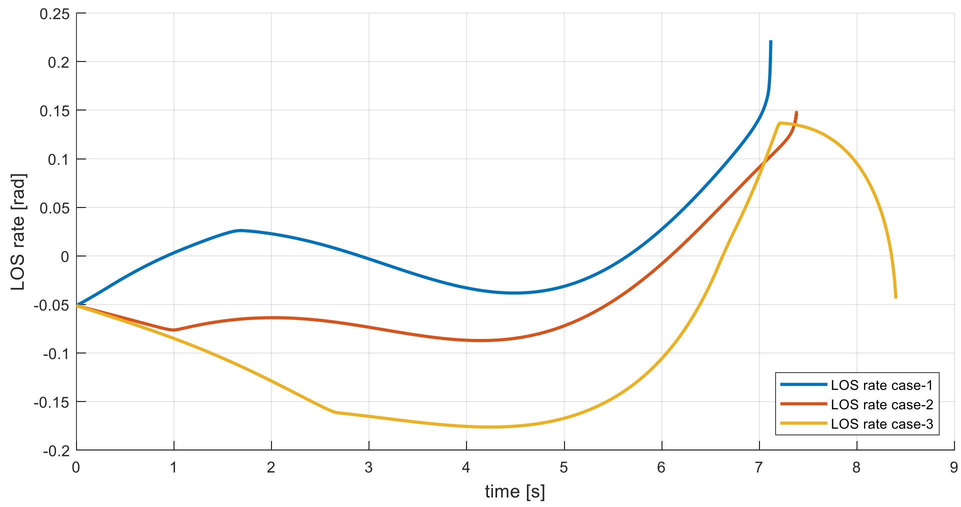

Figure 5.23. LOS angular rates

Impact angle errors and miss distances are tabulated at Table 5.8:

Table 5.8. Impact angle errors and miss distances

	Miss distances [m]	Impact angle errors [deg]
Case - 1	0.020	0.488
Case - 2	0.036	0.242
Case - 3	0.079	0.318

By looking at Figure 5.18 similar flight trajectories with previous scenarios are observed. In Figure 5.20 the guidance command at case-3 hit the limit as it appeared in the first scenario. If the sliding variable of case-3 in Figure 5.21 is compared with the first scenario, sliding variable gets further away from the sliding surface in this scenario. Existence of both actuator and target maneuver obstructs convergence process of the sliding variable. LOS angular rates in Figure 5.23 diverges towards to interception time. Nevertheless, the sliding mode guidance law preserves its precision with low impact angle errors and miss distances as seen in Table 5.8 and Figure 5.22. Results of this scenario consolidates the robustness property of the guidance law.

5.2.4. Effect of Range and Receding Target

In this scenario, SMGL performance is evaluated on various target ranges. Five distinct target ranges are considered. Target moves away from the missile as it is in previous scenarios. Both actuator failure and target maneuvers are included in the engagement geometry. The scenario inputs are shown in Table 5.9.

Table 5.9. Scenario input parameters

Missile's initial position [x y] (m)	[0 600]
Missile's initial flight path angle (deg)	0
Missile velocity (m/s)	250
Target's initial position [x y] (m)	40
Target's initial orientation angle (deg)	0
Target velocity (m/s)	25
Target normal acceleration (m/s^2)	2sin(t)
Multiplicative uncertainty and disturbance term	1 - 0.15sin(t)
Additive uncertainty and disturbance term	20sin(t)

In Table 5.10, initial conditions are presented for different cases:

Table 5.10. Initial conditions of the cases

Case - 1	Initial target position [x y] = [1000 10] m
Case - 2	Initial target position [x y] = [1500 10] m
Case - 3	Initial target position [x y] = [2000 10] m
Case - 4	Initial target position [x y] = [2500 10] m
Case - 5	Initial target position [x y] = [3000 10] m

Resultant simulation outputs are presented in Figures 5.24-5.28:

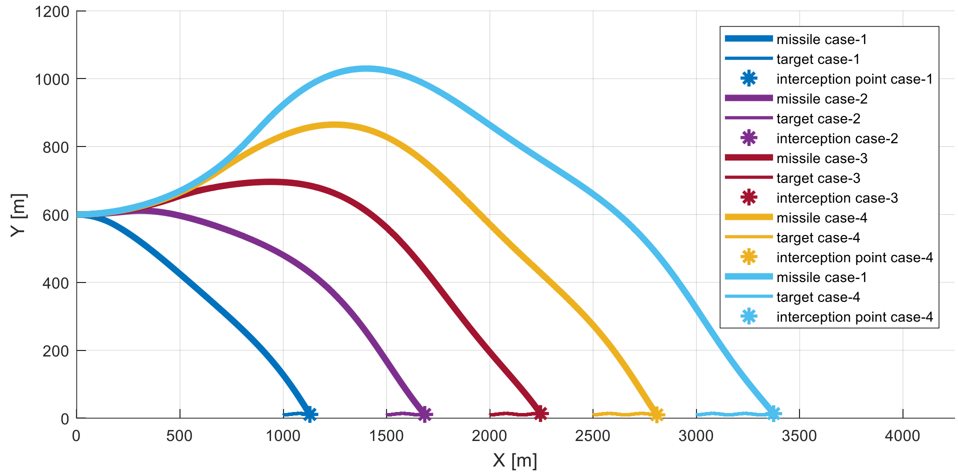

Figure 5.24. Missile and target trajectories

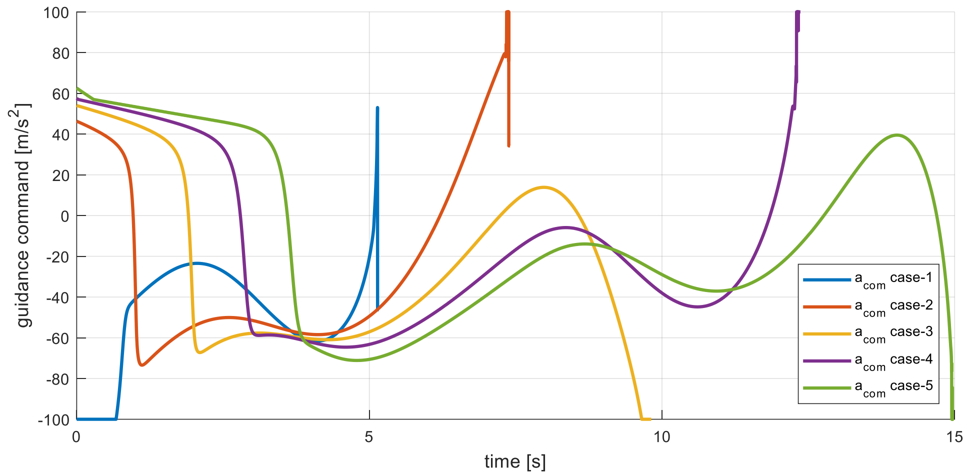

Figure 5.25. Guidance commands

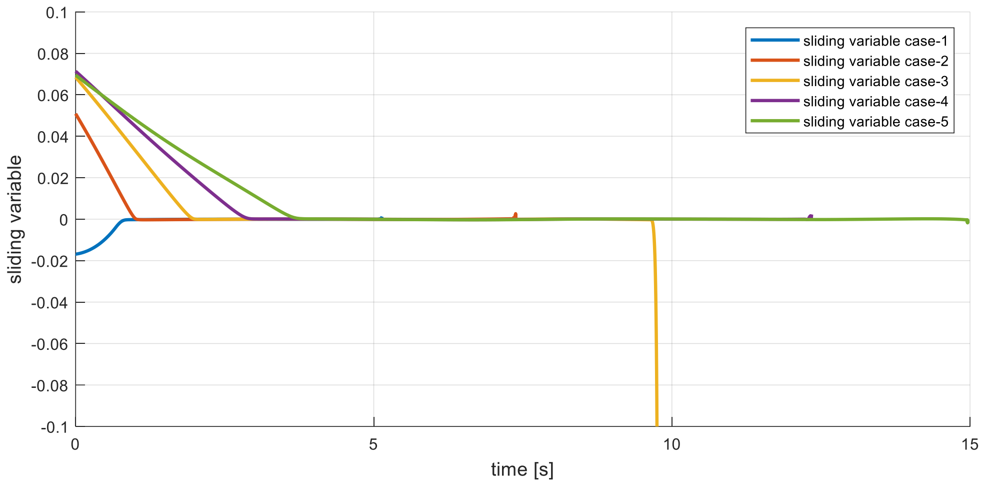

Figure 5.26. Sliding variables

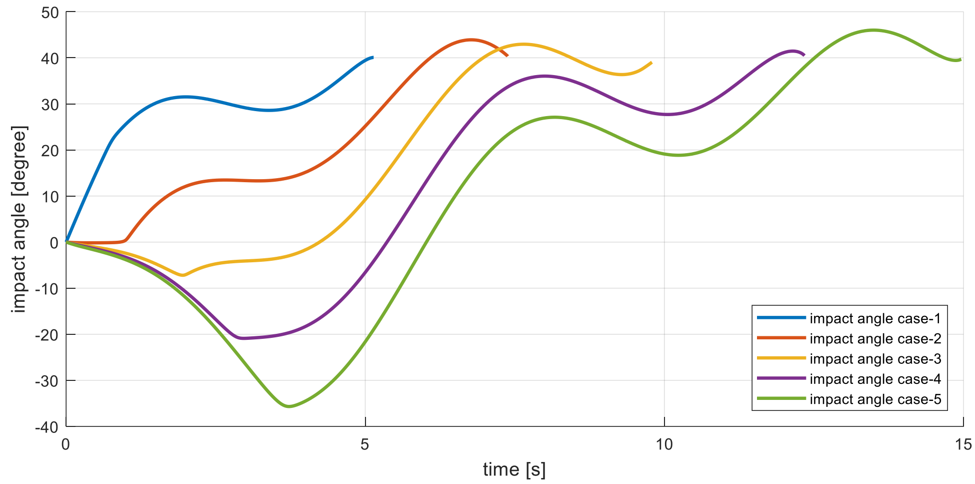

Figure 5.27. Impact angles

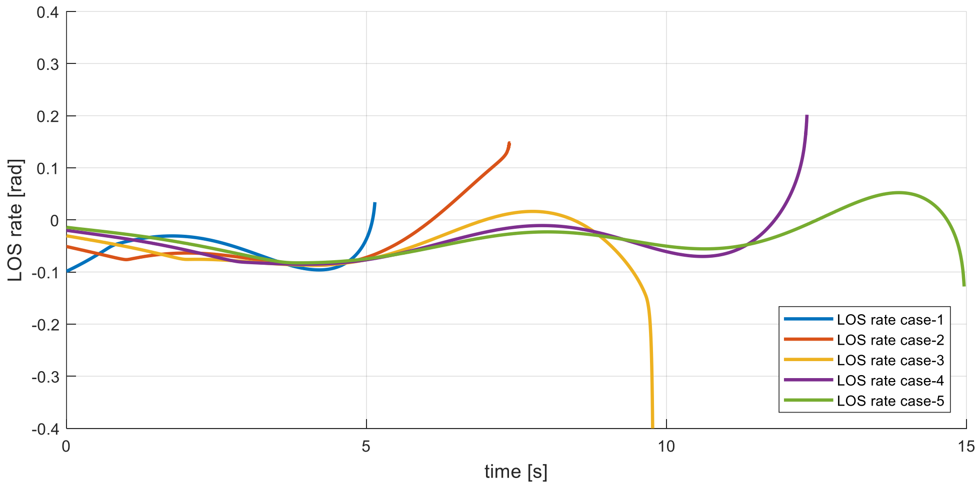

Figure 5.28. LOS angular rates

Impact angle errors and miss distance are tabulated at Table 5.11:

Table 5.11. Impact angle errors and miss distance and

	Miss distances [m]	Impact angle errors [deg]
Case - 1	0.017	0.077
Case - 2	0.012	0.365
Case - 3	0.027	0.960
Case - 4	0.079	0.471
Case - 5	0.082	0.298

From Figure 5.24, maximum altitude gained by the missile rises as the initial range between missile and target increases. Figure 5.25 illustrates proper guidance commands for different ranges. Guidance command is only limited beginning of the terminal guidance in case-1. As seen in Figure 5.27, desired impact angles are achieved. LOS angular rates tend to diverge at the end of the flight, in Figure 5.28. However, magnitudes of the LOS angular rates are kept sufficiently low and precision on interceptions is ensured. Table 5.13 presents the accuracy of the SMGL with sufficiently low miss distances and impact angles.

5.2.5. Effect of Range and Approaching Target

In this scenario target approaches to the missile direction. Approaching target scenarios expedites the engagement kinematics. If the controller is not capable of responding against faster kinematics, sliding variable may not converge rapidly. Thus, it is important to check sliding mode performance against faster kinematics. Similar with previous scenario, five different target ranges are considered along with actuator failure and target maneuver. Cases of the scenarios are chosen from Table 5.10. Scenario inputsare shown in Table 5.12:

Table 5.12. Scenario input parameters

Missile's initial position [x y] (m)	[0 600]
Missile's initial flight path angle (deg)	0
Missile velocity (m/s)	250
Desired impact angle (deg)	40
Initial target orientation angle (deg)	0
Target velocity (m/s)	-25
Target normal acceleration (m/s^2)	2sin(t)
Multiplicative uncertainty and disturbance term	1 - 0.15sin(t)
Additive uncertainty and disturbance term	20sin(t)

The resultant scenario outputs are demonstrated in Figures 5.29-5.32:

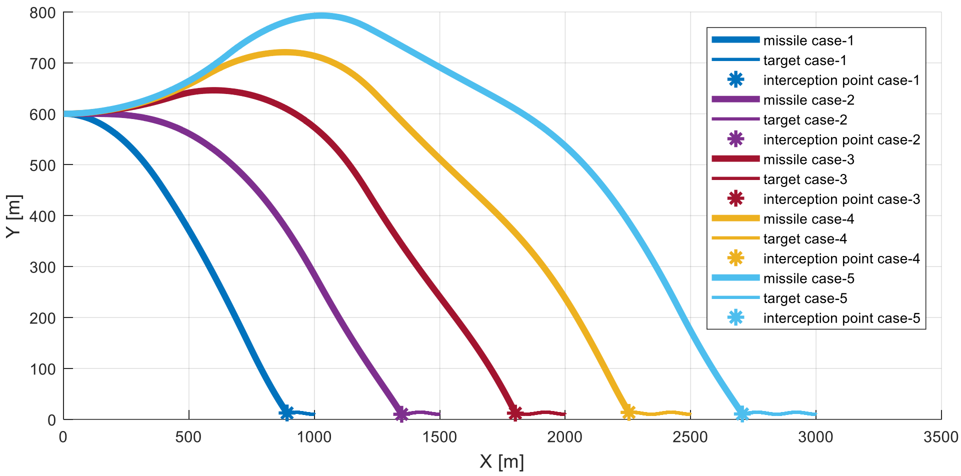

Figure 5.29. Missile and target trajectories

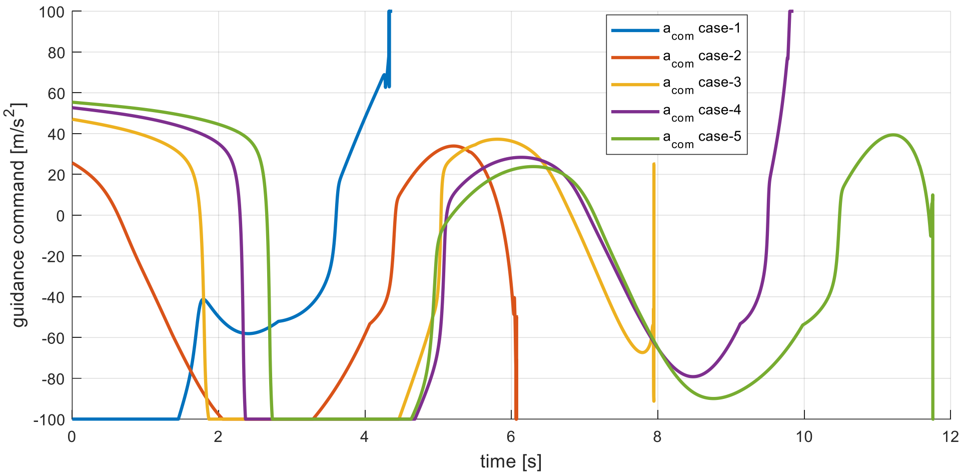

Figure 5.30. Guidance commands

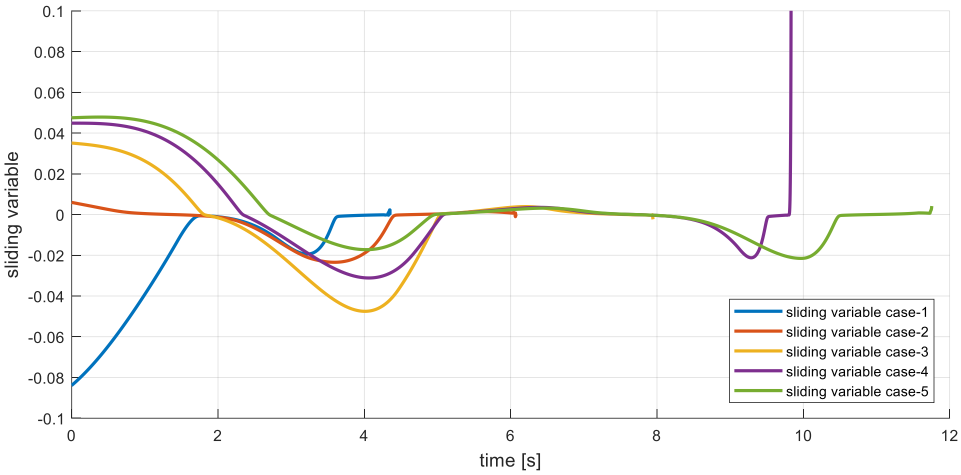

Figure 5.31. Sliding mode variables

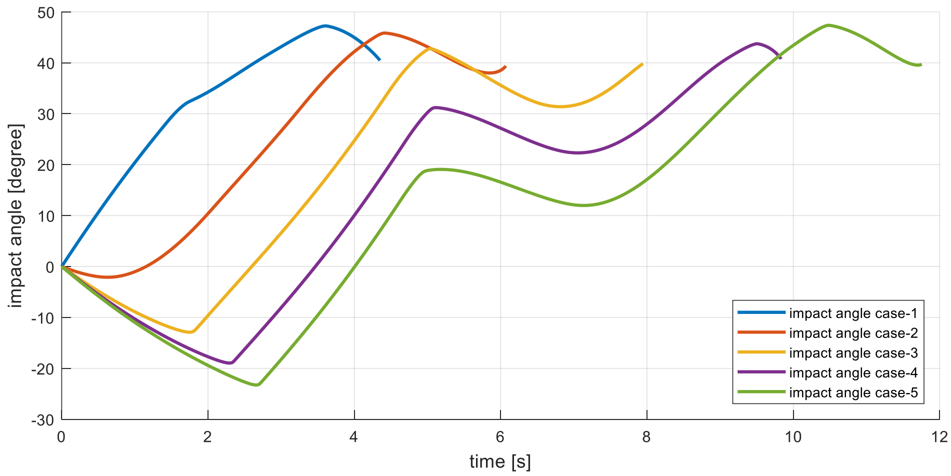

Figure 5.32. Impact angles

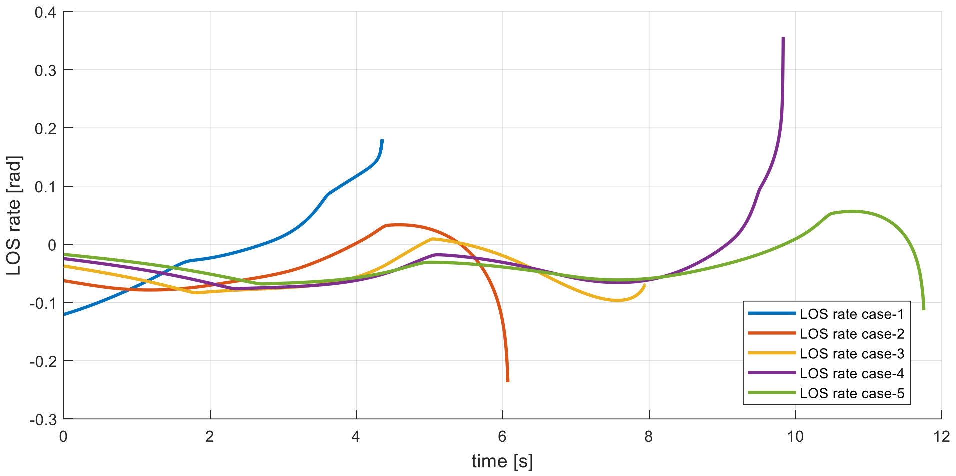

Figure 5.33. LOS angular rates

Impact angle errors and miss distance and are tabulated at Table 5.13:

Table 5.13. Impact angle errors and miss distances

	Miss distances [m]	Impact angle errors [deg]
Case - 1	0.033	0.472
Case - 2	0.128	0.688
Case - 3	0.027	0.175
Case - 4	0.079	0.762
Case - 5	0.082	0.287

In Figure 5.28, maximum altitudes gained are lower than the previous scenario, because target approaches to the missiles direction and range decreases faster. By looking Figures 5.29 and 5.30, it is clear that approaching target scenario is harder than receding target scenario. Since the engagement kinematics are faster, missile needs larger guidance commands. In all cases, guidance commands are limited in a time interval. From Figure 5.31 it can be seen that, fluctuations of sliding variables are increased with respect to receding target case. In case-4 and case-5 sudden fluctuations are observed, just before the impact time. In Figure 5.33, LOS rate diverges with higher magnitudes. Nevertheless, impact angle error and miss distance values are still acceptable, as seen from Figure 5.32 and Table 5.13.

5.3. SLIDING-MODE GUIDANCE LAW WITH LOS RATE ESTIMATION

In previous scenarios, the SMGL performance is examined under disturbance and uncertainties against a maneuvering target. By including noise on measurement data LOS angle, effect of noisy data on the system is observed in. Afterwards, intended use of approximate signum function is discussed. Finally, the SMGL performance is analyzed by using Monte Carlo method. The sliding mode differentiator parameters are optimized in order to achieve minimum estimation error through the flight. The cost function to be minimized for parameter optimization is described in Equation (48):

$$CF = \int_0^{t_f} (\hat{\dot{\lambda}} - \dot{\lambda})^2 dt \tag{48}$$

5.3.1. Maneuvering Target with Actuator Failures Scenario

In this simulation study, maneuvering target scenario is discussed in absence of LOS rate. Besides the target maneuver, bounded actuator failure effects are also included to test the guidance law performance under tough conditions. Scenario initial conditions are listed in Table 5.13:

Table 5.13. Scenario initial conditions

Missile's initial position [x y] (m):	[0 600]
Missile's initial flight path angle (°):	0
Missile velocity (m/s):	250
Desired impact angle (°):	40
Initial target position [x y] (m):	[1500 10]
Initial target orientation angle (°):	0
Target velocity (m/s):	25
Target normal acceleration (m/s^2):	2sin(t)
Multiplicative uncertainty and disturbance term :	1 - 0.15sin(t)
Additive uncertainty and disturbance term :	20sin(t)

The guidance algorithm parameters are specified at Table 5.14:

Table 5.14. Guidance algorithm parameters

ε	γ	k_1	k_2	k_3	L	M_1	M_2	M_3	T	e	d_1
500	1.25	0.5	35	50	0.02	14	6	5	0.1	0.015	0.01

In figures 5.34-5.39 the outputs of the simulation scenario are shown:

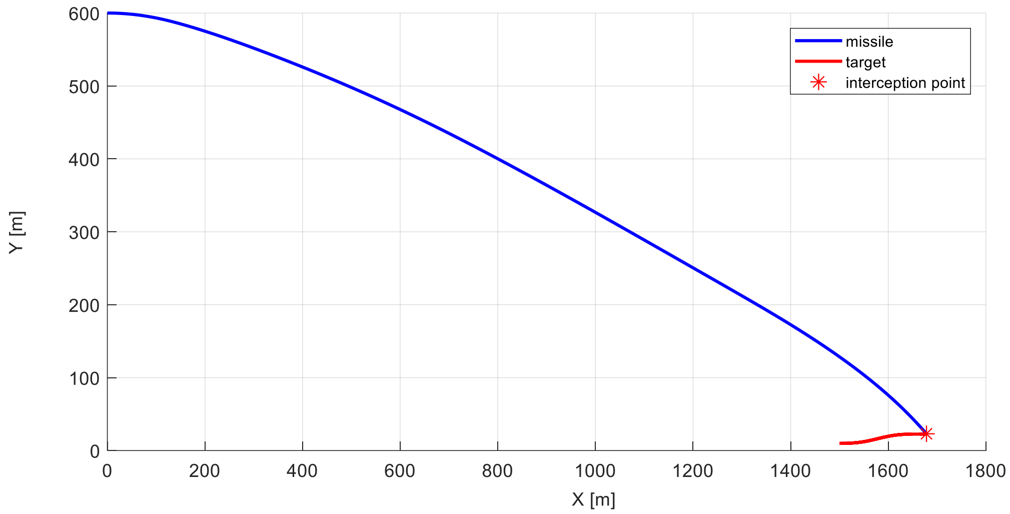

Figure 5.34. Missile and target trajectories

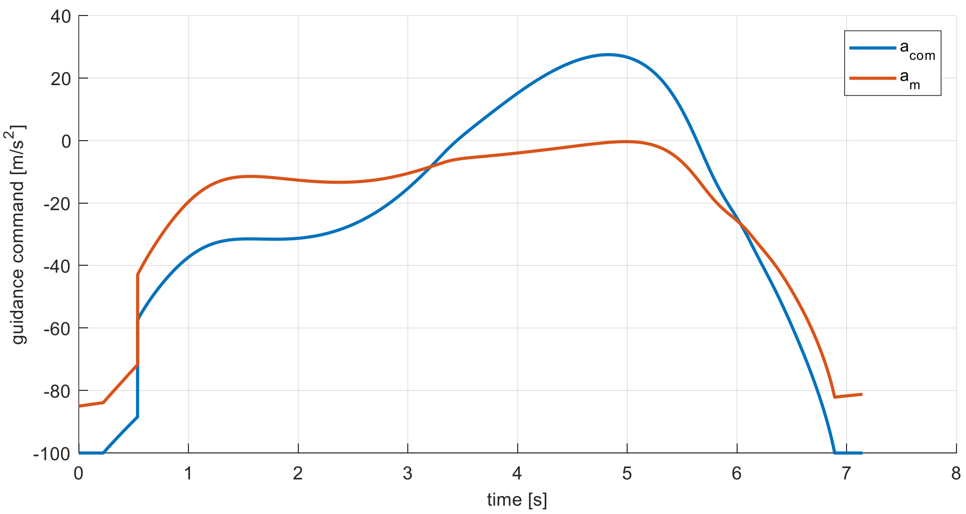

Figure 5.35. Guidance command and actual missile acceleration

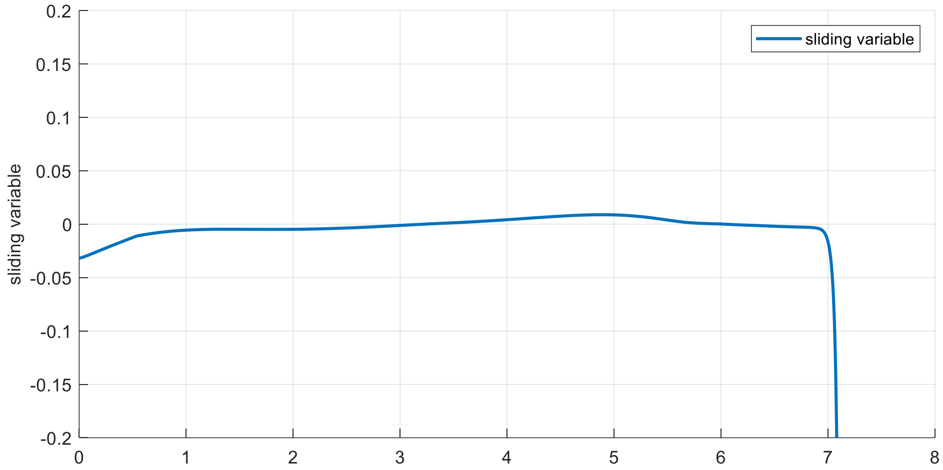

Figure 5.36. Sliding variable

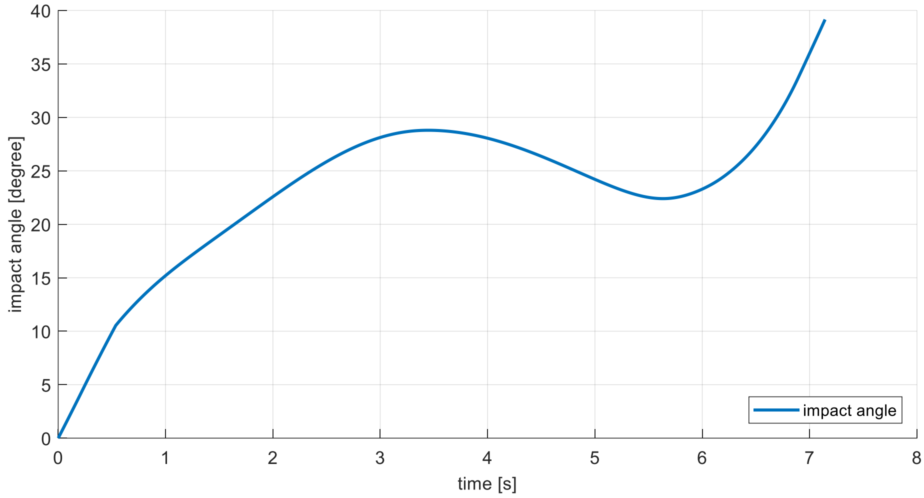

Figure 5.37. Impact angle

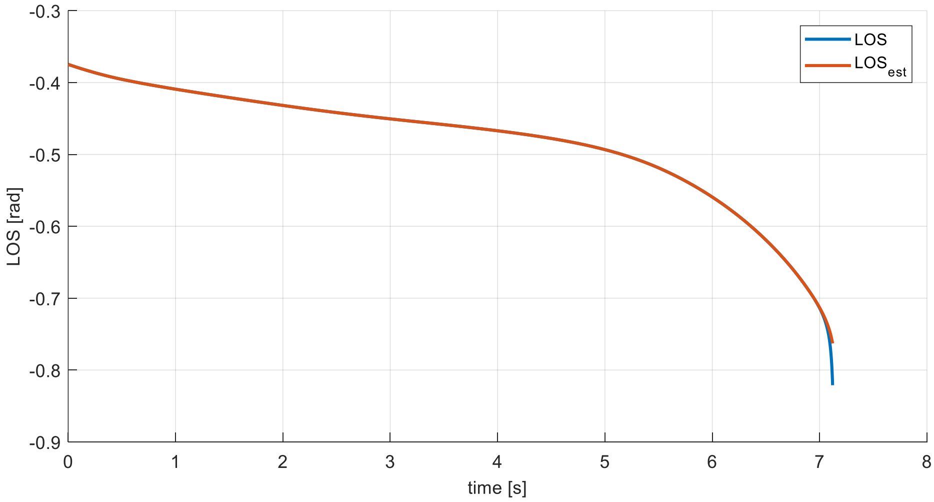

Figure 5.38. LOS angle and estimated LOS angle

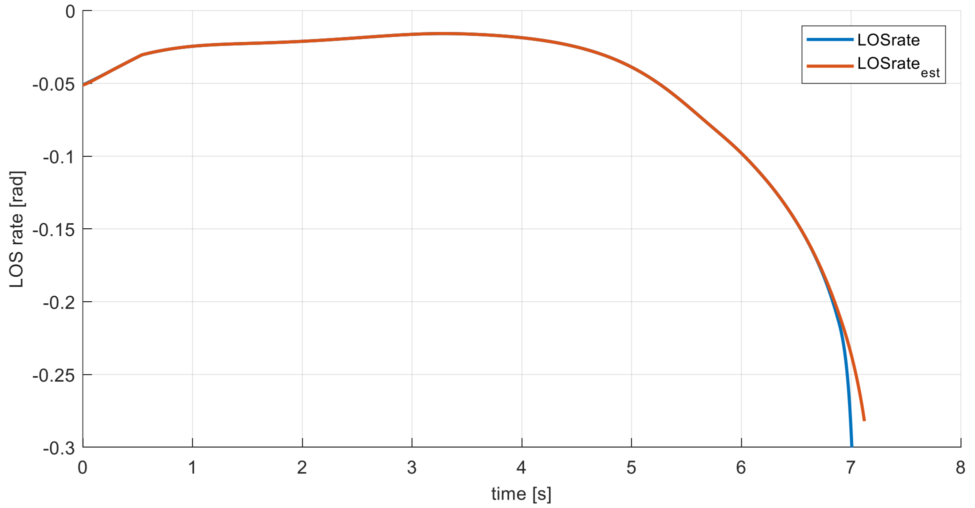

Figure 5.39. LOS rate and estimated LOS rate

As a result of the simulation, miss distance is found as 0.529 m. The missile the desired impact angle with an error of 0.75˚. A smooth trajectory of the missile is illustrated at Figure 5.34. Figure 5.37 shows that; desired impact angle is achieved at final time of the flight. As seen in Figure 5.36, the sliding variable rapidly converges around zero. Since the unknown target acceleration, uncertainty and disturbance terms are changing in time; sliding variable tends to escape from zero. However, the proposed guidance law keeps the sliding variable in a small region around zero. Because sliding variable is away from the zero at the beginning of the scenario, high amounts of guidance command is generated in Figure 5.35. Even though, the actual missile acceleration is saturated as shown in Figure 5.35, the sliding variable converges around zero in a short span of time. Guidance commands fluctuates over the flight as a response against sinusoidal disturbance and uncertainty terms. Since the disturbance and uncertainty effects on missile acceleration are canceled by guidance commands, resultant actual missile acceleration displays more stable profile than the guidance command. The sliding mode differentiator estimates LOS angle and its derivative with perfect performance through the flight, as it can be seen in figures 5.39 and 5.40.

5.3.2. Maneuvering Target with Actuator Failures and Measurement Noise

In order to evaluate the guidance performance under real world conditions, measurement noise on LOS angle is considered in this section. Noise is included on LOS angle as an additive Gaussian distributed parameter of three sigma 0.1˚. The simulation parameters and guidance algorithm gains are taken same as in previous scenario, in the sake of compare two results. The results of the simulation are shown at Figures 5.40-5.45:

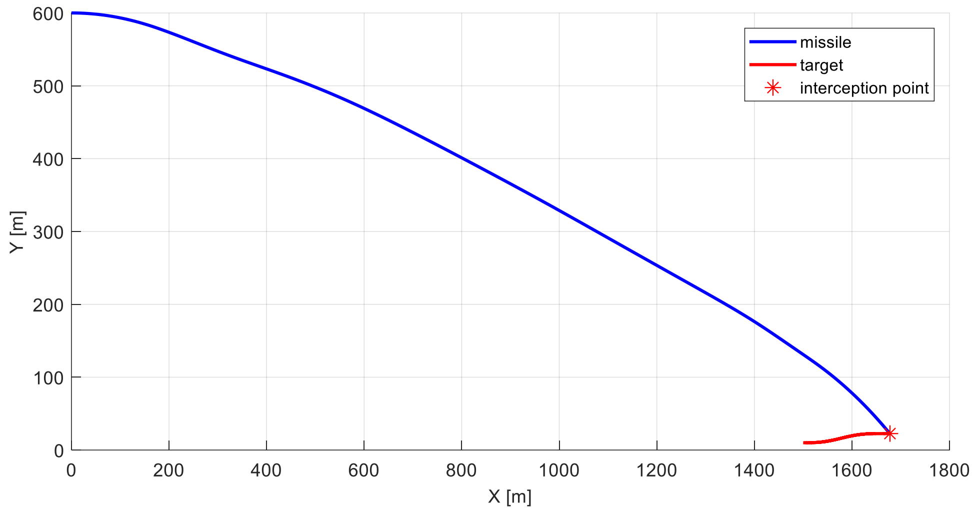

Figure 5.40. Missile and target trajectories

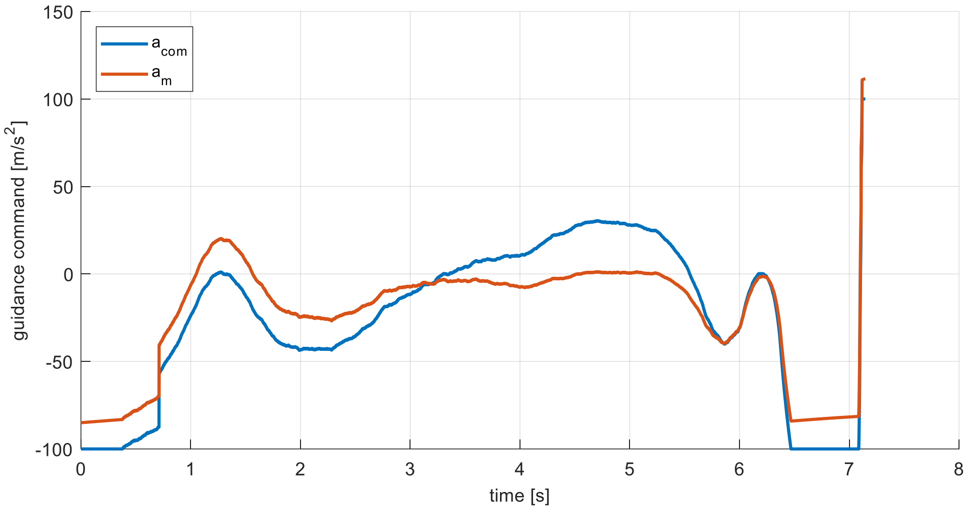

Figure 5.41. Guidance command and actual missile acceleration

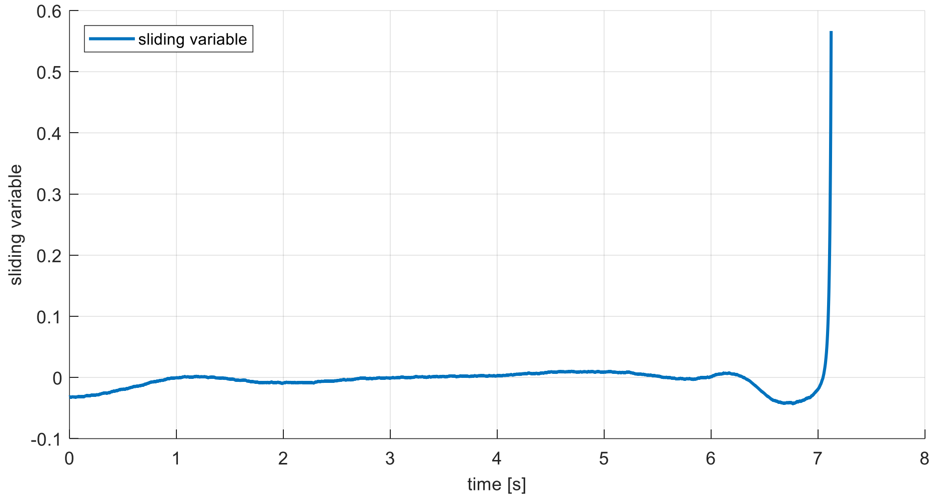

Figure 5.42. Sliding variable

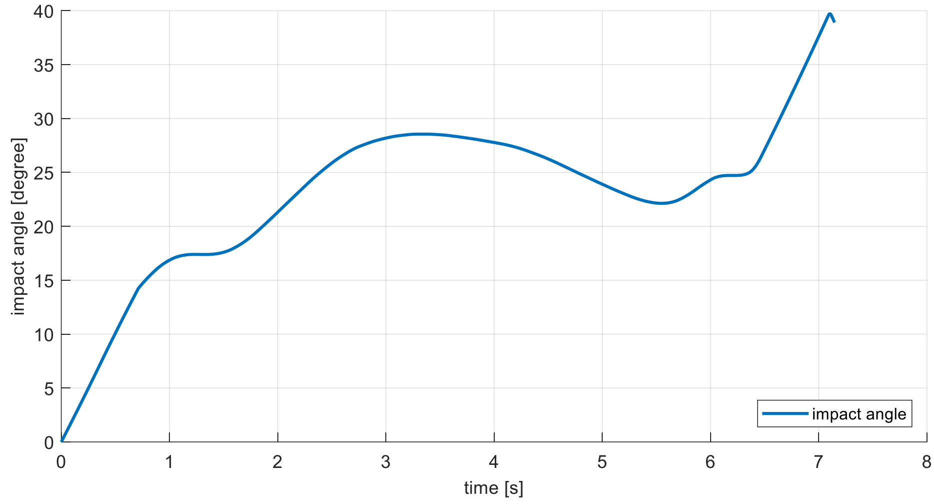

Figure 5.43. Impact angle

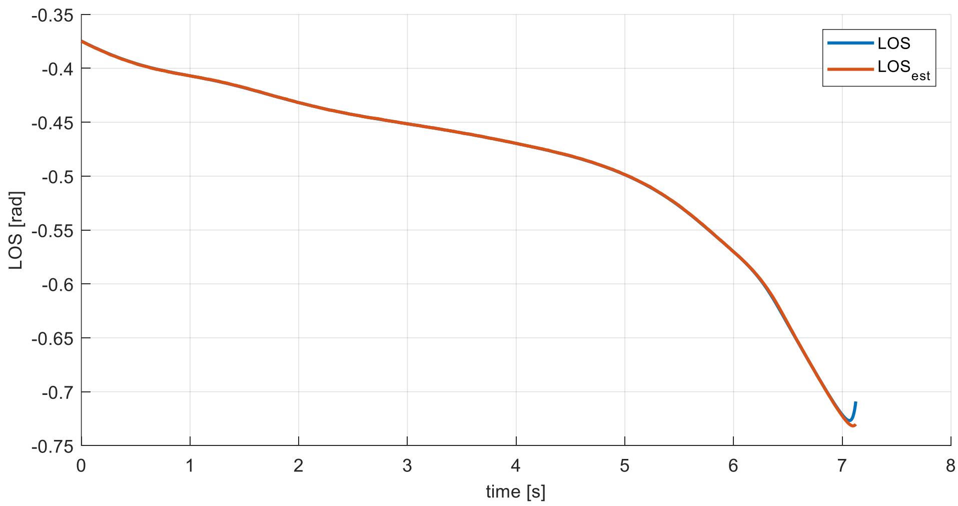

Figure 5.44. LOS angle and estimated LOS angle

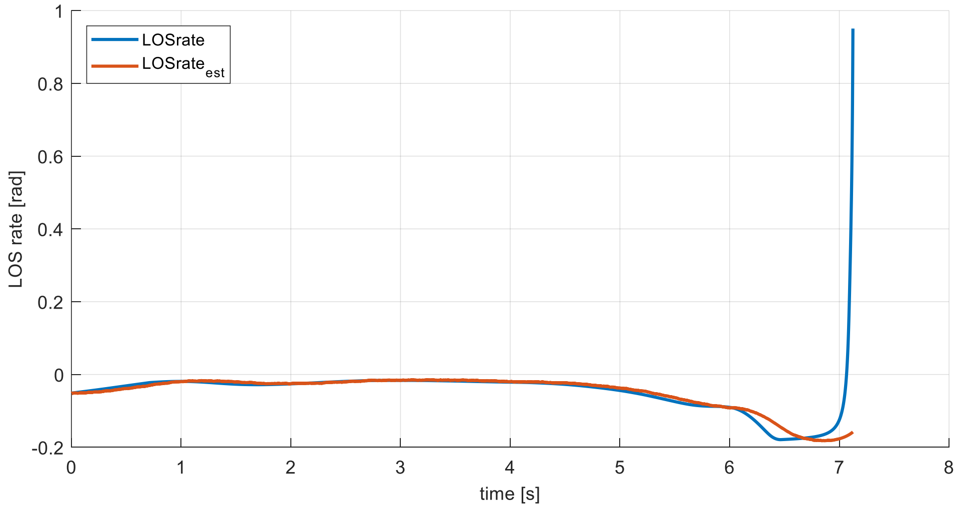

Figure 5.45. LOS rate and estimated LOS rate

Despite the fact that there exists noise on LOS measurement alongside with disturbance and uncertainties, the resultant impact angle error is 1.09˚m and miss distance takes the value of 0.112 m. As shown at Figure 5.40, the missile keeps its usual trajectory. Impact angle error can be observed from Figure 5.43. Again, the sliding variable converges in to a small region around zero, as demonstrated in Figure 5.42. Figure 5.44 shows that; exact LOS angle and estimated LOS angle curves overlap, despite the noisy LOS angle data. Since, value of LOS rate changes faster just before the interception time in Figure 5.45, estimated LOS rate pursues the exact LOS rate with a time delay. This gives rise to increased fluctuations on sliding variable. Nevertheless, the guidance goals miss distance and impact angle are achieved with high performance.

From Figure 5.41, it can be seen that guidance command and actual target acceleration have a consistent profile as it was in the first section. To compare the simulations outputs of sections 5.3.1 and 5.3.2 some important figures are presented:

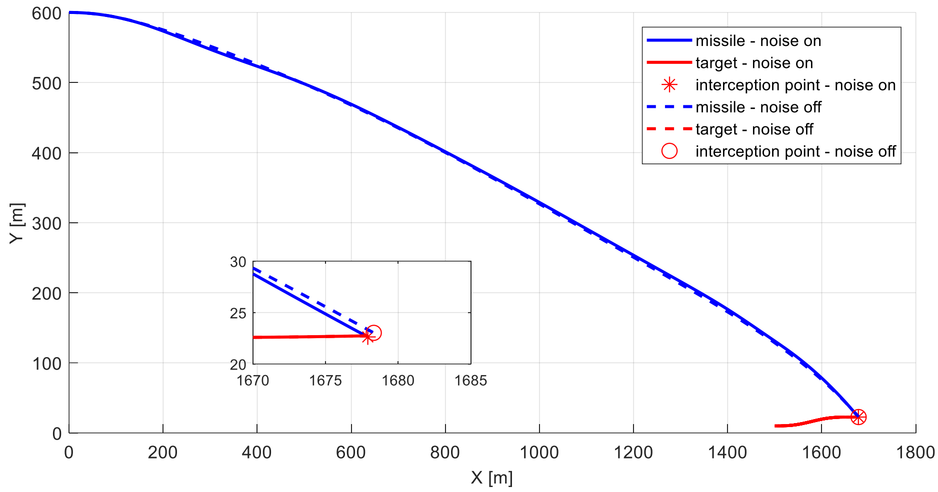

Figure 5.46. Missile and target trajectories

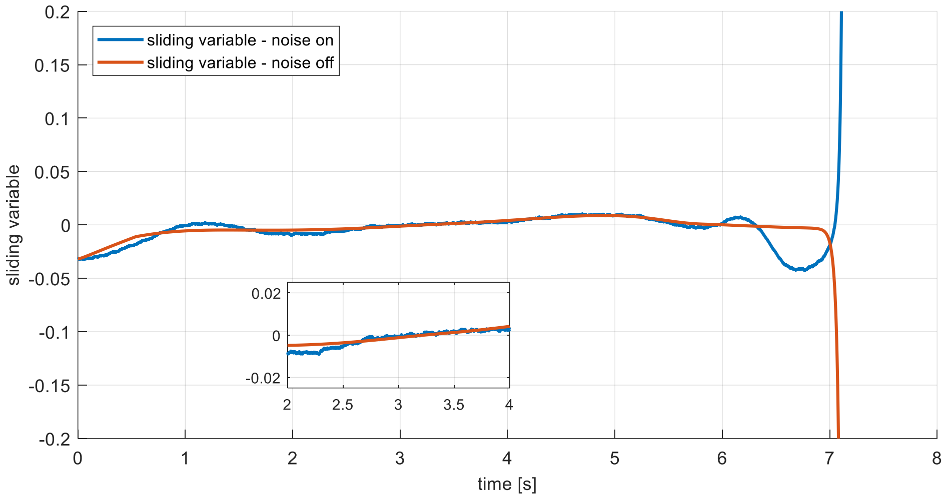

Figure 5.47. Sliding variables

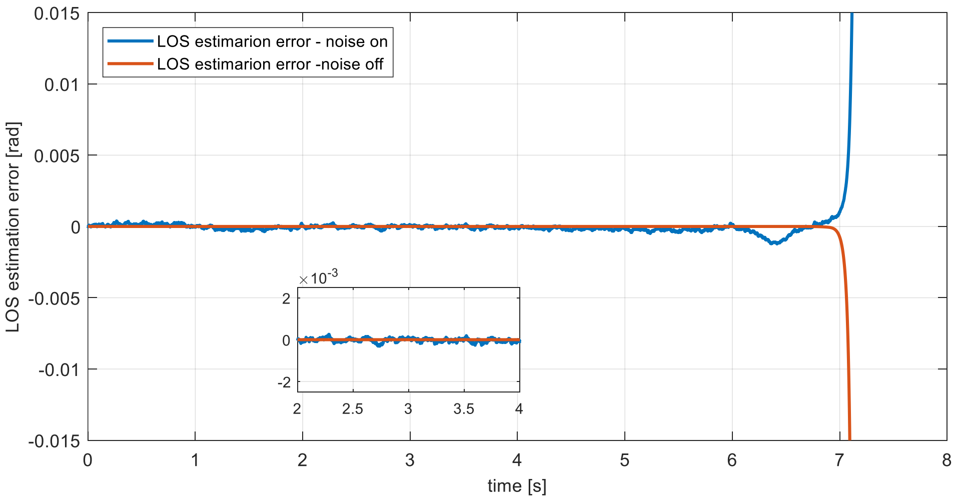

Figure 5.48. LOS angle estimation errors

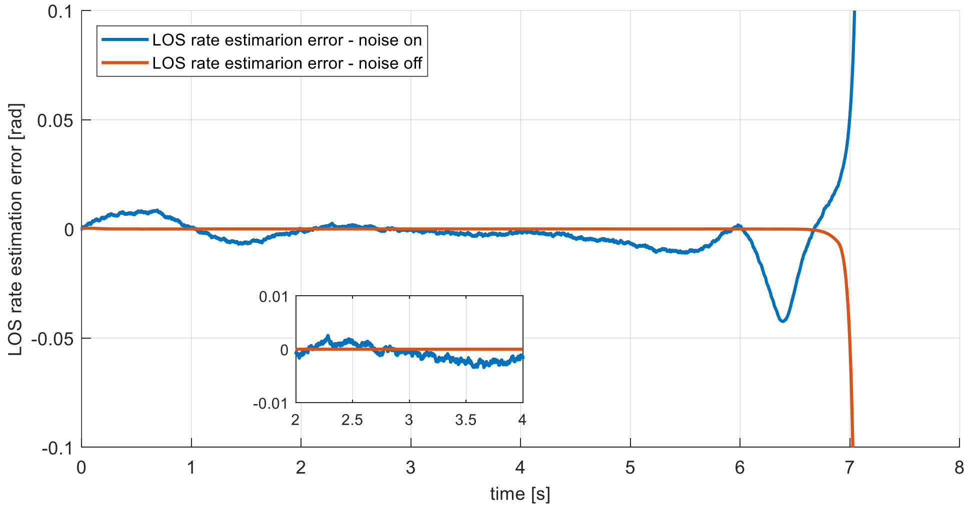

Figure 5.49. LOS angular rate estimation errors

As it is illustrated in Figure 5.46, noisy data does possess any significant impact on the trajectory of the missile. Amount of difference on the trajectory is directly related to estimation errors. LOS and LOS rate estimation errors in Figure 5.48 and Figure 5.49 are negligible in absence of noise. The error values are larger as expected, when the measurement noise is introduced. If noise is presented, the sliding variable converges around zero with a small amount of delay, as shown in Figure 5.47. As mentioned before, sliding variable fluctuations increases at the before the end of the flight. Even though there exist discrepancies between scenario outputs, estimation errors are low and sliding variable behaviors are very similar. Thus, the noise does not affect the guidance performance significantly with the help of sliding mode differentiator and approximate signum function.

5.3.3. Effect of Sigmoid Function

The switching function has a significant role on sliding mode controllers as mentioned before. In ideal case, where noise is absent and all guidance parameters are exact values; signum function provides the best solution as a switching function. In this part of the study, an estimated signum function is used to lower chattering and reduce the effects of existing noise. In order to clarify the reason why the estimated signum function is used in proposed guidance law, a brief analysis is introduced in this scenario. Four cases are considered in the analysis. In the first three cases, switching function is chosen as original signum function. It is assumed that exact values of LOS angle and its derivative are both accessible by the guidance algorithm in the first case so that, the SMD is not used. In the second case, estimated parameters are used in the absence of noise. Different from the second one, noise on LOS angular rate measurement is included in the third case. The last case is the scenario considered in section 5.3.3. On the purpose of comparing the cases clearly, scenario inputs are same with previous scenario for all cases. Result are presented in Figures 5.50-5.54:

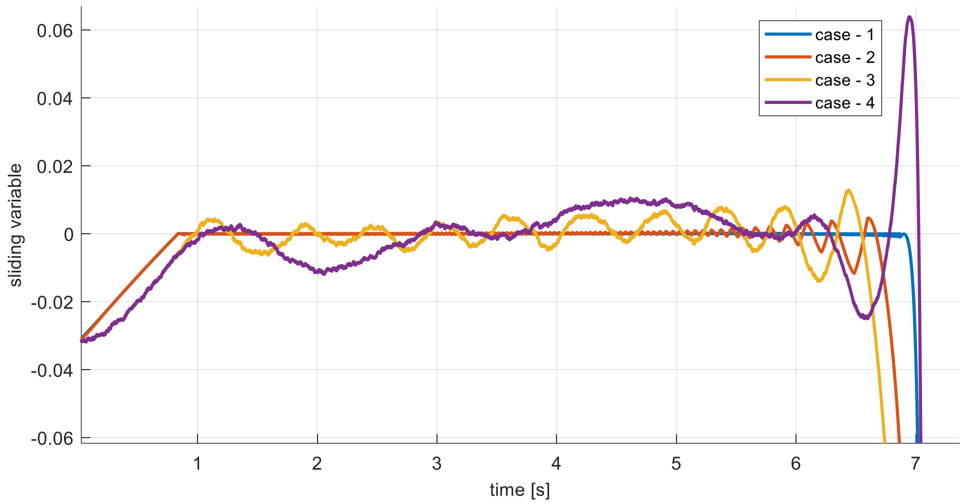

Figure 5.50. Sliding variables

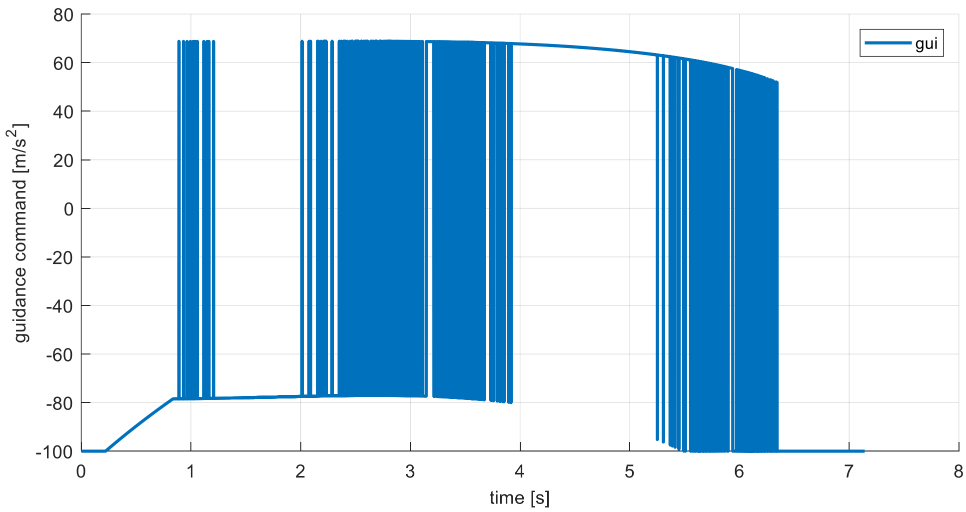

Figure 5.51. Guidance command (case-1)

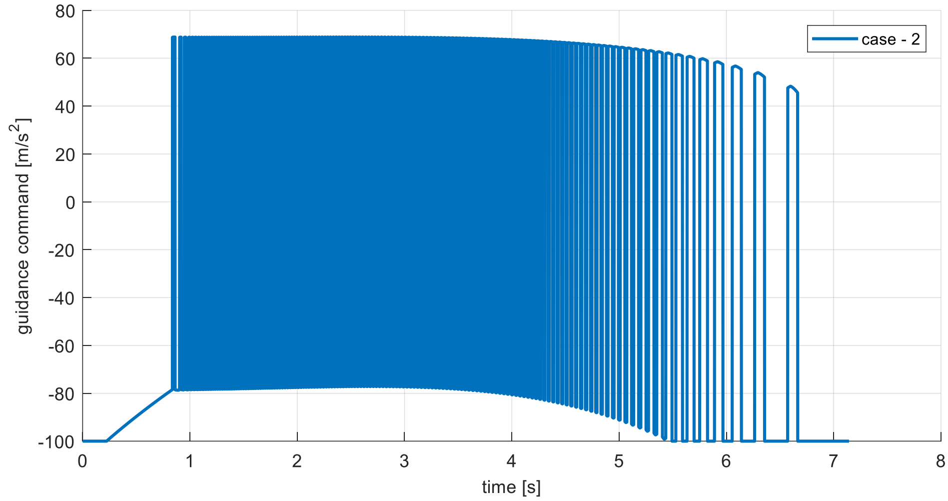

Figure 5.52. Guidance command (case-2)

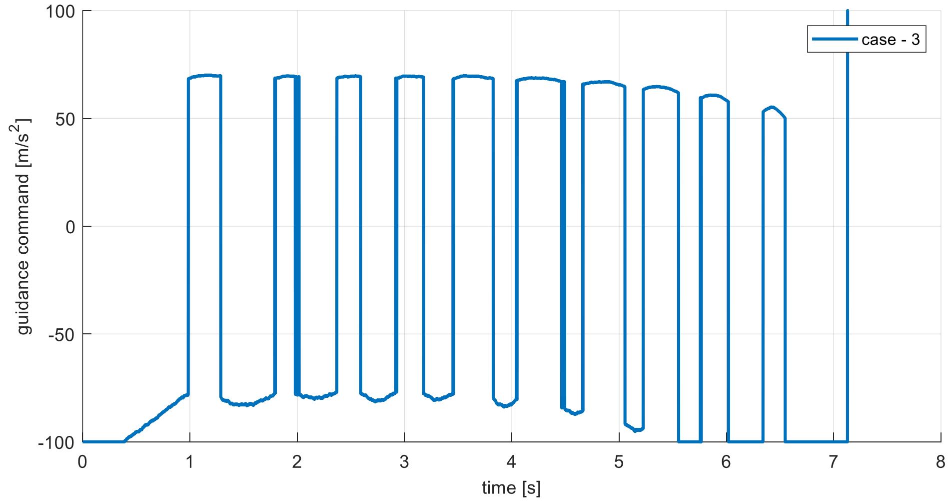

Figure 5.53. Guidance command (case-3)

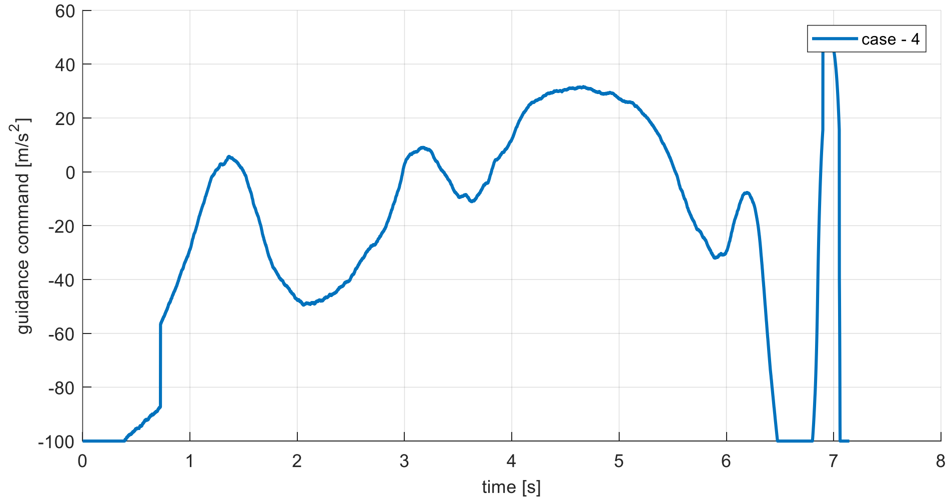

Figure 5.54. Guidance command (case-4)

In the first case, sliding variable converges and keeps on zero perfectly, as it can be observed in Figure 5.50. However, price of the perfect convergence is the chattering, which can be seen on Figure 5.51. High frequency discontinuous guidance commands are able to keep the sliding variable at zero, but these commands are not acceptable for a real system. When the sliding mode differentiator is introduced in the second case and even though the estimation errors are known to be low if noise is absent, fluctuation on sliding variable increases and they are observable just before the interception time, as presented in Figure 5.50. As a result, even more chattering than the first case is presented. This means that, use of estimated values on sliding variable is another cause of chattering phenomena. From Figure 5.52 it can be observed that, presence of noise increases the amplitudes of sliding variable fluctuations in the third case. Increase in amplitudes makes sliding variable to move away from the sliding surface slightly. Thus, sliding variable sign changes slower than the first two cases. As seen in Figure 5.53, this results in less chattering than the second case, but still high amount of chattering exists. It can be noted from Figure 5.50, when the approximate signum function is adopted in the fourth case, sliding mode dynamics gets slower around zero. However, Figure 5.54 demonstrates that; chattering is attenuated. Although the sliding mode dynamics are slow in a region around zero, the proposed guidance law presents sufficient performance.

5.3.4. Monte Carlo Study

It is important to analyze SMGL performance under a wide range of scenarios. Hence, a Monte Carlo Analysis is made to prove the SMGL's robustness property. Parameters on target acceleration, uncertainty and disturbance terms are chosen as normally distributed random numbers (rn) or uniformly distributed normal numbers (ru). Also, the seed values of LOS angle measurement noise and random numbers are generated randomly per scenario. Scenario inputs are indicated in Table 5.15:

Table 5.15. Scenario input parameters

Missile's initial position [x y] (m):	[0 600]
Missile's initial flight path angle (°):	0
Missile velocity (m/s):	250
Desired impact angle (°):	40
Initial target position [x y] (m):	[1500 10]
Initial target orientation angle (°):	0
Target velocity (m/s):	25
Target normal acceleration (m/s^2):	(rn2/3)sin(|rn1/3|t+ru2π)
Multiplicative uncertainty and disturbance term :	1-(rn0.15/3)sin(|rn1/3|t+ru2π)
Additive uncertainty and disturbance term :	(rn20/3)sin(|rn1/3|t+ru2π)
LOS angle measurement noise (°) :	rn(0.1/3)

The guidance algorithm parameters are displayed at Table 5.16:

Table 5.16. Guidance algorithm parameters

ε	γ	k_1	k_2	k_3	L	M_1	M_2	M_3	T	e	d_1
700	0.55	0.5	35	50	0.02	14	6	5	0.1	0.015	0.01

1000 runs of Monte Carlo Analysis are studied in this section. The resultant histograms of scenario goals are listed below:

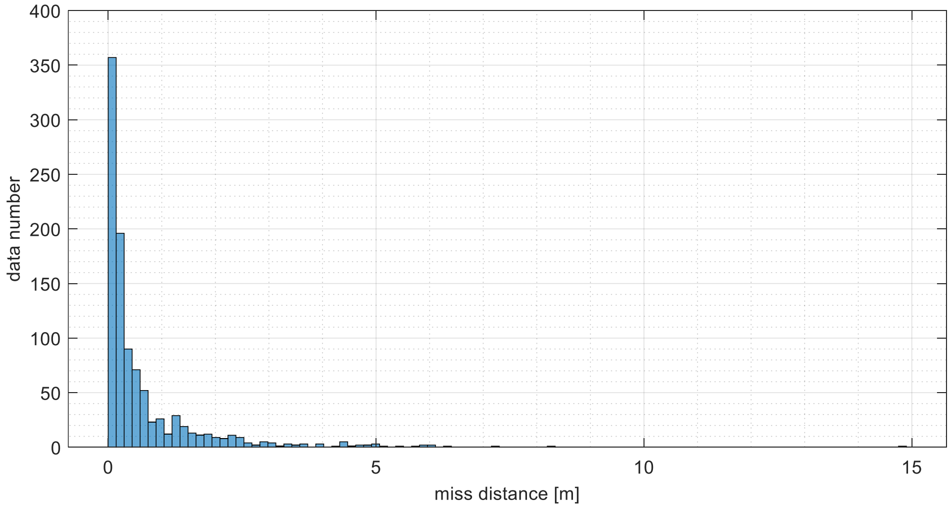

Figure 5.55. Histogram of miss distances

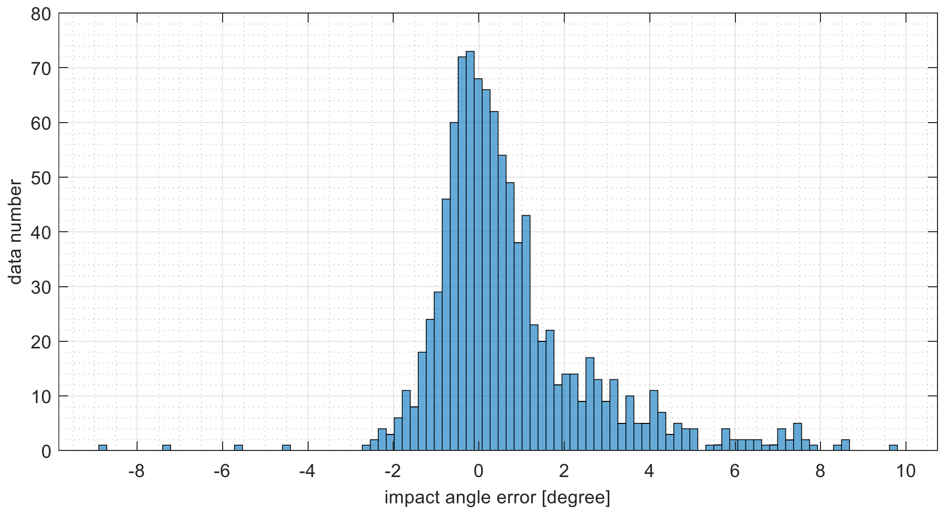

Figure 5.56. Histogram of impact angle errors

On this analysis, impact angle error below 5° and miss distance below 2 m is considered as a successful hit. In the Monte Carlo analysis results, %91 of the runs are successful. Figures 5.55 and 5.56 gives insight about distribution of impact angle and miss distance error data of Monte Carlo result. Standard deviations of impact angle and miss distance error are 1.112 m and 1.883° respectively. The guidance law ensures great performance with high hit probability and low errors on guidance goals, despite all varying and tough conditions on the missile-target engagement geometry.

6. RESULTS AND CONCLUSION

In this thesis, actuator fault tolerant SMGL is introduced with impact angle and acceleration considerations. As mentioned before, actuator failures and target accelerations act as a disturbance on missile-target engagement kinematics. The presence of actuator failure may prevent the actuation of guidance commands properly. Thus, desired normal missile acceleration is not achieved in presence of actuator failures. Discrepancies between guidance command and actual missile acceleration reduce the precision of the missile. Target acceleration is considered as unknown data for the missile computer. If guidance law is not robust against target acceleration, successive interceptions may not be possible over maneuvering targets. Different from the sliding mode guidance laws in the literature, the SMGL gains the missile to the ability to perform successful interceptions with desired impact angles in presence of actuator failure and unknown target maneuver. The SMGL is capable of estimating LOS rate from noisy LOS angle data. Hence, the suggested SMGL is able to operate if LOS angular rate information is not accessible by the missile computer.

In SMGL design, a first order SMC structure is adopted. Selected sliding surface states contain desired impact angle error and LOS rate. During the flight, the states converge to zero as they slide along the sliding surface. Thus, sliding mode guidance law keeps the missile in collision course and forces the missile to achieve desired impact angle. In order to attenuate chattering, equivalent control method is applied and a sigmoid function is used instead of a discontinuous signum function in guidance design. LOS angular rate and LOS angle estimations are performed by using a second order SMD. For the purpose of mitigating chattering in presence of measurement noise, an approximate signum function is used instead of a sigmoid function.

In the first part of the simulation studies, the SMGL performance is tested under various scenarios. Effect of actuator failures and target motion is considered separately, in the first place. It is assumed that LOS angular rate information is accessible by the missile computer. Results in Table 5.4 and 4.6 show that, even though actuator failure or target maneuver is considered in the scenarios, the sliding mode guidance law ensures successful interceptions with various impact angles. Both target maneuver and actuator failure are discussed in the next scenario. Considering the derived SMGL is design to be robust against both disturbances, simulation results prove the robustness of the SMGL. From Table 5.8, impact angle errors and miss distances are considerably low for every desired impact angle. Figures 4.3, 4.12 and 4.18 show that magnitudes of guidance commands increase when actuator failure is presented. Higher guidance commands are needed against excessive disturbances. Sliding variable behaviors in these scenarios supports this conclusion. If Figure 4.4 and 4.19 are compared, similar sliding variable curves are observed. Especially if a higher impact angle of $60°$ is desired, the sliding variable tends to oscillate around the sliding surface. In Figure 4.13, the sliding variable converges to the sliding surface faster when only target maneuver is considered. However, Figure 4.6, 4.15 and 4.21 shows that LOS angular rates tend to diverge more if target maneuver is introduced. Hence, target maneuver has more impact on LOS angular rate profile. The SMGL performance is also tested for different ranges. Both receding and approaching targets are considered in this part of the study. Despite both actuator failure and target maneuver effects, the proposed sliding mode guidance law consolidates its robustness property for various engagement geometries. From Figure 4.23 and 4.28 it can be seen that generated guidance commands are higher in approaching target scenario. In approaching target scenarios, missile-target engagement kinematics expedites. Faster kinematics are challenging for the guidance law since LOS angular rate has more diverging behavior as shown in Figure 4.26 and 4.31. Nevertheless, Table 5.11 and 5.13 show that, precision on impact angle and miss distance achieved for every range and target motion.

In the second part of the simulation studies, the SMGL performance is evaluated when LOS angular rate is not accessible. In the first scenario, both actuator failure and maneuvering target are considered in absence of LOS angle measurement noise. Results of the scenario show that; successive interception is achieved without LOS angular rate information. In Figures 4.36 and 4.37, estimated values of LOS angle and its derivative are nearly identical with exact values. The sliding mode differentiator shows superior performance in absence of noise. If Figures 4.32-4.35 are compared with Figures 4.16-4.20 of the same scenario in the previous part, similarities in results can be observed. This is an expected result since estimated parameters are close to exact values. Only LOS angular rate exhibits distinct behaviors in between Figures 4.21 and 4.37. The discrepancy between LOS angular rates curves can be related to the impact angle errors and miss distances. The result of the next scenario shows the SMGL performance against measurement noise. In Figure 4.43, deviations between estimated and exact LOS angular rate can be observed. If Figure 4.40 is compared with Figure 4.34, the fluctuation amplitudes of the sliding mode variable are higher. The fluctuation is originated from estimation errors. Although measurement noise introduces a compelling effect on guidance performance, the missile achieves desired impact angle with low miss distance. In the last part of the study, a Monte Carlo analysis performed. A wide range of scenarios are tested under actuator failure, target maneuver and LOS angle measurement noise. Results in Figure 4.53 and 4.54 indicates that sliding mode guidance law provides a successful hit probability of %91. If it is considered that all Monte Carlo scenarios contain actuator failures, resultant successive hit probability proves the robustness of the SMGL.

In conclusion, this thesis proposes a robust SMGL against slowly moving targets. The performance is demonstrated with simulation studies. The guidance law is able to tolerate the effects of actuator failure and target maneuvers. Additionally, the guidance law is suitable for missiles with strapdown seekers, since LOS angular rate can be estimated from LOS angle measurements.

In order to extend this study, several subject are considered in the future work:

1. Instead of a kinematic model, a detailed dynamic model can be implemented to the simulation. Thus, the SMGL performance can be analyzed in more realistic scenarios.

2. Autopilot dynamics can be considered in guidance design to enhance the robustness property of the SMGL.

3. Seeker FOV constraints can be included in guidance law design.

4. Higher order SMC and SMD structures can be designed and compared with the proposed SMGL.

5. The guidance law can be analyzed for an air-to-air missile or surface-to-air or.
6. Proposed guidance law can be derived for three dimensions.

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