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Multiple Model Kalman and Particle Filters and Applications: A Survey

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Abstract: Kalman Filters (KF) is a recursive estimation algorithm, a special case of Bayesian estimators under Gaussian, linear and quadratic conditions. For non-linear systems, Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) provide first and higher order linearization approximations. Particle Filters (PF), on the other hand, are sequential Monte Carlo methods to provide estimations for non-linear non-Gaussian problems. For complex systems, Kalman or Particle Filter based single model filters may not be sufficient to model the system behaviour. Multiple Model (MM) Filters achieve more reliable estimates by using more than one filter with different models in parallel and the outputs of each filter are fused by assigning a probability to each filter. The most common methods used in the literature for multiple model estimation are Multiple Model Adaptive Estimation (MMAE) and Interacting Multiple Model (IMM). This paper presents an overview of the recent research on multiple model filters.

Keywords: Kalman Filter; Multiple Model; Particle Filter; EKF; UKF; IMM; MMAE

1. INTRODUCTION

Kalman Filter (KF) (Kalman (1960), also known as Linear Quadratic Estimator (LQE), predicts the future state of a system based on previous state. KF can be used to estimate the system parameters (even under noise) when the parameters cannot be measured directly. It aims at minimizing the error, inaccuracy and noise during estimation. KF consists of some mathematical equations used for recursive estimation and it minimizes the error covariance under specific conditions (Bishop and Welch (2001)). KF is a special case of Bayesian filtering under linear, quadratic and Gaussian conditions (Ho and Lee (1964)).

2. KALMAN AND PARTICLE FILTERS

2.1. Why Is The Kalman Filter Used?

KF has made it possible to estimate the future values of system parameters and the parameters that cannot be measured directly. Mohinder and Angus (2001) claim that it is possibly the greatest discovery in the twentieth century. One of the earliest use of KF was the navigation and control system of Apollo space shuttle developed at NASA Ames Research Center in the early 1960s (Mcgee and Schmidt (1985)). KF provides a fast and efficient solution to the problem of processing noisy data combined with errors and inaccuracies. As KF needs to keep history of only the previous state, it is fast and requires small memory which makes it useful for real-time estimation.

2.2. How Does The Kalman Filter Work?

State Dynamics (Process Model):

$$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1} + \boldsymbol{w}_{k-1} \quad (1)$$

Output Equation (Measurement Model):

$$\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \quad (2)$$

KF is mainly defined by (1) and (2). System is modelled by the linear functions of the state parameters where state is the combination of parameters that describe the system model at a specific time. Current state is affected by the combination of previous state, control inputs and noise and the measurements are affected by the state parameters and noise. The dependence between the state parameters are given in a covariance matrix $\boldsymbol{P}$, which describes the correlation between the parameters. The variable definitions are given in Table 1.

For the prediction of state parameters at time t;

$$\hat{\boldsymbol{x}}_k = \boldsymbol{F}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1} \quad (3)$$

State covariance matrix is obtained as:

$$\boldsymbol{P}_k = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}\boldsymbol{F}_{k-1}^T + \boldsymbol{Q}_{k-1} \quad (4)$$

After the predictions of state and covariance using the values from previous estimates, Kalman Gain $\boldsymbol{K}_k$ is calculated as

$$\boldsymbol{K}_k = \boldsymbol{P}_k(\boldsymbol{H}_k\boldsymbol{P}_k\boldsymbol{H}_k^T + \boldsymbol{R}_k)^{-1} \quad (5)$$

State prediction is then corrected using the Kalman Gain $\boldsymbol{K}_k$ and the error $(\boldsymbol{y}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_k)$ as:

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k + K_k(\boldsymbol{y}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_k) \quad (6)$$

Finally, the covariance matrix is updated as:

$$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_k \quad (7)$$

Where $\boldsymbol{I}$ is the identity matrix.

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Table 1. Kalman Filter Variable Definitions

Symbol	Variable	Definition
$\boldsymbol{x}$	State	Parameters of interest (e.g., position, velocity)
$\boldsymbol{y}$	Output	Measurements from sensors (e.g., acceleration)
$\boldsymbol{u}$	Control Input	Inputs that are affecting the state (e.g., force applied by throttle or brake)
$\boldsymbol{w}$	Process Noise	Random errors with zero mean multivariate normal distribution
$\boldsymbol{v}$	Measurement Noise	Zero mean Gaussian white noise
$\boldsymbol{F}$	State Transition Matrix	Effect of state parameters at time t-1 on parameters at time t
$\boldsymbol{G}$	Control Input Matrix	Effect of control inputs on state parameters
$\boldsymbol{H}$	Observation Matrix	Transforms state parameters to measurement domain
$\boldsymbol{P}$	State Covariance Matrix	Describes the correlation between the state parameters
$\boldsymbol{Q}$	Process Noise Covariance	
$\boldsymbol{R}$	Measurement Noise Covariance	

2.3. Kalman Filter Types

2.3.1. Extended Kalman Filter (EKF)

Extended Kalman Filter (EKF) is the modified version of KF for use in non-linear systems. The non-linear system is defined as differentiable functions and linearized by using Taylor Series expansion. After linearization, KF is applied to this linearized model for estimation. Usually, the first order EKF is used but higher order EKF can be obtained by using higher order terms of Taylor series expansion.

2.3.2. Unscented Kalman Filter (UKF)

Unscented Kalman Filter is an improvement over EKF. Since the nonlinear system model is approximated using Jacobian matrices in the EKF, calculation may be costly and it may not be easy to obtain accurate results for the highly nonlinear systems due to linearization. Mcgee and Schmidt (1985) claims that approximation of a probability distribution is easier than approximation of an arbitrary nonlinear function and states that UKF is based on this principle. According to LaViola (2003), UKF captures the mean and covariance estimates with a deterministic sampling approach instead of linearization of Jacobian matrices. Therefore, UKF is considered more robust and more accurate (Mageswari et al. (2012)).

2.3.3.Robust Kalman Filters (RKF)

Robust Kalman Filter (RKF) (Xie et al. (1994)) addresses uncertain discrete-time systems. RKF tries to solve the parameter uncertainties in state and output matrices by providing an upper-bound guarantee for the variance of filtering error. Riccati equation or linear matrix inequality based methods provide an upper bound for error covariance for linear and nonlinear systems (Xiong et al. (2012)).

2.3.4.Cubature Kalman Filter (CKF)

Cubature Kalman Filter (CKF) (Arasaratnam and Haykin (2009)) is defined as an approximate Bayesian filter for discrete-time nonlinear filtering problems. Spherical-radial cubature rule is used for computation of multivariate moment integrals encountered in the nonlinear Bayesian filter and to provide a systematic solution to high-dimensional nonlinear filtering problems. CKF is better in terms of divergence and dimensionality compared to EKF, UKF, and Quadrature Kalman Filter. Being derivative-free, CKF is advantageous in cost calculation and 3rd-degree CKF is claimed to be optimal.

2.4. Particle Filters (PF)

Similar to KF, Particle Filters (PF) are also Bayesian. PF use sequential Monte Carlo method for state estimation. In PF, continuous distributions are approximated (López-Salcedo et al. (2014)) and the posterior probability is updated using random variables (particles) (Chen (2013), Wang et al. (2012)). There is no need for a functional approximation or linearization when using PF method; in contrast to this advantage, PFs require more computational power (Doucet and Johansen (2009)). PF is considered as an effective algorithm for solving nonlinear and non-Gaussian state space problems and it is advantageous in overcoming the shortcomings of easy divergence, low tracking accuracy and large error in conventional linearized methods such as EKF and UKF (Fei et al. (2008)). PFs rely on importance sampling and, as a result, require the design of proposal distributions that can approximate the posterior distribution reasonably well (Van Der Merwe et al. (2001)).

3. MULTIPLE MODEL FILTERS

In cases where the system is complex and it is not easy to model the system behaviour, multiple filters (each with a different system model) can run in parallel. The outputs of each filter are then fused to obtain more reliable estimates. Commonly used multiple model estimation methods in the literature are multiple model adaptive estimation (MMAE) (Hanlon and Maybeck (2000)) and interacting multiple model (IMM) (Seah and Hwang (2009)). The basic block diagrams of MMAE and IMM are presented in Fig. 1 and Fig. 2 respectively. Besides some differences, these methods rely on fusing the state estimation results of multiple filters by assigning a probability to each.

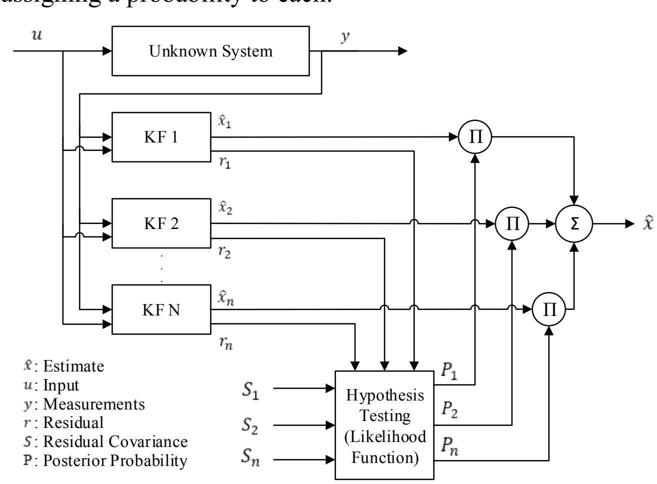

Fig. 1. MMAE Process (Martins (2006))

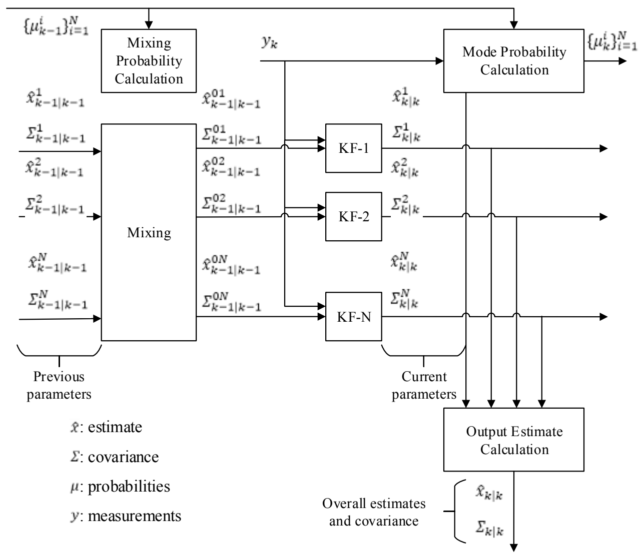

Fig. 2. IMM Process (Orguner (2013))

3.1.Multiple Model Adaptive Estimation (MMAE)

3.1.1. Kalman Filter Based MMAE

Kang-hua et al. (2007) use Multiple Model Adaptive Estimation (MMAE) method for MEMS-IMU/GPS Integrated Navigation System. Multiple KFs are run in parallel using different dynamic or stochastic models. To overcome the problem of converging to using only one of the filters, Tang et al. propose a modified version of MMAE (generalized residual Multiple Model Kalman Filter) which uses two sub-filters; first to estimate attitude and gyro errors, whereas the second for the position and velocity errors as well as the accelerometer biases. Authors claim that proposed method obtains a position accuracy <5m, velocity accuracy <0.1m/s and attitude error <0.5deg. Comparing the performance of proposed method with the classical KF, they claim to have improved the performance of classical KF.

Li et al. (2014) criticize MMAE for the excessive competition behaviour. Authors state that, because of excessive competition behaviour of MMAE among parallel models (filters), weight of the model closest to the true model increases to 1 rapidly, leaving the weight of other filters as 0 and approximating the multiple model to a single model. To overcome this behaviour, authors propose modifying MMAE with an exponential decay item introduced in the probability density function (PDF). A time-varying penalty value with an exponential decay item is introduced in the PDF. Tri-axial position and velocity error are estimated using both MMAE and modified MMAE and the 100m position and 5m/s velocity errors are improved as 20m and 2m/s respectively.

Xiong et al. (2015) introduce Robust Multiple Model Adaptive Estimation (RMMAE) method. To minimize parameter identification, authors use robust filtering method, stating that "*robust filtering approach guarantees an upper bound to the estimation error covariance*" (Xiong et al. (2015)). Robust Kalman Filter (RKF) is used as sub-filter. It is claimed that computational cost of RKF is roughly same as the EKF. For performance comparison, position and velocity of a spacecraft are estimated using EKF, UKF, RKF, MMAE and RMMAE. Position accuracy of RMMAE is reported to be ~x4 better compared to EKF and UKF, ~50% better compared to RKF and ~10% better compared to MMAE. Although the processing time is ~x20-x22 slower than EKF, UKF and RKF, it is almost the same as MMAE. Since the performance of RKF is similar to RMMAE, authors perform another test and compare the angular position accuracy of RMMAE with RKF. It is observed that the accuracy of RKF is dependent on the uncertainty and RMMAE angular accuracy is ~x2 better than RKF. Authors conclude that RMMAE performance is superior to EKF, UKF, RKF, MMAE and it is more stable.

Kottath et al. (2015) propose an alternative MMAE method (MMAE with Filter Stripping (SMMAE)). The key idea of SMMAE is to start with a large number of models, assigning equal weights to each model, later iteratively updating the model weights and eliminating the ones with a low weight factor. This way, the proposed method initially benefits the advantage of having many models which increases the accuracy, and finally the most effective models would be chosen among all models. Attitude measurement is performed to test the performance of SMMAE and a ~25% improvement is reported in comparison to classical MMAE.

Another modified MMAE method by Kottath et al. (2016) is Window based MMAE (WMMAE). This time, authors suggest using Innovation Adaptive Estimation (IAE) blocks instead of EKF in the MMAE scheme. IAE uses EKF but "*with an inbuilt framework to adapt process and measurement noise covariance parameters (Q and R) on the basis of the innovation or the residual sequence*" (Kottath et al. (2016)). Overall performance of MMAE is improved by improving the performance of each model using IAE instead of EKF and adding window size as an unknown parameter for noise covariance calculation. Weights of several IAE models which have different window sizes are adjusted by the proposed scheme. Performance of WMMAE is tested using attitude measurement and on average x5 improvement in error is reported compared to classical MMAE.

3.1.2.Particle Filter Based MMAE

Zhao et al. (2017) introduce Multiple Model Unscented Particle Filter (MMUPF). Proposed method replaces UKF inside MMAE with Unscented Particle Filter (UPF) to improve the accuracy. Performance comparisons among UKF, MMUKF, UPF, and MMUPF are performed and best results in terms of accuracy (~10% improvement over MMUKF) and highest calculation cost are obtained with MMUPF among these methods.

3.2. Interacting Multiple Model (IMM)

3.2.1. Kalman Filter Based IMM

Gao et al. (2017) propose fusing Adaptive Fading UKF (AFUKF) and Robust UKF (RUKF). The fusing scheme used for the proposed method is Interacting Multiple Model Estimation-Based Adaptive Robust UKF (IMM-ARUKF). Markov chain is used for probability transition between AFUKF and RUKF. Comparing the algorithm's Root Mean Squared Error (RMSE) with classical UKF, AFUKF and RUKF, IMM-ARUKF is claimed to have a strong ability to inhibit the disturbances on filtering due to system model uncertainties (Gao et al. (2017)). Position error performance of the proposed method is claimed to be significantly better compared to classical UKF, AFUKF and RUKF.

Taking the linear KF based scalar-weight IMM (SIMM) and matrix-weight IMM (MIMM) proposed in Fu et al. (2010) as a reference, Gao et al. (2012) propose improving these filters for nonlinear systems by using EKF and UKF. Gao et. al (2012) introduce EKF-SIMM, EKF-MIMM, UKF-SIMM and UKF-MIMM as the improved IMM algorithms and compare their performance. UKF-MIMM is reported to provide the lowest position and velocity errors with the highest computational cost among other introduced algorithms.

Wan et al. (2010) propose using Cubature Kalman Filter (CKF) instead of UKF in the IMM algorithm. Their experiments in estimating the position and velocity shows that IMMCKF has slightly better accuracy compared to IMMUKF with a small improvement over computation time.

Sun and Shen (2014) propose Optimal Mode Transition Matrix IMM (OMTM-IMM) algorithm. The main idea of this work is to derive optimal mode transition probabilities so that the accuracy of initial state and state transition increases. The authors report better position and velocity accuracy compared to IMM and around 20% increase in computational cost.

Zhu et al. (2016) use fifth-degree CKF (5CKF) (Jia et al. (2013)) in an IMM and introduce Interacting Multiple Models Five Degree Cubature Kalman Filter (IMM5CKF). In order to improve accuracy and response time, the authors add 5CKF after IMM. The authors compare the position and velocity errors with previous algorithms (IMMUKF, IMMCKF, 5CKF and OMTM-IMM) and claim to improve the performance of the closest alternative (IMMCKF) by ~45% in velocity errors. The position error performance of IMM5CKF is similar to that of IMMCKF. Nearly 100% increase is observed in the computation cost.

Liu and Wu (2017) improve IMM5CKF further and introduce Interacting Multiple Model Fifth-Degree Spherical Simplex-Radial Cubature Filter (IMM5thSSRCKF). Fifth-degree spherical simplex-radial rule is used for improving the filtering accuracy. Performance of the proposed algorithm is compared to IMMUKF, IMMCKF, and IMM5CKF. Position accuracy, velocity accuracy and computation time are improved by around 5% compared to IMM5CKF.

3.2.2. Particle Filter Based IMM

Zhai et al. describe the advantage of PF based MMPF over KF based IMM as follows: "*The IMM method only approximates the target model distribution (usually a Gaussian mixture) with a single Gaussian distribution in the merging step. However, the MMPF framework directly approximates the true target model distribution and generates particles from this distribution.*" (Zhai et al. (2015)). Wang et al. (2012) propose combining IMM with PF in order to obtain more robust method (IMMPF) for 3D target tracking for underwater wireless sensor network. The authors argue that the state of a dynamic system with several modes that switch from one to another can be estimated using mode likelihoods and mode transition probabilities provided by the IMM. In addition to this argument, Guo et al. (2008) state that although the standard IMM filters are useful for target tracking with weak non-linearity, they are not capable of tracking strongly non-linear and non-Gaussian targets; thus, IMM in combination with PFs are suggested.

Wang et al. (2012) use three different target motion models, constant velocity (CV), constant acceleration (CA), and coordinated turn (CT) in order to model the moving patterns of the underwater target. Each model is filtered separately and the estimates are combined through probability update for an overall state estimation. In order to evaluate the performance of the proposed IMMPF algorithm, authors present the results of PF, EKF and UKF on a predefined simulation scenario using only CV model first. It is observed that PF estimates for position are more accurate compared to EKF and UKF. Secondly, a combined trajectory is simulated using the combination of CV, CT and CA and this time, the performances of PF and the proposed algorithm (IMMPF) are compared, again, in terms of position errors in each axis and combined position error. The results indicate that IMMPF yields higher tracking precision. However, since the simulation scenario sequentially combines the target motion models (CV, CT, CA), the performance of the algorithm under a more realistic and highly non-linear scenario where CV, CT, CA are added upon is unknown. In addition, there is no data on the performance comparison with other multiple model estimation methods like MMAE and IMM alone.

The method proposed in by Guo et al. (2008) is similar to IMMPF, but instead of using standard PF, UKF based Unscented Particle Filter (UPF) is used. Authors implement the proposed Interacting Multiple Model Unscented Particle Filter (IMMUPF) method on a ground tracking problem. The performance of IMMUPF is compared with IMMPF, IMMEKF and IMMUKF in terms of root mean square errors (RMSEs) of position, velocity and acceleration in x and y axes. The simulation scenario combines CA and CV modes. Results indicate that IMMUPF achieves lower error rates, whereas the computation duration is substantially higher.

Hong-tao and Feng-ju (2015) propose Least Square Interacting Multiple Model Unscented Particle Filter (LSIMMUPF) algorithm. Authors use Least Squares method to pretreat Pitching and Azimuth angle data from the sensors, UKF for generating density function, PF for processing nonlinear non-Gaussian data and IMM for automatic bandwidth adjustment. Position and velocity accuracy of LSIMMUPF is compared with LSIMM and an improvement of 21%-33% is reported in position and velocity accuracy with a x4 computational cost.

4.CONCLUSION

KF is a widely used algorithm for the estimation problems. Its restriction of linear and Gaussian models have been overcome with the introduction of EKF, UKF and CKF. EKF and UKF provide first and higher order linearization approximations respectively for solving non-linear problems whereas CKF uses spherical-radial cubature rule to provide a solution for high-dimensional non-linear filtering problems. RKF provides an upper bound for estimation error covariance to handle model uncertainties. PF, on the other hand, deals with the restrictions of KF by solving non-linear systems with non-Gaussian noise (Ko et al. (2012)). Although the computational complexity of PF is higher, limiting the number of particles provides a better computational performance in addition to more accurate results compared to KF based solutions (Boers and Driessen (2003)). Multiple model filters are introduced for complex systems where it is not easy to model the system parameters. Multiple KFs run in parallel with different system models and the outputs are then fused to obtain more reliable estimates by assigning a probability to each filter. The most common methods used in the literature for multiple model estimation are Multiple Model Adaptive Estimation (MMAE) and Interacting Multiple Model (IMM). In MMAE, the residuals of the multiple filters are used to form the adaptive weights whereas IMM improves MMAE by reducing the complexity through mixing the initial condition of each filter at each time-step and obtaining lower mode estimation delays (Hwang et al. (2003)). There are both fixed structure (FSMM) and variable structure multiple models (VSMM). A basic taxonomy for the multiple model filters is presented in Fig.3. In this work, recent research on multiple model filters are reviewed and results of these works are presented. Table 2 presents an overview of the methods referenced in this paper. The methods, underlying multiple models, sub-filter types and claimed accuracy improvements in terms of position, velocity and attitude against the compared methods are provided in Table 2. Accuracy improvements in multiple axis are averaged and approximated. In general, it is observed that MMAE is used for navigation applications, whereas IMM is used for target tracking problems. Each of the methods have their own advantages and limitations. Since performance comes with a computational sacrifice, the method to be used should be chosen carefully, considering the characteristics of the system and computational resources available.

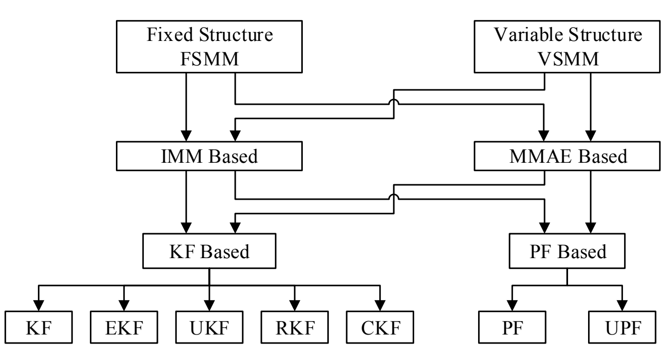

Fig. 3. Basic Taxonomy for Multiple Model Kalman and Particle Based Filters

Table 2. Accuracy Improvements of MM Methods

	Ref	Name	Sub Filter	Improvement %			Compared To
				Pos	Vel	Att	
MMAE	Kang-hua(2007)	Mod.MMAE	KF	45	95	10	KF
	Li(2014)	Mod.MMAE	EKF	80	60	-	MMAE
	Xiong(2015)	RMMAE	RKF	10	-	-	MMAE
	Kottath(2015)	SMMAE	EKF	-	-	25	MMAE
	Kottath(2016)	WMMAE	EKF	-	-	80	MMAE
	Zhao(2017)	MMUPF	UPF	-	-	10	MMUKF
IMM	Gao(2017)	IMM-ARUKF	AFUKF RUKF	55	-	-	UKF
				45	-	-	AFUKF
				40	-	-	RUKF
	Gao(2012)	EKF-SIMM EKF-MIMM UKF-SIMM UKF-MIMM	EKF UKF	55	75	-	IMMEKF
				25	45	-	IMMUKF
	Wan(2010)	IMM-CKF	CKF	15	20	-	IMMEKF
				1	1	-	IMMUKF
	Sun(2014)	OMTM-IMM	KF	50	80	-	IMM
	Zhu(2016)	IMM5CKF	CKF	5	45	-	IMMUKF
				2	45	-	IMMCKF
				35	75	-	OMTMIMM
	Liu(2017)	IMM5SSRCKF	CKF	3	5	-	IMM5CKF
	Wang(2012)	IMMPF	PF	40		-	PF
	Guo(2008)	IMMUPF	UPF	70	65	-	IMMUKF
				20	35	-	IMMPF
	Hong-tao(2015)	LSIMMUPF	UPF	20	30	-	LSIMM

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