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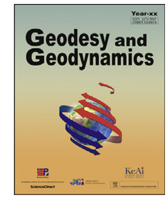

NVIS HF signal propagation in ionosphere using calculus of variations

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ABSTRACT

Modeling Near Vertical Incidence Sounding (NVIS) High Frequency (HF) signal propagation in the ionosphere is important. Because, ionosondes which are special types of radars probing the ionosphere with certain HF frequencies (between 2 and 30 MHz), work mostly in NVIS mode (where elevation angle is between 89 and 90°). In this work, we are going to propose a new method for NVIS wave propagation in the ionosphere by discretizing the NVIS wave propagation path into mediums in which the refractive index changes linearly, where we solve the ray propagation in each medium analytically using calculus of variations and use Snell's Law at medium changes. The main advantage of the proposed solution is the reduced computational complexity and time. This algorithm can be used to simulate and compare the behavior of vertical ionosondes together with other ray tracing algorithms.

1. Introduction

Ionosphere is the layer of the atmosphere that lies between 60 km and 1000 km above Earth surface and has a great importance in High Frequency (HF) radio and satellite communications [1]. Ionosondes are the special type of radars that are utilized for measuring the local real-time ionosphere operating in Near Vertical Incidence Sounding (NVIS) mode [2]. Ionosondes emit pulses, chirp or other waveforms in the shortwave frequency (i.e., HF frequency) range of 2–30 MHz, to measure the group delay of the return signal bounced back from the ionosphere and generate virtual reflection heights versus frequency graphs called ionograms.

As ionosphere is very important for long-distance communications, an empirical model of the ionosphere called International Reference Ionosphere (IRI) model is developed [3,4]. IRI calculates electron density, ion composition, ion and electron temperatures of the ionosphere for time and position at an altitude ranging from 60 to 1500 km based on available and reliable observations. IRI extended to the plasmasphere (IRI-Plas) is a more recently developed version that covers the plasmasphere region up to the Global Positioning System (GPS) orbital height of 20,200 km [5]. IRI-Plas also enables inputting Total Electron Content (TEC) for updating and scaling the ionospheric coefficients during the computation of desired outputs [6,7]. Online IRI-Plas service is available at the IONOLAB website (www.ionolab.org).

Wave propagation in the inhomogeneous ionosphere in terms of ray tracing is applicable as the dimension of the irregularities in the ionosphere is generally larger than the wavelength of the HF wave. Initial ray tracing depends on the Haselgrove equation set [8,9]. There are also new methods based on variational methods [10] and Snell's law [11].

In this study, we are going to propose a new method for NVIS wave propagation in the ionosphere using the exact solution of two-dimensional wave propagation with a one-dimensional linear refractive index change, based on calculus of variations [12]. In this prospect, Section 2 will introduce the parametric wave propagation equations developed for the two-dimensional wave propagation, Section 3 will give a brief summary of the coordinate systems used in this study, Section 4 will explain the calculation of refractive indices in the ionosphere, Section 5 will present Snell's law of refraction, and Section 6 will explain the local approximation of three-dimensional wave propagation into two-dimensional wave propagation in order to utilize of the parametric wave propagation equations developed in Section 2. Finally, Section 7 will present the proposed NVIS wave propagation algorithm in full detail.

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2. Two-dimensional wave propagation using calculus of variations

Let us consider a light wave, with a propagation vector shown in Fig. 1, entering into a medium with a linear refractive index changing only in the y-direction.

Thus, a parametric two-dimensional light wave propagation path based on the Fermat's Principle of Least Time is already developed in [12] using the Calculus of Variations as

$$y(x) = \frac{1}{\beta}\left[1 - \cos\phi_0 \cosh(\beta x \sec\phi_0 - \zeta_0)\right] \tag{1}$$

where refractive index changes linearly in the y-direction as

$$\eta(y) = \eta_0(1 - \beta y) \tag{2}$$

with η_0 being the initial refractive index at $y = 0$, ϕ_0 is the elevation angle (measured from the x-axis) of the incident wave at the medium entrance point $(0, 0)$. Here, ζ_0 is a constant of form

$$\zeta_0 = \beta x_r \sec\phi_0 \tag{3}$$

where $p_r = (x_r, y_r)$ is the reflection point, and x_r and y_r are given by

$$x_r = \frac{\cos\phi_0}{\beta}\ln(\sec\phi_0 + \tan\phi_0) \tag{4}$$

$$y_r = \frac{1 - \cos\phi_0}{\beta}. \tag{5}$$

respectively.

An example normalized plot is shown in Fig. 2 for $\phi_0 = \pi/3$ and $\beta = 0.5$.

We can also develop an equation for $x(y)$ from (1) for $x \in [0, x_r]$ as

$$x(y) = 2x_r - \frac{\cos\phi_0}{\beta}\left(\operatorname{arccosh}\left(\frac{1 - \beta y}{\cos\phi_0}\right) + \zeta_0\right) \tag{6}$$

and for $x \in [x_r, 2x_r]$ as

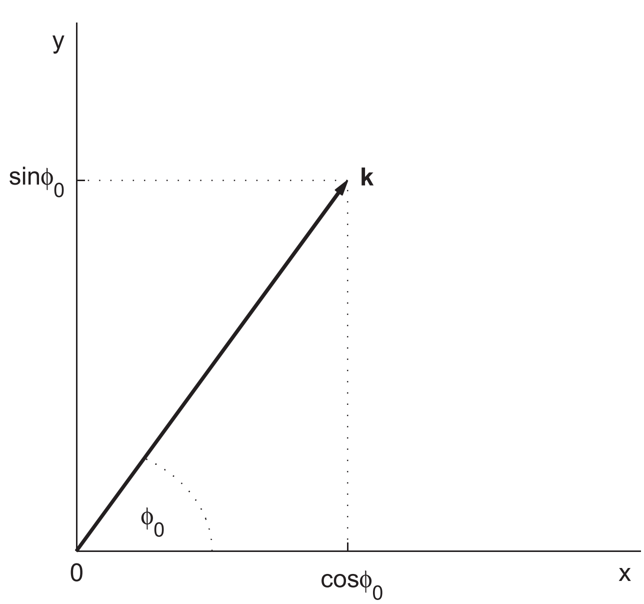

Fig. 1. Initial propagation vector entered into the medium.

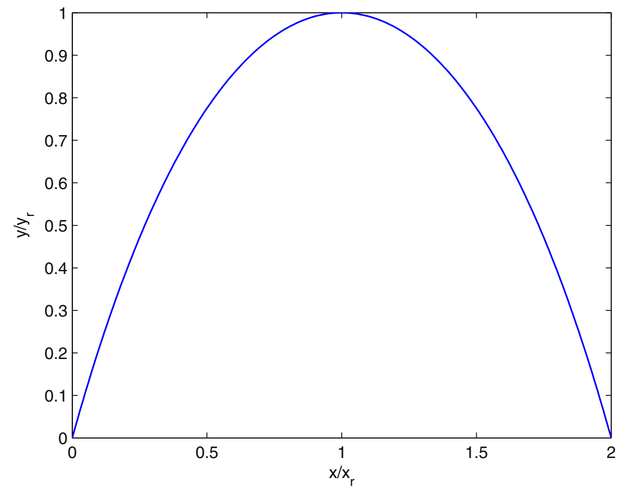

Fig. 2. Plot of y/y_r versus x/x_r for $\phi_0 = \pi/3$ and $\beta = 0.5$.

$$x(y) = \frac{\cos\phi_0}{\beta}\left(\operatorname{arccosh}\left(\frac{1 - \beta y}{\cos\phi_0}\right) + \zeta_0\right). \tag{7}$$

As the derivative of $y(x)$ gives the tangent of the propagation angle, let us first express the derivative of $y(x)$ with respect to x as

$$y'(x) = \sinh(\zeta_0 - \beta x \sec\phi_0). \tag{8}$$

Using (8) above, normalized propagation vector $\mathbf{k}(x)$ is obtained as

$$\mathbf{k}(x) = [\operatorname{sech}(\zeta_0 - \beta x \sec\phi_0),\ \tanh(\zeta_0 - \beta x \sec\phi_0)]. \tag{9}$$

Similarly, unnormalized propagation vector $\tilde{\mathbf{k}}(x)$ where the x-component remains the same will be given by

$$\tilde{\mathbf{k}}(x) = [\cos\phi_0, \cos\phi_0 \sinh(\zeta_0 - \beta x \sec\phi_0)]. \tag{10}$$

where $\mathbf{k}(x) = \tilde{\mathbf{k}}(x)/\left\|\tilde{\mathbf{k}}(x)\right\|$. Thus, only y-component of $\tilde{\mathbf{k}}(x)$ needs to be calculated whenever it is necessary. Notice that $\mathbf{k}(0) = [\cos\phi_0, \sin\phi_0]$ and $\mathbf{k}(0) = \tilde{\mathbf{k}}(0)$.

In the NVIS wave propagation, we are going to assume that refractive index propagation will change only in the up-direction in the medium as the elevation angle of the transmission vector will be between 89 and 90°. Then, we are going to approximate the refractive index change with a linear equation. Thus, we will be able to use the results presented in (1)–(10).

3. Geographic coordinate systems

There are three main global geographic coordinate systems known as geodetic, geocentric and Earth-Centered Earth-Fixed (ECEF) coordinate systems, respectively. Geodetic coordinate system is the standard coordinate system used in our daily life, e.g., cartography, geodesy, and navigation, and it is currently governed by the W9S84 standard. Geocentric coordinate system is an earth-centered spherical coordinate system (defined by latitude, longitude and radius), and ECEF is the geocentric cartesian coordinate system defined by X, Y and Z axes. There also local coordinate systems known as East-North-Up (ENU) and Azimuth-Elevation-Range (AER) coordinate systems centered at a particular location. In AER coordinate system, azimuth is measured clockwise from local north (i.e., true north). Transformation between these coordinate systems are available in the MATLAB Mapping Toolbox [13]. We are going to refer these coordinate systems in the subscript of a given variable or parameter and assume that it is converted into this coordinate system correctly.

4. Refractive index in ionosphere

The refractive index η in the ionosphere can be calculated with the widely used Appleton-Hartree formula [9,11] given by

$$\eta^2 = 1 - \frac{2X(1-jZ-X)}{2(1-jZ)(1-jZ-X) - Y^2\sin^2\vartheta \pm \sqrt{Y^4\sin^4\vartheta + 4(1-jZ-X)^2 Y^2\cos^2\vartheta}} \tag{11}$$

where

$$X = N_e q^2 \Big/ \left(\varepsilon_0 m\omega^2\right) = f_N^2 \Big/ f^2 \tag{12}$$

$$Y = qB/(m\omega) = f_H/f \tag{13}$$

$$Z = f_v/f \tag{14}$$

and $j = \sqrt{-1}$, N_e is the electron density, f_N is the plasma frequency, f is the wave frequency, B is the magnitude of the geomagnetic field (earth's magnetic field), ϑ is the angle between the wave propagation vector and the direction of the geomagnetic field, f_H is the electron cyclotron frequency, and f_v is the electron collision frequency, q is the electron charge, m is electron mass, and ε_0 is the free space dielectric constant. When the incident wave enters the ionosphere (i.e., at the entrance point), it is divided into two waves known as ordinary and extraordinary waves. The '±' sign in the numerator denotes that plus sign is used for the refractive index of ordinary waves and the minus sign is used for the refractive index of extraordinary waves [11]. Note that we will always use and refer to the real part of the refractive index η in our algorithm and calculations.

In order to calculate the refractive index η, we will obtain the electron density N_e, ion densities, electron and ion temperatures from the IRI-Plas profile output generated by running the IRI-Plas model at a given date, time and location (geocentric latitude and longitude). Normally, IRI-Plas is run with TEC input in order to reflect a more accurate state of the ionosphere. Also, the magnitude and direction of the geomagnetic field are calculated for a given geocentric location according to the recent IGRF model [14]. If there are any ionosonde measurements available, then electron density profiles developed from ionosonde measurements can also be used in the calculation of ionosphere refractive indices.

5. Snell's law

Snell's law (or the law of refraction) is a formula used to describe the relationship between the incidence angle φ_1 and refraction angle φ_2, when light or other waves passing through a boundary between two different isotropic media, as shown in Fig. 3.

Incidence are refraction angles are measured from the surface normal $\mathbf{n}_2$ at the entrance point p_2. Using Snell's law, we can express the sine of the refraction angle φ_2 as

$$\sin\varphi_2 = \frac{\eta_1}{\eta_2}\sin\varphi_1 \tag{15}$$

where φ_1 is the incidence angle, η_1 is the refractive index of the incoming wave medium and η_2 is the refractive index of the outgoing wave medium. Consequently, the refracted wave propagation vector $\mathbf{k}_2$ is given by

$$\mathbf{k}_2 = \frac{\eta_1}{\eta_2}\mathbf{k}_1 - \left(\frac{\eta_1}{\eta_2}\cos\varphi_1 - \cos\varphi_2\right)\mathbf{n}_2 \tag{16}$$

where $\mathbf{k}_1$ is the incidence wave propagation vector and $\mathbf{n}_2$ is the surface normal pointing into the outgoing medium [15]. Note that, $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{n}_2$ are all normalized vectors. Also, outward-pointing surface normal $\mathbf{n}_2$ for a spherical surface (e.g., an ionosphere layer) is given by

$$\mathbf{n}_{2(\mathrm{ecef})} = \left[\cos\left(\chi_{2(\mathrm{geoc})}\right)\cos\left(\lambda_{2(\mathrm{geoc})}\right), \cos\left(\chi_{2(\mathrm{geoc})}\right)\sin\left(\lambda_{2(\mathrm{geoc})}\right), \sin\left(\chi_{2(\mathrm{geoc})}\right)\right] \tag{17}$$

where $\chi_{2(\mathrm{geoc})}$ is the geocentric latitude, $\lambda_{2(\mathrm{geoc})}$ is the geocentric longitude at $p_{2(\mathrm{geoc})} = (\chi_{2(\mathrm{geoc})}, \lambda_{2(\mathrm{geoc})}, h_{2(\mathrm{geoc})})$ with $h_{2(\mathrm{geoc})}$ being the geocentric altitude.

Note that, if $\sin\varphi_2 > 1$, then total reflection will occur, and the reflected propagation vector $\mathbf{k}_2$ can be calculated as in [15].

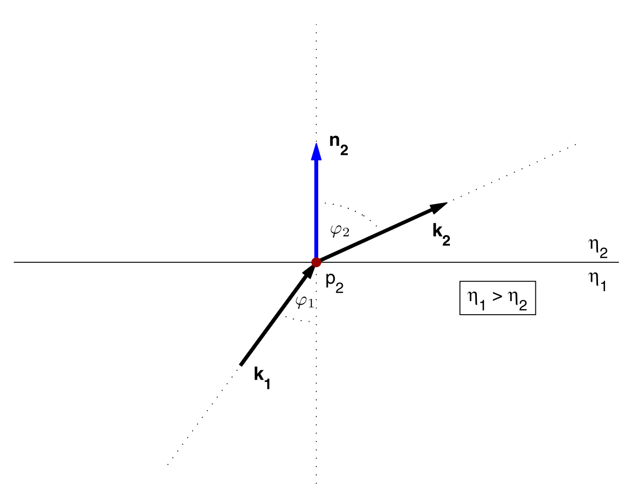

Fig. 3. Refraction of light at the interface between two media of different refractive indices with $\eta_1 > \eta_2$.

6. Local two-dimensional approximation of NVIS wave propagation

We are going to develop a discretized NVIS HF wave propagation algorithm based on the two-dimensional wave propagation model presented in Section 2. In order to utilize the model developed in Section 2, we will discretize the approximate ray path with appropriate step sizes L_i (e.g., $L_i = 5$ km). Also, we will use the local ENU coordinate system at each discretization point. Because the up-direction is in the same direction as the outward-pointing surface normal of the spherical ionosphere layer, and north–east plane (i.e., the tangent plane) represents the surface of the ionosphere layer within the local vicinity. Once the propagation vector $\mathbf{k}_{enu}$ in local ENU coordinates is obtained at each discretization point, then the up-direction will be the y-direction and the direction obtained by the projection of the propagation vector $\mathbf{k}_{enu}$ onto the tangent plane will be the x-direction. Thus, we will be able to use equations 1–10 at each discretization step.

Given a geocentric point $p_{i(\mathrm{geoc})} = (\chi_{i(\mathrm{geoc})}, \lambda_{i(\mathrm{geoc})}, h_{i(\mathrm{geoc})})$, refractive index η_i and refracted propagation vector $\mathbf{k}_{i(\mathrm{enu})} = \tilde{\mathbf{k}}_{i(\mathrm{enu})} = [k_{i(\mathrm{east})}, k_{i(\mathrm{north})}, k_{i(\mathrm{up})}]$ in ENU coordinates at that point, we approximate the refractive index slope β_i as

$$\beta_i \cong \frac{1}{L_i}\left(1 - \frac{\eta_i'}{\eta_i}\right) \tag{18}$$

where η_i' is the refractive index at point $p_i' = (\chi_{i(\mathrm{geoc})}, \lambda_{i(\mathrm{geoc})}, h_{i(\mathrm{geoc})} + \alpha_i L_i)$. Here, $\alpha_i = \mathrm{sign}(k_{i(\mathrm{up})})$[1] i.e, $\alpha = 1$ if the direction is up and $\alpha = -1$ if the direction is down. Note that, refractive indices are calculated using (11) as explained in Section 4. Then, y- and x-directions, $\hat{y}_i$ and $\hat{x}_i$, are given by

$$\hat{y}_i = \left[0, 0, k_{i(\mathrm{up})}\right] \Big/ \left|k_{i(\mathrm{up})}\right| \tag{19}$$

$$\hat{x}_i = \left[k_{i(\mathrm{east})}, k_{i(\mathrm{north})}, 0\right] \Big/ \sqrt{k_{i(\mathrm{east})}^2 + k_{i(\mathrm{north})}^2} \tag{20}$$

Once we determine β, x- and y-directions, we can utilize the model developed in Section 2 using the approximation algorithm given below. Here, the derived algorithm calculates the exit point p_{i+1}' of the medium, the propagation vector $\mathbf{k}_{i+1}'$ and the refractive index η_{i+1}' at this exit point.

1. Calculate the reflection point x_r and y_r using (4) and (5) with $\phi_0 = k_{i(\mathrm{elevation})}$ and $\beta = |\beta_i|$,
2. Calculate the reflection index η_r using (2) with $y = y_r$ and $\beta = |\beta_i|$,
3. Calculate the the exit point $x(L_i)$
   (a). if $\beta_i < 0$
      i. Calculate $x(L_i)$ using (6) with $y = -L_i$ and $\beta = |\beta_i|$. Note that $x(L_i)$ will be negative.
   (b) else
      i. if $\eta_i' > \eta_r$, calculate $x(L_i)$ using (6) with $y = L_i$ and $\beta = \beta_i$
      ii. else, calculate $x(L_i)$ using (7) with $y = L_i$ and $\beta = \beta_i$
4. Calculate unnormalized exit-point propagation vector $\tilde{\mathbf{k}}_{i+1}'$ using (10) as

$$\tilde{\mathbf{k}}_{i+1}' = \cos\phi_0\left[k_{i(\mathrm{east})}, k_{i(\mathrm{north})}, 0\right] + \cos\phi_0 \sinh(\zeta_0 - |\beta_i| x(L_i) \sec\phi_0)\left[0, 0, k_{i(\mathrm{up})}\right] \tag{21}$$

5. Calculate normalized exit-point propagation vector $\mathbf{k}_{i+1}'$ as $\mathbf{k}_{i+1}' = \tilde{\mathbf{k}}_{i+1}' / \left\|\tilde{\mathbf{k}}_{i+1}'\right\|$ and convert it to ECEF coordinates.
6. Calculate exit-point refractive index η_{i+1}' using (2) with $y = L_i$ and $\beta = \beta_i$.
7. Calculate exit-point $p_{i+1(\mathrm{enu})}'$ in ENU coordinates as

$$p_{i+1(\mathrm{enu})}' = \mathrm{sign}(\beta_i) x(L_i)\hat{x} + L_i \hat{y}, \tag{22}$$

then convert to the geocentric coordinate $p_{i+1(\mathrm{geoc})}' = (\chi_{i+1(\mathrm{geoc})}, \lambda_{i+1(\mathrm{geoc})}, h_{i+1(\mathrm{geoc})})$.

Selection of step size L_i is important in order to ensure that refractive index changes linearly in the up-direction, i.e., (18) is correct, and result of local coordinate calculations do not deviate from the global coordinates.

7. NVIS wave propagation algorithm

In this section, we are going to develop the whole wave propagation algorithm by dividing the propagation in three parts: propagation from earth to ionosphere, propagation inside ionosphere and propagation from ionosphere to earth.

Firstly, we determine a transmit location on the Earth surface, normally expressed by the geodetic latitude $\chi_{tx(\mathrm{geod})}$ and longitude $\lambda_{tx(\mathrm{geod})}$ coordinates, i.e., $P_{tx(\mathrm{geod})} = (\chi_{tx(\mathrm{geod})}, \lambda_{tx(\mathrm{geod})})$, at a given date and time. An HF wave with a certain frequency f between 2 MHz and 30 MHz will be transmitted in a certain direction defined by the azimuth ψ_{tx} and elevation θ_{tx}, i.e., $\mathbf{k}_{tx(\mathrm{aer})} = [\psi_{tx}, \theta_{tx}, 1]$. Note that, in this study we are only concerned with NVIS propagation, so $89^\circ \le \theta_{tx} \le 90^\circ$. We also need to set the minimum height $h_{\min}$ and maximum height $h_{\max}$ of the ionosphere. Typically, $h_{\min} = 80$ km and $h_{\max} = 500$ km.

7.1. Propagation from earth to ionosphere

In this section, we are going to determine the entrance point p_1 into the ionosphere, i.e., the intersection point with the lowest layer of the ionosphere at height $h_1 = h_{\min}$. Propagation medium between the Earth surface and the ionosphere will be referred to as air and assumed to have a refractive index of one, i.e., $\eta_{air} = 1$. So, the ray reaches to the lowest ionosphere layer without any refraction, i.e., propagation vector stays the same, i.e., $\mathbf{k}_0 = \mathbf{k}_{tx}$.

Entrance point p_1 is calculated by shooting a ray in direction $\mathbf{k}_{tx(\mathrm{ecef})}$ from the transmission point $P_{tx(\mathrm{ecef})}$ and calculating its intersection with the sphere of radius $r = R_{\mathrm{earth}} + h_1$.

7.2. Propagation inside ionosphere

In this section, we are going to determine the exit point p_∞ from the ionosphere. The propagation algorithm should be carried out from start to end either for an ordinary or an extraordinary wave, where only the refractive index calculation changes accordingly as explained in Section 4.

Algorithm is given by:

- Initialization
  - $\eta_0 = \eta_\infty = \eta_{air} = 1$
  - $\mathbf{k}_0 = \mathbf{k}_{tx}$
  - p_1 = ionosphere entrance point calculated in Section 7.1
  - $h_1 = h_{\min}$
  - p_∞ = null
  - $i = 1$
- Repeat
  1. Enter into the medium with linear refractive index using Snell's Law
     (a) Calculate refractive index η_i at point p_i using (11)

[1] $\mathrm{sign}(x) = 1$ for $x \ge 0$ and $\mathrm{sign}(x) = -1$ for $x < 0$.

(b) Calculate the refraction angle φ_i using (15) and check for total reflection
i. If total reflection occurs (should not normally occur)
A. Change L_i and restart the step.
ii. Else
A. Calculate refracted propagation vector $\mathbf{k}_i$ using (16)
2. Propagate in the medium with linear refractive index according to Section 6
(a) Calculate the exit-point values p'_{i+1}, η'_{i+1} and $\mathbf{k}'_{i+1}$ of the medium with linear refractive index, using the the algorithm developed in Section 6.
(b) Set $p_{i+1} = p'_{i+1}$, $\eta_i = \eta'_{i+1}$ and $\mathbf{k}_i = \mathbf{k}'_{i+1}$.
3. If $h_{i+1} \le h_{\min}$ or $h_{i+1} \ge h_{\max}$, then set $p_\infty = p_{i+1}$ and $\mathbf{k}_\infty = \mathbf{k}_i$, and exit
4. Increment i, i.e., $i = i + 1$
- Until p_∞ is not null

7.3. Propagation from ionosphere to earth

If the exit point p_∞ from the ionosphere is at the top ionosphere layer, i.e., the propagation vector does not point to the Earth surface, then the transmitted wave will not reach to the Earth, rather it will propagate to the outer space.

However, if the exit point p_∞ from the ionosphere is at the bottom ionosphere layer, then we can determine the receiver point P_{rx} on the Earth surface. The ray reaches from exit point p_∞ from the ionosphere to the Earth surface without any refraction, i.e., propagation vector stays the same, i.e., $\mathbf{k}_{rx} = \mathbf{k}_\infty$.

Receiver point P_{rx} on the Earth surface can be calculated by shooting a ray in direction $\mathbf{k}_{\infty(\mathrm{ecef})}$ from the transmission point $p_{\infty(\mathrm{ecef})}$ and calculating its intersection with the sphere of radius $r = R_{\mathrm{earth}}$.

Thus, we calculated the ray propagation path from P_{tx} to P_{rx}.

8. Conclusion

In this study, we proposed an NVIS HF wave propagation algorithm using calculus of variations and discretization along the height of the ionosphere. The main advantage of the proposed solution is the reduced computational complexity and time. This algorithm can be used to simulate and compare the behavior of vertical ionosondes together with other ray tracing algorithms.

Acknowledgments

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