

5<sup>th</sup> World Conference on Educational Sciences - WCES 2013

## A study on teaching proof to 7<sup>th</sup> grade students

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### Abstract

This study has been prepared with the purpose to investigate proof teaching on early grades. The purpose of this study is to investigate proof on 7<sup>th</sup> grades and 7<sup>th</sup> grade students' performance on proof. Data have been obtained during lessons focused on proof teaching by video recording. At the end of the research it is seen that proof can be adapted to this elementary grade and students could prove depending on their mathematical knowledge, their abstraction capacity, and their algebra knowledge.

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Selection and/or peer-review under responsibility of Academic World Education and Research Center.

*Keywords:* Proof, verification, proof teaching, proof for all grades;

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### 1. Introduction

Mathematics is intrinsically about proof because it deals not only with what actually truth is, but also why it is true and also it convince other people that it is true (Almedia, 1996). Proof is necessary for meaningful learning and it prevents rote learning. For this reason it should be seen as an important part of mathematics teaching process, it shouldn't be seen as only a special topic in the curriculum. Proof teaching is valuable for mathematics teaching process not only for reaching the correct mathematical statement, but also for knowing and doing mathematics correctly, for building the basis of mathematical thinking, and for understanding, using and improving the mathematical knowledge (Hanna and Jahnke, 1996; Kitcher, 1984; Polya, 1981). It should be "a natural, ongoing part of classroom discussions, no matter what topic is being studied" (NCTM, 2000, p.342).

The number of studies supporting that proof should have a central part of all grade levels of the mathematics education, that proof is important at knowing and doing mathematics, and that it should be incorporated into the mathematical experiences of elementary students is increased rapidly (Stylianides, 2007; Ball & Bass, 2003; Ball etc., 2002; Hanna, 1995; NCTM, 2000, Zack, 1997; Lampert, 1990; Tall, 1999). Despite this attributed importance, proof teaching takes mostly part in high school and university level, in the process of mathematics education. For example, students in Turkey encounter with proof, especially in geometry courses, intensively in 9th grade in secondary education and also in further grades.

As it is mentioned before, proof is important for learning mathematics. It prevents rote learning, provides conceptual and meaningful learning. So proof teaching should be adapted to all grade levels and should be central to

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student's school mathematical experiences. But it shouldn't be forgotten that process of studying proof with lower grade students is not so easy. The questions, "What is proof?", "What could count as a proof in the early grades?" and "What is the difference between proof and verification?" come into prominence at that point. Sometimes, every verification can be accepted as a proof or justification by using examples can be count as a "proof scheme" (Harel & Sowder, 1998).

Students usually tend to accept empirical arguments as proofs of mathematical generalizations (Stylianides & Stylianides, 2009). At that point Stylianides (2007) suggested a definition for the act of proving that is embedded in the classroom context in order to clarify what actually proof is; proof is mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

1. It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justifications;
2. It employs forms of reasoning (modes of argumentation) that are valid and known to, or within the conceptual reach of, the classroom community; and last
3. It is communicated with forms of expression (modes of argument representation) that are appropriate and known to, or within the conceptual reach of, the classroom community.

For Stylianides (2007), the main difference between proof and empirical arguments lies in the modes of argumentation. At one of his article, he described this difference more clearly:

Empirical arguments provide inconclusive evidence by verifying its truth only for a proper subset of all the cases covered by the generalization, whereas proofs provide conclusive evidence for its truth by treating appropriately all cases covered by the generalization. Thus, we may say that, in contrast to empirical arguments, proofs offer secure methods for validation of mathematical generalizations (Stylianides & Stylianides, 2009).

In the view of these discussions the purpose of this study is to investigate proof on 7<sup>th</sup> grades and their performance on proving. In particular, this study is conducted to find the answer of the following questions:

1. Is "proof" really a strange concept for 7th graders?
2. Can 7<sup>th</sup> grade students prove?
3. Can 7<sup>th</sup> grade student's ability/capacity on proving be improved?

## 2. Method and procedure

To answer the above research questions, several stages were followed. First of all content of the lessons were constructed from mind game puzzles and the 6<sup>th</sup> and 7<sup>th</sup> grade mathematics topics; odd and even numbers, consecutive numbers, and divisibility rules. Mind game puzzles were used in order to keep students interest at lessons and also try to give the logic of proof to them. Later, these lesson plans were presented to the expert opinion. According to their suggestions and criticism, lesson plans were finalized.

This research held in a private elementary school in Ankara, with 16 7<sup>th</sup> grade students at second semester of 2011-2012 educational years. It took 2 months, one hour lesson in a week.

First lesson was started by questioning what proof and proving were. After that discussion, a brief summary about algebraic notation and properties of numbers were given to the students. Then the class started to deal with mathematical proof examples.

During the whole lessons, teacher took an active role by guiding students. All the lessons were video recorded.

## 3. Findings and discussion

### 3.1. Is "proof" a strange concept for 7th graders?

"Proof" is a strange word for the 7<sup>th</sup> grade students within the scope of mathematics lesson. But they have been very familiar with proving in their daily life, especially while playing games.

Lessons started with talking about proof and proving. Students hadn't got any opinion about proof as a part of mathematics course and so the first aim was to build it by using daily life situations. Answers given by students was tried to connect to what proof was during the discussions.

At the first lesson, the meaning of "proof" was asked to the students. Some of their responses were:

- "Trying to verify something."
- "Trying to convince my friends that I'm telling the truth."
- Student: I think proof means showing the truth.

Teacher: To show whom?

Student: To my friends.

Teacher: Why? What happens if you don't show?

Student: No, if I don't, they don't believe me.

As it seen from these responses students usually used the words "verification", "truth", "convince someone" when explaining "proof".

Then, the class started to talk about proving. The responses of the question "how can you prove something?" were as follows:

- "I present them something, may be evidences."
- "Go and ask someone who knows, for example my Mom."
- "I talk to him, I tell him what I believe and some other truths, truths known by someone else, and lastly I make him to believe me."

Consulting to an authority, showing concrete evidence and talking in order to convince someone were the most common ones that took parts at students' responses.

All of these students' responses show that students are not so far away from the idea of proof.

### 3.2. 1. How can mathematical proof be applied to the 7th grade and can 7th grade students prove?

Many studies reveal that students tend to use concrete examples when they prove (Fischbein, 1982; Knuth & Sutherland, 2004; Özer & Arıkan, 2002; Stylianides & Stylianides, 2009; Stylianides, 2007; Zaskis, Liljedahl & Chernoff, 2008). Similar to these studies, at the beginning of the lessons, students tended to use examples when they tried to prove. Some of them gave only one example, but some of them didn't content with a single example and preferred several examples. When the discussion focused on the claim "The difference of two numbers which are multiples of 3 is also divided by 3", the class started to argue on "Is an empirical argument enough for reaching mathematical generalizations?". But it was seen that they were still continuing on using examples (All the names are pseudonyms);

Ayşe: Let's take 9 and 3, then I get 6 and yes 6 is divided by 3.

Teacher: Thank you Ayşe, as you see [to the class] she picks only two numbers and she gets the result. But is it enough to make a generalization? What if there exist a number of pairs may be they can be very big numbers, whose difference is not divided by 3?

Ozan: At that time... Let's take 2 times and 3 times of 3, and then 4 times and 5 times of it ... Yeah, it is correct [he is calculating on his notebook]. Now we take 10 and 12 times of 3 in order to get bigger numbers ... again we get a number, mmm, yes it is again divided by 3.

After seeing that the class continued persistently on accepting empirical arguments as proofs, "The Circles and Spots Problem" which was adapted from Mason (Stylianides & Stylianides, 2009) used for challenging students reliance on empirical arguments. This question made them more suspicious on empirical arguments and they mostly started to prove sometimes by using algebra, sometimes by using only numbers and their logics.

For example, after dealing with the difference between justification by examples and proof by the help of “The Circles and Spots Problem”, students who were reminded algebra at the beginning of the lessons and who were capable of using algebra, started to discuss on a proof as follows;

*“The sum of two odd numbers is even.”*

Berk: I take two numbers, let’s say  $k$  and  $k$ . And then the sum will be  $2k$ .

Teacher: Is  $k$  always odd? My claim is ‘the sum of every two odd number is even’.

Berk: No, it is not always odd. For example, if I take 2 as  $k$ ... The sum is 4; it is an even number but mmm... Yeah, this example doesn’t verify your claim.

Teacher: [to class] In this case, how can we choose these numbers appropriately? [Mert raise his hands.]

Mert: We can take  $2k - 1$  and  $2m + 1$ , these numbers are always odd. Then sum of these numbers... We get  $2k + 2m$ , which is always even, no matter what  $k$  and  $m$  will be.

Furthermore, students could also reach to the generalization by using more creative ways. After the discussion above, Ali claimed that there exists another way to prove and he started to describe it on blackboard;

Ali: I take the odd numbers which are between 1 and 10, and then I’ll sum all the pairs. [Then he wrote at blackboard]

$$1+1=2 \quad 3+5=8 \quad 5+7=12 \quad 7+9=16$$

$$1+3=4 \quad 3+7=10 \quad 5+9=14$$

$$1+5=6 \quad 3+9=12$$

$$1+7=8$$

$$1+9=10$$

Ali: We see that the sum of two odd digits is always even, since I tried all the possible pairs. It does not matter if I pick two very very big odd numbers, it does not matter which number you pick, all of their units digit must be one of them, 1 to 9. And so, very very big odd numbers sums units digits must be one of these sums units digit. As we see they are all even.

Teacher: Why are you looking at unit’s digit?

Ali: Because when we look at a number is odd or even, we must look at its unit’s digit. If it is a multiple of 2, it is an even number. Whether how big two numbers you add, the result is always odd if the unit’s digit of the sum is not 0 or any multiple of 2. Namely, I prove this by using unit’s digit.

#### 4. Conclusion

To conclude, the research was focused on 7<sup>th</sup> graders first encounter with proof by the help of an active role of the teacher. The active role means not to content with student’s response only, at the same time guide them to reach the appropriate proof.

“Proof” is a strange word for the 7<sup>th</sup> grade students within the scope of mathematics lesson. But students are not so far away from the idea of proof in their daily life experience.

First of all, at the beginning of the study it was seen that students had a tendency to use empirical arguments for validating mathematical generalization. Then after focusing on the transition from empirical arguments to proof by using “The Circles and Spots Problem” which was adapted from Mason (Stylianides & Stylianides, 2009), the students started to use different ways when they tried to prove.

At the end of the research it was seen that students could understand what actually proof was, could produce proof depending on their mathematical knowledge, their abstraction capacity, and their algebra knowledge.

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