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EXAMINATION OF INFANT MORTALITY RISK IN TURKEY WITH SPATIO-TEMPORAL BAYESIAN MODELS

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I dedicate this thesis to my dear youngest brother Merdol KILIÇ who passed away in November 2012.

ABSTRACT

Kılıç Yıldırım, S., Examination of Infant Mortality Risk in Turkey with Spatio-temporal Bayesian Models, Hacettepe University Graduate School of Health Sciences Doctor of Philosophy Thesis in Biostatistics, Ankara, 2020. In this thesis, it was aimed to determine relative risk (RR) of infant mortality for each province in Turkey between 2009 and 2017 years by including the concepts of space, time and space-time interaction, to obtain risk maps, and to examine the risk factors affecting. Spatio-temporal Bayesian models were implemented to estimate RR with integrated nested Laplace approximation in R software. Structured spatial and temporal random effects on RR were modeled with Gaussian Markov random fields, using intrinsic conditional autoregressive structure and random walk model, respectively. The best model was determined according to deviance information criterion (DIC). The major contribution to variability of RR explained with the best model was from unstructured spatial and structured temporal interaction random effect. From 2009 to 2017 the number of provinces with high RR, decreased. From 2009 to 2017 in each year consistently; significant risk areas clustered in eastern and southeastern Anatolia regions. Effects of gross domestic product (GDP) per capita, percentage of mothers aged under 20 and percentage of mothers aged over 39 on RR of infant mortality were examined with generalized linear model without concepts of space and time and with the best spatio-temporal Bayesian model. As GDP per capita increased, RR decreased for generalized model and spatio-temporal model. Whereas percentage of mothers aged under 20 and percentage of mothers aged over 39 increased, RR increased for generalized model. But percentage of mothers aged under 20 and percentage of mothers aged over 39 had no effect on RR for spatio-temporal model. Spatio-temporal Bayesian model can be more preferable than generalized model, because of having lower DIC than generalized model. Therefore, while determining the factors that may have an effect on RR of infant mortality, it is also important to consider the effects of space, time and space-time interaction.

Key words: Integrated nested Laplace approximation, spatio-temporal model, infant mortality, Gaussian Markov random field, structured effect.

ÖZET

Kılıç Yıldırım, S., Türkiye'de Bebek Ölüm Riskinin Mekan-zamansal Bayesci Modeller ile İncelenmesi, Hacettepe Üniversitesi Sağlık Bilimleri Enstitüsü Biyoistatistik Programı Doktora Tezi, Ankara, 2020. Bu tezde 2009 ve 2017 yılları arasında Türkiye'de illerdeki bebek ölüm göreli riskini mekan, zaman ve mekan-zaman etkileşimi ile belirlemek, risk haritaları elde etmek ve etkileyen risk faktörlerini incelemek amaçlanmıştır. R yazılımında bütünleşik iç içe Laplace yaklaşımı ile göreli riski kestirmek için mekan-zamansal Bayesci modeller uygulanmıştır. Göreli risk üzerindeki yapılandırılmış mekansal ve zamansal rasgele etkiler, sırasıyla içsel koşullu otoregresif yapı ve rasgele yürüyüş modeli kullanılarak Gauss Markov rasgele alanları ile modellenmiştir. En iyi model sapma bilgi kriterine göre belirlenmiştir. En iyi modelle açıklanan göreli riskin değişkenliğe; en büyük katkı yapılandırılmamış mekansal ve yapılandırılmış zamansal etkileşimi rasgele etkisinden kaynaklanmaktadır. 2009'dan 2017'ye, yüksek göreli riske sahip illerin sayısı azalmıştır. 2009’dan 2017’ye her yıl sürekli olarak önemli risk alanları Doğu ve Güneydoğu Anadolu bölgelerinde kümelenmiştir. Kişi başına gayrisafi yurt içi hasıla (GSYH), 20 yaş altı annelerin yüzdesinin ve 39 yaş üstü annelerin yüzdesinin göreli risk üzerindeki etkileri, mekan ve zaman kavramları olmadan genelleştirilmiş doğrusal model ve en iyi mekan-zamansal Bayesci model ile incelenmiştir. Genelleştirilmiş model ve mekan-zamansal model için kişi başına GSYH arttıkça, göreli risk azalmıştır. Genelleştirilmiş model için 20 yaş altı annelerin yüzdesi ve 39 yaş üstü annelerin yüzdesi arttıkça, göreli risk artmıştır. Fakat mekan-zamansal model için 20 yaş altı annelerin yüzdesinin ve 39 yaş üstü annelerin yüzdesinin göreli risk üzerinde etkisi olmamıştır. Mekan-zamansal Bayesci model; genelleştirilmiş modelden daha düşük sapma bilgi kriterine sahip olduğundan, genelleştirilmiş modelden daha çok tercih edilebilir. Bu nedenle, bebek ölüm göreli riskini etkileyebilecek faktörler belirlenirken; mekan, zaman ve mekan-zaman etkileşiminin etkilerini de dikkate almak önemlidir.

Anahtar kelimeler: Bütünleşik iç içe Laplace yaklaşımı, mekan-zamansal model, bebek ölüm, Gauss Markov rasgele alanı, yapılandırılmış etki.

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LIST OF ABBREVIATION

DIC	Deviance information Criterion
$\bar{D}$	Mean of Deviance
INLA	Integrated Nested Laplace Approximation
$iCAR$	Intrinsic Conditional Autoregressive
GMRF	Gaussian Markov Random Field
GDP	Gross Domestic Product
MCMC	Markov Chain Monte Carlo
p_D	Effective Number of Parameters
rw1	Random Walk Model of Order 1
rw2	Random Walk Model of Order 2
sd	Standart Deviation
SMR	Standardized Mortality Ratio

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1. INTRODUCTION

With the increase in the accessibility of data including information about space and time, there are many applications of spatio-temporal models in recent studies of different fields. Spatio-temporal models are used in the assessment of risk in the health field (mortality, incidence, suicide etc.). It is important to consider the data according to geographical location and time in spatio-temporal models. Areas which are close to each other may be similar with respect to the risk. This similarity can be also seen for time points. Spatio-temporal models include spatial effect, temporal effect, and spatio-temporal interaction effect to estimate risk and to determine risk factors. To reveal spatial and spatio-temporal differences, risk maps are used (1, 2).

If uncertainty in estimations is taken into account by Bayesian approaches, these models are called Bayesian spatio-temporal models. In the Bayesian approach; it is assumed that the observed data has a certain distribution and the parameter of the distribution is unknown. To express the uncertainty about the unknown parameter, a prior distribution is specified. This prior distribution is combined with the likelihood function of observed data to obtain the posterior distribution of the unknown parameter. In the Bayesian approach, all inference procedures are made based on the posterior distribution. To summarize the posterior distribution, Monte Carlo method is used. But in cases where the parameter is high-dimensional and the posterior distribution is difficult to sample directly, Markov Chain Monte Carlo (MCMC) methods are used to summarize the posterior distribution. Bayesian approaches have been used in spatial and spatio-temporal modeling with MCMC methods becoming known. Since MCMC method is a sampling strategy from posterior distribution, convergence of posterior samples should occur. Time and extensive simulations are required for convergence. During Bayesian inference with MCMC methods; the convergence problem and long-term calculations may be encountered, so Integreated Nested Laplace Approximation (INLA) method is used as an alternative to MCMC methods. (2, 3, 4)

In this thesis, relative risk of infant mortality for provinces from 2009 to 2017 in Turkey was estimated by spatio-temporal Bayesian models with using INLA.

1.1. Hypotheses of This Thesis

The hypotheses for modeling relative risk of infant mortality were:

- Geographical location of provinces and time might have an effect on relative risk of infant mortality.
- Geographic location and time interaction might have an effect on relative risk of infant mortality.

1.2. Objectives of This Thesis

The objectives of this thesis were:

- To specify spatio-temporal structure of relative risk of infant mortality in Turkey by using different spatio-temporal Bayesian models,
- To estimate relative risks of infant mortality for provinces and years.
- To present map of relative risks of infant mortality for years based on posterior means of the estimated relative risks of infant mortality for provinces and years,
- To present risk clusters based on posterior distributions of the relative risks of infant mortality for provinces and years,
- To determine effects of gross domestic product per capita ($) for provinces and years and percentage mothers aged under 20 and over 39 for provinces and years on relative risk of infant mortality.

It is clear that risk map will be useful in assessing the effectiveness of health policies and determining future health policies.

1.3. Organization of This Thesis

General information: Types of spatial data, the fundamental structure of Bayesian spatial and spatio-temporal models for areal data, literature review are given.

Material and method: The data and spatio- temporal Bayesian models used for estimation of relative risk of infant mortality are expressed.

Results: The results are given with posterior means and maps.

Discussion: Obtained results are discussed.

Conclusion and recommendation: The conclusions are expressed based on spatio-temporal structure of relative risk of infant mortality and factors which have an effect on relative risk of infant mortality. The recommendations to institutions are given.

2. GENERAL INFORMATION

2.1. Bayesian Inference

Bayesian inference is the determination of the probability distribution of the data set which is studied and summarization of the results based on posterior distribution of unknown parameter which belongs to the distribution of the data set (5).

2.1.1. Likelihood Function

Observations in data set y are represented with $y_1, \ldots\ldots, y_n$. $\pi(.)$ is used to express probability distribution function. The likelihood function of the data set y is the joint probability of $y_1, \ldots\ldots, y_n$ with unknown parameter (θ). The data set is modeled using a probability distribution function $\left(\pi(y_i|\theta)\right)$. With the assumption that the observations of the data set are independent, the likelihood function (6) is expressed as follows:

$$L(\theta|y) = \prod_{i=1}^{n} \pi(y_i|\theta) \tag{2.1}$$

2.1.2. Prior Distribution

In Bayesian approach; parameter of the probability model of a data set is unknown and treated as uncertain. The uncertainty in the values of the unknown parameter is specified by a prior distribution before a data set is observed (6).

Prior distribution for unknown parameter is determined according to whether or not to have information about the unknown parameter. If there is no prior information or the prior information is difficult to obtain about the unknown parameter, it is preferable to use prior distribution which is noninformative (vague, flat) prior distribution. The range and behaviour of the unknown parameter have an effect on choosing the non-informative priors. Prior distribution on positive axis is chosen for unknown variance parameter. Hence; Gamma, inverse Gamma or uniform families are appropriate prior distributions for unknown variance parameter. For the parameters found in the finite range, for example the coefficient of regression, the zero-mean Gaussian distribution is the appropriate prior distribution (2, 6, 7).

If the previous investigation about the unknown parameter is available, the result of the previous investigation is used to determine the prior distribution of the unknown parameter. This prior distribution is informative prior distribution.

If prior and posterior distributions of the unknown parameter belong to same family, this type of prior distribution is called conjugate prior. For example; the data set follows a Binomial distribution. The unknown parameter of this distribution is probability of the occurence of outcome. The prior distribution of the unknown parameter is specified as Beta distribution. After combining likelihood function of the data set with the prior distribution, posterior distribution of this unknown parameter is determined as Beta distribution. Therefore the prior distribution of this unknown parameter is conjugate prior (2, 6).

2.1.3. Posterior Distribution

The prior distribution is updated with the likelihood function of the data set to get posterior distribution. So the posterior distribution of unknown parameter $\left(\pi(\theta|y)\right)$ is calculated as follows,

$$\pi\ (\theta|y) \propto L(\theta|y) * \ \pi(\theta) \tag{2.2}$$

where $\pi(\theta)$ is prior distribution of the unknown parameter (θ) and $L(\theta|y)$ is likelihood function of the data set (2, 6).

2.1.4. Summary Statistics of Posterior Distribution

Posterior mean, posterior median and credibility interval are summary statistics of posterior distribution.

Posterior mean for a continuous parameter (θ) is calculated as follows

$$\mathrm{E}\ (\theta|y) = \int_{\theta\epsilon\Theta} \theta\pi(\theta|y)d\theta \tag{2.3}$$

where θ takes all possible values from Θ. If parameter is discrete, sum is used instead of integral (3).

Median of posterior distribution $(\theta_{0.5})$ which divides probability distribution into two equal parts is expressed as follows (3):

$$\pi\ (\theta \leq \theta_{0.5}|y) = 0.5 \text{ and } \pi\ (\theta \geq \theta_{0.5}|y) = 0.5 \tag{2.4}$$

The 95% credibility interval (2) includes interval between 0.025 quantile $(\theta_{0.025})$ and 0.975 quantile $(\theta_{0.975})$ which are defined as follows:

$$\pi\ (\theta \leq \theta_{0.025}|y) = 0.025 \text{ and } \pi\ (\theta \geq \theta_{0.975}|y) = 0.025 \tag{2.5}$$

2.2. Spatial Data

When a particular subject is examined in terms of space, a data set $\{y(s_1), \dots., y(s_n)\}$ which is analysed must be collected from each spatial unit $\{s_1, \dots., s_n\}$. The observation $y(s_i)$ $(i = 1, \dots., n\,)$ taken from each spatial unit is considered as a random variable, so the spatial data are obtained as a result of this random (stochastic) process. The spatial data indexed by space where D is a subset of $\mathbb{R}^d$ is expressed as follows:

$$Y(s) = \{y(s), s \in D\} \tag{2.6}$$

where D can be a continuous surface or a countable collection of d-dimensional units of space (3).

In spatial literature there are three types of spatial data: areal and lattice data, point-referenced (or geostatistical) data, spatial point pattern data. These types of data are described as follows:

- Areal and lattice data: The observation $y(s_i)$ is collected from each area s_i $(i = 1, \dots., n)$ which is defined by boundary. If the area is irregular and defined by administrative (district, region, county, etc.) boundry, the corresponding data set is called areal data. If the area is regular, the corresponding data set is called lattice data. The obtainment of the data set process is evaluated as spatially discrete random process; for this process $y(s_i)$ represents the gathered or averaged value for each s_i (3, 8).

- Point-referenced (or geostatistical) data: If the observation $y(s_i)$ is collected from each point s_i $(i = 1, \ldots.., n)$ which is represented by latitude and longitude the corresponding data set is point-referenced (or geostatistical) data. The obtainment of the data set is specified as spatially continuous process (3, 8).
- Spatial point pattern data: At the location s_i $(i = 1, \ldots.., n)$, the occurence of the event is represented by $y(s_i)$. If the event occurs $y(s_i) = 1$; otherwise $y(s_i) = 0$. If the observation $y(s_i)$ is collected from each location s_i $(i = 1, \ldots \ldots, n)$, the corresponding data set is called spatial point pattern data (3, 8).

2.3. Spatio-temporal Data

When a particular subject is investigated in terms of space and time; spatio-temporal data set $y = \{y(s_1, t_1), \ldots\ldots, y(s_n, t_T)\}$ which is analysed must be collected from n spatial units (point, area or location) $(s_1, \ldots\ldots, s_n)$ and T time points $(t_1, \ldots \ldots., t_T)$. The spatio-temporal data is expressed as follows (3):

$$Y(s,t) = \{y(s,t), (s,t) \in D \in \mathbb{R}^2 * \mathbb{R}\} \qquad (2.7)$$

2.4. Latent Gaussian Models

Observations in the data set y are represented with $y_1, \ldots\ldots, y_n$. When the observation (y_i) $(i = 1, \ldots, n)$ is assumed to follow one of the distributions of exponential family (Gaussian, gamma, exponential, binomial, Poisson); the unknown parameter (θ_i) of the distribution is modeled with structured additive regression model through a specific link function $g(.)$. The structured additive predictor η_i which is unknown parameter with link function $g(\theta_i)$ is modeled as:

$$\eta_i = g(\theta_i) = \alpha + \sum_{m=1}^{M} \beta_m x_{mi} + \sum_{l=1}^{L} f_l(z_{li}) + \epsilon_i \qquad (2.8)$$

In this model f_l functions represent the nonlinear effect of covariates, time trends, space, time and space-time interaction, β_m represent the linear effect of covariates. Assume that α, β_m, f_l and η_i are gathered under Gaussian Markov Random field x. Each component of $x = \{\eta_i, \alpha, \beta_m, f_l\}$ is assigned a Gaussian prior to get Latent Gaussian model. Note that η_i is included in x instead of ϵ_i. ψ is termed as hyperparameter which is parameter of this latent process (4, 9,1 0).

Latent Gaussian models are considered as a three-stage hierarchical model which are observed data, Gaussian Markov random field, hyperparameters:

Observed Data (y): The distribution of observed data set $\big(y = (y_1, y_2, \dots \dots y_n)\big)$ is calculated according to assumption is that each observation (y_i) is conditionally independent given each Gaussian Markov random field x_i and hyperparameters (ψ_1). The distribution of observed data set $\big(\pi(y\,|x, \psi_1)\big)$ is expressed with likelihood function as follows:

$$\pi(y\,|x, \psi_1\,) = \prod \pi(y_i|x_i, \psi_1) \qquad (2.9)$$

When the observations of the data set are evaluated with taking the concept of space into account in some applications, the observations are correlated. Because of this, the observations of the data set would not unconditionally independent (7).

Gaussian Markov Random Field (GMRF) $(\boldsymbol{x})$: The GMRF (x) is a random vector which has a Gaussian (Normal) distribution with mean μ and covariance (inverse pericision matrix) $\Sigma = Q^{-1}$.

$$x|\psi_2 \sim \mathcal{N}(\mu, Q^{-1}(\psi_2)) \qquad (2.10)$$

x includes η_i, α, β_m and f_l. It is assumed that the distribution of x is Gaussian with zero mean and covariance (inverse precision matrix $Q^{-1}(\psi_2)$) (4, 9).

The precision matrix of x ; (Q),, includes precision matrix of each element of x. Precision matrix of each element of x depends on neighboring structure and unknown precision (inverse variance) of each element of x (9).

Let $g = (g_1, g_2, \ldots\ldots, g_n)$ is a GMRF. The conditional independence structure in g is determined according to markov properties of GMRF. Indirect graphs are used to express this independence structure. For undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$; the set of nodes (1,2,….n) in the graph is $\mathcal{V}$ and the set of edges $\{i, j\}$ is $\mathcal{E}$. The neighbors of node i are $ne(i) = \{j \in \mathcal{V} : \{i, j\} \in \mathcal{E}\}$.

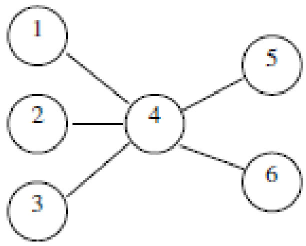

Figure 2.1. Undirected graph with 6 nodes (12).

The structure for Figure 2.1. is given as follows:

$\mathcal{V} = \{1,2,3,4,5,6\}$ and $\mathcal{E} = \{(1,4), (2,4), (3,4), (4,5), (4,6)\}$

$ne(1) = 4$, $ne(2) = 4$, $ne(3) = 4$, $ne(5) = 4$, $ne(6) = 4$,

$ne(4) = \{1,2,3,5,6\}$

The local Markov property:

$$g_i \perp g_{-\{i,ne(i)\}} | \, g_{ne(i)} \quad for \;\; every \, i \in \mathcal{V} \tag{2.11}$$

For this property g for i th area (node i) is independent from all g's for the other areas in where area i and $ne(i)$ aren't included.

The pairwise Markov property:

$$g_i \perp g_j | g_{-ij} \quad if \; \{i, j\} \notin \mathcal{E} \text{ and } \; i \neq j \tag{2.12}$$

For this property if there is no edge between node i and node j, Q_{ij} is zero (because of conditional independence structure). If the edge between node i and node j is $\{i, j\} \in \mathcal{E}$, node i and node j are neighbors (i~j). Hence Q_{ij} is not zero (11, 12).

Hyperparameters $(\boldsymbol{\psi} = (\boldsymbol{\psi_1}, \boldsymbol{\psi_2})$**:** Prior distribution is assigned to hyperparameter (4, 9).

2.5. Integrated Nested Laplace Approximation (INLA)

The joint posterior distribution of GMRF and hyperparameters is calculated as follows (10):

$$\pi(x, \psi | y)\ \alpha\ \ \pi(x, \psi)\ \pi(y|x, \psi) \alpha\ \ \pi(x|\psi)\pi(\psi) \prod_{i=1}^{n} \pi(y_i|x_i, \psi)$$

$$\alpha\ \pi(\psi)\ |Q(\psi)|^{n/2} exp \left[-\frac{1}{2} x^T Q\ (\psi)\ x + \sum_{i=1}^{n} \log\{ \pi(y_i|x_i, \psi) \right] \tag{2.13}$$

Posterior distributions of GMRF and hyperparameters are calculated with following integrals (10):

$$\pi(x_i|y) = \int \pi\ (x_i|\psi, y)\ \pi\ (\psi|y)\ d\psi \quad i = 1, 2, \dots., n \tag{2.14}$$

$$\pi\big(\psi_j|y\big) = \int \pi\ (\psi|y)\ d\psi_{-j} \quad j = 1, 2, \dots., m \tag{2.15}$$

If the dimension of vector of hyperparameters is small, it is possible to get posterior distribution by using Equation 2.14 and Equation 2.15. Otherwise the posterior distribution of GMRF becomes approximate posterior distribution by INLA as follows:

$$\tilde{\pi}(x_i|y) = \sum \tilde{\pi}\big(x_i\big|\psi^{(k)}, y\big)\ \tilde{\pi}\ \big(\psi^{(k)}\big|y\big)\ \triangle\ \psi^{(k)} \tag{2.16}$$

The INLA method is examined in two main sections to obtain approximate posterior distributions:

- Obtain $\tilde{\pi}(\psi|y)$, explore $\tilde{\pi}(\psi|y)$ to determine good points $\psi^{(k)}$ and calculate area weights $\triangle\ \psi^{(k)}$
- Obtain $\tilde{\pi}(x|\psi, y)$ and calculate for good points $\psi^{(k)}$

$\tilde{\pi}(\psi|y)$; which is equivalent to Tierney and Kadane's Laplace approximation of a posterior distribution is obtained with the Formula 2.17,

$$\tilde{\pi}\,(\psi|y)\alpha\ \frac{\pi\,(y|x,\psi)\pi\,(x|\,\psi)\,\pi\,(\psi)}{\pi_G\,(x|\,\psi,y)}\Big|_{x=x^*(\psi)} \tag{2.17}$$

where $x^*(\psi)$ is the mode of $\pi(x|\psi, y)$ for a given ψ (2,10).

The denominator of Formula 2.17 is Gaussian approximation of Formula 2.18.

$$\pi(x|\psi, y) \propto\ exp\{-\frac{1}{2}\,x^T Q\,x + \sum_{i=1}^{n} g_i\,(x_i)\} \tag{2.18}$$

where $g_i(x_i) = \log\{\pi(y_i|x_i, \psi)\}$. The mode of $\pi(x|\psi, y)$ is obtained with Newton-Raphson method. This Gaussian approximation is the result of matching mode and curvature of $\pi(x|\psi, y)$ at the mode (10).

The strategy to explore $\tilde{\pi}(\psi|y)$ is determined according to the dimension of vector of hyperparameters. Grid strategy is used when the dimension is lower than 4, else central composite design is used (2). $\log\tilde{\pi}(\psi|y)$ is optimised with Quasi-Newton method with respect to ψ to locate the mode of $\tilde{\pi}(\psi|y)$. With grid strategy it is determined grid of points for ψ covering the region where great part of probability mass of $\tilde{\pi}(\psi|y)$ is located. Central composite design is a type of response surface design. In this design points are laid in space to perform a second order approximation to response variable, and then the response surface is estimated at each point (18). With central composite designs, the factorial or fractional design is expanded with adding star (axial) points used for estimating curvature and center point (19). A second order model is obtained for response surface with curvature. The shape of response surface and region where the optimal response occurs are determined with central composite design (20, 21).

After having good point $\psi^{(k)}$, the marginal posterior for each hyperparameter $(\pi(\psi_j|y))$ is calculated using interpolation strategy according to the strategy used to explore $\tilde{\pi}(\psi|y)$.

$\tilde{\pi}(x_i|\psi, y)$ can be obtained with three types of approximations which are examined as follows:

Using Gaussian Approximation:

$\tilde{\pi}_G(x|\psi, y)$ is computed to obtain $\tilde{\pi}(\psi|y)$. Gaussian approximation to $\pi(x_i|\psi, y)$ is $\pi_G(x_i|\psi, y)$. $\mu_i(\psi)$ (mean of $\pi_G(x_i|\psi, y)$) is obtained from mean of $\pi_G(x|\psi, y)$ and $\sigma_i^2(\psi)$ (variance of $\pi_G(x_i|\psi, y)$) is calculated from precision matrix of $\pi_G(x|\psi, y)$ (10).

Using Laplace Approximation:

The Laplace approximation is expressed with Formula 2.19.

$$\tilde{\pi}_{LA}(x_i|\psi, y)\alpha \left.\frac{\pi(x,\psi,y)}{\tilde{\pi}_{GG}(x_{-i}|x_i, \psi, y)}\right|_{x_{-i}= x_{-i}^*(x_i,\psi)} \tag{2.19}$$

The expression $\tilde{\pi}_{GG}$ in the denominator of Formula 2.19 is the Gaussian approximation to $x_{-i}|\ x_i, \psi, y$ and $x_{-i}^*(x_i, \psi)$ is its mode. As the denominator of Formula 2.19 is calculated for each value of x_i and ψ and its precison matrix depends on x_i and ψ, it is difficult to compute. Hence the difference between log-density of $\tilde{\pi}_{LA}(x_i|\psi\ , y)$ and $\tilde{\pi}_G(x_i|\psi\ , y)$ is calculated at selected points and the cubic spline is fitted to difference. And this cubic spline is normalized by applying quadrature integration. The density of $\tilde{\pi}_{LA}(x_i|\psi, y)$ is expressed as follows (10);

$$\tilde{\pi}_{LA}(x_i|\psi, y)\alpha\ \mathcal{N}\{x_i; \mu_i(\psi), \sigma_i^2(\psi)\} * \exp\{cubic\ spline\ (x_i)\} \tag{2.20}$$

Using Simplified Laplace Approximation:

Simplified Laplace approximation $\tilde{\pi}_{SLA}(x_i|\psi, y)$ is obtained with serial Taylor expansion of $\tilde{\pi}_{LA}(x_i|\psi, y)$ around x_i=$\mu_i(\psi)$. With simplified Laplace approximation, this expansion makes correction on Gaussian approximation for location and skewness. The accuracy and short computation time make this approximation standart option (2, 3).

2.6. Disease Mapping

The spatial pattern of disease is determined with disease mapping. The mortality of disease for each geographical region is specified with standardized mortality ratio (SMR) for disease mapping. Direct use of SMR for large geographical regions (countries or states) is reliable, but not for small regions like counties, provinces. The SMR for each region is equal to observed number of deaths for each region is divided by expected number of deaths for each region. It is calculated without taking variability of different population sizes for different regions (the expected number for the large population size is larger than for the small population size) and spatial patterns of regions (number of observed deaths is 0; SMR is 0 no risk). Hence direct use of SMR doesn't reflect the relative risk of disease. It is reliable to use the smooted esimates of SMR which is produced the result hierarchical Bayes approach which accounts spatial dependency (15, 16).

2.7. Spatial Bayesian Models

(y_i) represents the observed number of cases in area i $(i = 1,2, \ldots., n)$, the observed number of cases for each area is assumed to have a Poisson distribution with mean $(E_i\theta_i)$. The expected number of cases for each area (E_i) is calculated and the unknown relative risk for each area (θ_i) is modeled with structured additive model by using link function (log) (12).

$$\eta_i = \log(\theta_i) = \alpha + \sum_{l=1}^{L} f_l(z_{li}) \tag{2.21}$$

2.7.1. Besag Model

The structured spatial random effect $f_1(z_{1i}) = u_i$ is modeled with intrinsic conditional autoregressive structure $(iCAR)$ in Besag model.

$$\eta_i = \log(\theta_i) = \alpha + f_1(z_{1i}) \tag{2.22}$$

$$u_i|u_{-i},\tau_u \sim \mathcal{N}\left(\frac{1}{n_i}\sum_{j\sim i}u_j\,,\frac{1}{n_i}\tau_u^{-1}\right) \tag{2.23}$$

where;

n_i; the number of neighbors of area i,

$j\sim i$; j and i are neighbors,

τ_u; is unkown precision of structured spatial random effect (3, 11,12).

The main consideration to construct intrinsic conditional autoregressive structure is based on independent increment. Area i and area j are neighbors, a normal increment is defined as follows:

$$(u_i - u_j) \sim \mathcal{N}(0, \tau_u{}^{-1}) \tag{2.24}$$

The assumption of independent increments provides the density for vector u=($u_1, u_2, \ldots., u_n$) with Equation 2.25.

$$\pi(u|\tau_u)\;\alpha\;\tau_u^{(n-1)/2} exp\left(\frac{\tau_u}{2}\sum_{i\sim j}(u_i - u_j)^2\right) = \tau_u^{(n-1)/2} exp\left(-\frac{1}{2}u^T Q\, u\right) \tag{2.25}$$

where $Q = \tau_u R$.

The joint distribution is expressed as follows

$$u|\tau_u \sim \mathcal{N}(0, \frac{1}{\tau_u}\; R^{-1}) \tag{2.26}$$

where τ_u is unknown precision and R is structure matrix of structured spatial random effect. If area i and area j are neighbors (i~j), R_{ij} is -1. If i is equal to j, R_{ij} is the number of neighbors to area i. And otherwise R_{ij} is zero (10, 13).

2.7.2. Besag-York-Mollie (BYM) Model

The BYM model was revealed by Besag et al. for disease mapping based on areal data.

$$\eta_i = \log(\theta_i) = \alpha + f_1(z_{1i}) + f_2(z_{2i}) \tag{2.27}$$

The BYM model included two random effects as structured spatial effect and unstructured spatial effect. $f_1(z_{1i}) = u_i$ and $f_2(z_{2i}) = v_i$ are two spatial random effect; structured, unstructured respectively. So it is seen that the BYM model is the combination of the Besag model with unstructured spatial random effect (3, 12, 13).

The unstructured spatial effect v_i reflects heterojenity effects between areas (14). The unstructured spatial effect is modeled with an exchangeable Gaussian prior. Gaussian prior follows identically and independently Normal distribution with zero mean and unknown precision of unstructured spatial random effect (τ_v) (2) given as follows:

$$v_i \sim \mathcal{N}(0, 1/\tau_v) \tag{2.28}$$

If the observation from each area i (i=1,…n) is modeled with BYM, the 2n*2n precision matrix of $\left((u+v), u\right)$ is given by Equation 2.29,

$$Q = \begin{pmatrix} \tau_v I & -\tau_v I \\ -\tau_v I & \tau_u R + \tau_v I \end{pmatrix} \tag{2.29}$$

where τ_u and τ_v are unknown precisions and I is identity matrix (12).

2.8. Spatio-temporal Models

(y_{it}) is the observed number of cases in area i $(i = 1, 2, \ldots., n)$ and time point t $(t = 1, 2, \ldots, T)$. The observed number of cases for each area and time point is assumed to have a poisson distribution with parameter $(E_{it}\theta_{it})$. Expected number of cases (E_{it}) for each area and time point is calculated and unknown relative risk for each area and time point (θ_{it}) is modeled with structured additive model using link function (log). For this case the spatial model is exdended to spatio-temporal model by including temporal effect as given with Equation 2.30. This model can be parametric or nonparametric.

$$\eta_{it} = \log(\theta_{it}) = \alpha + u_i + v_i + temporal_t \tag{2.30}$$

The sequence of T graphs is presented as in Figure 2.2. The black node is x_{it}. Spatial neighbors of x_{it} are spatially close $\{x_{it}: i{\sim}j\}$ so there are 4 neighbors. Addition to spatial neighbors, neighbors of x_{it} by taking the time concept into account are the previous time step $x_{(i,t-1)}$ and next time step $x_{(i,t+1)}$ of black node x_{it} (11).

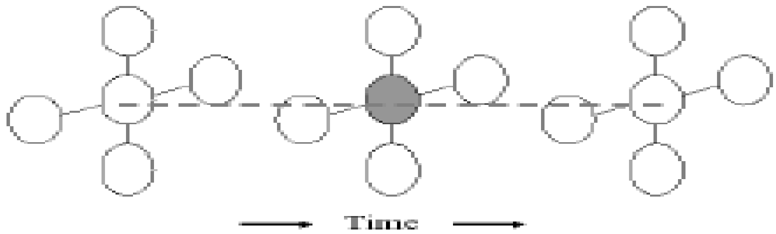

Figure 2.2.The neighborhood structure in a spatio-temporal pattern (11).

2.8.1. Parametric Spatio-temporal Bayesian Model

Parametric model contains area-wide temporal trend (β) as a linear effect of time and the differential trend (ζ_i) for each area i $(i = 1, 2, \dots, n)$ which represents difference between area-wide (β) trend and area-specific trend. If the differential trend (ζ_i) is under 0, the area-wide trend is determined to be less than the area-specific trend. If the differential trend (ζ_i) is above 0, the area-wide trend is determined to be more than the area-specific trend.

$$\eta_{it} = \log(\theta_{it}) = b_0 + u_i + v_i + (\beta + \zeta_i) * t \tag{2.31}$$

It is assumed that differential trend (ζ_i) is modeled as independent with Gaussian prior zero mean with unknown precision (3).

$$\zeta_i \sim \mathcal{N}\left(0, 1/\tau_\zeta\right) \tag{2.32}$$

2.8.2. Nonparametric Spatio-temporal Bayesian Models

In parametric spatio-temporal Bayesian model; temporal trend is examined with linear effect of time (fixed effect), but in nonparametric spatio-temporal Bayesian models; temporal trend is examined with structured temporal effect (random effect) and unstructured temporal effect (random effect). The structured and unstructured temporal effects are added to spatial model to have a spatio-temporal model, as given in Equation 2.33.

$$\eta_{it} = \log(\theta_{it}) = b_0 + u_i + v_i + \gamma_t + \phi_t \tag{2.33}$$

The structured temporal effect is modeled with random walk according to neighboring structure of time. With the assumption of independent incements ($\Delta\gamma_t$), the random walk model of order 1 (rw 1) for vector γ= (γ_1,.....,γ_T) is expressed as follows:

$$\triangle\, \gamma_t = \gamma_t - \gamma_{t-1} \sim \mathcal{N}\left(0, \tau_\gamma^{-1}\right) \tag{2.34}$$

$$\pi\left(\gamma|\tau_\gamma\right) \alpha\; \tau_\gamma{}^{(n-1)/2} \exp\left\{-\ \frac{\tau_\gamma}{2} \sum (\triangle\ \ \gamma_t)^2\right\} = \tau_\gamma^{(t-1)/2} \exp\{-\frac{1}{2}\gamma^T \mathcal{Q}\, \gamma\} \tag{2.35}$$

where $\mathcal{Q} = \tau_\gamma R$ and R is structure matrix represents neighboring structure of γ and τ_γ is unknown precision.

With the assumption of independent second order incements ($\Delta\gamma_t^2$), the random walk model of order 2 (rw 2) for vector γ= (γ_1,.....,γ_T) is expressed as follows:

$$\triangle\, \gamma_t^2 = \gamma_t - 2\gamma_{t+1} +\ \gamma_{t+2} \sim \mathcal{N}\left(0, \tau_\gamma^{-1}\right) \tag{2.36}$$

$$\pi\left(\gamma|\tau_\gamma\right) \alpha\; \tau_\gamma{}^{(n-2)/2} \exp\{-\ \frac{\tau_\gamma}{2} \sum (\triangle^2\ \gamma_t)^2\} = \tau_\gamma^{(t-2)/2} \exp\{-\frac{1}{2}\gamma^T \mathcal{Q}\, \gamma\} \tag{2.37}$$

where $\mathcal{Q} = \tau_\gamma R$ and R is structure matrix represents neighboring structure of γ and τ_γ is unknown precision.

The joint distribution for γ= (γ_1,.....,γ_T) is given as follows:

$$\gamma|\tau_\gamma \sim \mathcal{N}(0, \frac{1}{\tau_\gamma}\ R^{-1}) \tag{2.38}$$

The unstructured temporal effect is modeled with an exchangeable Gaussian prior; identically and independent distributed with zero mean and unknown precision (τ_{ϕ}) (2) given as follows:

$$\phi_i \sim \mathcal{N}(0, 1/\tau_{\phi}) \tag{2.39}$$

The spatio-temporal interaction effect (δ_{it}) is included in model according to combinations of spatial effect (structured, unstructured) and temporal effect (structured, unstructured) (four types of spatio-temporal interaction). With the model given equation 2.40, any differences in the time trends for different areas and any differences in spatial trends for different time points can be determined.

$$\eta_{it} = \log(\theta_{it}) = b_0 + u_i + v_i + \gamma_t + \phi_t + \delta_{it} \tag{2.40}$$

The structure matrix of spatio-temporal interaction effect is defined by using Kronecker product (⊗)(11).

The Kronecker product of two matrixes is calculated with Equation 2.41 using matrix A and matrix B (2).

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}; A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & a_{13}B \\ a_{21}B & a_{22}B & a_{23}B \\ a_{31}B & a_{32}B & a_{33}B \end{pmatrix} \tag{2.41}$$

Type 1 interaction:

For the model given by Equation 2.40; it is assumed that δ_{it} is interaction of unstructured spatial effect and unstructured temporal effect. The structure matrix of δ is defined with Equation 2.42 where I is identity matrix.

$$R_{\delta} = R_v \otimes R_{\phi} = \mathrm{I} \otimes \mathrm{I} = \mathrm{I} \tag{2.42}$$

The precision matrix of δ (Q) is $\tau_{\delta}\ R_{\delta}$; where τ_{δ} is unknown precision (inverse of variance). With assumption that all interaction effects are independent without spatial and temporal structured effect; the density of $\delta = (\delta_{11}, \delta_{12}, \dots \dots \delta_{nT})'$ is expressed with Equation 2.43.

$$\pi(\delta|\tau_\delta)\ \alpha\ exp\left(-\frac{\tau_\delta}{2}\sum_{i=1}^{n}\sum_{t=1}^{T}(\delta_{it})^2\right) = exp\left(-\frac{1}{2}\ \delta^T Q\ \delta\right) \quad (2.43)$$

The distribution of joint interaction effects δ is given as follows (2, 22):

$$\delta|\ \tau_\delta \sim \mathcal{N}(0, \frac{1}{\tau_\delta} R_\delta^{-1}) \quad (2.44)$$

Type 2 interaction:

For the model given by Equation 2.40; the assumption is the interaction of unstructured spatial effect and structured temporal effect is δ_{it}. For Clayton's rule each δ_i= (δ_{i1},……,δ_{iT})' (i =1,…,n) is modeled with a random walk, independent from other areas. Each area has different temporal trend, but do not have any spatial structure. $\delta = (\delta_{11}, \delta_{12}, \delta_{13} \ldots, \delta_{1T}, \delta_{21}, \delta_{22}, \ldots.., \delta_{2T} \ldots \ldots, \delta_{nT})'$ and structure matrix of interaction is defined with Equation 2.45.

$$R_\delta = R_V \otimes R_\gamma = \mathrm{I} \otimes R_\gamma \quad (2.45)$$

The precision matrix of interaction; Q, is $\tau_\delta\ R_\delta$; where τ_δ is unknown precision (inverse of variance). The density of δ is expressed as follows:

$$\pi(\delta|\tau_\delta)\ \alpha\ exp\left(-\frac{\tau_\delta}{2}\sum_{i=1}^{n}\sum_{t=2}^{T}\left(\delta_{it} - \delta_{i,t-1}\right)^2\right) = exp\left(-\frac{1}{2}\ \delta^T Q\ \delta\right) \quad (2.46)$$

The distribution of δ is given as follows (2, 22):

$$\delta|\ \tau_\delta \sim \mathcal{N}(0, \frac{1}{\tau_\delta} R_\delta^{-1}) \quad (2.47)$$

Type 3 interaction:

It is assumed that δ_{it} is an interaction of structured spatial effect and unstructured temporal effect for the model given by Equation 2.40. For Clayton's rule each δ_t= (δ_{1t},……,δ_{nt})$'$ t=1,…,.T is modeled with an intrinsic autoregressive structure, independent from other time points. Each time point has different spatial trends but do not have any temporal structure.

$\delta = (\delta_{11}, \delta_{21}, \delta_{31} \ldots, \delta_{n1}, \delta_{12}, \delta_{22}, \ldots\ldots, \delta_{n2} \ldots\ldots, \delta_{nT})'$ and structure matrix of interaction is defined with Equation 2.48.

$$R_\delta = R_\phi \otimes R_u = \mathrm{I} \otimes R_u \quad (2.48)$$

The precision matrix of interaction; Q, is $\tau_\delta\, R_\delta$; where τ_δ is unknown precision (inverse of variance). The density of δ is expressed as follows

$$\pi(\delta|\tau_\delta)\ \alpha\ exp\left(-\frac{\tau_\delta}{2}\sum_{t=1}^{T}\sum_{i\sim j}\left(\delta_{it}-\delta_{jt}\right)^2\right) = exp\left(-\frac{1}{2}\ \delta^T Q\ \delta\right) \quad (2.49)$$

The distribution of δ is given as follows (2, 22):

$$\delta|\ \tau_\delta \sim \mathcal{N}(0, \frac{1}{\tau_\delta} R_\delta^{-1}) \quad (2.50)$$

Type 4 interaction:

For the model given by Equation 2.40; it is assumed that δ_{it} is an interaction of structured spatial effect and structured temporal effect. Hence each area has temporal trend with taking account temporal pattern of neighboring areas. The temporal trends are probable to be similar for neighboring areas.

$\delta = (\delta_{11}, \delta_{12}, \delta_{13} \ldots, \delta_{1T}, \delta_{21}, \delta_{22}, \ldots\ldots, \delta_{2T} \ldots\ldots, \delta_{nT})'$ and structure matrix of interaction effect is

$$R_\delta = R_u \otimes R_\gamma \quad (2.51)$$

The precision matrix of interaction Q is $\tau_\delta\, R_\delta$; where τ_δ is unknown precision (inverse variance). The density of δ is expressed with Equation 2.52.

$$\pi(\delta|\tau_\delta)\ \alpha\ exp\left(-\frac{\tau_\delta}{2}\sum_{t=2}^{T}\sum_{i\sim j}\left(\delta_{it}-\delta_{jt}-\delta_{i,t-1}+\delta_{j,t-1}\right)^2\right) = exp\left(-\frac{1}{2}\ \delta^T Q\ \delta\right) \quad (2.52)$$

The distribution of δ is expressed as follows (2, 22):

$$\delta|\tau_\delta \sim N(0, \frac{1}{\tau_\delta}\ R_\delta{}^{-1}) \quad (2.53)$$

2.9. Model Selection

As the unknown parameter is modeled, the posterior distribution of this unknown parameter is obtained. Deviance (D) is calculated taking into account only likelihood of the data for this unknown parameter θ (2).

$$D(\theta) = -2\log\big(\pi(y|\theta)\big) \tag{2.54}$$

As deviance takes different values it is evaluated as a random variable. So mean of the deviance $\left(\bar{D} = E_{\theta|y}\big(D(\theta)\big)\right)$ which quantifies model fitting is calculated. The effective number of parameters (p_D) is calculated for quantifying model complexity with the equation as follows:

$$p_D = E_{\theta|y}\big(D(\theta)\big) - D\left(E_{\theta|y}(\theta)\right) = \bar{D} - D(\bar{\theta}) \tag{2.55}$$

where $D(\bar{\theta}) = D\left(E_{\theta|y}(\theta)\right)$ is the deviance of the posterior mean of parameter.

The common measure of model selection for Bayesian models is deviance information criterion (DIC) which is composed of $\bar{D}$ and p_D. So the DIC is expressed as follows (2):

$$DIC = \bar{D} + p_D \tag{2.56}$$

Models are evaluated according to DIC. Therefore the model with lowest DIC is selected.

2.10. Literature Review for Spatial and Spatio-temporal Bayesian Modeling

In recent articles, effects of air pollution ($PM_{2.5}$) on low birth weight in Los Angeles was examined by Coker et al. (23), stomach cancer in Libya was studied by Alhdiri et al. (24), the association between smoking and cardiovascular diseases was examined by Lee et al. (25), HIV and HSV-2 among women in Kenya was studied by Okango et al. (26), Malaria in Malaysia was investigated by Samat et al. (27), health insurance claims for chronic obstructive pulmonary disease in Northeastern Germany was examined by Kaulh (28), social risk factors for Rotavirus infections in Berlin was investigated by Wilking et al. (29), pancreatic cancer mortality by gender-age-period was examined by Etxeberria et al. (30) and neck pain prevalence in Iran was evaluated by Ghorbanpour et al. (31) with spatial modeling. Participation rates in the mammography screening program in the city of Dortmund was examined by Lemke et al. (32), variation of HIV infection in Kenya was investigated by Tonui et al. (33) and Tuberculosis in Kenya was examined by Kipruto et al. (34) with spatio-temporal modeling. Although there are many studies about spatio-temporal Bayesian models, the studies in which the observed number of cases for each area is assumed to have Poisson distribution and models were implemented with R-INLA package are examined.

- **Spatial Modeling**

In the first study; geographical variations in cervical cancer risk in San Luis Potosi State (Mexico) were examined with spatial Bayesian model by Teran-Hernandez et al. (35). Cervical risk for each municipality was estimated with using four models. The first model contained only a set of covariates (unemployment, single female percentage, marginalization index (measurement of deprivation and lack of basic socio-economic resources), positive screening index, index of accessibility to health services, the lack of coverage of Cervical Cancer-Screening Programme). The spatial unstructured random effect and a set of covariates were included in the second model. The third model contained spatial structured random effect and a set of covariates. The spatial structured random effect, spatial unstructured random effect and a set of covariates were included in the fourth model. Spatial structured random effect was modeled with intrinsic conditional autoregressive. The analysis was implemented with R-INLA package. Model fitting was determined with DIC. For the forth model the lack of coverage of Cervical Cancer-Screening Programme, Marginalisation Index and lack of accessibility to health services were determined as significant. An each unit increase in the lack of coverage of Cervical Cancer- Screening Programme, Marginalisation Index and lack of accessibility to health services was resulted as increased in cervical cancer risk by 17%, 5% and 1% respectively (35).

In the second study; ovarian cancer mortality risk among women who lived near Spanish industries was investigated at a municipal level (8098 Spanish towns) by Garcia-Perez et al. (36). To estimate overian cancer mortality risk, BYM model was implemented with R-INLA package. The effect of industrial groups and pollutant substances were examined. As industrial groups; refineries and coke ovens, glass and mineral fibers, ceramic, fertilizers, pharmaceutical products, urban waste-water treatment plants, paper and wood production, food and beverage sector were found to increase ovarian cancer mortality risk. As pollutant substances; carcinogens, metals, polycyclic aromatic chemicals, persistent organic pollutants, trichloroethylene and benzene were found to increase ovarian cancer mortality risk (36).

- **Spatio-temporal modeling**

In the first study; morbidity and lethality risk of human leptospirosis for 27 states in Brazil from 2000 to 2016 were investigated by Santos Baquero et al. (37). The morbidity and lethality risk were modeled with spatial-temporal Bayesian models. All models were applied with INLA using R programme. The structured and unstructured spatial effect together was modeled with modified BYM. Structured temporal effect was modeled with rw1. In the models spatio-temporal interactions were constructed according to combination of spatial effect (unstructured, structured) and temporal effect (structured, unstructured). The best model was determined according to DIC. The model with unstructured spatial and temporal effect interaction is the best model for morbidity risk. The model with unstructured spatial and structured temporal interaction effect is best for lethality risk. The main effect of morbidity risk was spatial effect, and spatial-temporal interaction (unstructured spatial and structured temporal effect inteaction) was the main effect on the lethality risk. The incrase in soil moisture, the proportion of households in poverty, precipitation and the decrease in the proportion of urban households was determined as risk factors of Leptospirosis morbidity. The increase in temperature and the proportion of households with proper collection of waste was determined as preventive factors of Leptospirosis morbidity. The number of illiterate individuals had no effect on Leptospirosis morbidity risk. The number of dengue cases had no effect on lethality risk (37).

In the second study; Waruru et al. (38) investigated spatio-temporal trend for mother-to-child transmission of HIV up to infancy for 12 districts in western Kenya during pre-Option B+ from 2007 to 2013 with spatio-temporal Bayesian models. Option B+ is a prevention approach for mother to child transmission of HIV which includes lifelong treatment of all HIV positive pregnant mothers. Infant and maternal factors were included as covariates in model to determine their linear effects on the risk. First, the effects of covariates on the transmison of HIV risk were evaluated with nonspatial generalized model. Second a spatial model without covariates, third a spatio-temporal model without covariate and forth a spatial model with covariates were applied. Finally a spatio-temporal model with covariates (early diagnosis (<8 weeks after birth), age of child at specimen collection, infant ever having breastfed, use of single dose nevirapine, maternal antiretroviral treatment status) were implemented. These models were carried out with INLA package in R. DIC was used to specify the fitting model for the data. The best model to explain time and geographical variation was the fifth model because of lowest DIC. For this model transmission of HIV risk up to infancy during pre-Option B+ gradually decreased from 2007 to 2013. Only two districts (Siaya and Suba) had higher risk in 2013 (38).

In the third study; Helbich et al. (39) evaluated risk and protective factors on suicide risk in 402 districts of Germany from 2007 to 2011 with spatio-temporal Bayesian models. Data about average annual income per person, annual unemployment rates and annual population density from 2007-2011 for each district was provided by General Federal Statistical Office. Data about number of general practitioners, psychiatrists and psychotherapists per 100000 persons, depression prevalence (in %) for each district in 2011 was obtained from Central Research Institute of Ambulatory Health Care. Parametric and nonparametric spatio- temporal Bayesian models were implemented to estimate suicide risk. Linear trend for study area-wide and differential component which signified difference between study area-wide trend and area-specific trend were included in parametric model. In the first parametric model; the linear effects of covariates on suicide risk were determined. Covariates; income, unemployment rate, population density changed in time, the remaining covariates were constant in time. But for the second parametric model the covariates changed in time were considered as nonlinear. So the second model contained these covariates which were modeled with second–order random walk. These covariates were included in parametric model as linear. The structured spatial effect with neighboring structure was modeled with $iCAR$ for the parametric and nonparametric models. The unstructured spatial effect was assumed to follow a Gaussian distribution with zero mean. The third model non parametric model contained covariates as linear effect. And the temporal structured effect was modeled with second–order random walk. As a result; parametric spatio-temporal model with evaluation of effect of covariates as unlinear was the best model according to DIC. Area-wide trend showed visible increment in every year from 2007 to 2011 in the suicide risk. It was determined every area-specific trend was different from area-wide trend. Income, unemployment and population density had a non linear effect on suicide risk. Income and population density influenced the suicide risk negatively but unemployment had positive effect on the suicide risk. The depression prevalence, psychiatrists, psychotherapists had no effect on suicide risk, but general practitioners had positive effect on suicide risk (36).

In the fourth study; Librero et al. (40) investigated hospitalization risks for Percutaneous Coronary Intervention (PCI), Colectomy in Colorectal Cancer (CCC) and Chronic Obstructive Pulmonary Disease (COPD) from 2002 to 2013 within two-level geographical structure; Autonomous Communities (AC) and Health Care Areas(HA) with spatio- temporal Bayesian models. The temporal structured random effect was modeled with rw1. For space (AC level) and time interaction effect, it was assumed that the interaction had a normal distribution with zero mean and unknown precision. It was determined how much variability was explained by random effects spatial AC level, spatial HA level, temporal, spatio (AC level)-temporal. Most of the variation in hospitalization risk was explained at the HA level for each condition; PCI, CCC and COPD. Hospitalization risk was more homogenous at the AC level in COPD. The temporal trend for hospitalization risk for PCI, CCC and COPD occured respectively upward, inverted V shape, downward from 2002 to 2013 (40).

In the fifth study; Ma et al. (41) assessed spatio-temporal patern of Bacillary Dysentery (BD) and determined effects of socieconomic factors on BD risk in 180 counties of Sichuan Province from 2004 to 2012. BD risk was modeled with the spatio-temporal Bayesian models which were implemented with INLA package in R. The first model contained unstructured and structured spatial effect and temporal effect. The second model contained unstructured and structured spatial effect, temporal effect and soci-economic variables (per capita gross domestic product, medical and technical personnel per thousand persons). The structured spatial effect was modeled with $iCAR$. The structured temporal effect was modeled through a time neighboring structure. The unstructured spatial and temporal effect had Gaussian distribution with zero mean. The second model is the best model because of lower DIC. The linear effect of socio-economic variables (per capita gross domestic product, medical and technical personnel per thousand persons) were examined with the model. An increase of 1000 yuan in per capita GDP decreased the BD risk by 2%. Increasing one person in the medical and technical personnel per thousand persons decreased BD risk by around 1% (41).

In the sixth study; Sparks (42) examined the disparities Hispanic and non-Hispanic for the digestive, respiratory cancer risk in Texas from 2000 to 2008 with spatio-temporal Bayesian models. The relative difference of cancer incidence in Hispanic and non-Hispanic was examined as linear difference variable in the model. All models were applied with INLA package in R taking into account the risk factors such as differences in access to health services, differences in socio-economic status and working area which may cause inequalities. The primary objective of this study was to determine the relative difference between the incidence of cancer in Hispanic and non-Hispanic of each county and year. Three spatio-temporal models contained an intercept for each cancer type, a mean difference between two ethnicity for each cancer type, risk factors for each county and etchnicity, spatial effect for each cancer type, unstructured temporal effect for each cancer type and unstructured spatial and temporal effect interaction as spatio-temporal effect. These models were different from each other according to a slope of a mean difference between two ethnicity for each cancer type. The slope was determined according to type of cancer in the first model. In the second model unstructured random slope for each county and cancer type was examined. The third model was different from the others by containing a structured slope (a spatial conditionally autoregressive random slope) for each county and cancer type. For the first model the difference between Hispanic and non-Hispanic was same across the state, for the second one difference is vary between counties according to unstructured random slope, and for the last one difference is vary between counties according to structured random slope. The third model was determined as a best model according to DIC. Exceedence probabilities for critical level used for clustering risk. When the probability was high, this indicated statistically important difference between Hispanic and non-Hispanic cancer incidence. The greatest disparity between the Hispanic and non-Hispanic in digestive and respiratory cancers was found in the east of Texas. This structure was found in 2000 and continued until 2008 (42).

In the seventh study; the risk of Zika Virus Disease (ZVD) and dengue were evaluated in parallel during the 2015-2016 ZVD outbreak for one department and one city of Colombia (separately in department of Santander and City of Bucaramanga) using Bayesian spatio-temporal models by Bello et al. (43). The time measure was represented by epidemiological week, the geographic units were represented by census sector (city level) and municipality (departmental level). The age, sex and address of dengue and zika cases were included in the data which was obtained from the public health surveillance system. Dengue or ZVD risk for each census sector and epidemiological week were modeled with five spatio-temporal models with INLA package in R. The first model included structured spatial, structured temporal and unstructured temporal effects but no interaction. The other ones included structured spatial effect, structured temporal effect and spatio-temporal interaction effect. The spatio-temporal interaction effects were included in the model according to combinations of spatial effect (structured) and temporal effect (structured, unstructured). Leroux structured spatial effect was modeled with conditional autoregressive (CAR). Structured temporal effect was modeled with random walk 1. In the department of Santander, the model with unstructured spatial and structured temporal interaction was the best model for dengue risk and ZVD risk. For this model, the dengue or ZVD risk in one municipality was highly associated with the same municipality during previous epidemiological weeks. In the city of Bucaramanga, the model with structured temporal and structured spatial interaction was the best model for dengue risk and ZVD risk. For this model, the dengue or ZVD risk in one census sector was highly associated with its neighboring census sectors in the same epidemiological week and its neighboring census sectors in the previous epidemiological week. (43).

3. MATERIAL and METHOD

Data about the number of infant deaths, the number of live births, percentage live-born infants' mothers aged under 20, percentage live-born infants' mothers aged over 39, gross domestic product per capita was obtained from Turkish Statistical Institute. The data are based on 81 provinces of Turkey and years from 2009 to 2017. Shape file of Turkey was obtained from Global Administrative Areas.

The spatio-temporal models were applied with INLA package of the R Statistical Programming Language 3.5.1. For spatio-temporal models a graph which determines neigborhood structure for each spatial unit is obtained. For this if polygon shape file is available, to load this shape file readShapePoly function is used in maptools package in R Statistical Programming. Poly2nb and nb2INLA functions in spdep package are used to transform shape file to adjacency matrix.

It was assumed that observed number of infant deaths y_{it} for each province (i=(1,2,….,81)) and year (t=1,2,…….,9) followed a Poisson distribution,

$$y_{it}|\theta_{it} \sim Poisson(E_{it}\theta_{it}) \tag{3.1}$$

The expected number of infant deaths E_{it} for each province (i=(1,2,….,81)) and year (t=1(2009), 2(2010),…….,9(2017)) is expressed as follows,

$$E_{it} = R_{it} \frac{\sum_{it} y_{it}}{\sum_{it} R_{it}} \tag{3.2}$$

where R_{it} is the number of live births for province i and year t.

Estimated relative risk of infant mortality for province i in year t (θ_{it}) was interpreted as ratio of the infant mortality rate of province i in year t to the average infant mortality rate of whole Turkey. Relative risk of infant mortality (θ_{it}) for each province (i=(1,2,….,81)) and year (t=1,2,…….,9)) was an unknown parameter of the poisson distribution. Relative risk of infant mortality was modeled with parametic and nonparametric spatio-temporal Bayesian models using INLA. The structures of models which were implemented are given in section 2 of the thesis (general information).

Parametric spatio-temporal Bayesian model

Parametric spatio-temporal model is referred to as Model 1 in next sections. The precision (τ) is inverse of variance (1/variance).

$$log(\theta_{it}) = b_0 + u_i + v_i + (\beta + \zeta_i) * t \quad (3.3)$$

$x = \{b_0, \beta, u = (u_1, .., u_{81}), v = (v_1, .., v_{81}), \zeta = (\zeta_1, .., \zeta_{81})\}$ is Gaussian Markov random field and $\psi = (\tau_u, \tau_v, \tau_\zeta)$ is vector of hyperparameters which are given with precision of structured spatial random effect (τ_u), unstructured spatial effect (τ_v) and differential trend (τ_ζ).

The priors for elements of x are assumed to be as

$u|\tau_u \sim \mathcal{N}(0, \frac{1}{\tau_u} {R_u}^{-1})$,

$v|\tau_v \sim \mathcal{N}(0, \frac{1}{\tau_v}\ I)$,

$\zeta|\tau_\zeta \sim \mathcal{N}(0,\ \frac{1}{\tau_\zeta}\ I)$,

where R_u is structure matrix of structured spatial effect and I is identity matrix.

The model given with Equation 3.3 was implemented using R-INLA. Structured spatial effect (u_i) and unstructured spatial effect (v_i) was modeled using BYM model. The differential trend (ζ_i) is modeled with iid model. Priors are assigned by default in R-INLA. b_0 has a Gaussian prior with zero mean and zero precision. β has prior as $\beta \sim \mathcal{N}(0, 1000)$.

The priors for elements of ψ are

$\log Gamma\ \tau_u \sim \log Gamma\ (1{,}0.0005)$,

$\log Gamma\ \tau_v \sim \log Gamma\ (1{,}0.0005)$,

$\log Gamma\ \tau_\zeta \sim \log Gamma\ (1{,}0.00005)$.

Nonparametric spatio-temporal Bayesian model without spatio-temporal interaction

Nonparametric Bayesian spatio-temporal model without spatio-temporal interaction is referred to as Model2 in the next sections.

$$log(\theta_{it}) = b_0 + u_i + v_i + \gamma_t + \phi_t \tag{3.4}$$

$x = \{b_0, u = (u_1, .., u_{81}), v = (v_1, .., v_{81}), \gamma = (\gamma_1, .., \gamma_9), \phi = (\phi_1, .., \phi_9)\}$ is Gaussian Markov random field and $\psi = (\tau_u, \tau_v, \tau_\gamma, \tau_\phi)$ is vector of hyperparameters which are given with precision of structured spatial random effect (τ_u), unstructured spatial effect (τ_v), structured temporal effect (τ_γ) and unstructured temporal effect (τ_ϕ).

The priors for elements of x are assumed to be as

$u|\tau_u \sim \mathcal{N}(0, \frac{1}{\tau_u}\ {R_u}^{-1})$,

$v|\tau_v \sim \mathcal{N}(0, \frac{1}{\tau_v}\ I)$,

$\gamma|\tau_\gamma \sim \mathcal{N}(0, \frac{1}{\tau_\gamma}\ {R_\gamma}^{-1})$,

$\phi|\tau_\phi \sim \mathcal{N}(0, \frac{1}{\tau_\phi}\ I)$,

where R_u is structure matrix of structured spatial effect, R_γ is structure matrix of structured temporal effect and I is identity matrix.

The model given with Equation 3.4 was implemented using R-INLA. Structured spatial effect (u_i) and unstructured spatial effect (v_i) was modeled using BYM model. The structured temporal effect for each province (γ_t) was modeled using rw1 and rw2. The unstructured temporal effect for each province (ϕ_t) was modeled using iid model. Priors are assigned by default in R-INLA. b_0 has a Gaussian prior with zero mean and zero precision.

The priors for elements of ψ are

$\log Gamma\ \tau_u \sim \log Gamma\ (1{,}0.0005)$,
$\log Gamma\ \tau_v \sim \log Gamma\ (1{,}0.0005)$,
$\log Gamma\ \tau_\gamma \sim \log Gamma\ (1{,}0.00005)$,
$\log Gamma\ \tau_\phi \sim \log Gamma\ (1{,}0.00005)$.

Nonparametric spatio-temporal models with spatio-temporal interaction

$$log(\theta_{it}) = b_0 + u_i + v_i + \gamma_t + \phi_t + \delta_{it} \tag{3.5}$$

$x = \{b_0, u = (u_1, .., u_{81}), v = (v_1, .., v_{81}), \gamma = (\gamma_1, .., \gamma_9), \phi = (\phi_1, .., \phi_9),$ $\delta = (\delta_{1\,1}, .., \delta_{81\,1}, \dots\dots \delta_{1\,9}, .., \delta_{81\,9})\}$ is Gaussian Markov random field and $\theta = \{\tau_u, \tau_v, \tau_\gamma, \tau_\phi, \tau_\delta\}$ is vector of hyperparameters which are given with precision of structured spatial random effect (τ_u), unstructured spatial effect (τ_v), structured temporal effect (τ_γ), unstructured temporal effect (τ_ϕ) and spatio-temporal interaction effect (τ_δ).

The priors for elements of x are assumed to be as

$u|\tau_u \sim \mathcal{N}(0, \frac{1}{\tau_u}\ {R_u}^{-1})$,

$v|\tau_v \sim \mathcal{N}(0, \frac{1}{\tau_v}\ I)$,

$\gamma|\tau_\gamma \sim \mathcal{N}(0, \frac{1}{\tau_\gamma}\ {R_\gamma}^{-1})$,

$\phi|\tau_{\phi} \sim \mathcal{N}(0, \frac{1}{\tau_{\phi}}\ I)$,

$\delta|\tau_{\delta} \sim \mathcal{N}(0, \frac{1}{\tau_{\delta}}\ {R_{\delta}}^{-1})$.

where R_u is structure matrix of structured spatial effect, R_γ is structure matrix of structured temporal effect, R_δ is structure matrix of spatio-temporal interaction effect and I is identity matrix.

Priors are assigned by default in R-INLA. b_0 has a Gaussian prior with zero mean and zero precision. The priors for elements of ψ are

$\log Gamma\ \tau_u \sim \log Gamma\ (1{,}0.0005)$,
$\log Gamma\ \tau_v \sim \log Gamma\ (1{,}0.0005)$,
$\log Gamma\ \tau_\gamma \sim \log Gamma\ (1{,}0.00005)$,
$\log Gamma\ \tau_\phi \sim \log Gamma\ (1{,}0.00005)$
$\log Gamma\ \tau_\delta \sim \log Gamma\ (1{,}0.00005)$.

In next sections; nonparametric spatio-temporal model which includes unstructured spatial effect and unstructured temporal effect interaction, unstructured spatial effect and structured temporal effect interaction, structured spatial effect and unstructured temporal effect interaction, structured spatial effect and structured temporal effect interaction is referred to respectively as model3, model4, model5 and model6.

The structured temporal effect was modeled with rw1 and rw2 in nonparametric spatio-temporal Bayesian models. So the nonparametric spatio-temporal Bayesian models were examined according to rw1 and rw2

The model with the lowest DIC was selected to determine the spatio-temporal structure of relative risk of infant mortality in Turkey.

The posterior means of random effects of the model and estimated relative risk of infant mortality, posterior distributions of precisions (1/variance) of random effects were obtained.

To compare the contribution of the variability of each random effect (spatial, temporal or spatio-temporal interaction random effects) to variability of relative risk of infant mortality explained by spatio-temporal model; n values drawn from the posterior distribution of 1/precision of random effect (variance of random effect). Value of ith random effect $(i = 1, \dots, m)$ for j $(j = 1, \dots, n)$ is represented with random effect ij. The contribution of $i\ th$ random effect to the variability of relative risk of infant mortality explained by spatio-temporal model is calculated as percentage:

$$contribution\ of\ i\ th\ random\ effect = 100 * \left(\frac{1}{n}\sum_{j=1}^{n}\frac{random\ effect\ ij}{\sum_{i=1}^{m} random\ effect\ ij}\right) \quad (3.6)$$

The contribution of spatial, temporal and spatio-temporal interaction random effects to variability of relative risk of infant mortality explained by spatio-temporal model were calculated.

The posterior means of estimated relative risks of infant mortality for provinces and years were used to map the relative risk of infant mortality.

To identify areas where the relative risk is higher than a certain critical level, the probability of exceeding this critical level for each province and year is calculated from the posterior distribution of the estimated relative risk of infant mortality. Areas with high probability can be identified as at risk. The critical level was taken as 1 for this study. And as a result significant risk clusters were specified for the relative risk of infant mortality in Turkey.

The fixed effect of percentage of live-born infants' mothers aged under 20, percentage of live-born infants' mothers over 39 and gross domestic product per capita (GDP per capita) on relative risk of infant mortality were evaluated in this thesis. The generalized linear model without the concepts of space and time and model 4 (rw1) by adding gross domestic product per capita ($), percentage of live-born infants' mothers aged under 20 and percentage of age of live-born infants' mothers aged over 39 for each province and year were implemented.

Genaralized linear model is given as follows;

$$log(\theta_{it}) = b_0 + b_1(GDP\ per\ capita/1000)_{it} + b_2(\%\ mothers\ aged\ under\ 20)_{it} + b_3(\%\ mothers\ aged\ over\ 39)_{it} \tag{3.7}$$

Spatio-temporal model (Model 4 a (rw1)) is given as follows;

$$log(\theta_{it}) = b_0 + u_i + v_i + \gamma_t + \phi_t + \delta_{it} + b_1(GDP\ per\ capita/1000)_{it} + b_2(\%\ mothers\ aged\ under\ 20)_{it} + b_3(\%\ mothers\ aged\ over\ 39)_{it} \tag{3.8}$$

where i=1,2,….,81 and t=1,2,…….,9.

4. RESULTS

The neighborhood structure of each province is given in Table 4.1.

Table 4.1. The neighborhood structure of each province.

Province	ID	ID of adjacent province	Province	ID	ID of adjacent province
Çanakkale	1	15 28 73	Kahramanmaraş	42	4 5 34 48 55 64 72
Çankırı	2	3 10 22 44 47 50	Kütahya	43	6 15 19 24 32 56 77
Çorum	3	2 9 47 50 67 70 80	Karabük	44	2 16 22 47 81
Adana	4	37 42 48 58 62 64	Karaman	45	11 54 58
Adıyaman	5	27 34 42 55 68	Kars	46	7 12 31 38
A.karahisar	6	23 26 32 39 43 54 77	Kastamonu	47	2 3 16 44 70
Ağrı	7	21 31 38 46 60 78	Kayseri	48	4 42 61 62 72 80
Aksaray	8	10 52 54 61 62	Kilis	49	34
Amasya	9	3 67 74 80	Kırıkkale	50	2 3 10 52 80
Ankara	10	2 8 22 32 50 52 54	Kırklareli	51	28 73
Antalya	11	23 39 45 54 58 59	Kırşehir	52	8 10 50 61 80
Ardahan	12	13 31 46	Kocaeli	53	24 40 66 79
Artvin	13	12 31 65	Konya	54	6 8 10 11 32 39 45 58 62
Aydın	14	26 41 56 59	Malatya	55	5 27 29 30 42 72
Balıkesir	15	1 24 41 43 56	Manisa	56	14 15 26 41 43 77
Bartın	16	44 47 81	Mardin	57	17 27 68 69 71
Batman	17	21 27 57 60 69	Mersin	58	4 11 45 54 62
Bayburt	18	30 31 33 65 75	Muğla	59	11 14 23 26
Bilecik	19	22 24 32 43 66	Muş	60	7 17 20 21 27 31
Bingöl	20	27 29 30 31 60 76	Nevşehir	61	8 48 52 62 80
Bitlis	21	7 17 60 69 78	Niğde	62	4 8 48 54 58 61
Bolu	22	2 10 19 25 32 44 66 81	Ordu	63	35 67 72 74
Burdur	23	6 11 26 39 59	Osmaniye	64	4 34 37 42
Bursa	24	15 19 43 53 66 79	Rize	65	13 18 31 75
Düzce	25	22 66 81	Sakarya	66	19 22 24 25 53
Denizli	26	6 14 23 56 59 77	Samsun	67	3 9 63 70 74
Diyarbakır	27	5 17 20 29 55 57 60 68	Şanlıurfa	68	5 27 34 57
Edirne	28	1 51 73	Siirt	69	17 21 57 71 78
Elazığ	29	20 27 30 55 76	Sinop	70	3 47 67
Erzincan	30	18 20 29 31 33 35 55 72 76	Şırnak	71	36 57 69 78
Erzurum	31	7 12 13 18 20 30 46 60 65	Sivas	72	30 35 42 48 55 63 74 80
Eskişehir	32	6 10 19 22 43 54	Tekirdağ	73	1 28 40 51
Gümüşhane	33	18 30 35 75	Tokat	74	9 63 67 72 80
Gaziantep	34	5 37 42 49 64 68	Trabzon	75	18 33 35 65
Giresun	35	30 33 63 72 75	Tunceli	76	20 29 30
Hakkâri	36	71 78	Uşak	77	6 26 43 56
Hatay	37	4 34 64	Van	78	7 21 36 69 71
Iğdır	38	7 46	Yalova	79	24 53
Isparta	39	6 11 23 54	Yozgat	80	3 9 48 50 52 61 72 74
İstanbul	40	53 73	Zonguldak	81	16 22 25 44
İzmir	41	14 15 56			

Spatial random effects were modeled with GMRF using intrinsic autoregressive structure. For this modeling the neighborhood structures of provinces were specified with adjacency matrix. Related with Table 4.1, adjacency matrix for 81 provinces was obtained as shown in Figure 4.1. The neighborhood structure of each province is seen in adjacency matrix. In the row and column of the matrix, the provinces are identified by ID (identifier for province). Neighbors of each province are indicated by a black square.

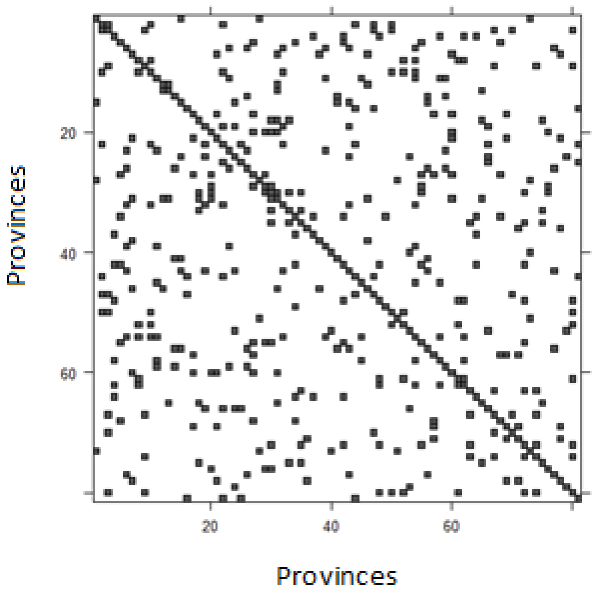

Figure 4.1. Adjacency matrix for 81 provinces in Turkey.

DIC, number of effective parameters and mean of deviance for parametric and nonparametric models are given in Table 4.2. Nonparametric models with spatio-temporal interaction (Model3, Model4, Model5, and Model6) have lower DIC when compared with parametric model. Model 2 was modeled relative risk without spatio-temporal interaction effect. When different combinations of spatio-temporal interaction effect were added to model 2 to form model 3, 4, 5, 6; these combinations reduced the DIC of model 2. Greatest reduction occured in model 4(rw1). From the table it is seen that the model 4(rw1) is the best model with the lowest DIC. Therefore the interaction of structured temporal effect and unstructured spatial effect is the best spatio-temporal interaction random effect to model relative risk of infant mortality.

Table 4.2. DIC, number of effective parameters, mean of deviance for parametric and nonparametric models.

Model	DIC	p_D	$\overline{D}$			
Parametric						
Model1	6293.24	131.05	6162.19			
	rw1			rw2		
Nonparametric	DIC	p_D	$\overline{D}$	DIC	p_D	$\overline{D}$
Model2	6677.33	81.39	6595.93	6677.39	81.27	6596.12
Model3	5775.11	410.50	5364.60	5775.38	412.57	5362.80
Model4	5750.91	340.02	5410.89	5852.59	300.59	5552.00
Model5	5771.31	369.23	5402.08	5771.3	369.22	5402.08
Model6	5758.35	313.46	5444.88	5833.21	303.39	5529.82

Table 4.3 presents the posterior summary statistics for hyperparameters. The contribution of random effects to the variance explained by model 4 (rw1) is determined with using the posterior distribution of hyperparameters.

Table 4.3. Posterior summary statistics for hyperparameters (precision of random effects).

Precision (1/variance)	mean	sd	0.025quantile	0.05quantile	0.975quantile
τ_v	1858.86	1777.49	124.71	1339.20	6553.76
τ_u	1873.94	1837.97	141.20	1336.96	6791.76
τ_γ	287.16	136.80	94.52	263.81	618.89
τ_ϕ	20221.84	19287.65	1842.96	14682.76	72022.67
τ_δ	125.98	15.62	98.54	124.81	159.83

Table 4.4 gives contribution of model 4(rw1)'s random effects to the variance explained by model 4 (rw1). The greatest contribution to the variance explained by model 4 (rw1) was from the spatio-temporal interaction effect (55.1%). The contributions of structured temporal effect, unstructured spatial effect, structured spatial effect and unstructured temporal effect to the explained variance were 28%, 8.2%, 7.9% and 0.8%, respectively. Spatio-temporal variability and structured temporal variability were more effective than other random effects to express variance explained by model 4 (rw1).

Table 4.4. Contribution of model 4(rw1)'s random effects to the variance explained by model 4 (rw1).

random effects	%
unstructured spatial effect	8.2
structured spatial effect	7.9
unstructured temporal effect	0.8
structured temporal effect	28.0
unstructured spatial and structured temporal interaction effect	55.1

In model4 (rw1), structured and unstructured spatial random effects were examined together with BYM model. Hence, posterior means of the sum of structured and unstructured spatial random effects $(v + u)$ for provinces are shown in Table 4.5.

Table 4.5. Posterior means of the sum of unstructured and structured spatial random effects $(v + u)$ for provinces.

Province	mean	Province	mean
Adana	-0.00000012	K.maraş	-0.00000007
Adıyaman	0.00000005	Karabük	-0.00000028
A.karahisar	-0.00000026	Karaman	-0.00000015
Ağrı	0.00000017	Kars	0.00000015
Aksaray	-0.00000021	Kastamonu	-0.00000024
Amasya	-0.00000017	Kayseri	-0.00000018
Ankara	-0.00000031	Kırıkkale	-0.00000022
Antalya	-0.00000025	Kırklareli	-0.00000037
Ardahan	0.00000014	Kırşehir	-0.00000024
Artvin	0.00000008	Kilis	0.00000025
Aydın	-0.00000023	Kocaeli	-0.00000029
Balıkesir	-0.00000029	Konya	-0.00000025
Bartın	-0.00000022	Kütahya	-0.00000026
Batman	0.00000017	Malatya	0.00000000
Bayburt	0.00000000	Manisa	-0.00000025
Bilecik	-0.00000030	Mardin	0.00000018
Bingöl	0.00000009	Mersin	-0.00000018
Bitlis	0.00000018	Muğla	-0.00000024
Bolu	-0.00000034	Muş	0.00000014
Burdur	-0.00000022	Nevşehir	-0.00000022
Bursa	-0.00000031	Niğde	-0.00000021
Çanakkale	-0.00000029	Ordu	-0.00000010
Çankırı	-0.00000028	Osmaniye	0.00000001
Çorum	-0.00000025	Rize	0.00000003
Denizli	-0.00000025	Sakarya	-0.00000028
Diyarbakır	0.00000006	Samsun	-0.00000017
Düzce	-0.00000023	Siirt	0.00000022
Edirne	-0.00000035	Sinop	-0.00000015
Elazığ	0.00000006	Sivas	-0.00000015
Erzincan	-0.00000006	Şanlıurfa	0.00000014
Erzurum	0.00000007	Şırnak	0.00000026
Eskişehir	-0.00000030	Tekirdağ	-0.00000032
Gaziantep	0.00000007	Tokat	-0.00000012
Giresun	-0.00000007	Trabzon	0.00000000
Gümüşhane	0.00000000	Tunceli	0.00000005
Hakkari	0.00000030	Uşak	-0.00000022
Hatay	0.00000003	Van	0.00000022
Iğdır	0.00000019	Yalova	-0.00000027
Isparta	-0.00000019	Yozgat	-0.00000025
İstanbul	-0.00000030	Zonguldak	-0.00000025
İzmir	-0.00000024		

The temporal effect was examined separately as structured and unstructured in model4 (rw1). The posterior means of the structured temporal random effect and unstructured temporal random effect for years are given in Table 4.6.

Table 4.6. Posterior means of structured temporal (γ) and unstructured temporal (ϕ) random effects for years.

structured temporal random effect		unstructured temporal random effect	
year	mean	year	mean
2009	0.22333297	2009	0.00401860
2010	0.09573644	2010	-0.00295230
2011	0.07025211	2011	-0.00056585
2012	0.06175310	2012	0.00202623
2013	-0.01667424	2013	-0.00201672
2014	-0.02341656	2014	0.00193422
2015	-0.09763600	2015	-0.00107752
2016	-0.13413434	2016	0.00011521
2017	-0.17904295	2017	-0.00148213

The posterior means of structured temporal effects from 2009 to 2017 are given with dashed line in Figure 4.2. It is seen from Table 4.6 that the posterior means of structured temporal effects decreased from 2009 to 2017. Furthermore posterior means of structured temporal effects were negative from 2013 to 2017. From 2009 to 2017 the posterior means of structured temporal random effect showed downward trend. The posterior means of unstructured temporal effect from 2009 to 2017 are given with solid line in Figure 4.2. It is seen from Figure 4.2 that the posterior means of unstructured temporal effects were around zero from 2009 to 2017.

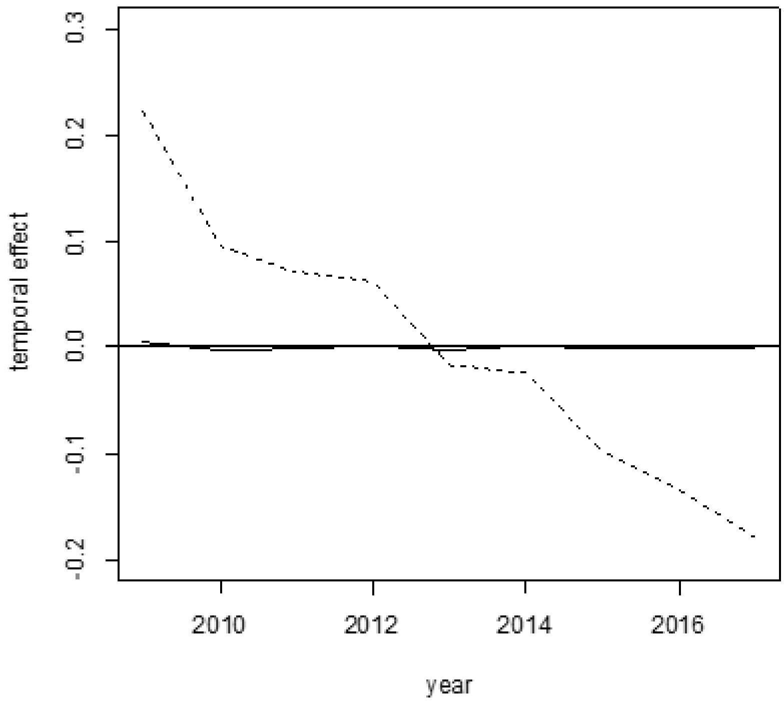

Figure 4.2. Posterior means of structured (dashed line) and unstructured (solid line) temporal random effect.

Posterior means of spatio-temporal interaction random effects for provinces and years are given in Table 4.7. The spatio-temporal interaction random effect was interaction of unstructured spatial effect and structured temporal effect for model 4 (rw1). The means of spatio-temporal interaction random effects of Çanakkale, Isparta, Karaman, Sakarya, Aksaray, Uşak, Afyonkarahisar, Aydın, Balıkesir, Denizli, Eskişehir, Ordu and Sinop increased relative risks of infant mortality of these provinces in 2009. As years passed from 2010 to 2017, the posterior means of interaction random effects of Çanakkale, Isparta, Karaman, Sakarya, Aksaray, Uşak, Afyonkarahisar, Aydın, Balıkesir, Denizli, Eskişehir, Ordu and Sinop decreased relative risks of infant mortality of these provinces. Conversely, the posterior means of spatio-temporal interaction random effects of Burdur, Iğdır, Kars, Mersin and Zonguldak decreased relative risks of infant mortality of these provinces in 2009. As years passed from 2010 to 2017, the means of interaction effects of Burdur, Iğdır, Kars, Mersin and Zonguldak increased relative risks of infant mortality of these provinces.

From 2009 to 2017; the posterior means of spatio-temporal interaction random effects of Çankırı, Çorum, Amasya, Ankara, Antalya, Artvin, Bartın, Bayburt, Bilecik, Bolu, Bursa, Düzce, Edirne, Erzincan, Gümüşhane, Giresun, İstanbul, İzmir, Karabük, Kastamonu, Kırklareli, Kırşehir, Kocaeli, Muğla, Nevşehir, Rize, Samsun, Tekirdağ, Trabzon, Tunceli, Tunceli, Yalova and Yozgat decreased relative risks of infant mortality of these provinces. Conversely; from 2009 to 2017 the posterior means of spatio-temporal interaction random effects of Adana, Adıyaman, Ağrı, Ardahan, Batman, Bingöl, Bitlis, Diyarbakır, Elazığ, Erzurum, Gaziantep, Hakkari, Hatay, Kahramanmaraş, Kütahya, Kilis, Malatya, Mardin, Muş, Osmaniye, Şanlıurfa, Siirt, Şırnak, Tokat, Van increased relative risks of infant mortality of these provinces.

The posterior mean of spatio-temporal interaction random effect of Konya decreased relative risk of infant mortality of Konya in 2015, in the other years the posterior means increased the relative risks. In 2012 and 2014; the posterior means of spatio-temporal interaction random effects of Sivas decreased relative risk of infant, in the other years the posterior means of interaction effects increased relative risks of Sivas. The posterior means of spatio-temporal interaction effect of Manisa decrased relative risks of infant mortality of Manisa in 2013, 2014 and 2015, in the other years, the posterior means of interaction effects increased the relative risks of Sivas.

The spatio-temporal interaction random effect of model 4(rw1) is interaction of structured temporal random effect and unstructured spatial effect. Hence each province has temporal trend, which is modeled with rw1 without structured spatial effect, different from other provinces. The spatio-temporal interaction random effect of the province reflects temporal trend of each province. As can be seen in Figure 4.2 the posterior means of main structured temporal random effect (γ_{it}) had downward trend from positive to negative. This downward trend from positive to negative was determined in Karaman and Çanakkale with spatio-temporal interactions of these provinces (Figure 4.3).

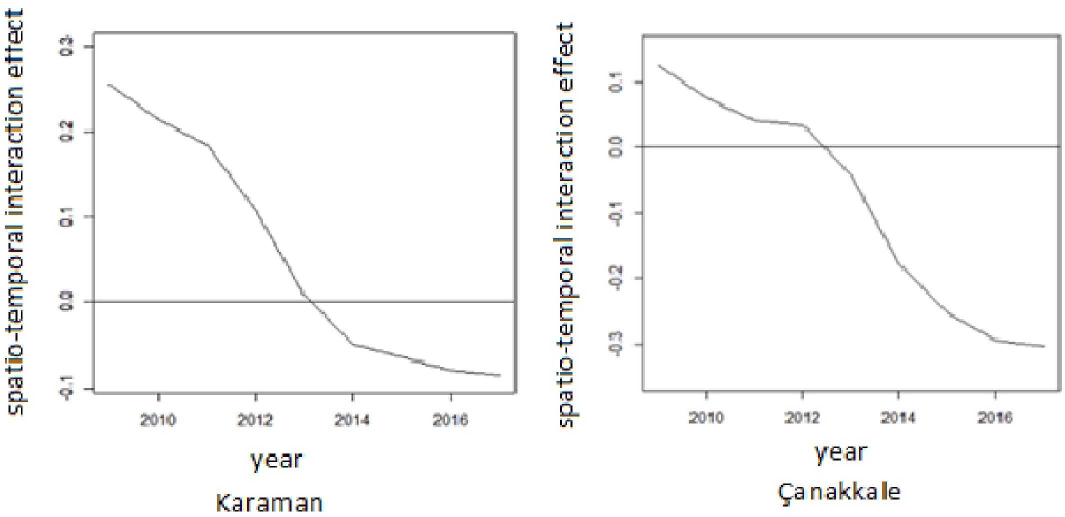

Figure 4.3. Posterior means of spatio-temporal random effects for Karaman and Çanakkale.

Table 4.7. The posterior means of spatio-temporal interaction effects (δ) for provinces and years.

	Year								
Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Adana	0.111	0.105	0.131	0.090	0.239	0.207	0.073	0.060	0.034
Adıyaman	0.236	0.206	0.220	0.214	0.195	0.172	0.191	0.266	0.235
A.karahisar	0.089	0.048	0.083	0.148	0.110	0.005	-0.006	-0.013	-0.026
Ağrı	0.058	0.313	0.541	0.455	0.431	0.373	0.424	0.413	0.391
Aksaray	0.175	0.137	0.107	0.041	0.016	0.031	0.026	-0.021	-0.007
Amasya	-0.193	-0.212	-0.204	-0.180	-0.150	-0.045	-0.106	-0.156	-0.214
Ankara	-0.278	-0.311	-0.242	-0.292	-0.300	-0.376	-0.298	-0.307	-0.233
Antalya	-0.135	-0.178	-0.263	-0.239	-0.246	-0.178	-0.145	-0.181	-0.189
Ardahan	0.119	0.114	0.107	0.136	0.120	0.093	0.083	0.091	0.093
Artvin	-0.167	-0.136	-0.096	-0.126	-0.164	-0.188	-0.193	-0.192	-0.155
Aydın	0.003	0.059	0.003	-0.028	-0.096	-0.127	-0.142	-0.166	-0.180
Balıkesir	0.028	-0.040	-0.100	-0.018	-0.167	-0.237	-0.198	-0.254	-0.230
Bartın	-0.188	-0.183	-0.185	-0.198	-0.211	-0.192	-0.235	-0.212	-0.225
Batman	0.043	0.100	0.203	0.286	0.414	0.390	0.329	0.303	0.287
Bayburt	-0.211	-0.225	-0.248	-0.232	-0.183	-0.132	-0.113	-0.121	-0.113
Bilecik	-0.270	-0.269	-0.240	-0.191	-0.155	-0.135	-0.103	-0.078	-0.042
Bingöl	0.248	0.308	0.330	0.372	0.368	0.362	0.389	0.483	0.429
Bitlis	0.019	0.058	0.095	0.190	0.297	0.328	0.383	0.362	0.413
Bolu	-0.265	-0.249	-0.276	-0.242	-0.226	-0.264	-0.264	-0.293	-0.318
Burdur	-0.029	-0.008	-0.043	-0.063	-0.025	0.038	0.023	0.027	0.050
Bursa	-0.092	-0.163	-0.167	-0.259	-0.274	-0.205	-0.210	-0.148	-0.226
Çanakkale	0.126	0.076	0.041	0.034	-0.043	-0.178	-0.253	-0.294	-0.303
Çankırı	-0.170	-0.177	-0.194	-0.175	-0.147	-0.138	-0.139	-0.114	-0.165
Çorum	-0.052	-0.093	-0.114	-0.160	-0.183	-0.158	-0.164	-0.220	-0.245
Denizli	0.055	-0.017	-0.019	-0.059	-0.144	-0.172	-0.102	-0.084	-0.110
Diyarbakır	0.163	0.150	0.188	0.259	0.294	0.274	0.180	0.141	0.106
Düzce	-0.184	-0.104	-0.013	0.026	-0.062	-0.048	-0.056	-0.049	-0.088
Edirne	-0.349	-0.338	-0.368	-0.376	-0.377	-0.403	-0.423	-0.412	-0.414
Elazığ	0.203	0.167	0.121	0.198	0.256	0.176	0.189	0.225	0.222
Erzincan	-0.113	-0.108	-0.153	-0.164	-0.146	-0.177	-0.203	-0.201	-0.178
Erzurum	0.059	0.045	0.094	0.083	0.278	0.257	0.312	0.317	0.346
Eskişehir	0.048	-0.083	-0.106	-0.108	-0.080	-0.132	-0.232	-0.229	-0.324
Gaziantep	0.462	0.426	0.395	0.366	0.471	0.503	0.466	0.407	0.388
Giresun	-0.175	-0.153	-0.141	-0.135	-0.169	-0.159	-0.170	-0.192	-0.170
Gümüşhane	-0.054	-0.089	-0.144	-0.183	-0.186	-0.155	-0.182	-0.216	-0.220
Hakkari	0.382	0.348	0.311	0.275	0.330	0.376	0.406	0.480	0.543
Hatay	0.017	0.059	0.149	0.175	0.089	0.094	0.078	0.144	0.166
Iğdır	-0.014	-0.031	-0.069	-0.045	0.047	0.104	0.090	0.091	0.079
Isparta	0.134	0.197	0.177	0.165	0.077	0.007	-0.008	-0.006	-0.019
İstanbul	-0.162	-0.202	-0.351	-0.286	-0.263	-0.170	-0.163	-0.155	-0.162
İzmir	-0.038	-0.143	-0.077	-0.085	-0.288	-0.167	-0.153	-0.242	-0.252
K.maraş	0.080	0.112	0.184	0.159	0.221	0.158	0.210	0.225	0.176
Karabük	-0.218	-0.244	-0.293	-0.335	-0.384	-0.435	-0.452	-0.459	-0.458
Karaman	0.256	0.215	0.185	0.108	0.008	-0.049	-0.064	-0.080	-0.086

Table 4.7. (cont.) The posterior means of spatio-temporal interaction effects (δ) for provinces and years.

	Year								
Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Kars	-0.170	0.009	0.130	0.152	0.232	0.223	0.275	0.225	0.132
Kastamonu	-0.135	-0.135	-0.182	-0.157	-0.178	-0.202	-0.212	-0.233	-0.220
Kayseri	-0.247	-0.167	-0.105	-0.034	-0.046	0.005	0.057	-0.035	-0.031
Kırıkkale	-0.057	-0.076	-0.025	0.029	0.049	0.095	0.068	0.109	0.097
Kırklareli	-0.467	-0.459	-0.429	-0.413	-0.351	-0.381	-0.388	-0.344	-0.285
Kırşehir	-0.235	-0.200	-0.218	-0.284	-0.310	-0.304	-0.270	-0.248	-0.199
Kilis	0.355	0.366	0.400	0.490	0.638	0.680	0.627	0.576	0.604
Kocaeli	-0.143	-0.187	-0.253	-0.189	-0.205	-0.212	-0.221	-0.234	-0.218
Konya	0.344	0.366	0.251	0.175	0.104	0.013	-0.015	0.016	0.071
Kütahya	0.306	0.294	0.171	0.130	0.108	0.074	0.079	0.088	0.090
Malatya	0.401	0.365	0.332	0.424	0.306	0.212	0.122	0.106	0.075
Manisa	0.144	0.091	0.006	0.023	-0.087	-0.072	-0.013	0.131	0.179
Mardin	0.254	0.329	0.281	0.293	0.332	0.389	0.311	0.360	0.403
Mersin	-0.006	-0.039	-0.060	0.036	0.090	0.157	0.100	0.042	0.062
Muğla	-0.040	-0.080	-0.101	-0.214	-0.278	-0.315	-0.275	-0.189	-0.237
Muş	0.427	0.318	0.291	0.250	0.261	0.298	0.413	0.459	0.488
Nevşehir	-0.271	-0.241	-0.226	-0.177	-0.149	-0.177	-0.136	-0.142	-0.079
Niğde	-0.055	-0.015	-0.056	-0.034	-0.098	-0.075	-0.084	-0.074	-0.081
Ordu	0.051	0.045	0.048	-0.027	-0.128	-0.158	-0.156	-0.111	-0.018
Osmaniye	0.086	0.135	0.189	0.145	0.142	0.107	0.135	0.178	0.120
Rize	-0.267	-0.219	-0.230	-0.207	-0.269	-0.255	-0.248	-0.164	-0.110
Sakarya	0.173	0.093	0.049	-0.023	-0.128	-0.161	-0.119	-0.109	-0.022
Samsun	-0.072	-0.074	-0.151	-0.084	-0.111	-0.088	-0.151	-0.189	-0.182
Siirt	0.228	0.289	0.433	0.419	0.402	0.406	0.418	0.406	0.386
Sinop	0.085	0.087	0.063	0.022	-0.026	-0.069	-0.099	-0.115	-0.133
Sivas	0.017	0.004	0.019	-0.021	0.011	-0.037	0.038	0.014	0.021
Şanlıurfa	0.230	0.358	0.376	0.336	0.400	0.533	0.630	0.631	0.605
Şırnak	0.318	0.286	0.347	0.370	0.380	0.470	0.488	0.495	0.427
Tekirdağ	-0.071	-0.144	-0.180	-0.218	-0.193	-0.150	-0.238	-0.240	-0.157
Tokat	0.364	0.331	0.311	0.167	0.068	0.088	0.093	0.093	0.062
Trabzon	-0.287	-0.253	-0.233	-0.165	-0.115	-0.098	-0.113	-0.142	-0.085
Tunceli	-0.152	-0.140	-0.132	-0.147	-0.160	-0.146	-0.152	-0.166	-0.143
Uşak	0.116	0.097	0.117	0.147	0.118	0.021	0.017	-0.077	-0.145
Van	0.126	0.096	0.089	0.022	0.415	0.470	0.498	0.404	0.324
Yalova	-0.396	-0.389	-0.339	-0.321	-0.312	-0.279	-0.247	-0.200	-0.136
Yozgat	-0.158	-0.152	-0.147	-0.212	-0.218	-0.136	-0.108	-0.126	-0.147
Zonguldak	-0.421	-0.379	-0.316	-0.228	-0.212	-0.171	-0.106	-0.018	0.026

Posterior means of spatio- temporal interaction effects are given as maps from 2009 to 2017 in Figure 4.4. In Figure 4.4 the positive spatio-temporal interaction random effect is categorized as (0.01, 1) with black, the negatif spatio-temporal effect is categorized as (-1,-0.01) with light grey. As years passed from 2009 to 2017 the number of provinces with spatio-temporal effect which had negative posterior mean, increased. The numbers of provinces with negative means for 2009, 2010, 2011, 2012, 2013, 2014, 2017 were 41, 43, 43, 44, 46, 46, 48, 49, 48 respectively.

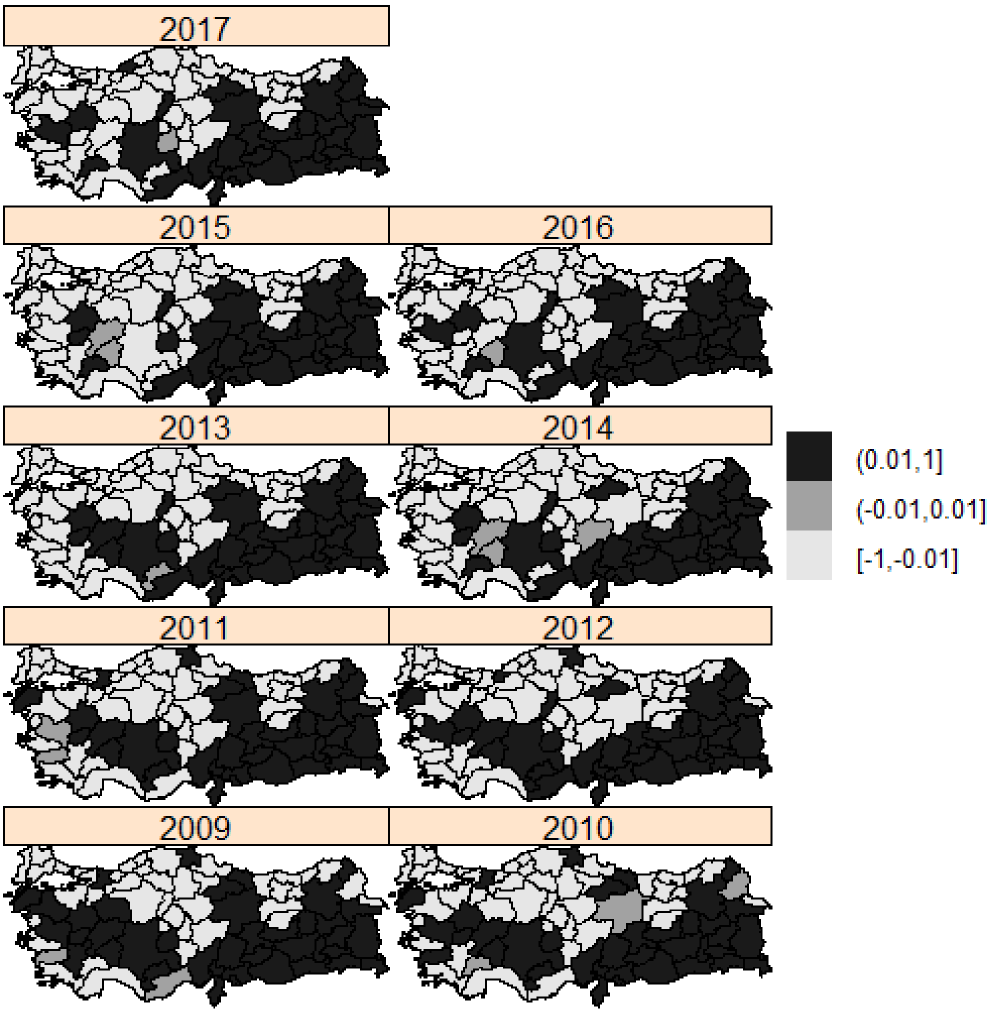

Figure 4.4. Posterior means of spatio-temporal interaction random effects.

Relative risk of the infant mortality was estimated by model 4 (rw1). Posterior means of estimated relative risks of infant mortality is presented with Table 4.8.

- From 2009 to 2017 relative risk of infant mortality of Amasya, Ankara, Bayburt, Bilecik, Bolu, Edirne, Karabük, Kayseri, Kırklareli, Kırşehir, Nevşehir, Rize, Trabzon, Yalova, Zonguldak were less than 1. From 2009 to 2017 these provinces had infant mortality rates which were less than the average infant mortality rate of whole Turkey. From 2009 to 2017 relative risk of infant mortality of Adıyaman, Ağrı, Batman, Bingöl, Bitlis, Elazığ, Erzurum, Gaziantep, Hakkari, Kilis, Mardin, Muş, Şanlurfa, Siirt, Şırnak and Van were greater than 1. These provinces had infant mortality rates which were greater than the average infant mortality rate of whole Turkey. The other provinces had relative risks of infant mortality greater than 1 in 2009. But as years passed from 2010 to 2017 their relative risks decreased under 1.
- For 2009 and 2010 in respect to infant mortality Gaziantep was the most risky province and Kırklareli was the least risky province.
- For 2011 Ağrı had the greatest relative risk of infant mortality and Kırklareli had the smallest relative risk of infant mortality.
- For 2012 in respect to infant mortality Kilis was the most risky province and Kırklareli was the least risky province.
- For 2013, 2014, 2015 and 2017 in respect to infant mortality Kilis was the most risky province and Karabük was the least risky province.
- For 2016 Şanlıurfa had the greatest relative risk of infant mortality and Karabük had the smallest relative risk of infant mortality.

Table 4.8. Posterior means of estimated relative risk of infant mortality for provinces and years.

	Year								
Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Adana	1.346	1.169	1.173	1.119	1.196	1.155	0.935	0.891	0.829
Adıyaman	1.526	1.294	1.283	1.267	1.146	1.116	1.053	1.096	1.015
A.karahisar	1.319	1.106	1.118	1.187	1.053	0.945	0.865	0.830	0.782
Ağrı	1.277	1.441	1.769	1.613	1.450	1.365	1.330	1.269	1.185
Aksaray	1.438	1.209	1.146	1.068	0.958	0.970	0.894	0.824	0.799
Amasya	0.997	0.854	0.842	0.857	0.813	0.900	0.784	0.721	0.651
Ankara	0.912	0.771	0.808	0.764	0.698	0.645	0.645	0.617	0.635
Antalya	1.053	0.882	0.791	0.805	0.737	0.786	0.753	0.700	0.664
Ardahan	1.366	1.186	1.151	1.178	1.067	1.036	0.951	0.925	0.887
Artvin	1.026	0.924	0.940	0.906	0.803	0.782	0.721	0.697	0.692
Aydın	1.210	1.118	1.033	0.995	0.856	0.828	0.755	0.712	0.671
Balıkesir	1.241	1.013	0.932	1.006	0.797	0.742	0.714	0.652	0.638
Bartın	1.005	0.881	0.859	0.843	0.766	0.779	0.691	0.684	0.646
Batman	1.259	1.164	1.262	1.362	1.425	1.387	1.209	1.138	1.068
Bayburt	0.985	0.848	0.809	0.817	0.790	0.829	0.783	0.751	0.724
Bilecik	0.925	0.809	0.813	0.848	0.809	0.824	0.788	0.780	0.774
Bingöl	1.548	1.435	1.433	1.486	1.363	1.350	1.285	1.364	1.234
Bitlis	1.229	1.117	1.132	1.238	1.269	1.305	1.277	1.206	1.213
Bolu	0.929	0.824	0.783	0.805	0.754	0.724	0.670	0.629	0.587
Burdur	1.176	1.049	0.988	0.963	0.922	0.979	0.893	0.866	0.848
Bursa	1.099	0.895	0.871	0.789	0.716	0.765	0.705	0.724	0.639
Çanakkale	1.370	1.139	1.074	1.061	0.904	0.788	0.677	0.628	0.595
Çankırı	1.022	0.886	0.851	0.862	0.817	0.822	0.760	0.753	0.685
Çorum	1.146	0.961	0.920	0.873	0.786	0.803	0.740	0.675	0.629
Denizli	1.274	1.036	1.010	0.966	0.816	0.792	0.786	0.773	0.719
Diyarbakır	1.418	1.223	1.242	1.325	1.264	1.235	1.041	0.966	0.891
Düzce	1.006	0.951	1.018	1.052	0.887	0.897	0.824	0.801	0.737
Edirne	0.854	0.754	0.715	0.705	0.648	0.630	0.572	0.559	0.533
Elazığ	1.478	1.245	1.162	1.248	1.217	1.122	1.051	1.052	1.003
Erzincan	1.081	0.949	0.886	0.871	0.817	0.790	0.713	0.690	0.675
Erzurum	1.279	1.102	1.131	1.111	1.244	1.215	1.189	1.152	1.134
Eskişehir	1.266	0.971	0.926	0.920	0.870	0.824	0.691	0.669	0.581
Gaziantep	1.911	1.612	1.526	1.474	1.508	1.553	1.385	1.260	1.180
Giresun	1.015	0.906	0.896	0.896	0.798	0.803	0.736	0.695	0.679
Gümüşhane	1.149	0.969	0.895	0.856	0.786	0.809	0.729	0.681	0.649
Hakkari	1.768	1.493	1.406	1.348	1.312	1.369	1.307	1.359	1.383
Hatay	1.225	1.116	1.194	1.218	1.029	1.032	0.940	0.969	0.946
Iğdır	1.192	1.023	0.963	0.980	0.990	1.045	0.953	0.922	0.871
Isparta	1.381	1.285	1.230	1.208	1.019	0.948	0.864	0.836	0.790
İstanbul	1.024	0.860	0.723	0.768	0.724	0.792	0.738	0.718	0.681
İzmir	1.160	0.912	0.953	0.939	0.706	0.795	0.746	0.659	0.623
K.maraş	1.305	1.178	1.237	1.200	1.175	1.100	1.073	1.052	0.956
Karabük	0.975	0.829	0.771	0.735	0.645	0.611	0.556	0.534	0.512
Karaman	1.561	1.308	1.241	1.142	0.952	0.896	0.818	0.778	0.739

Table 4.8. (cont.) Posterior means of estimated relative risk of infant mortality for provinces and years.

	Year								
Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Kars	1.018	1.064	1.173	1.193	1.189	1.175	1.147	1.054	0.917
Kastamonu	1.057	0.923	0.860	0.876	0.791	0.769	0.706	0.668	0.647
Kayseri	0.942	0.891	0.927	0.989	0.899	0.944	0.921	0.811	0.778
Kırıkkale	1.143	0.979	1.006	1.056	0.992	1.036	0.933	0.939	0.887
Kırklareli	0.760	0.669	0.673	0.680	0.666	0.644	0.592	0.598	0.606
Kırşehir	0.958	0.866	0.830	0.773	0.693	0.696	0.667	0.659	0.661
Kilis	1.725	1.522	1.538	1.673	1.787	1.858	1.632	1.498	1.472
Kocaeli	1.045	0.874	0.799	0.847	0.768	0.760	0.697	0.665	0.645
Konya	1.699	1.518	1.322	1.218	1.045	0.951	0.857	0.853	0.860
Kütahya	1.639	1.415	1.223	1.166	1.051	1.013	0.943	0.918	0.879
Malatya	1.800	1.517	1.434	1.564	1.280	1.162	0.983	0.934	0.865
Manisa	1.391	1.154	1.035	1.047	0.864	0.874	0.859	0.958	0.959
Mardin	1.554	1.464	1.363	1.372	1.312	1.386	1.187	1.203	1.200
Mersin	1.197	1.013	0.969	1.061	1.031	1.099	0.961	0.875	0.853
Muğla	1.159	0.973	0.932	0.827	0.715	0.687	0.661	0.696	0.634
Muş	1.849	1.448	1.377	1.314	1.224	1.266	1.315	1.330	1.307
Nevşehir	0.923	0.830	0.823	0.859	0.814	0.789	0.761	0.731	0.744
Niğde	1.143	1.039	0.974	0.991	0.855	0.873	0.801	0.781	0.742
Ordu	1.270	1.103	1.081	0.997	0.830	0.803	0.745	0.752	0.789
Osmaniye	1.314	1.206	1.244	1.184	1.086	1.046	0.996	1.005	0.906
Rize	0.926	0.848	0.820	0.834	0.721	0.730	0.681	0.715	0.722
Sakarya	1.433	1.156	1.081	1.001	0.829	0.801	0.772	0.753	0.785
Samsun	1.122	0.979	0.885	0.941	0.843	0.860	0.748	0.695	0.669
Siirt	1.515	1.407	1.587	1.557	1.410	1.411	1.322	1.261	1.181
Sinop	1.318	1.153	1.100	1.049	0.921	0.880	0.791	0.753	0.707
Sivas	1.227	1.058	1.050	1.003	0.953	0.906	0.905	0.852	0.820
Şanlıurfa	1.516	1.505	1.498	1.430	1.404	1.600	1.632	1.576	1.467
Şırnak	1.657	1.402	1.456	1.481	1.378	1.504	1.417	1.377	1.230
Tekirdağ	1.124	0.913	0.861	0.824	0.778	0.809	0.686	0.661	0.686
Tokat	1.735	1.467	1.406	1.210	1.010	1.027	0.956	0.923	0.855
Trabzon	0.906	0.819	0.816	0.869	0.840	0.852	0.778	0.730	0.738
Tunceli	1.048	0.925	0.910	0.890	0.809	0.818	0.754	0.719	0.704
Uşak	1.357	1.163	1.159	1.187	1.062	0.961	0.887	0.780	0.697
Van	1.367	1.159	1.125	1.046	1.427	1.503	1.431	1.256	1.109
Yalova	0.816	0.717	0.737	0.745	0.692	0.713	0.682	0.691	0.704
Yozgat	1.030	0.906	0.890	0.829	0.759	0.822	0.782	0.742	0.694
Zonguldak	0.793	0.723	0.752	0.816	0.763	0.793	0.784	0.826	0.825

In 2009, relative risks of infant mortality for many provinces were greater than 1. From 2009 to 2017 the number of dark colored provinces which had relative risk greater than 1, decreased (Figure 4.5).

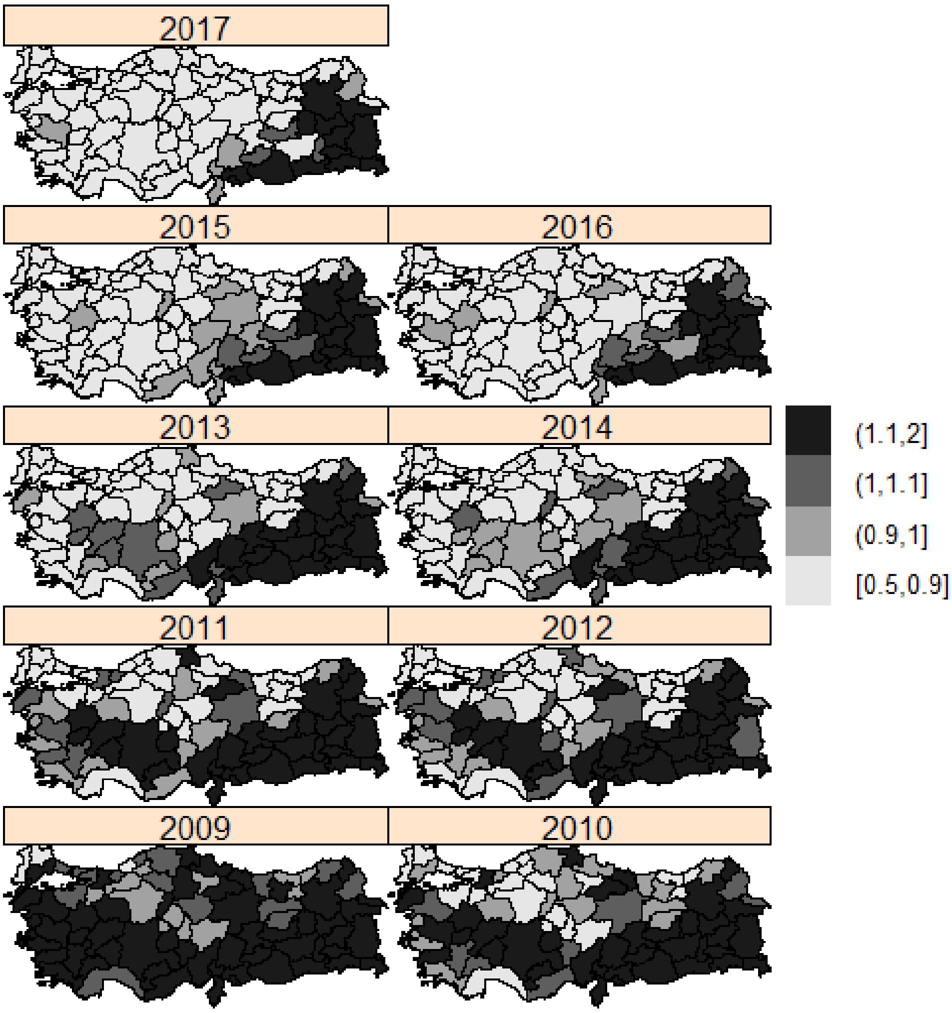

Figure 4.5. Posterior means of estimated relative risk of infant mortality.

Table 4.9 presents the posterior probability of exceeding 1 $\left(Pr(\theta > 1)\right)$ for relative risks of infant mortality for provinces and years.

- Zonguldak had the least exceedence probability with 0.002 and Afyonkarahisar, Adana, Adıyaman, Ağrı, Aksaray, Balıkesir, Batman, Bingöl, Çanakkale, Denizli, Diyarbakır, Elazığ, Erzurum, Gaziantep, Hakkari, Hatay, Isparta, İzmir, Kahramanmaraş, Karaman, Kilis, Konya, Kütahya, Malatya, Manisa, Mardin, Mersin, Muş, Ordu, Osmaniye, Sakarya, Siirt, Şanlıurfa, Şırnak, Tokat, Van had the greatest exceedence probability with 1.00 in 2009.
- Ankara, İstanbul, Zonguldak, Kırklareli had the least exceedence probability with 0.00 and Adana, Adıyaman, Ağrı, Bingöl, Diyarbakır, Elazığ, Gaziantep, Hakkari, Isparta, Kahramanmaraş, Kilis, Konya, Kütahya, Malatya, Mardin, Muş, Siirt, Şanlıurfa, Şırnak, Tokat, Van had the greatest exceedence probability with 1.00 in 2010.
- Ankara, İstanbul, Antalya, Kocaeli, Kırklareli, Zonguldak, Edirne, Bursa had the least exceedence probability with 0.00 and Adana, Adıyaman, Ağrı, Batman, Bingöl, Diyarbakır, Gaziantep, Hakkari, Hatay, Kahramanmaraş, Kilis, Konya, Malatya, Mardin, Muş, Osmaniye, Siirt, Şanlıurfa, Şırnak, Tokat, Van had the greatest exceedence probability with 1.00 in 2011.
- Ankara, İstanbul, Bursa, Antalya, Kırklareli, Edirne, Kocaeli had the least exceedence probability with 0.00 and Adıyaman, Ağrı, Batman, Bingöl, Bitlis, Diyarbakır, Elazığ, Gaziantep, Hakkari, Hatay, Kahramanmaraş, Kilis, Konya, Malatya, Mardin, Muş, Siirt, Şanlıurfa, Şırnak had the greatest exceedence probability with 1.00 in 2012.
- Ankara, Antalya, Balıkesir, Bursa, Çorum, Denizli, Edirne, İstanbul, İzmir, Karabük, Kırklareli, Kırşehir, Kocaeli, Muğla, Rize, Tekirdağ, Yalova, Zonguldak had the least exceedence probability with 0.00 and Adana, Ağrı, Batman, Bingöl, Bitlis, Diyarbakır, Erzurum, Gaziantep, Hakkari, Kahramanmaraş, Kilis, Malatya, Mardin, Muş, Siirt, Şanlıurfa, Şırnak, Van had the greatest exceedence probability with 1.00 in 2013.

- Ankara, Antalya, Balıkesir, Bolu, Bursa, Denizli, Edirne, İstanbul, İzmir, Karabük, Kırklareli, Kırşehir, Kocaeli, Muğla, Ordu, Rize , Sakarya, Tekirdağ, Yalova had the least exceedence probability with 0.00 and Erzurum, Adana, Muş, Bingöl, Bitlis, Hakkari, Van, Batman, Kilis, Ağrı, Diyarbakır, Siirt, Şırnak, Mardin, Gaziantep, Şanlıurfa had the greatest exceedence probability with 1.00 in 2014.
- Ankara, Antalya, Aydın, Balıkesir, Bartın, Bolu, Bursa, Çanakkale, Çorum, Denizli, Edirne, Erzincan, Eskişehir, Giresun, İstanbul, İzmir, Karabük, Kastamonu, Kırklareli, Kırşehir, Kocaeli, Konya, Muğla, Ordu, Rize , Sakarya, Tekirdağ, Trabzon, Yalova, Zonguldak had the least exceedence probability with 0.00 and Bingöl, Batman, Mardin, Hakkari, Bitlis, Siirt, Muş, Van, Kilis, Ağrı, Şırnak, Gaziantep, Şanlıurfa had the greatest exceedence probability with 1.00 in 2015.
- Amasya, Ankara, Antalya, Aydın, Balıkesir, Bolu, Bursa, Çanakkale, Çorum, Denizli, Edirne, Erzincan, Eskişehir, Giresun, İstanbul, İzmir, Karabük, Kastamonu, Kayseri, Kırklareli, Kırşehir, Kocaeli, Konya, Muğla, Nevşehir, Ordu, Rize, Sakarya, Tekirdağ, Trabzon, Yalova, Yozgat had the least exceedence probability with 0.00 and Siirt, Mardin, Bingöl, Hakkari, Ağrı, Kilis, Muş, Van, Şırnak, Gaziantep, Şanlıurfa had the greatest exceedence probability with 1.00 in 2016.
- Adana, Amasya, Ankara, Antalya, Aydın, Balıkesir, Bolu, Bursa, Çanakkale, Çorum, Denizli, Edirne, Erzincan, Eskişehir, Giresun, İstanbul, İzmir, Karabük, Kastamonu, Kayseri, Kırklareli, Kırşehir, Kocaeli, Muğla, Sakarya, Samsun, Tekirdağ, Trabzon, Uşak, Yozgat had the least exceedence probability with 0.00 and Şırnak, Mardin, Kilis, Hakkari, Muş, Gaziantep, Şanlıurfa had the greatest exceedence probability with 1.00 in 2017.

Table 4.9. Posterior probability of exceeding 1 (Pr(θ>1)) for relative risks of infant mortality for provinces and years.

	Year								
Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Adana	1.000	1.000	1.000	0.998	1.000	1.000	0.045	0.003	0.000
Adıyaman	1.000	1.000	1.000	1.000	0.992	0.974	0.813	0.939	0.577
A.karahisar	1.000	0.952	0.969	0.998	0.796	0.169	0.010	0.002	0.001
Ağrı	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997
Aksaray	1.000	0.997	0.978	0.823	0.260	0.322	0.056	0.005	0.005
Amasya	0.473	0.033	0.019	0.030	0.006	0.104	0.003	0.000	0.000
Ankara	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Antalya	0.857	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ardahan	0.994	0.936	0.905	0.939	0.719	0.613	0.309	0.246	0.178
Artvin	0.561	0.222	0.261	0.157	0.015	0.009	0.001	0.001	0.003
Aydın	0.998	0.972	0.702	0.458	0.004	0.001	0.000	0.000	0.000
Balıkesir	1.000	0.576	0.110	0.530	0.000	0.000	0.000	0.000	0.000
Bartın	0.496	0.108	0.062	0.042	0.004	0.007	0.000	0.001	0.001
Batman	1.000	0.997	1.000	1.000	1.000	1.000	1.000	0.989	0.846
Bayburt	0.434	0.093	0.039	0.043	0.022	0.058	0.025	0.016	0.016
Bilecik	0.241	0.019	0.016	0.039	0.012	0.020	0.007	0.008	0.015
Bingöl	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.992
Bitlis	0.998	0.955	0.974	1.000	1.000	1.000	1.000	0.998	0.995
Bolu	0.233	0.019	0.003	0.007	0.001	0.000	0.000	0.000	0.000
Burdur	0.929	0.679	0.433	0.323	0.169	0.391	0.103	0.068	0.068
Bursa	0.986	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Çanakkale	1.000	0.950	0.822	0.774	0.088	0.001	0.000	0.000	0.000
Çankırı	0.554	0.115	0.049	0.061	0.019	0.023	0.004	0.006	0.001
Çorum	0.956	0.277	0.109	0.024	0.000	0.001	0.000	0.000	0.000
Denizli	1.000	0.719	0.560	0.265	0.000	0.000	0.000	0.000	0.000
Diyarbakır	1.000	1.000	1.000	1.000	1.000	1.000	0.852	0.183	0.003
Düzce	0.510	0.258	0.576	0.733	0.056	0.076	0.007	0.004	0.001
Edirne	0.060	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Elazığ	1.000	1.000	0.992	1.000	0.999	0.964	0.773	0.769	0.502
Erzincan	0.747	0.279	0.085	0.057	0.011	0.005	0.000	0.000	0.000
Erzurum	1.000	0.964	0.990	0.975	1.000	1.000	0.999	0.995	0.977
Eskişehir	0.999	0.317	0.116	0.094	0.016	0.002	0.000	0.000	0.000
Gaziantep	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Giresun	0.547	0.105	0.078	0.078	0.002	0.003	0.000	0.000	0.000
Gümüşhane	0.855	0.371	0.140	0.065	0.010	0.022	0.002	0.001	0.001
Hakkari	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Hatay	1.000	0.995	1.000	1.000	0.742	0.765	0.075	0.234	0.127
Iğdır	0.965	0.596	0.305	0.385	0.432	0.700	0.264	0.163	0.077
Isparta	1.000	1.000	0.998	0.995	0.590	0.229	0.029	0.014	0.006
İstanbul	0.877	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
İzmir	1.000	0.006	0.090	0.040	0.000	0.000	0.000	0.000	0.000
K.maraş	1.000	1.000	1.000	1.000	1.000	0.977	0.927	0.842	0.203
Karabük	0.394	0.034	0.004	0.001	0.000	0.000	0.000	0.000	0.000
Karaman	1.000	0.999	0.996	0.945	0.264	0.092	0.010	0.003	0.002

Table 4.9. (cont.) Posterior probability of exceeding 1 (Pr(θ>1)) for relative risks of infant mortality for provinces and years.

	Year								
Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Kars	0.575	0.803	0.990	0.995	0.994	0.989	0.970	0.743	0.159
Kastamonu	0.702	0.165	0.033	0.052	0.002	0.001	0.000	0.000	0.000
Kayseri	0.136	0.011	0.059	0.401	0.015	0.112	0.045	0.000	0.000
Kırıkkale	0.901	0.392	0.512	0.727	0.445	0.645	0.205	0.240	0.125
Kırklareli	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Kırşehir	0.333	0.069	0.024	0.003	0.000	0.000	0.000	0.000	0.000
Kilis	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Kocaeli	0.802	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Konya	1.000	1.000	1.000	1.000	0.859	0.108	0.000	0.000	0.001
Kütahya	1.000	1.000	0.998	0.988	0.757	0.561	0.194	0.121	0.066
Malatya	1.000	1.000	1.000	1.000	1.000	0.995	0.377	0.130	0.020
Manisa	1.000	0.998	0.745	0.809	0.002	0.005	0.002	0.197	0.230
Mardin	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Mersin	1.000	0.604	0.238	0.906	0.746	0.985	0.185	0.002	0.001
Muğla	0.982	0.320	0.122	0.001	0.000	0.000	0.000	0.000	0.000
Muş	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Nevşehir	0.196	0.015	0.009	0.031	0.006	0.002	0.001	0.000	0.003
Niğde	0.941	0.687	0.349	0.435	0.016	0.033	0.002	0.001	0.001
Ordu	1.000	0.939	0.888	0.467	0.002	0.000	0.000	0.000	0.002
Osmaniye	1.000	0.999	1.000	0.997	0.903	0.755	0.463	0.516	0.095
Rize	0.207	0.028	0.008	0.014	0.000	0.000	0.000	0.000	0.001
Sakarya	1.000	0.994	0.910	0.494	0.001	0.000	0.000	0.000	0.000
Samsun	0.977	0.333	0.011	0.123	0.001	0.003	0.000	0.000	0.000
Siirt	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.985
Sinop	0.993	0.921	0.835	0.681	0.181	0.090	0.011	0.005	0.003
Sivas	0.998	0.808	0.773	0.505	0.220	0.061	0.061	0.010	0.006
Şanlıurfa	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Şırnak	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Tekirdağ	0.951	0.068	0.007	0.001	0.000	0.000	0.000	0.000	0.000
Tokat	1.000	1.000	1.000	0.998	0.547	0.644	0.246	0.131	0.032
Trabzon	0.083	0.001	0.001	0.013	0.003	0.006	0.000	0.000	0.000
Tunceli	0.588	0.274	0.227	0.175	0.047	0.059	0.018	0.011	0.014
Uşak	0.999	0.968	0.970	0.986	0.772	0.295	0.068	0.002	0.000
Van	1.000	1.000	0.997	0.836	1.000	1.000	1.000	1.000	0.983
Yalova	0.040	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.001
Yozgat	0.637	0.078	0.047	0.004	0.000	0.004	0.001	0.000	0.000
Zonguldak	0.002	0.000	0.000	0.002	0.000	0.001	0.000	0.007	0.016

Posterior probabilities exceeding 1 for relative risk of the infant mortality for provinces are given with maps from 2009 to 2017 in Figure 4.6. The categories of probabilities are shown with shades of gray in maps. In 2009 many of provinces had exceedence probability between 0.95 and 1. There were many significant risk areas in 2009. As years passed from 2009 to 2017 the number of provinces which have exceedence probability between 0.95 and 1, decreased. From 2009 to 2017 in each year consistently; Erzurum, Van, Siirt, Bingöl, Bitlis, Ağrı, Şırnak, Mardin, Kilis, Hakkari, Muş, Gaziantep, Şanlıurfa had exceedence probability between 0.95 and 1. Therefore from 2009 to 2017 in each year consistently; significant risk areas clustered in eastern and southeastern Anatolia regions.

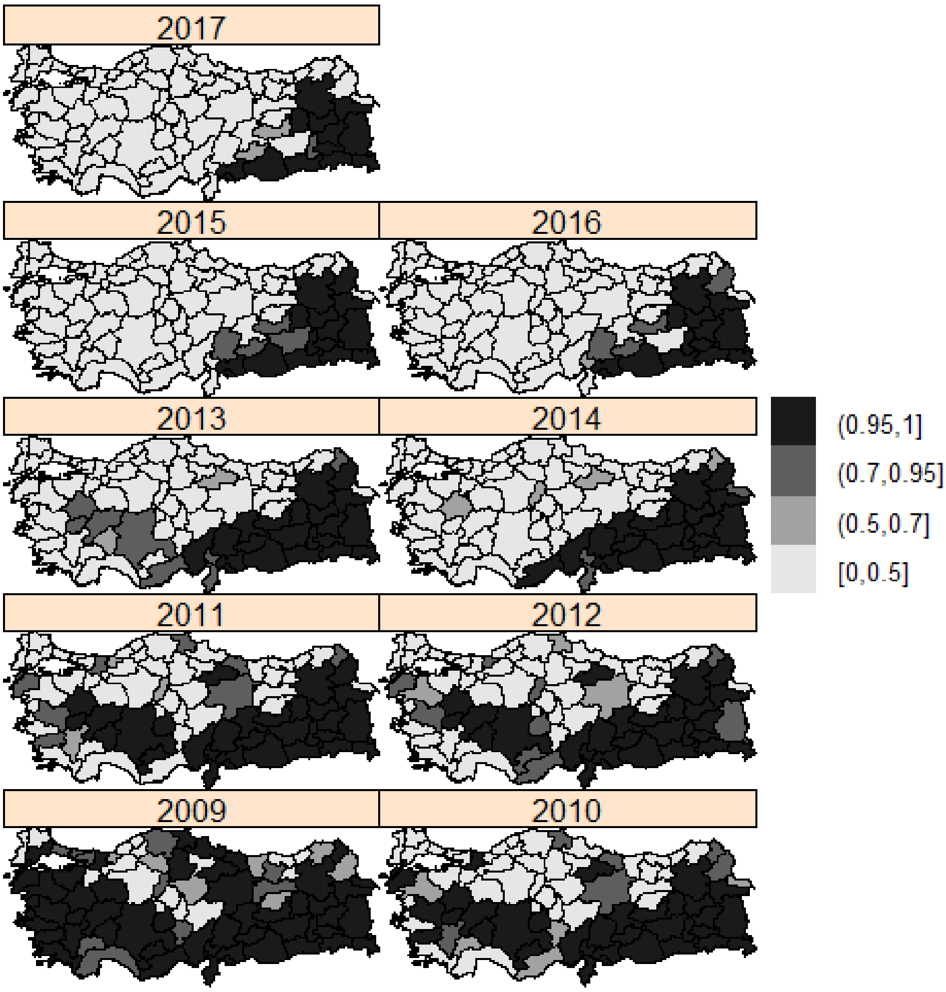

Figure 4.6. Posterior probability of exceeding 1 (Pr(θ>1)) for relative risks of infant mortality for provinces and years.

Percentage of mothers aged over 39, percentage of mothers aged under 20 and gross domestic product per capita which are thought to have an effect on relative risk of infant mortality were examined. Maps of the gross domestic per capita ($) from 2009 to 2017 are presented in Figure 4.7. In the figure provinces in the highest category of GDP per capita and in the lowest category of GDP per capita are colored with black and light grey, respectively. From 2009 to 2014 the number of provinces with black colored increased, from 2015 to 2017 the number provinces with black colored decreased. From 2009 to 2014 the number of provinces with ligh grey colored decreased, from 2015 to 2017 the number provinces with ligh grey increased. In each year from 2009 to 2017 consistently; Ağrı, Van, Şanlıurfa were in the lowest category of GDP per capita and İstanbul, Ankara, İzmir, Kocaeli, Bursa, Tekirdağ, Bolu, Antalya, Bilecik were in the highest category of GDP per capita.

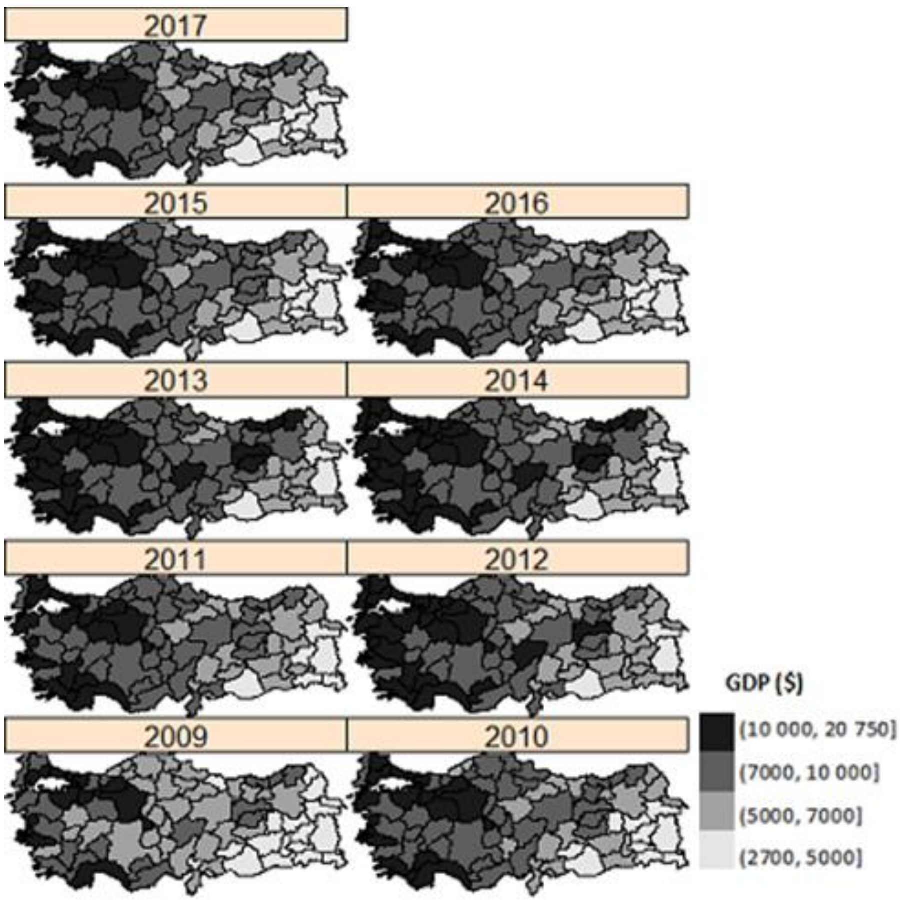

Figure 4.7. GDP per capita ($) in Turkey from 2009 to 2017.

Maps of percentage of live-born infants’ mothers aged under 20 from 2009 to 2017 are given in Figure 4.8. In the figure provinces in the highest category of mother aged under 20 as percentage and in the lowest category of mother aged under 20 as percentage are colored with black and light grey, respectively. In 2009 Ağrı, Ardahan, Muş, Kars, Yozgat, Kırıkkale and Niğde were in the highest category. In 2010 Ağrı and Kars were in the highest category. In 2011 Ağrı was in the highest category. From 2012 to 2017 there were no province in the highest category. As year passed from 2009 to 2017 the number of dark colored provinces decreased. From 2012 to 2017 in each year consistently; Iğdır, Kilis, Bitlis, Ağrı and Muş were in category (10, 15] with dark grey color.

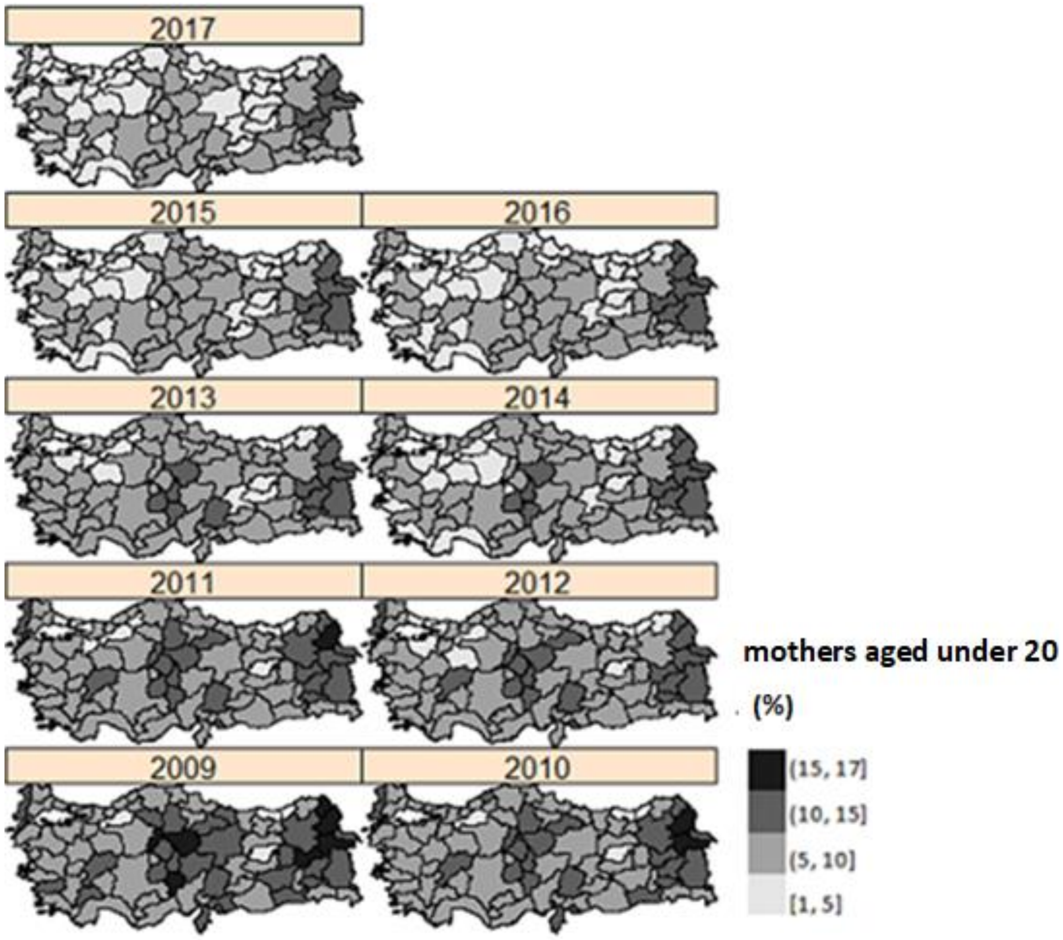

Figure 4.8. Percentage of live-born infants' mothers aged under 20 in Turkey from 2009 to 2017.

Maps of percentage of live-born infants' mothers aged over 39 from 2009 to 2017 are presented in Figure 4.9. In the figure provinces in the highest category of mother aged over 39 as percentage and in the lowest category of mother aged over 39 as percentage are colored with black and light grey, respectively. In 2009 many of the provinces were in the lowest category. As years passed from 2010 to 2017 the number of provinces with grey colored increased.

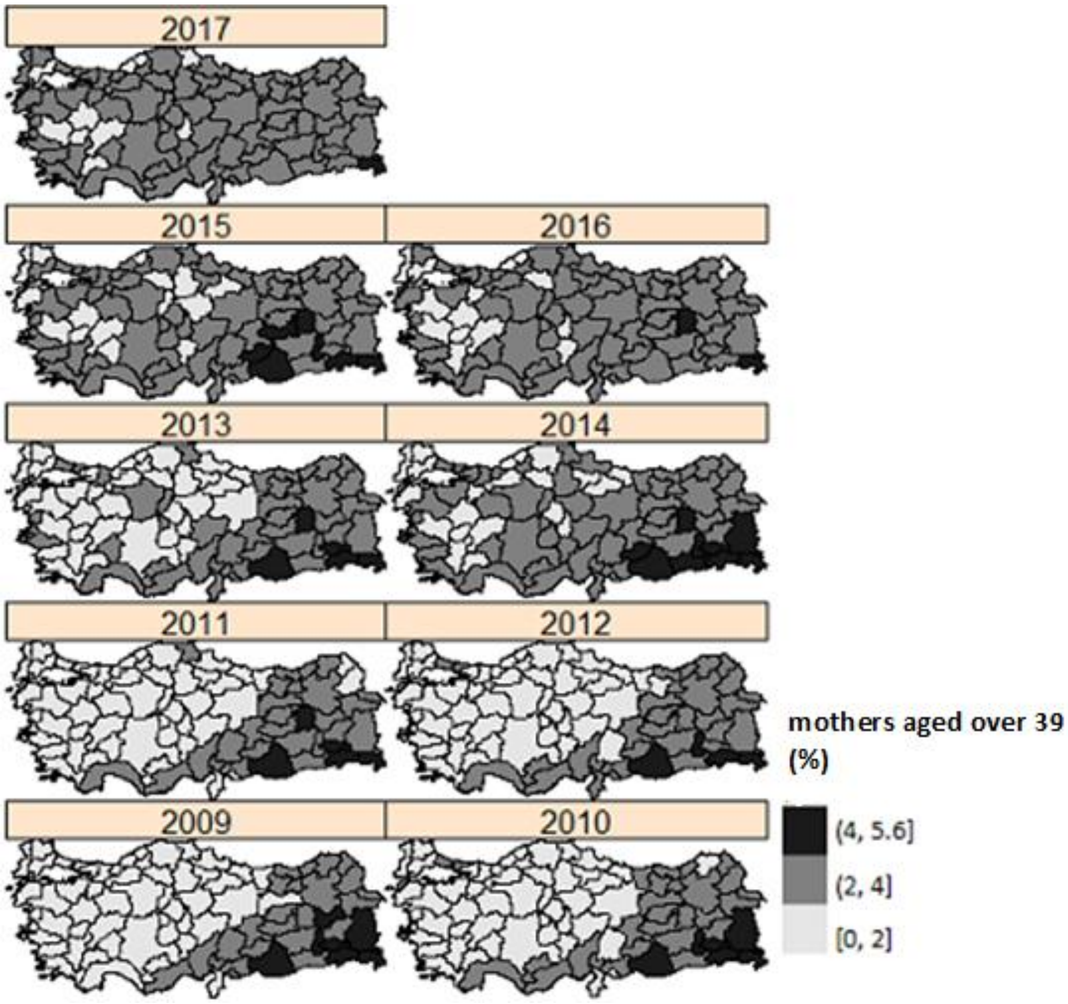

Figure 4.9. Percentage of live-born infants' mothers aged over 39 in Turkey from 2009 to 2017.

The fixed effects of percentage of mothers aged under 20, percentage of mothers aged over 39 and per capita gross domestic product ($) on relative risk of infant mortality were evaluated with generalized linear model given with Equation 3.7 and model 4 a (rw1) given with Equation 3.8. DIC of the model, posterior means of fixed effects and effects of percentages of mothers and GDP per capita on RR of infant mortality with credibility interval (Cl) are given in Table 4.10. DIC of model 4 a (rw1) (5755.53) is lower than DIC of generalized linear model (9366.52). For generalized model, an increase of 1000 $ in gross domestic product per capita reduced the relative risk of infant mortality by 2.3%. An increase in the percentage of mothers aged under 20 increased the relative risk of infant mortality by 3.8%. An increase in the percentage of mothers aged over 39 increased the relative risk of infant mortality by 6%. For spatio-temporal Bayesian model, an increase of 1000 $ in the gross domestic product per capita reduced the relative risk of infant mortality by 2.8%. Percentage of mothers aged under 20 (RR=1.011; 95% Cl (0.986, 1.037) and percentage of mothers aged over 39 (RR=0.985; 95% Cl (0.940, 1.032)) had no effect on the relative risk of infant mortality.

Table 4.10. DIC, posterior means of fixed effects and effects of percentages of mothers and GDP on RR of infant mortality with credibility interval

Generalized linear model (DIC=9366.52)			
	mean	RR	%95 Cl of RR
% mothers aged under 20	0.037	1.038	(1.035, 1.040)
% mothers aged over 39	0.058	1.060	(1.053, 1.067)
GDP per capita/1000 ($)	-0.023	0.977	(0.975, 0.979)
Model 4 a (rw1) (DIC=5755.53)			
	mean	RR	%95 Cl of RR
% mothers aged under 20	0.011	1.011	(0.986, 1.037)
%mothers aged over 39	-0.015	0.985	(0.940, 1.032)
GDP per capita/1000 ($)	-0.029	0.972	(0.946, 0.998)

From the results; it is seen that using spatio-temporal Bayesian model is suitable for determining risky provinces in Turkey in terms of infant mortality with taking account space, time and space-time interaction.

5. DISCUSSION

Disease mapping is generally formed by identifying the risks of different geographic regions which are determined by standardized mortality or incidence ratio. Since direct use of this ratio is not suitable especially for small regions, relative risk of infant mortality was estimated with spatio-temporal Bayesian models in this study.

Although there are many studies in the literature revealing the spatio-temporal structure of diseases with spatio-temporal Bayesian models, there is no study that determines the spatio-temporal structure of relative risk of infant mortality in Turkey with Bayesian method.

The change in infant mortality rate from 1988 to 2008 at five-year intervals was given as a percentage in a study by Yalçın et al. (44). Infant mortality rate and related factors for 7 provinces in northeastern Turkey were examined with Chi-square analysis, analysis of variance (ANOVA), and multinomial regression in a study by Vançelik et al. (45). Risk factors of infant mortality at district-level in nine states of India were determined with spatial regressions by Grupta et al. (46). Effects of racial and poverty segration on infant mortality in the United States were assessed with spatial Bayesian regression using MCMC method by Sparks et al. (47).

In this thesis, unlike the literature, infant mortality in Turkey was examined in terms of relative risk. Parametric and non-parametric spatio-temporal Bayesian models were implemented to estimate relative risk of infant mortality. Maps for relative risk of infant mortality from 2009 to 2017 were presented. In this thesis, answers were sought about what the probabilities of provinces' relative risks of infant mortality were and where and when the significant risk clusters of infant mortality were. Maps of exceedence probabilities from 2009 to 2017 were presented. Clusters of exceess risk were specified.

In parametric model; the temporal effect was considered as a constant effect with a linear trend. In nonparametric models, the temporal effect was considered as a random effect. Structured spatial and temporal effect was examined by considering the neighborhood structure. Heterogeneity arising from the location on the basis of the province and the heterogeneity arising from the time on the basis of year was examined with unstructured spatial effect and unstructured temporal effect, respectively. Spatial-temporal interaction effect was examined in the model as different combinations of structured and unstructured spatial and temporal effects. All spatio-temporal Bayesian models were applied with using INLA. After the best model was selected; contribution of spatial, temporal and spatio-temporal interaction effects to variability of relative risk of infant mortality were determined.

The fixed effects of percentage of live-born infants' mothers aged under 20, percentage of live-born infants' mothers aged over 39 and gross domestic product per capita, which are thought to have an effect on relative risk of infant mortality, were examined with both the generalized linear model and the best spatio-temporal Bayesian model. Generalized linear model estimated relative risk of infant mortality without the spatial, temporal and spatio-temporal interaction random effects. Both models tested the hypothesis that percentage of live-born infants' mothers aged under 20, percentage of live-born infants' mothers aged over 39 and the gross domestic product per capita had no effect on the risk of infant mortality. In the generalized linear model, all three null hypotheses were rejected. However, in the spatio-temporal Bayesian model, age-related hypotheses were not rejected and hypothesis about per capita gross domestic product was rejected.

6. CONCLUSION and RECOMMENDATIONS

In these study; nonparametric and parametric Bayesian spatio-temporal models were applied to estimate relative risk of infant mortality for provinces in Turkey from 2009 to 2017. Nonparametric models included spatio-temporal Bayesian model without spatio-temporal interaction and spatio-temporal Bayesian models with different spatio-temporal interactions. It was determined that there was a decrease in DIC with the addition of different combinations of interaction to the nonparametric spatio-temporal Bayesian model where the interaction effect was not included. The spatio-temporal models with spatio-temporal interaction have more performance to estimate relative risk of infant mortality than the spatio-temporal model without interaction. In conclusion; nonparametric Bayesian model with structured temporal and unstructured spatial interaction random effect is the best model to reveal spatio-temporal pattern of relative risk of infant mortality for provinces in Turkey from 2009 to 2017. Unstructured spatial and structured temporal interaction random effect and structured temporal random effect were found to be more effective than spatial effect to explain spatio-temporal variability of relative risk of infant mortality.

From 2009 to 2017 in each year consistently; Erzurum, Van, Siirt, Bingöl, Bitlis, Ağrı, Şırnak, Mardin, Kilis, Hakkari, Muş, Gaziantep, Şanlıurfa had probability of exceeding 1 for relative risk of infant mortality between 0.95 and 1. Therefore from 2009 to 2017 in each year consistently; significant risk areas clustered in eastern and southeastern Anatolia regions. It is important to produce health policies that will save these regions from this structure. In addition, it is very important that the increase in gross domestic product per capita in these regions is taken into consideration by decision-makers as a factor reducing relative risk of infant mortality.

As percentage of mothers aged under 20 and percentage of mothers aged over 39 increased, relative risk of infant mortality increased for generalized model. The percentage of mothers aged under 20 and percentage of mothers aged over 39 had no effect on relative risk of infant mortality for spatio-temporal Bayesian model.

But spatio-temporal Bayesian model can be more preferable than generalized linear model, because of having lower DIC than generalized linear model. Therefore, while determining the factors that may have an effect on relative risk of infant mortality, it is also important to consider the effects of space, time and space-time interaction.

When planning the study, firstly, it was wanted to work with point reference data. However, since point reference data could not be obtained from Turkish Statistical Institute due to the scope of the protection of personal data law and related legislation, it was not possible to work on point reference data. Therefore, it was decided to work with areal data (province basis). Subsequently, some data were requested from the Ministry of Health and related units on provincial basis that could have an effect on relative risk of infant mortality. But these data could not be obtained. It would be beneficial for institutions to support such studies both in determining and evaluating health policies.

7. REFERENCES

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8. APPENDICES

APPENDIX-1: Infant mortality rate (‰) for provinces in Turkey from 2009 to 2017

Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Adana	15.0	12.9	13.3	11.8	13.9	13.2	10.0	10.0	9.1
Adıyaman	17.3	14.0	14.5	14.2	12.8	12.1	11.2	13.2	11.0
A.karahisar	15.1	11.5	12.1	14.4	12.5	9.5	9.6	9.3	8.5
Ağrı	12.3	16.2	22.0	17.5	16.4	14.4	15.3	14.2	13.0
Aksaray	16.7	13.3	13.4	11.2	9.9	11.2	10.7	8.1	9.1
Amasya	11.7	8.6	8.9	9.4	6.7	15.3	8.3	8.3	5.3
Ankara	10.2	8.4	9.2	8.4	7.9	6.9	7.3	6.7	7.2
Antalya	11.9	10.0	8.4	9.1	7.9	8.9	8.6	7.7	7.4
Ardahan	15.7	13.0	10.3	16.6	12.5	10.4	9.1	10.7	9.9
Artvin	9.7	9.5	15.0	10.7	7.9	7.7	7.6	5.5	9.9
Aydın	13.0	13.5	11.2	11.4	9.2	9.1	8.5	7.8	7.3
Balıkesir	14.5	11.2	9.0	13.4	8.1	7.3	8.8	6.5	7.3
Bartın	11.1	10.0	10.1	9.6	6.5	12.2	3.8	9.7	6.2
Batman	13.5	12.5	14.2	14.8	17.2	15.8	13.2	12.6	11.7
Bayburt	12.5	9.9	5.0	6.2	8.2	12.6	11.3	6.6	8.8
Bilecik	10.4	7.5	8.1	10.1	9.6	8.8	9.0	8.2	10.2
Bingöl	15.9	16.8	15.5	17.6	15.2	14.3	12.8	18.5	12.4
Bitlis	13.2	12.4	11.8	13.6	15.2	14.2	15.3	12.4	14.2
Bolu	9.8	10.7	6.5	9.7	10.2	6.8	8.5	6.9	5.5
Burdur	12.4	13.9	10.3	8.3	9.1	14.1	9.1	8.8	10.4
Bursa	12.5	9.7	10.0	8.6	7.7	8.7	7.7	8.5	6.9
Çanakkale	16.5	12.2	11.3	13.5	11.4	7.5	6.7	6.3	6.4
Çankırı	11.8	10.2	7.6	9.1	10.0	9.8	6.9	12.6	4.6
Çorum	13.4	10.3	10.7	9.3	7.9	9.5	9.2	6.9	6.5
Denizli	14.9	10.8	11.6	11.2	8.5	8.0	9.2	9.0	7.7
Diyarbakır	15.8	13.5	13.7	14.9	14.2	14.0	11.4	10.7	9.8
Düzce	9.3	10.2	12.6	14.9	7.4	10.5	8.8	10.0	7.2
Edirne	9.2	9.6	7.3	7.7	7.9	6.9	5.4	6.6	5.8
Elazığ	17.0	13.9	11.3	14.2	15.4	11.3	11.4	12.3	11.1
Erzincan	11.9	12.5	8.5	8.6	10.9	8.7	6.8	6.8	8.4
Erzurum	14.4	11.8	13.1	10.8	15.5	13.0	13.6	12.6	12.9
Eskişehir	16.0	9.2	10.0	9.9	10.7	9.8	6.4	8.7	5.3
Gaziantep	21.4	17.9	17.0	16.1	17.0	17.5	15.5	13.9	13.1
Giresun	10.8	10.2	10.1	11.1	7.7	9.6	8.4	6.5	8.1
Gümüşhane	15.0	12.0	8.8	7.2	6.4	12.8	8.5	5.7	6.8
Hakkari	20.4	16.6	15.6	13.2	14.8	15.6	13.6	15.4	16.8
Hatay	13.5	12.2	13.6	14.0	11.1	11.6	10.1	11.0	10.6
Iğdır	13.8	11.9	9.0	9.1	11.9	13.6	10.2	10.6	9.3
Isparta	14.1	16.1	13.5	15.2	10.9	9.3	9.2	9.6	8.4
İstanbul	11.4	9.6	7.9	8.6	8.0	8.9	8.2	8.0	7.6
İzmir	13.2	9.7	10.8	10.9	7.1	9.1	8.5	7.2	6.9
K.maraş	14.4	12.9	14.3	12.9	13.7	11.6	12.2	12.1	10.3
Karabük	12.2	10.2	8.2	8.7	7.1	5.2	5.6	5.5	5.6
Karaman	18.7	14.1	15.4	13.5	9.2	8.6	9.1	8.3	7.9
Kars	8.3	12.8	14.8	12.3	14.9	11.9	14.8	12.7	8.1
Kastamonu	11.8	11.5	7.5	11.1	8.9	8.1	8.2	6.3	7.5

Province	2009	2010	2011	2012	2013	2014	2015	2016	2017
Kayseri	10.1	10.0	10.3	11.5	9.7	10.5	11.0	8.5	8.7
Kırıkkale	13.4	8.4	11.0	13.2	9.9	14.3	7.8	12.4	9.3
Kırklareli	8.3	6.5	8.0	6.0	10.6	6.4	4.8	6.1	8.6
Kırşehir	9.3	11.8	11.3	7.0	6.2	6.7	7.9	6.2	9.4
Kilis	18.8	15.8	14.6	16.2	24.4	24.6	18.0	13.3	17.5
Kocaeli	11.9	9.8	8.3	9.8	8.5	8.5	7.8	7.3	7.2
Konya	18.8	17.4	14.6	13.6	11.7	10.4	9.3	9.4	9.8
Kütahya	18.5	17.6	12.1	12.7	11.9	10.6	10.4	10.3	9.8
Malatya	20.4	16.8	14.7	19.6	14.0	12.9	10.2	10.6	9.3
Manisa	15.9	13.1	10.8	12.5	8.8	9.5	9.0	11.3	11.0
Mardin	16.8	17.0	14.8	15.1	14.5	16.2	12.5	13.4	13.6
Mersin	13.5	11.2	10.2	12.0	11.4	12.8	10.7	9.4	9.6
Muğla	13.4	10.6	11.4	8.7	7.6	6.8	6.8	9.2	6.5
Muş	21.7	15.2	15.5	14.1	13.3	13.3	15.4	15.0	14.9
Nevşehir	9.6	9.5	8.2	10.2	10.6	6.9	9.8	6.1	10.2
Niğde	12.0	13.1	9.5	12.8	7.6	10.4	8.5	9.1	8.1
Ordu	14.2	12.1	13.1	11.5	8.3	8.5	7.7	7.7	10.0
Osmaniye	14.0	13.3	15.2	12.7	12.5	10.9	10.9	12.6	9.2
Rize	9.1	11.0	8.2	11.9	5.8	8.4	5.2	8.9	9.6
Sakarya	16.7	12.5	12.3	11.5	8.5	8.3	8.9	7.7	9.5
Samsun	12.5	11.4	8.8	11.2	9.0	10.2	8.1	7.4	7.5
Siirt	16.1	14.5	19.9	17.4	15.4	15.6	15.1	14.2	12.8
Sinop	14.7	13.9	13.1	12.1	9.8	9.2	8.0	8.5	6.7
Sivas	13.8	11.4	12.4	10.3	11.6	8.5	11.4	9.1	9.2
Şanlıurfa	16.6	17.0	16.8	15.7	15.5	17.9	18.4	17.6	16.3
Şırnak	18.8	14.7	16.5	16.6	14.6	17.3	15.9	16.1	13.1
Tekirdağ	13.4	9.7	9.6	8.5	8.5	10.2	6.9	6.6	8.3
Tokat	19.8	16.1	17.5	12.8	9.4	11.7	10.7	10.8	8.9
Trabzon	9.7	9.2	8.5	9.9	9.7	9.9	8.8	7.1	8.9
Tunceli	10.5	10.3	13.3	9.8	5.2	11.7	9.3	3.1	10.7
Uşak	15.7	11.8	12.6	14.9	13.6	8.3	12.3	7.9	5.8
Van	15.4	12.8	12.8	9.7	17.3	16.9	16.4	13.9	12.0
Yalova	8.9	5.8	9.8	8.8	6.6	8.0	7.0	7.0	10.2
Yozgat	11.4	10.1	11.1	8.2	6.7	10.2	9.6	8.3	7.2
Zonguldak	8.3	7.7	8.0	10.2	8.1	8.4	8.3	10.0	10.0