Education and Science tedmem

Vol 40 (2015) No 177 141-159

Deficits in Basic Number Competencies May Cause Low Numeracy in Primary School Children *

Sinan Olkun [1], Arif Altun [2], Sakine Göçer Şahin [3], Zeynep Akkurt Denizli [4]

Abstract

There are two hypotheses about why individuals have mathematics learning difficulties (MLD). The core deficit hypothesis claims that disorders in number module which was designated for processing quantities either at approximate or exact levels cause learning difficulties in mathematics. The access deficit hypothesis on the other hand posits that the reason behind MLD is not deficits in processing quantities but deficits in connecting quantities to symbols or vice versa. To test these two hypotheses, we designed dot enumeration, symbolic number comparison, and mental number line tasks. Participants were 487 students from 1st to 4th grades selected from 12 different schools in a mid-Anatolian, large metropolitan city in Turkey. Students were given a curriculum based arithmetic achievement test and they were divided into four groups as MLD risk, low achieving, typical achieving, and high achieving based on the achievement test scores. Results showed that there were large significant differences both among groups and grades. The largest difference was observed in canonic dot counting tasks from first through fourth grade. While Arabic number comparison tasks were important at first and second grade, MNL tasks became more important at the third and fourth grade. We conclude that the results provided evidence for both core deficit hypothesis and access deficit hypothesis. Numerical efficiency changes very little from first to fourth grade. Future research should consider testing for unique contributions of exact and approximate number systems and access to symbols as well as mapping their neural correlates.

Keywords

MLD
Basic number competencies
Dot enumeration
Symbolic number comparison
Mental number line
Core deficit hypothesis
Access deficit hypothesis

Article Info

Received: 04.20.2014
Accepted: 10.24.2014
Online Published: 02.15.2015

DOI: 10.15390/EB.2015.3287

* Funding for this work was provided by the Scientific and Technological Research Council of Turkey (TÜBİTAK), under the grant number 111K545

[1] TED University, Faculty of Education, Department of Primary Education, Turkey, sinan.olkun@tedu.edu.tr
[2] Hacettepe University, Faculty of Education, Computer Education and Instructional Technology, Turkey, altunar@hacettepe.edu.tr
[3] Hacettepe Üniversitesi, Faculty of Education, Department of Educational Sciences, Turkey, sahing@hacettepe.edu.tr
[4] Ankara Üniversitesi, Faculty of Education, Department of Primary Education, Turkey, zakkurt@ankara.edu.tr

Introduction

Mathematical skills are necessary in everyday life as well as in many professional, academic and scientific fields. Yet, many people have considerable difficulty in learning mathematics in schools. For some researchers, approximately 5% of the school age children have mathematics learning difficulties or dyscalculia (Shalev & Gross-Tsur, 2001). For others, the figure changes from 6% to 14% (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005) depending on the criteria used for determining dyscalculia. Why these students are having such a difficulty is a main concern for researchers.

Compared to their age cohorts, students with dyscalculia have more difficulty in acquiring numbers, number words, calculations and other number related concepts. The fact that some students have normal intelligence, and have normal academic attainment in other areas but yet underachieve in arithmetic implies that dyscalculia is a specific learning disability. Therefore, recent research has focused on basic number competencies rather than general cognitive functions such as semantic and working memory.

Several hypotheses have been proposed for the epidemiology of dyscalculia. These hypotheses stem from the nature of the knowledge students with dyscalculia have severe difficulty learning or cannot learn at all. More specifically, number can be dealt with either approximate or exact level. Therefore, mental or internal representations of numbers could be exact or approximate. A student having a deficit in either system might also have difficulty in learning about numbers. Additionally, external representations of numbers can be analog or symbolic. A student having difficulty transcoding among these representations might have difficulty in attaching numerical meaning quantities and symbols.

Genetic, neurobiological, and epidemiologic evidences indicate that dyscalculia, like other learning disabilities, is a brain-based disorder (Shalev, 2004). According to Butterworth and Laurillard (2010), it is clear from recent research that very basic domain-specific core deficits or deficit in the number module can severely reduce the capacity to learn arithmetic. Number module or number system is considered one of the several units in human cognition.

Core Systems of Knowledge in Human Cognition

Human cognition is endowed with a small number of separable units for processing any kind of knowledge (Spelke & Kinzler, 2007). These are objects, actions, numbers, space, and possibly social partners. Humans are believed to be born with these systems. These core foundations are thought to be used to deal with new, flexible skills and belief systems, which are possibly interacting with each other for representing and acting on different types of knowledge (Olkun, Altun, Cangöz, Gelbal, & Sucuoğlu, 2012). Actions for example, might have both numerical and spatial attributes such as number of traces. Similarly, objects may have spatial as well as numerical qualities. Further discussions of the core systems of knowledge are beyond the scope of this paper and can be found in Spelke and Kinzler (2007). Our focus, instead, is on the system that constitutes the construction of numbers, number concepts and calculations.

Core Systems of Number in Human Cognition

Based on research with human infants and adults, Feigenson, Dehaene, and Spelke (2004a) proposed that human cognition has a separate core system for representing number. Some researchers (Klahr & Wallace, 1976; Strauss & Curtis, 1981) claimed that the two aspects of number, namely counting and estimation are dependent on subitizing, which is a rapid apprehension of the numerosity of small sets usually smaller than four. Although not explicitly stated, this assumption implies that there is only one system for processing number. Recent research (McCrink & Wynn, 2004; Xue & Spelke, 2000) has revealed that this system has at least two sub-systems possibly representing two different aspects of number at the conceptual level. One of the two separate systems of number, called approximate number system or ANS, deals with numbers, usually large numbers (>4) with approximation, while the other system, called exact number system or ENS, engages in representing numbers usually small numbers (≤4) exactly (Feigenson, Dehaene, & Spelke, 2004b). Yet, the two subsystems are considered to be functioning independently (Feigenson et al., 2004a).

From several days after birth, human beings, and even some animal species, has an innate capacity to determine the number of items in a set at a glance when the number of items is less than four, which is called subitizing (Antell & Keating, 1983). If the number of items is more than four and the person has a limited time to decide the numerosity of the set on the other hand, a different system, called ANS, is activated. If enough time is provided, then counting or other calculation procedures and strategies are used to determine the exact numerosity of larger sets. By its very nature, ANS works based on contextual and/or perceptual estimation while the ENS works on such mental actions as subitizing, counting, and calculations (Olkun et al., 2012). The nature of the numerical task and the time available for doing the task determines which system will engage in solving the numerical problems. Basically, if the numerical magnitude is visually presented and sufficiently small, usually 4 or less, then the ENS is assumed to be activated.

Some researchers believe that the main causes of dyscalculia is a core deficit in ANS (Mazzocco, Feigenson, & Halberda, 2011) while some others propose that the core deficit might lie in subitizing or exact number system (Landerl, Bevan, & Butterworth, 2004; Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009). There are evidences to support both of the positions. However, there is a limited number of studies investigating the relationships between these two subsystems and their unique contributions on dyscalculia.

A series of experiments by Lipton and Spelke (2003) showed that infants can discriminate not only small visual quantities but also large quantities both in visual and auditory modalities. Precision on these tasks was depended on a ratio between the two numbers to be discriminated, called Weber fraction. Evidence from studies investigating these two number systems suggests that both systems may function independently. For example, Lemer, Dehaene, Spelke, and Cohen (2003) found that quantity deficits (deficit in approximate system or ANS) show more impaired in subtraction than in multiplication, and severe slowness in approximation, and associated impairments in subitizing and numerical comparison tasks, both with Arabic digits and with arrays of dots. Verbal deficits (deficits in verbal or exact system or ENS), on the other hand, show more impaired in multiplication than in subtraction, have intact approximation abilities, and show preserved processing of non-symbolic numerosities.

The tasks used for measuring the capacity of ENS include but not limited to determining the number of dots in a collection as fast as possible. The number of dots ranges from 3 to 9 and the subjects are required to say aloud the number or touch a corresponding Arabic number. Since the task is very easy to answer, nearly all items are answered correctly. However, the elapsed time to answer each item varies depending on the strategy used by individuals. For example, to enumerate a set of seven dots, a person with a good subitizing and arithmetic skill may subitize the set as 3 and 4 first. Then, add them together to find seven, while another person with a weak subitizing skill may try to count all the dots one by one. Still another person with good subitizing skill but weak arithmetic skill may subitize four dots and counts on the other three. Consequently, the elapsed time to answer the question will not be the same for these three persons. Therefore, latency for doing these tasks might be a good predictor of learning difficulties in mathematics.

Access Deficit Hypothesis (ADH)

Some researcher claimed that the major reason behind MLD is not deficits in ANS or ENS but rather in accessing magnitudes from symbols or vice versa. For example, Rousselle and Noel (2007) found that children with MLD were only impaired when comparing Arabic digits (i.e., symbolic number magnitude) but not when comparing collections (i.e., non-symbolic number magnitude). Desoete, Ceulemans, De Weerdt, and Pieters (2012), on the other hand, found that Arabic number (AN) comparison at kindergarten was predictively related to procedural calculation two years later whereas non-symbolic skills in kindergarten were predictively related to arithmetical achievement one year later and fact retrieval two years later. In addition, they found that children with MLD already had deficits in non-symbolic and symbolic AN comparison in kindergarten, whereas in grade 2 the deficits in processing symbolic information remained.

These mixed findings suggest that children with mathematics learning disabilities might have difficulty both in accessing number magnitude from symbols and in processing numerosities in different modalities. Purely symbolic and purely non-symbolic number comparisons have produced different results and found to be related to different arithmetic skills later on, suggesting that there might be unique contributions of each process on learning arithmetic (Desoete, Ceulemans, Roeyers, & Huylebroeck, 2009). These discussions indicate a need for investigating the processing of quantities in different modalities in relation to achievement in school mathematics.

The root reason behind the mathematical learning disorder is explained by two different hypothesis as core deficit and access deficit. Therefore it is measured by simple numerical tasks such as dot counting, symbolic number comparison (numerical Stroop), analog magnitude comparison, and estimating relative magnitude of numbers (Butterworth, 1999; Desoete et al., 2012; Heine et al., 2010). Counting, magnitude comparison, and mental number line estimation tasks are thought to be related to core deficit hypotehis (Landerl et al., 2004) while symbolic number comparison task was considered to support access deficit hypotehis (Gilmore, McCarthy, & Spelke, 2010). Students with mathematics disorder are thought to have difficulty with one or more of these tasks. In this research, analog quantities, Arabic numerals, and mental number lines will be used to represent number externally. Specifically, dot counting tasks were used to measure the ENS. Mental number line estimation tasks were used to measure the ANS. Finally, symbolic number comparison tasks were used to measure the access to symbols.

Although many studies investigated the relationship between mathematics achievement and basic numerical competencies, studies investigating the relationships among them and unique contributions of each competency over the achievement in mathematics in elementary grades are scarce. It is especially important to distinguish between MLD and low achievement in mathematics both in terms of diagnosis and treatment. We hypothesize with Butterworth (2010) that while the major reason behind MLD is a core deficit, including access deficit in the number module the major reason behind low achievement in mathematics is bad or inappropriate teaching. Therefore, this current study is an attempt to address the complex relationships among basic numerical competencies and mathematics achievement in the number domain from first through fourth grades.

The specific research questions of this study were:

1. Can basic number competencies tests be used to explain primary school students' mathematics achievements?
2. Can basic number competencies tests be used to differentiate students with very low, low, normal, and high math achievers in the primary school?

Methods

Participants

Participants were 481 students, selected from 12 elementary schools, located four different SES locations with an intention to draw a representative sample of students from 1st to 4th grade within a metropolitan area in the mid-Anatolia. Twelve schools (3 from each of the 4 sub regions in the greater city), 4 classrooms from each grade level, and 11 students from each classroom were randomly selected through a lottery technique. Initially, 132 students from each grade level (a total of 528 students) were determined to include in the study. Due to the failure to complete any of the tests used to collect the data, some students excluded from the study. An additional six students were also excluded since they were diagnosed with some sort of general learning disorders and/or mainstreamed in regular classrooms. The final sample from each grade level was 125, 126, 121, and 109 (481 in total) from 1st, 2nd, 3rd, and 4th grades respectively.

Testing materials

Five separate tests were administered to the students. The first one is the mathematics achievement test developed by Fidan (2013) based on the number domain of the current Turkish state curriculum (MOE, 2005). It was a curriculum based test developed for this study. There were different achievement tests for each grade level. The tests contained 13, 15, 16, and 24 items for the first, second, third, and fourth grade respectively. All the questions in the tests were open-ended, short answer form. The content, construct and criterion referenced validity of the tests were examined by various methods. The reliability of the tests estimated by KR-20 reliability method and for each grade level and all coefficents were high enough. The reliability coeficients were 0.80, 0,92, 0,93, and 0,96 for the 1st, 2nd, 3rd, and 4th grade respectively. The math achievement test is an untimed test but the administration took one class hour (approximately 40 minutes) for the students.

The remaining 4 tests consisted of neuropsychological tasks intended to be developed in this study. They were administered individually through a tablet PC. Both accuracy and latency were recorded in a data file in the tablet PC. The developed tests consist of dot counting, estimating relative magnitude of numbers, and symbolic number comparisons. Dot counting tasks (CDC and RDC) are related to ENS while mental number line tasks were related to ANS. Both ENS and ANS is considered to be within core deficit hypotehis. Symbolic number comparison tasks on the other hand are used to support access deficit hypothesis.

The first test included Canonic Dot Counting tasks (CDC). Dots ranging from 3 to 9 were arranged into dice or domino like patterns. Students are requested to enter their responses by touching a number ordered left to right from 1 to 9. There are 14 items in this test. The second test, Random Dot Counting, RDC, was similar to the first test except that the dots were arranged in pseudo random order in which the same number was not asked consecutively. The reason for using two separate tests for dot counting is that the time for students to enumerate randomly-ordered dots would be different from the ones lined up in the form of dominoes because of the enumeration strategies students used so that we can differentiate slow learners or slow processors from the fast ones. Sample items from the CDC and RDC tests are provided in Figure 1.

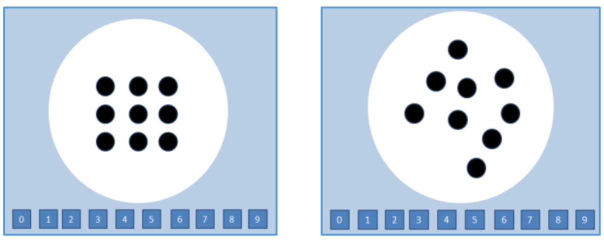

Figure 1. Sample Items from the CDC and RCD Tests

Sample items from the CDC (left) and RCD Tests (right) are presented in Figure 1. Subjects were asked to quicly and accurately determine the number of dots presented and touch the corresponding number below. According to Clements (1999), children learn domino like arrangements from first grade on, changing their perceptual subitizing into conceptual subitizing, in other words forming units of units for dot counting. We hypothesized that if there is a math learning disability; this conceptual subitizing would be delayed reflecting itself as a delayed response time.

The third test, Symbolic Number Comparison, (SNC) consisted of Arabic number comparison tasks arranged in accordance with numerical Stroop paradigm. Numbers from 3 to 9 were arranged in a pseudo random order. Students were requested to enter their answer by touching the numerically larger number. No physical comparison tasks were included. Only numerical comparison tasks with a distance 1 and 2 were asked. The numbers to be compared in the test were arranged in three different forms as congruent (5-7), neutral (5-7) and incongruent (5-7). There were totally 24 items, 8 congruent, 8 neutral and 8 incongruent in this test. Correct answers were equally distributed on both sides. Students especially with math disorder are distracted by the physical size of the number while comparing numerical magnitudes (size-congruity effect) (Girelli, Lucangeli, & Butterworth, 2000; Rubinsten & Henik, 2006). Since these tasks require connecting the symbols with the magnitudes they represents they are used to support access deficit hypothesis (Attridge, Gilmore, & Inglis, 2009). There were totally 24 items, 8 congruent, 8 neutral and 8 incongruent in this test. A sample item from the SNC test is presented in Figure 2. In the task presented in Figure 2, the numeral 9 is written with smaller font than the numeral 3 (incongruent). Students are expected to touch the numeral 9 without being distracted from the physically larger numeral 3.

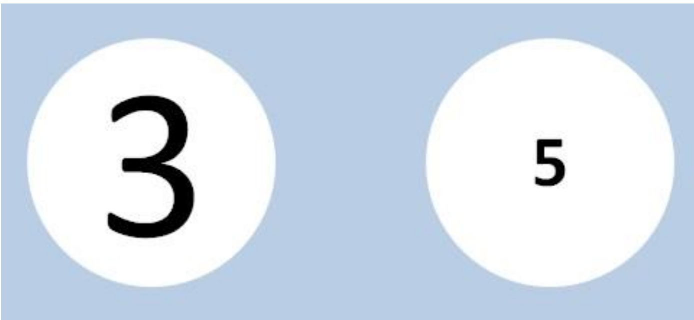

Figure 2. A Sample Item from the SNC Test

The fourth test, Mental Number Line, (MNL) was consisted of number placement tasks. A typical number line is a horizontal or vertical line with zero on the left end and 10 (MNL-1), 20 (MNL-2), 100 (MNL-3), or 1000 (MNL-4) on the other end. Students are requested to place the numbers shown one at a time on the number line by drawing a hash mark on the number line. In experiments, number lines were placed horizontally on the screen. With this test, students would be able to touch the relative place of numbers and move their finger on the screen to adjust the finer place. When touched, a vertical short line appeared on the horizontal number line and moved as the students moved their fingers. No timing was recorded for this test. Only the absolute values of the difference between the estimation and to be estimated numbers were recorded in number to position tasks.

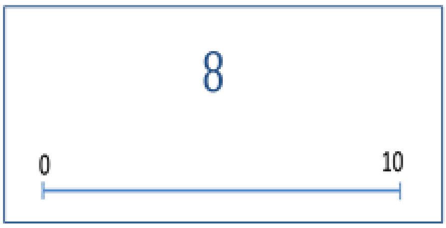

Figure 3. A Sample Item from 0-10 Number Line Test (MNL1)

A sample item from 0-10 number line test (MNL1) is presented in Figure 3. In this task students were asked to place the number 8 in the number line given.

Analysis

In the analysis, raw scores were gathered from the math achievement tests. In each grade level, students were placed in four groups based on their math achievement scores. The prevalence of dyscalculia or MLD was reported from 5.9% to 13.8% according to the formula used (Barbaresi et al., 2005). These formulas are based on IQ scores and standard achievement scores. These formulas involve regression and inequality which change according to each grade level. In our study the lowest 10% of the students were placed in the MLD risk group, 11-25% in low achievement, 26-95% in typical achievement and >95% in high achievement group based on the cutoff points (See Table 1). While forming these groups, the individuals who got the same score and below the cut off point were included in the groups. For instance, if the 10% of the group consists of 48 individuals that got 3 points or below, hovewer if the individuals from 49 to 60 also got the same score, in that case the cut off point was determined as the 60th individual's score.

Finally, teachers' opinions about the students were also gathered to make sure that students' math scores reflected their general situations in mathematics and students had no other learning difficulties. Defining the MLD risk group was loose in this study so this was another limitation of the current study. Therefore, we used MLD risk group instead of MLD for the lowest achiever group.

Table 1. Group Sizes and Percentages in Each Grade Level Formed Based on Math Achievement Scores

	1st grade		2nd grade		3rd grade		4th grade		**Total**
Groups	**N**	**%**	**N**	**%**	**N**	**%**	**N**	**%**	
MLD risk	20	16,0	13	10,3	15	12,4	11	10,1	59
Low Achievement	17	13,6	34	27,0	18	14,9	30	27,5	99
Normal Achiever	84	67,2	64	50,8	77	63,6	65	59,6	290
High Achiever	4	3,2	15	11,9	11	9,1	3	2,8	33
Total	125	100,0	126	100,0	121	100,0	109	100,0	481

MLD: Math Learning Disability

We calculated Inverse Efficiency Scores, IES (Bruyer & Brysbaert, 2011) for CDC, RDC, and SNC tests. Bruyer and Brysbaert (2011) suggested that IES is a better dependent variable when there is a high correlation between response time and percentage of incorrect answers, and the percentage of correct answers was high. IES is calculated through dividing total time individuals spend responding the test items into the percentage of correct answers. We also calculated absolute error scores, AES for MNL tests. All these scores should be in an inverse relationship with math achievement scores.

The relationship between AES values of MNL and IES values of CDC, RDC, SNC tests and math achievement scores was examined with correlational analyses. Nonparametric tests were used for group comparisons. Finally, regression analysis was run to predict math achievement scores. Before starting the analysis, we investigated whether the data meet the assumptions of the analysis to be made. For the regression it has been proved that the data meet multicollinearity, homoscedasticity assumptions. The outliers were removed as data points if an individual datum was too different than the remaining of the data. Although some test scores didn't meet normality assumption since there were anough cases in each group, it was assumed that all these scores were normally distributed according to central limit theorem. In case of inadequate number of individuals in each group, the nonparametric tests were applied instead of ANOVA and t test.

Reliability and validity

The reliability (Cronbach Alpha) measures of the tests have been presented in Table 2. As seen in the table, except MNL2, all other tests have reliability measures of over .70, which is considered enough for psychological testing. Since the items in the tests are usually used in testing neuropsychological performances about number processing (Desoete et al., 2009; Landerl et al., 2004; Siegler & Booth, 2004), they were considered as valid measures. In addition, very high correlations between math achievement scores and the tests as well as among the tests were observed. These findings can be considered as an indicator of validity of measures.

Table 2. Reliability Measures

	Number of items	**Cronbach Alpha**
CDC	14	0.92
MNL1	11	0.75
MNL2	11	0.66
MNL3	11	0.72
MNL4	11	0.96
RDC	14	0.90
SNC	24	0.93

CDC: Canonic Dot Counting, MNL: Mental Number Line
RDC: Random Dot Counting, SNC: Symbolic Number Comparison

Findings

Correlations among the basic number processing tests

We begin presenting data about correlations among the tests used. Apart from math achievement test appropriate for each grade level, we used 4 different basic number processing tests. The correlations among the tests are depicted in Table 3 and almost all correlations are significant at p<.001 level. Math achievement scores and other test scores are negatively correlated while the four basic number processing tests are positively correlated to each other.

Table 3. Correlations Among the Tests

	Grade	N	CDC -IES	RDC -IES	SNC -IES	MNL -AES
MAT	1	125	-.356***	-.331***	-.449***	-.547***
	2	126	-.560***	-.431***	-.393***	-.297***
	3	121	-.532***	-.429***	-.404***	-.457***
	4	109	-.552***	-.418***	-.271**	-.567***
CDC-IES	1	125		.849***	.243**	.306***
	2	126		.594***	.329***	.457***
	3	121		.638***	.441***	.458***
	4	109		.579***	.418***	.519***
RDC-IES	1	125			.423***	.350***
	2	126			.675***	.289***
	3	121			.710***	.447***
	4	109			.546***	.313***
SNC-IES	1	125				.567***
	2	126				.159
	3	121				.375***
	4	109				.357***

MAT: Math Achievement Test, CDC-IES: Canonic Dot Counting Inverse Efficiency Score,
RDC-IES: Random Dot Counting Inverse Efficiency Score, SNC-IES: Symbolic Number Comparison Inverse Efficiency Score,
MNL-AES: Mental Number Line Absolute Error Score.

Regression analyses

A regression analysis was computed for each grade level to compute the explanatory power of the variance in math achievement scores. In order to observe if any differences exist between the grades, grade level analyses were conducted. Table 4 shows the relevant values for the first graders. As seen in the table, CDC, RDC SNC and MNL tests altogether accounted 37% of the total variance in math achievement scores ($R=0.609$, $R^2=0.37$, $F_{(4,120)}=17.724$ $p<.000$). According to the standardized regression coefficients (Beta), the importance levels of predicting the math achievement scores were MNL, CDC, SNC and RDC sequentially. Yet, only CDC, SNC and MNL tests are found to be significant in explaining the math achievement scores.

Table 4. Regression Analysis for explaining First Graders' Math Achievement Scores

Variable	B	SE	Beta	t	p	Partial r
Intercept	10.550	.720		14.655	.000	
CDC IES	-2.30E-005	.000	-.357	-2.497	.014	-.222
RDC IES	1.24E-005	.000	.204	1.349	.180	.122
SNC IES	-3.40E-005	.000	-.236	-2.465	.015	-.220
MNL-AES	-.008	.002	-.375	-4.170	.000	-.356

$R=0.609$ $R^2=0.371$ $F_{(4,120)}=17.724$ $p=0.000$

Table 5 shows the relevant regression values for the second graders. As seen in the table, CDC, RDC SNC and MNL tests altogether accounted almost 37% of the total variance in math achievement scores (R=0.605, R^2=0.366, $F_{(4,121)}$=17.431 p<.000). According to the standardized regression coefficients (Beta), the importance levels of predicting the math achievement scores were CDC, SNC, RDC, and MNL sequentially. However, only CDC and SNC tests are found significant in explaining the math achievement scores of second graders.

T Table 5. Regression Analysis for explaining Second Graders' Math Achievement Scores

Variable	B	SE	Beta	t	p	Partial r
Intercept	18.702	1.496		12.498	.000	
CDC IES	.000	.000	-.477	-4.894	.000	-.407
RDC IES	8.29E-006	.000	.040	.341	.733	.031
SNC IES	-6.60E-005	.000	-.255	-2.580	.011	-.228
MNL-AES	-.002	.003	-.049	-.607	.545	-.055

R=0.605 R^2=0.366 $F_{(4,121)}$=17.431 p=0,000

Table 6 shows the relevant regression values for the third graders. As depicted in the table, CDC, RDC SNC and MNL tests altogether accounted almost 36% of the total variance in math achievement scores (R=0.601, R^2=0,36, $F_{(4,121)}$=16.386 p<.000). According to the standardized regression coefficients (Beta), the importance levels of predicting the math achievement scores were CDC, MNL SNC and RDC respectively. However, only CDC and MNL tests are important variables in explaining the second graders' math achievement scores.

Table 6. Regression Analysis for explaining Third Graders' Math Achievement Scores

Variable	B	SE	Beta	t	p	Partial r
Intercept	18.889	1.872		10.090	.000	
CDC IES	-8.07E-005	.000	-.370	-3.716	.000	-.326
RDC IES	1.19E-005	.000	.043	.346	.730	.032
SNC IES	.000	.000	-.182	-1.715	.089	-.157
MNL-AES	-.001	.000	-.238	-2.767	.007	-.249

R=0.601 R^2=0.361 $F_{(4,121)}$=16.386 p=0.000

We presented relevant regression values for the fourth graders in Table 7. As depicted in the table, CDC, RDC SNC and MNL tests altogether accounted almost 43% of the total variance in math achievement scores (R=0.656, R^2=0.43, $F_{(4,121)}$=19.602 p<.000). According to the standardized regression coefficients (Beta), the importance levels of predicting the math achievement scores were MNL CDC, RDC and SNC sequentially. However, only CDC and MNL tests are important variables in explaining the fourth graders' math achievement scores.

Table 7. Regression Analysis for explaining Fourth Graders' Math Achievement Scores

Variable	B	SE	Beta	t	p	Partial r
Intercept	26.875	2.327		11.547	.000	
CDC IES	.000	.000	-.277	-2.743	.007	-.260
RDC IES	-6.11E-005	.000	-.180	-1.803	.074	-.174
SNC IES	8.08E-005	.000	.085	.929	.355	.091
MNL-AES	-.003	.001	-.396	-4.494	.000	-.403

R=0.656 R^2=0.43 $F_{(4,121)}$=19.602 p=0.000

Group differences

We looked at the data if the scores students gained on four different tests are useful in discriminating the groups formed based on math achievement scores as MLD, LA, TA, and HA. Since the number of students in some of the groups are very few we used nonparametric tests (Kruskal Vallis test) for group comparisons. We present both visual depiction of test scores and statistical comparisons. Means of the every test scores for each grade level have been presented in details in the graphs.

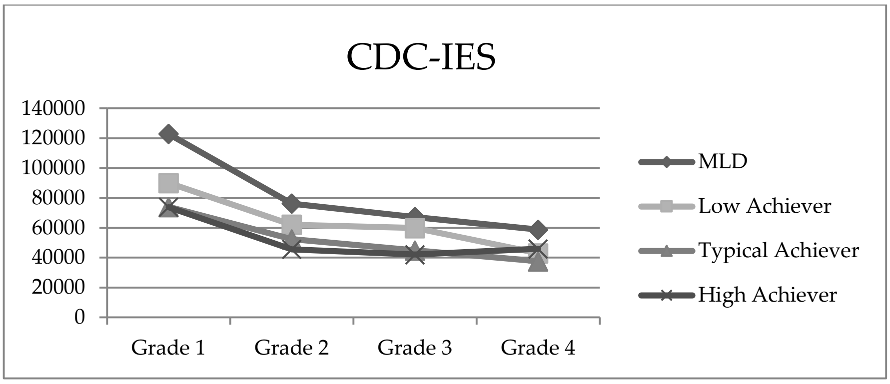

Figure 4. CDC-IE Scores of Subgroups from 1st through 4th Grade

As seen in Figure 4, the CDC-IE Scores of the MLD groups are consistently above of all other groups. MLD group is followed by LA, TA, and HA groups respectively. Statistical analysis showed that there are significant differences between groups from first through fourth grade (see Table 8 for details). CDC-IES is useful in discriminating MLD groups from the other groups especially TA and upper groups. It is also useful in discriminating TA groups from the other upper groups.

Table 8. Group Comparisons based on CDC-IES From First Through Fourth Grade

Grades	Groups	N	Mean	DF	χ^2	p	Sig. Differences b/w
1	MLD	20	86.05	3	16.196	0.001	MLD- TA
	LA	17	78.47				LA - TA
	TA	84	55.35				
	HA	4	42.75				
2	MLD	13	107.92	3	38.910	0.000	MLD- LA, TA, HA
	LA	34	78.53				LA – TA, HA
	TA	64	52.97				
	HA	15	35.87				
3	MLD	15	86.87	3	29.708	0.000	MLD- TA, HA
	LA	18	90.56				LA – TA, HA
	TA	77	51.92				
	HA	11	40.91				
4	MLD	11	87.82	3	20.304	0.000	MLD- LA, TA
	LA	30	62.68				LA - TA
	TA	65	45.28				
	HA	3	68.33				

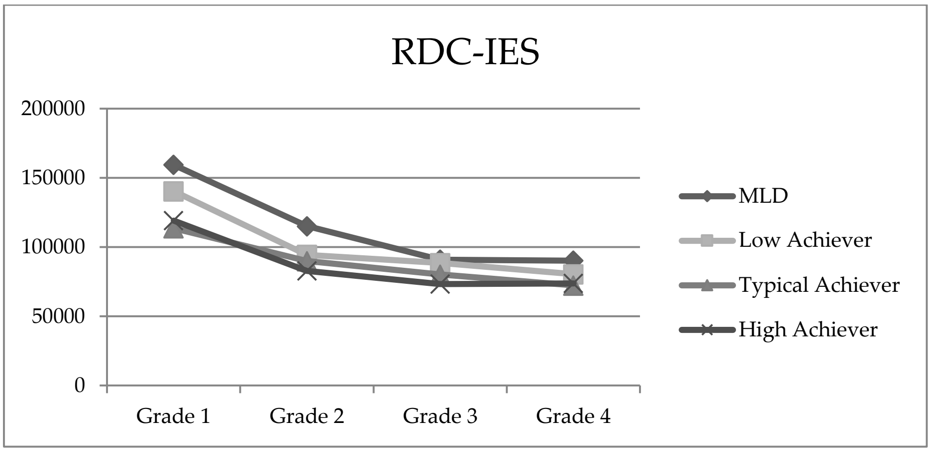

Figure 5. RDC-IE Scores of Subgroups from 1st through 4th Grade

As seen in Figure 5, RDC-IE scores of the MLD groups are consistently higher than the other groups. Similar to CDC-IE scores, RDC-IE scores are also good at discriminating MLD groups from TA and upper groups. Again it is good at discriminating LA groups from upper groups from first through fourth grades. It is also good at discriminating MLD from the LA at second grade (see Table 9 for details).

Table 9. Group Comparisons based on RDC-IES From First Through Fourth Grade

Grades	Groups	N	Mean	DF	χ^2	p	Sig. Differences b/w
1	MLD	20	80.97	3	11.397	0.010	MLD- TA LA - TA
	LA	17	78.24				
	TA	84	56.30				
	HA	4	49.00				
2	MLD	13	97.15	3	19.770	0.000	MLD- LA, TA, HA LA – TA, HA
	LA	34	70.91				
	TA	64	58.11				
	HA	15	40.53				
3	MLD	15	80.40	3	14.603	0.002	MLD- TA, HA LA – TA, HA
	LA	18	77.83				
	TA	77	56.53				
	HA	11	38.27				
4	MLD	11	75.82	3	11.422	0.010	MLD- LA, TA LA - TA
	LA	30	64.43				
	TA	65	47.18				
	HA	3	53.67				

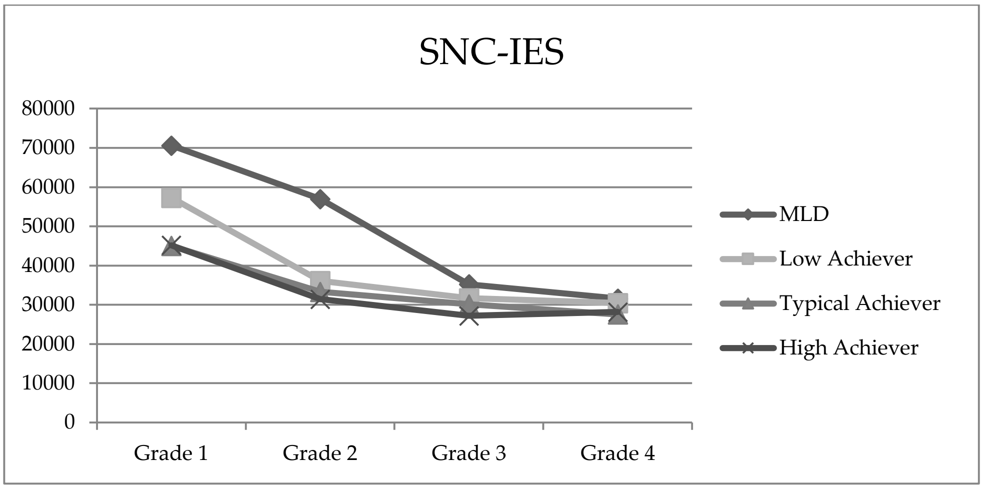

Figure 6. SNC-IE Scores of Subgroups from 1st through 4th Grade

As seen in Figure 6, SNC-IE scores of the MLD groups are consistently higher than the upper groups. Again, The LA groups got higher SNC-IE scores the TA and HA groups. The maximum difference between the MLD and upper groups occurred in second and first grade respectively. Statistical analysis of group differences showed that SNC-IE score is good at discriminating MLD groups from TA and upper groups. It is even good at discriminating MLD from LA only at the second grade. Again this score is also good at discriminating LA from the upper groups.

Table 10. Group Comparisons based on SNC-IES From First Through Fourth Grade

Grades	Groups	N	Mean	DF	χ^2	p	Sig. Differences b/w
1	MLD	20	93.25	3	24.501	0.000	MLD- TA LA - TA
	LA	17	79.82				
	TA	84	52.76				
	HA	4	55.25				
2	MLD	13	100.85	3	22.055	0.000	MLD- LA, TA, HA LA – TA, HA
	LA	34	72.12				
	TA	64	55.44				
	HA	15	46.00				
3	MLD	15	88.60	3	17.341	0.001	MLD- TA, HA LA – TA, HA
	LA	18	70.39				
	TA	77	57.06				
	HA	11	35.55				
4	MLD	11	72.45	3	9.024	0.029	MLD- LA, TA LA - TA
	LA	30	63.87				
	TA	65	47.88				
	HA	3	56.67				

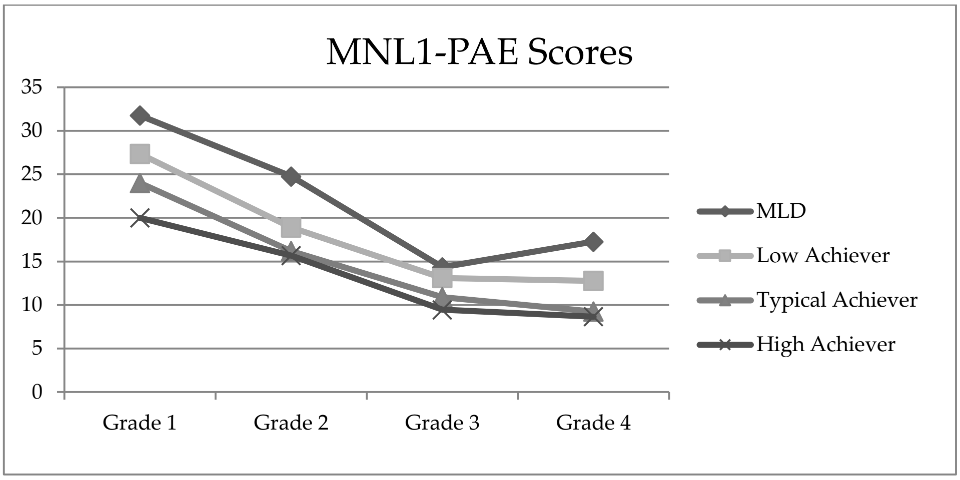

Figure 7. MNL1-AE Scores (0 to 10 number line) of Subgroups from 1st through 4th Grade

Figure 7 shows the graphical depiction of the absolute error scores (AES) obtained from MNL1 (the 0-10 number line). The MLD groups' total absolute errors are consistently higher than that of the other upper groups. The differences between the groups approached significance (χ_3^2 =7.757, p=.051) for the first grade and discriminated MLD from other groups at the second grade (χ_3^2 =11.708, p=.008) but no differences occurred at third and fourth grade.

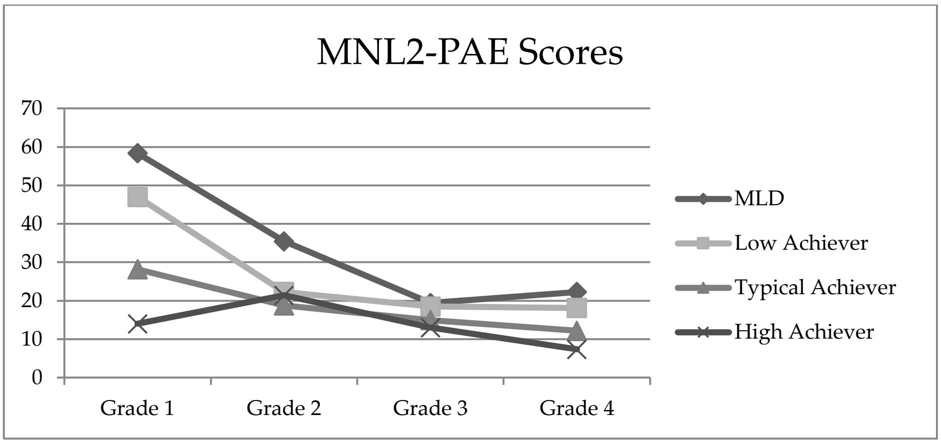

Figure 8. MNL2-AE Scores (0 to 20 number line) of Subgroups from 1st through 4th Grade

Figure 8 shows the graphical depiction of the absolute error scores (AES) obtained from MNL2 (the 0-20 number line). Total absolute error scores obtained from MNL2 discriminate MLD from TA, and HA and discriminates LA from TA and HA (χ_3^2 =27.067, p=.000) at the first grade. No differences were detected at the second grade. It is also good at discriminating LA from HA (χ_3^2 =8.497, p=.037) at the third grade, MLD from TA, LA from TA and HA, and TA from HA (χ_3^2 =16.479, p<0.001) at the fourth grade.

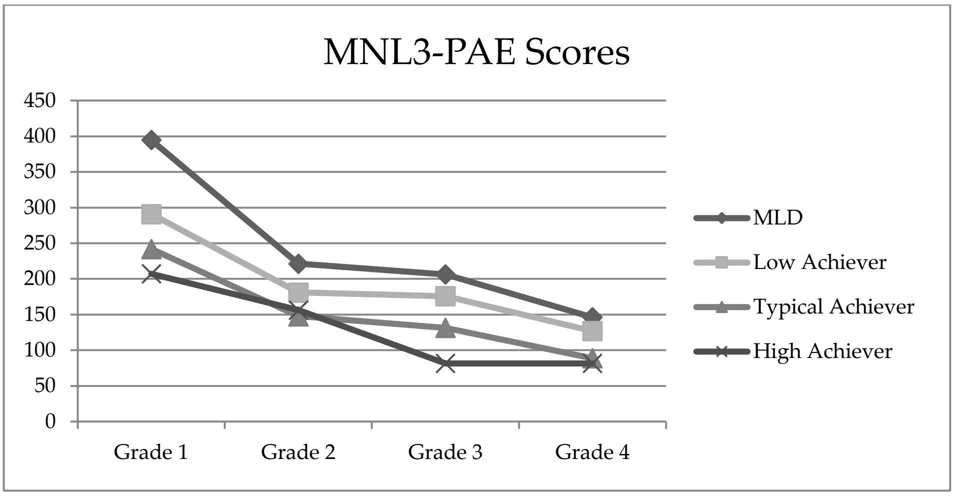

Figure 9. MNL3-AE Scores (0 to 100 number line) of Subgroups from 1st through 4th Grade

Figure 9 shows the graphical depiction of the absolute error scores (AES) obtained from MNL3 (the 0-100 number line). As seen from the Figure, group differences are almost consistent from first through fourth grade. Statistical analysis of the group differences showed that there are significant differences between MLD and other three upper groups (χ^2_3 =28.905, p=.000) at the first grade. It discriminates MLD from TA and HA, and LA from TA at the second grade (χ^2_3 =14.621, p=.002). It discriminates MLD from TA and HA, LA from TA and HA, and TA from HA at the third grade (χ^2_3 =23.815, p=.000). Statistically significant differences were also detected between MLD and TA, and LA and TA at the fourth grade (χ^2_3 =16.008 p<0.001).

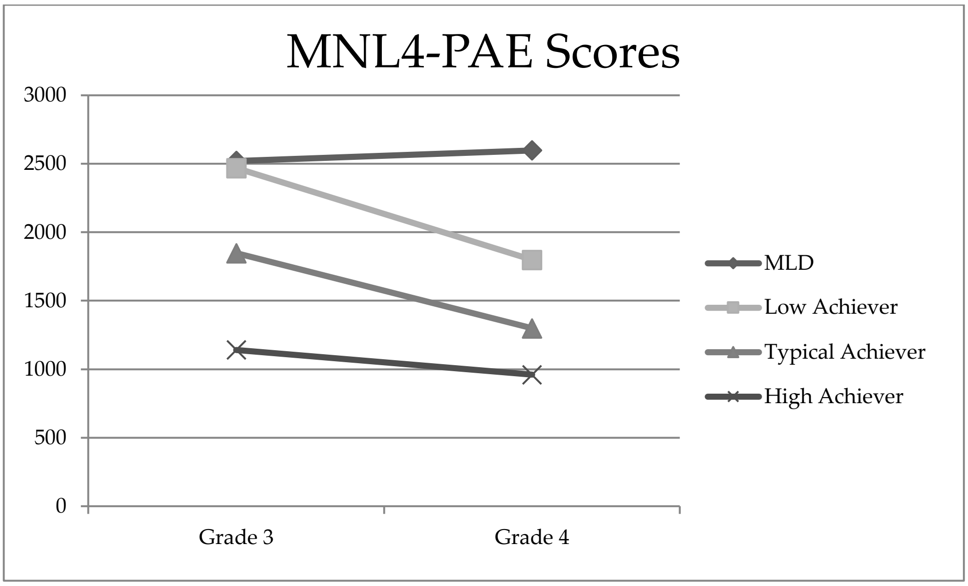

Figure 10. MNL4-AE Scores (0 to 1000 number line) of Subgroups from 1st through 4th Grade

MNL4 (0 to 1000 number line) was given only to the third and fourth grade students. Figure 10 shows the graphical depiction of the absolute error scores (AES) obtained from MNL4. As seen in Figure 10, there are almost consistent differences between the groups both at third and fourth grade. Statistical analysis of the differences showed that it discriminates MLD from TA and HA, LA from TA and HA, and TA from HA at the third grade (χ^2_3 =22.585, p<0.000). It sharply discriminates MLD from all other upper groups, LA from TA at the fourth grade (χ^2_3 =26.749, p<0.000).

Discussion and Conclusions

This study was designed to explore the complex relationships between elementary grade students' mathematics achievement and basic number processing abilities. The administered tests and the related tasks were found to be reliable and valid to explore the mathematical achievement. However, minor differences were observed from 1st through 4th grades. Similar results were obtained from the regression analysis. In the first grade, only CDC, SNC and MNL tests have explanatory power on math achievement as CDC being the most predictive. In the second grade, only CDC and SNC tests have explanatory power on math achievement scores with CDC being the most predictive. In the third grade, only CDC and MNL tests have explanatory power on math achievement with CDC being the highest predictive value. For the fourth graders, CDC and MNL tests have explanatory power on math achievement with MNL being the most predictive.

The fact that, in all grade levels, students with math disorder risk spent longer time finding the number of dots in CDC tasks supports the claim that they might have problems with their exact number system (ENS) or might have core deficit in their number module (Butterworth & Laurillard, 2010) explained in core deficit hypothesis (Landerl et al., 2004). It is also possible that these students might have disorders in their subitizing mechanisms (Landerl et al., 2004). The finding that 3rd and 4th grade students' total absolute errors in their mental number line estimations correlate negatively with their math achievement scores also support the idea that these students might have problems with their approximate number system (ANS) (Sasanguie, De Smedt, Defever, & Reynvoet, 2011). This finding also support the core deficit hypothesis.

Students with math disorder risk have also showed lower efficiency in symbolic number comparison (SNC) tasks in first and second grades. This finding supports the idea that students with math disorder risk might have difficulty in accessing magnitudes from symbols or vice versa, lending support to Access Deficit Hypothesis. (Gilmore et al., 2010).

The results of this study indicate that CDC is an important predictor of math achievement from first through fourth grade. While the SNC test was the second important predictor for the first and the second graders, the MNL test becomes a predictor for the fourth graders. These results show that CDC, SNC, MNL and RDC tests have a potential to be used as a screening tool to determine individual differences in mathematics. In order to test this hypothesis, group comparison tests were calculated to see if there are differences among the groups previously formed as MLD risk, LA, TA, and HA based on math achievement scores. Group comparisons showed that CDC test consistently discriminated MLD risk from the other groups especially TA and upper groups. In the second and fourth grade it also separated MLD risk from the LA groups. Why it has not discriminated MLD risk from LA in the first and third grade is an important concern. One possible explanation is that it was possible that some of the students who should be in LA group mistakenly placed in MLD risk group because of the low discriminative power of the math achievement test or wrong cutoff points or both. Finer groupings may have revealed better results.

Although relatively less consistently, RDC test also discriminated MLD groups from TA and HA groups from first through fourth grade. It also discriminates MLD risk group from LA in the second grade. Why is it that CDC is more precise than RDC in discriminating MLD from other groups? One possible explanation is that all students might have treated random dots as collections to be counted not imposing any groupings on them. Students in general enumerated canonically presented dots almost twice faster than the ones presented randomly. In enumerating the canonically arranged dots, upper achievement groups might have made finer groupings or conceptual subitizing (Clements, 1999) and used arithmetic operations with recalled facts so that they spent shorter time for enumeration.

SNC test was also good at discriminating MLD risk from TA and HA groups from first through fourth grade. It is also good at separating MLD from LA group at the second grade only. The discriminative power of SNC sharply decreases at third and fourth grade. Symbol reading seems especially to be important at the first and second grade.

Total absolute errors (TAE) calculated from MNL estimations revealed that MLD groups got consistently higher TAE scores than that of other upper achievement groups. However, only some of these differences reached statistical significance at certain grades. For example, MNL1 (0-10 number line) test separated MLD risk group from other groups at the border in the first grade, and significantly at the second grade. Again this might have occurred because of the possibility that some students at the LA group might have mistakenly placed in the MLD risk group making it difficult to separate these two groups. MNL2 (0-20 number line) is good at discriminating MLD risk group from TA and HA at the first grade, also good at separating LA from TA and HA at the first grade. It is also good at discriminating LA from TA and HA at both third and fourth grade. MNL3 (0-100 number line) test was more consistently good at separating MLD risk from other groups at the first grade and MLD from TA and HA at the second, third, and fourth grade. MNL4 (0-1000 number line) was asked only to third and fourth graders. This test has even more sharply discriminated the four subgroups from each other. Similar results were also reported in the literature. For example, Geary, Hoard, Nugent, and Byrd-Craven (2008) found that MLD children are less accurate in their number line placements than both LA and TA cohorts.

Taken together, these results provide evidence to support both of the positions claiming that MLD may result either from the deficit in the number module (either ANS or ENS or both) or in accessing magnitudes from symbols. These results also show that mathematical learning difficulties can be screened from first through fourth grade with tests containing dot counting, symbolic number comparisons and number line estimation tasks. Since these tasks are curriculum independent, the same tests can be used across grades. However, group norms are needed to report more confidently about MLD risk and low achievement in mathematics.

Instructional implications

It seems that basic number processing abilities such as dot counting, symbolic number comparison, and number line estimations have very strong relations to mathematical learning at the elementary grades. Therefore, it seems quite reasonable to train individuals' basic number processing skills to increase their math learning potentials. Kucian et al. (2011) trained a group of dyscalculic individuals with a custom-designed training program and found that the training leads to an improved spatial representation of the mental number line as well as a modulation of neural activation, which both facilitate processing of numerical tasks. Similarly, representations of numerical magnitudes on a number line have been found in correlation with and causally related to arithmetic learning in the first grade (Booth & Siegler, 2008). Presentations of individuals with relevant tasks tended to improve estimation ability (Siegler & Booth, 2004) and arithmetic problem solving abilities (Booth & Siegler, 2008).

Training to improve dot counting and subitizing abilities have also been found useful in math learning abilities. For example; Groffman (2009) trained individuals with subitizing tasks and found that this specific training improved both subitizing and math abilities. Similarly, Clements (1999) coined the term conceptual subitizing to denote the ability to recognize eight dots at a glance. He also claimed that conceptual subitizing plays an advanced organizing role and should be taught to children. To support these claims, we found that canonic dot counting tasks are more useful in discriminating MLD, LA and TA students from first through fourth grade consistently.

References

Antell, S. E., & Keating, D. P. (1983). Perceptions of numerical invariance in neonates. *Child Development*, *54*, 695-701.

Attridge, N., Gilmore, C., & Inglis, M. (2009). *Symbolic addition tasks, the approximate number system and dyscalculia.* Paper presented at the British Society for Research into Learning Mathematics.

Barbaresi, W. J., Katusic, S. K., Colligan, R. C., Weaver, A. L., & Jacobsen, S. J. (2005). Math learning disorder: incidence in a population-based birth cohort, 1976-82, Rochester, Minn. *Ambulatory Pediatrics, 5*(5), 281-289. doi: 10.1367/A04-209R.1

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79*(4), 1016-1031. doi: 10.1111/j.1467-8624.2008.01173.x

Bruyer, R., & Brysbaert, M. (2011). Combining speed and accuracy in cognitive psychology: is the inverse efficiency score (ies) a better dependent variable than the mean reaction time (rt) and the percentage of errors (pe)? *Psychologica Belgica, 51*(1), 5-13.

Butterworth, B. (1999). *The Mathematical Brain*. London: McMillian.

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences, 14*(12), 534-541. doi: 10.1016/j.tics.2010.09.007

Butterworth, B., & Laurillard, D. (2010). Low numeracy and dyscalculia: identification and intervention. *ZDM Mathematics Education, 42*(6), 527-539. doi: 10.1007/s11858-010-0267-4

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics, March*, 400-405.

Desoete, A., Ceulemans, A., De Weerdt, F., & Pieters, S. (2012). Can we predict mathematical learning disabilities from symbolic and non-symbolic comparison tasks in kindergarten? Findings from a longitudinal study. *Br J Educ Psychol, 82*(Pt 1), 64-81. doi: 10.1348/2044-8279.002002

Desoete, A., Ceulemans, A., Roeyers, H., & Huylebroeck, A. (2009). Subitizing or counting as possible screening variables for learning disabilities in mathematics education or learning. *Educational Research Review, 4*(1), 55-66. doi: 10.1016/j.edurev.2008.11.003

Feigenson, L., Dehaene, S., & Spelke, E. (2004a). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307-314. doi: 10.1016/j.tics.2004.05.002

Feigenson, L., Dehaene, S., & Spelke, E. (2004b). Origins and endpoints of the core systems of number. Reply to Fias and Verguts. *Trends in Cognitive Sciences, 8*(10), 448-449. doi: DOI 10.1016/j.tics.2004.08.010

Fidan, E. (2013). *İlkokul öğrencileri için matematik dersi sayılar öğrenme alanında başarı testi geliştirilmesi.* (Yayımlanmamış Yüksek Lisans Tezi), Ankara Üniversitesi, Eğitim Bilimleri Enstitüsü.

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of Number Line Representations in Children With Mathematical Learning Disability. *Developmental Neuropsychology, 33*(3), 277-299. doi: 10.1080/87565640801982361

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition, 115*(3), 394-406. doi: 10.1016/j.cognition.2010.02.002

Girelli, L., Lucangeli, D., & Butterworth, B. (2000). The development of automaticity in accessing number magnitude. *J Exp Child Psychol, 76*(2), 104-122. doi: 10.1006/jecp.2000.2564

Groffman, S. (2009). Subitizing: Vision Therapy for Math Deficits. *Optometry & Vision Development, 40*(4), 229-238.

Heine, A., Tamm, S., De Smedt, B., Schneider, M., Thaler, V., Torbeyns, J., & Jacobs, A. (2010). The numerical stroop effect in primary school children: A comparison of low, normal and high achievers. *Child Neuropsychology, 16*, 461-477. doi: 10.1080/09297041003689780

Klahr, D., & Wallace, J. G. (1976). *Cognitive development and information processing view*. Hillsdale, N.J. and New York: L. Erlbaum Associates.

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schonmann, C., Plangger, F., & von Aster, M. (2011). Mental number line training in children with developmental dyscalculia. *NeuroImage, 57*(3), 782-795. doi: 10.1016/j.neuroimage.2011.01.070

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: a study of 8-9-year-old students. *Cognition, 93*(2), 99-125. doi: 10.1016/j.cognition.2003.11.004

Lemer, C., Dehaene, S., Spelke, E., & Cohen, L. (2003). Approximate quantities and exact number words: dissociable systems. *Neuropsychologia, 41*(14), 1942-1958. doi: 10.1016/s0028-3932(03)00123-4

Lipton, J., & Spelke, E. (2003). Origins of number sense: Large-Number Discrimination in Human Infants. *Psychological Science, 14*(5), 396-401.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Dev, 82*(4), 1224-1237. doi: 10.1111/j.1467-8624.2011.01608.x

McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by nine-month-old infants. *Psychological Science, 15*(11), 776-781.

MOE. (2005). *İlköğretim 1-5 sınıflar matematik dersi öğretim programı*. Ankara: MEB.

Moeller, K., Neuburger, S., Kaufmann, L., Landerl, K., & Nuerk, H. C. (2009). Basic number processing deficits in developmental dyscalculia: Evidence from eye tracking. *Cognitive Development, 24*(4), 371-386. doi: 10.1016/j.cogdev.2009.09.007

Olkun, S., Altun, A., Cangöz, B., Gelbal, S., & Sucuoğlu, B. (2012). *Developing Tasks for Screening Dyscalculia Tendencies.* Paper presented at the E-Leader, Berlin.

Rousselle, L., & Noel, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*(3), 361-395. doi: 10.1016/j.cognition.2006.01.005

Rubinsten, O., & Henik, A. (2006). Double dissociation of functions in developmental dyslexia and dyscalculia. *Journal of Educational Psychology, 98*(4), 854-867. doi: 10.1037/0022-0663.98.4.854

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2011). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology, no-no*. doi: 10.1111/j.2044-835X.2011.02048.x

Shalev, R. S. (2004). Developmental Dyscalculia. *Journal of Child Neurology, 19*(10), 765-771.

Shalev, R. S., & Gross-Tsur, V. (2001). Developmental dyscalculia. *Pediatr Neurol, 24*(5), 337-342.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Dev, 75*(2), 428-444. doi: 10.1111/j.1467-8624.2004.00684.x

Spelke, E. S., & Kinzler, K. D. (2007). Core knowledge. *Developmental Science, 10*(1), 89-96. doi: 10.1111/j.1467-7687.2007.00569.x

Strauss, M. E., & Curtis, L. E. (1981). Infant perception of numerosity. *Child Development, 52*, 1146-1152.

Xue, F., & Spelke, E. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*, B1-B11.