THE EFFECT OF TURKISH MORTALITY IMPROVEMENTS ON THE COST OF ANNUITIES USING ENTROPY MEASURE

TÜRKİYE'DEKİ ÖLÜMLÜLÜK DEĞİŞİMLERİNİN ANÜİTE FİYATLARI ÜZERİNDEKİ ETKİSİNİN ENTROPİ ÖLÇÜSÜ

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ETHICS

In this thesis study, prepared in accordance with the spelling rules of Institute of Graduate School of Science and Engineering of Hacettepe University,

I declare that

- all the information and documents have been obtained in the base of the academic rules
- all audio-visual and written information and results have been presented according to the rules of scientific ethics
- in case of using other Works, related studies have been cited in accordance with the scientific standards
- all cited studies have been fully referenced
- I did not do any distortion in the data set
- and any part of this thesis has not been presented as another thesis study at this or any other university.

/ / 2014

BÜKRE YILDIRIM

ABSTRACT

THE EFFECT OF TURKISH MORTALITY IMPROVEMENTS ON THE COST OF ANNUITIES USING ENTROPY MEASURE

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Over the twentieth century there have been unprecedented and unexpected gradual decline in mortality rates due to considerable process of eliminating hazards to survival, which have increased the life expectancy at birth remarkably. Decrease in mortality rates often been underestimated by annuity and pension providers. Especially, mortality improvements for post-retirement ages have significant financial effect on the annuity and pension plans.

To correctly measure the effect of mortality rate improvements on the life expectancy or annuity costs many demographers use the idea of entropy. Entropy has also been used as a measure of the rectangularization of the survival function, variability of the age-at-death and the convexity of the survival curve.

In this thesis work, the Keyfitz definition of entropy is used to measure the effect of any changes in the force of mortality on the cost of a life annuity. Entropy parameters investigated to see how they affect the entropy measure. Firstly, Turkish period life tables are used to calculate the entropy measure for the cost of annuities. Then a mathematical model for the force of mortality is adopted to obtain a more theoretically satisfactory conclusion.

This master thesis is comprised of 5 chapters, mainly as; introduction, entropy in actuarial science, entropy values using Turkish life tables, entropy values and its behavior using a mathematical model and finally results and conclusions.

In Chapter 2; the formula for the entropy measure for life expectancy at birth is explained and then the corresponding formula for the life annuity is explained. In Chapter 3; the value of entropy measure H is calculated using the Turkish Life Mortality Tables over the period 1931-2015. In Chapter 4; the value of entropy measure is calculated with an assumption that mortality is following a Gompertz mortality model. Lastly, Chapter 5 discusses the overall conclusions and makes some recommendations.

Keywords: Entropy, force of mortality, annuity cost, rectangularization.

ÖZET

TÜRKİYE'DEKİ ÖLÜMLÜLÜK DEĞİŞİMLERİNİN ANNUİTE FİYATLARI ÜZERİNDEKİ ETKİSİNİN ENTROPİ ÖLÇÜSÜ

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20. yüzyıl boyunca beklenen yaşam süresini etkileyen tehlikelerin azaltılması sonucunda mortalite hızlarında, daha önce görülmemiş ve beklenmedik kademeli düşüşler olmuştur. Bu nedenle doğumda beklenen yaşam süresi önemli bir biçimde artmıştır. Günümüzde mortalite gelişmelerinin yavaşlamakta olduğuna dair ya da insan ömrünün belirli bir teorik üst limiti olduğuna dair kesin bir kanıt bulunmamaktadır.

Tarihsel süreç boyunca, gözlemlenen, mortalite hızlarındaki azalmaların arkasında son derece önemli gelişmeler bulunmaktadır. Bunlardan bazıları; daha besleyici gıdaların üretilmesi, temiz su kaynakları, atık imha alanındaki gelişmeler, yüksek yaşam standartları, daha iyi barınma ve çalışma alanları, motorlu araç güvenliğindeki gelişmeler, temel sağlık hizmetlerine erişimin kolaylaşması, aşılamanın yaygınlaşması ve tıbbi gelişmelerdir.

Türkiye'de toplam doğurganlık hızı Türkiye İstatistik Kurumu'nun hesaplamalarına göre 2012 yılı için 2.08'dir. Bu nüfus yenilenme hızı olarak kabul edilen 2.1 değerinin de altındadır. Nüfusun yaşlanmakta olduğu ve nüfustaki genç insan sayısının, yani çalışma gücüne katkı sağlayan kişi sayısının azalmakta olduğuna

şüphe yoktur. Uzun yaşama riski, azalan doğurganlık ve mortalite hızlarının etkisiyle daha da önemli bir hale gelmiştir.

Mortalite hızındaki süregelen bu düşüş beklenenden daha fazla olduğu için genellikle dikkate alınamamıştır. Dolayısıyla primler eksik tahmin edilmektedir. Özellikle emeklilik sonrası yaşlar için ortaya çıkan mortalite iyileşmelerinin emeklilik planlarını düzenleyen kuruluşlar ve anüite garantisi veren şirketler üzerinde önemli finansal etkileri olmuştur. Sosyal güvenlik sisteminin yapısı nedeniyle mevcut olan eksiklikler de mortalitedeki gelişmelerin etkisi ile birlikte kötüleşmektedir.

Entropi ilk olarak Alman fizikçi Rudolf Clasius tarafından 1865'de, Sadi Carnot ve Lord Kelvin'in önceki çalışmalarını da temel alarak tanıtılmıştır. Clasius sistemler arasındaki mükemel ısı transferlerinde bile kullanılabilir enerjinin bir kısmının kaybolmasının kaçınılmaz olduğunu bulmuştur ve bu kaybolmayı entropideki artış olarak tanımlamıştır. Entropi; termodinamiğin doğadaki en temel kanunlarından biri olan ikinci yasasıdır. Bu şu anlama gelir; tüm süreçler kaçınılmaz enerji kaybı (entropi artışı) nedeniyle $\%100$'den daha az verimle çalışmaktadır. Entropi kavramı modern bir bilim dalı olan bilgi kuramında da önemli bir role sahiptir ve bir mesajın içerdiği bilginin beklenen değerini ifade eder. Bilgi kuramının 1948'de Amerikalı matematikçi Claude E. Shannon'ın öncü çalışması "A Mathematical Theory of Communication" da bulunduğu kabul edilmektedir.

Geçtiğimiz yıllarda birçok demograf entropi ölçütünü toplumların mortalite karşılaştırması amacıyla kullanmıştır. İlk olarak Nathan Keyfitz entropi ölçütünü mortalite hızındaki herhangi bir değişimin beklenen yaşam süresi üzerindeki etkisini ölçmek için bir indeks olarak kullanmıştır. Yüksek entropi değeri düşük entropi değerine göre, beklenen yaşam süresinin mortalitedeki değişimlere daha duyarlı olduğunu göstermektedir.

Entropi kavramı birçok araştırmacı tarafından yaşam fonksiyonu için dikdörtgenselleşmenin bir ölçütü olarak da kullanılmıştır. Dikdörtgenselleşme geçtiğimiz yüzyılda yüksek mortaliteden düşük mortaliteye geçişin neredeyse tamamlandığı birçok gelişmiş ülkede ortaya çıkmıştır. Dikdörtgenselleşme ya da başka bir deyişle, yaşam eğrisinin artan yaşama olasılıkları ve beklenen yaşam süresine bağlı olarak düzleşmesi, yaşam fonksiyonunun gittikçe dikdörtgen bir şekil almasıdır. Yaşama olasılığının ileri yaşlara kadar bire yakın kalıp kısa bir zaman aralığı içinde sıfıra düşmesi sonucu dikdörtgen bir şekil ortaya çıkar. Mortalite hızlarındaki gelişim insanların daha ileri yaşlarda ölmesine neden olmuştur. Bu, ölüm yaşındaki değişkenliğin azalması ve ölümlerin yaşamın daha ileri yıllarında birikim göstermesi anlamına gelir.

Bu tez çalışmasında Keyfitz'in entropi tanımı, mortalitedeki değişimlerin anüite değerleri üzerindeki etkisinin bulunması için kullanılmıştır. Daha sonra entropide yaş, cinsiyet, mortalite değişimi ve faiz oranı gibi parametrelerinin etkileri incelenmiştir. Öncelikle 1931- 2015 yılları arasındaki Türkiye periyot hayat tabloları kullanılmış ve entropi anüite değerleri için hesaplanmıştır. Daha sonra, teorik olarak daha tatmin edici bir sonuca ulaşılabilmesi için mortalite hızı için Gompertz modeli kullanılmış, yeni hayat tablolarıyla işlemler tekrar edilmiştir.

Hesaplamalar entropinin cinsiyet değişkenine çok duyarlı olmadığını ancak yaş, yıl, faiz oranı ve mortalite değişimlerine karşı duyarlı olduğunu göstermiştir. Anüite ve entropi değerlerinin kullanılan faiz ile ters, yaş ile doğru orantılı olduğu görülmüştür.

Bu yüksek lisans tezi beş kısımdan oluşmaktadır. Bunlar başlıca: Giriş, Aktüerya biliminde entropi, Türkiye hayat tabloları kullanılarak entropi değerleri, Matematiksel model kullanılarak entropi değerleri ve son olarak Sonuç ve öneriler'dir. Bölüm 2'de; ilk olarak doğumda beklenen yaşam süresi için entropi ölçütü formülü açıklanmış daha sonra bu formül annüite değerleri için kullanılabilir hale getirilmiştir. 3. Bölüm'de entropi ölçütünün değeri, 1931-2015 Türkiye mortalite tabloları kullanılarak hesaplanmıştır. Bölüm 4'de entropi değerleri mortalitenin matematiksel bir modele uyduğu varsayımı altında hesaplanmış ve bu modelin parametrelerine göre davranışı incelenmiştir. son olarak Bölüm 5 elde edilen genel sonuçlar hakkında bilgi vermekte ve ileride yapılabilecek çalışmalar için önerilerde bulunmaktadır.

Anahtar Kelimeler: Entropi, mortalite hızı, anüite değeri, dikdörtgenselleşme.

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1. INTRODUCTION

Over the twentieth century there have been unprecedented and unexpected gradual decline in mortality rates due to considerable process of eliminating hazards to survival, which have increased the life expectancy at birth remarkably.

Aging is still an uncertain process and there is much debate as to whether there are upper limits to human longevity. However, it is clear that there is no conclusive evidence to suggest that current mortality improvements are tending to slow down and no fixed theoretical limit to human longevity is apparent today [1].

Going back in history, there exist a number of extremely important developments behind the observed reductions in mortality rates. These include; more nutritious foods, clean water supply, improvements in waste removal, improvements in sanitary conditions, higher standards of living, better housing and working conditions, improvements in motor vehicle safety, access to primary medical care for the general population, availability of immunizations, medical advances, etc. Throughout the years each of these developments will make a smaller impact to mortality improvement.

Further reductions in mortality will depend upon factors such as; development of new diagnostic and surgical techniques, emergence of new forms of disease, presence of environmental pollutants, cigarette smoking, improvements in exercise and nutrition, incidence of violence, ability and willingness to pay for the development of new treatments and technologies.

Over the years, this evolution has led death causes to change from infectious diseases, such as tuberculosis, diphtheria and cholera, to degenerative and chronic diseases, such as cancer and cardiovascular diseases which effect mostly older ages [2]. In other words, mortality from exogenous causes is mostly eliminated, and the remaining variability in the age at death is generally caused by genetic factors [3].

Figure 1.1. demonstrates the improvements in mortality rates of Turkey for females and males respectively over the years 1931 to 2015. It can be seen that mortality rates have fallen considerably. There are significant mortality improvements in some decades and almost no improvements in other decades.

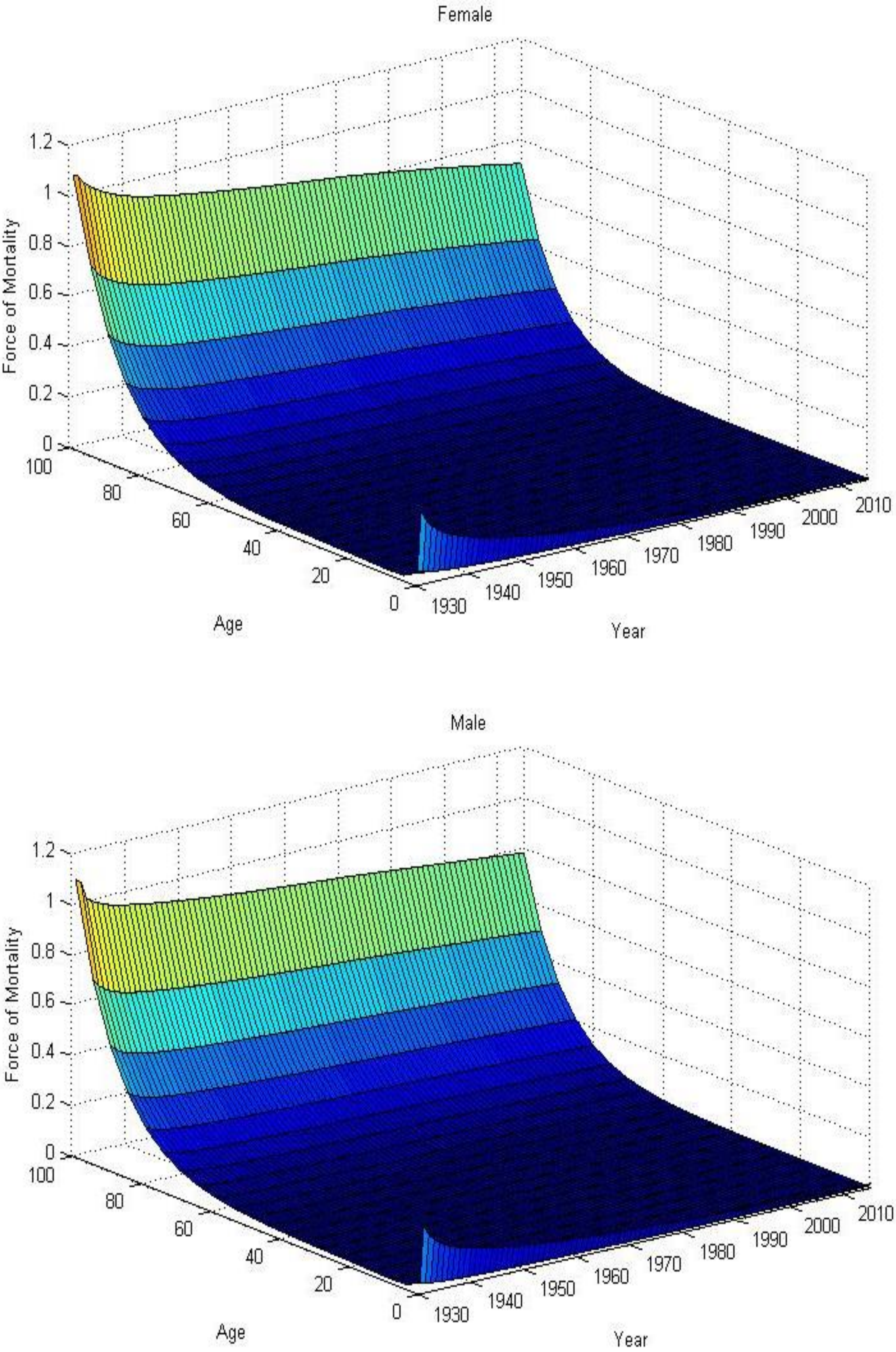


Fig. 1.1. Mortality rates of Turkey for females and males respectively over the years 1931 to 2015.

Life expectancy at birth in the early times among humans was likely 20 to 30 years [4] proven by tombstones inscription, skeletal remains, and genealogical records [5]. When the first population data in history began being collected in 1750 in the Nordic countries, life expectancy was around 35-40 years. Then rapid improvements began at the beginning of 20th century and life expectancy reached 60-65 years by the middle of the 20th century. Most of the life expectancy increase occurred in the last two centuries. The 20th century in particular has made a huge increase in life span compared to all other centuries. Since then life expectancy gains mostly have been due to medical factors which have reduced mortality among older persons. By the beginning of the 21st century, life expectancy at birth has reached about 70 years even 80 years in some countries, such as Japan [6].

In Europe, Eurostat states that the total fertility rate has dropped to 1.5 in 2012, which is below the rate of 2.1, considered to be the replacement rate of populations. In Turkey the total fertility rate for year 2012 is 2.08 according to calculations of Turkish Statistical Institute (TÜİK) [7]. Due to these reasons, without arguing that the world's population is getting older and younger people in population who constitutes the work force is declining with the contribution of declining fertility rates. The effect of longevity risk is even more important with the consequences of falling fertility rates and improving mortality. It has agitated the plan providers, especially for products offering annuity guarantees.

This persistent decrease in mortality rates was unexpected and as a result they have often been underestimated by annuity and pension providers. Especially, mortality improvements for post-retirement ages have a significant financial effect on the annuity and pension plans. Because of the tax-paid structure, the deficiencies of public finance are aggravated by mortality improvements. Also due to low fertility rates, the ratio between old people and young people decreased over time [2].

Which takes us to entropy: The term entropy in physical sciences evolves around the definitions of the thermodynamics [8]. In thermodynamics the second law states that in a system or process, entropy is the disorder or randomness. Thermodynamic entropy is a measure of how disordered a system is. A system which in an imbalanced state ,for example a hot region next to a cold region, will always tend to even out, with the heat flowing from hot to cold region until the systems even out.

Entropy was first introduced when the steam engine became science (thermodynamics) by the German physicist Rudolf Clasius in 1865, based on earlier work by Sadi Carnot and Lord Kelvin. Clasius found that even for perfect exchanges of heat energy between systems, a loss of useful energy is inevitable. He called this loss an increase in entropy. The increase in entropy is the amount of heat transfer divided by the absolute temperature. This is one of the basic laws of nature, known as the second law of thermodynamics. This means that all processes must operate at less than 100% efficiency due to the inevitable loss of heat energy, entropy rise [8].

The concept of entropy also plays an important role in the modern discipline of information theory. Information theory is generally considered to have been founded in 1948 by the American mathematician Claude E. Shannon in his pioneer work “A Mathematical Theory of Communication” [9]. In information theory, entropy quantifies the expected value of the information contained in a message.

To correctly measure the effect of mortality rate improvements on the life expectancy or annuity costs many demographers use the idea of entropy. During the past decades, the concept of entropy has been adopted by several demographers.

For example, Pollard [10] focused on the function of mortality rate and its effects on life expectancy. He used discrete differences in life expectancy at two moments in time, and analyzed the effect of difference between two mortality rates. The second approach by Keyfitz [11] introduced the concept of entropy into the field of demography as a measure of the elasticity of the life expectancy for proportional changes in mortality rates.

As stated by Keyfitz, entropy of a survival model is the time derivative of the life expectancy. He used the entropy measure as an index to measure the effect of any change in the force of mortality on life expectancy. A high value of entropy measure indicates that the life expectancy has a greater sensitivity to a change in the force of mortality than a lower value of entropy measure. So a high value of entropy measure means that future reductions in mortality rates would have an impact on life expectancy. On the other hand, a low value of entropy measure means that life expectancy is relatively ‘immune’ to future reductions in mortality rates. The particular advantage of the concept of entropy is that it allows the sensitivity of the life expectancy to changes in mortality rates to be summarized in a single figure index.

The concept of entropy has also been used as a measure of the rectangularization of the survival function by a number of researchers (e.g. Goldman and Lord [12], Nagnur [13], Nusselder and Mackenbach [3]). Rectangularization showed up over the past century in most developed countries of the world in which the transformation from high mortality to low mortality is nearly complete [13].

Rectangularization or the flattening of the survival curve is a trend toward a more rectangular shape of the survival function due to increased survival rate and expectation of life. A rectangular curve emerges when the probability of survival stays close to one up to old ages and then drops to zero over a short age range (sharp down slope) at very old ages, due to demographic transition from high to low mortality levels. Improvements of the mortality rates have caused people to die at older ages. This means that; the variability in the age at death declines and deaths are concentrated to the upper years of life. Also, a way of analyzing the rectangularization is to observe the temporal variations in the values of entropy.

Entropy is also a measure of the variability of the age-at-death and a measure of the convexity of the survival curve [5]. If the survival curve is completely rectangular, entropy is zero and everybody lives to a certain age and then dies at that age hence the variability in the age-at-death is zero. If the survival curve declines in a linear form with age and the number of deaths are equal at all ages, entropy is 0.5. If mortality is independent of age in other words the force of mortality is equal at all ages, entropy equals the unity.

In this thesis work, the Keyfitz definition of entropy is used to measure the effect of any changes in the force of mortality on the cost of a life annuity. Then we investigate some parameters of the entropy to see how they affect the entropy measure -for example; the effects of the age and gender of the pensioner at inception and the rate of interest. Firstly we use period life tables to calculate entropy measure from annuity values and life expectations. Then we consider a mathematical model for the force of mortality to obtain a more theoretically satisfactory conclusion.

This master thesis is comprised of 5 chapters, mainly as; introduction, entropy in actuarial science, entropy values using Turkish life tables, entropy values and its behavior using a mathematical model and finally results and conclusions.

In chapter 2; the formula for the entropy measure for life expectancy at birth is explained and then the corresponding formula for the life annuity is explained. In chapter 3; the value of entropy measure H is calculated using the Turkish Life Mortality Tables over the period 1931-2015. In chapter 4; the value of entropy measure is calculated with an assumption that mortality is following a Gompertz mortality model. Lastly, chapter 5 discusses the overall conclusions and makes some recommendations.

2. ENTROPY IN ACTUARIAL SCIENCE

In this chapter we take into account the formula of the entropy measure for life expectancy at birth and then introduce the corresponding formula for the life annuity. After brief explanations of some basic actuarial terms in Section 2.1, Section 2.2 presents the theory of entropy in life expectancy from Keyfitz [11] in 2.2.1., and the entropy measure for annuity costs in 2.2.2.

2.1. Notation

Some important terms which frequently used in the following sections will be defined basically here.

A life table shows fundamental parameters of a population for each age or age group, such as; the number of survivors, the number of deaths, the probability that they die or live to their next birthday and the life expectancy. It describes the mortality and survival pattern of a population. From this point forward, a number of fundamental statistics can be derived.

There are two types of life tables:

I. Period (static) life tables show the current probability of death (for people with different ages, in the current year). It is based on the mortality experience of an entire population.

II. Cohort, or generation, life tables are based on mortality experience over the lifetime of people from a given cohort (which usually covers one year) over the course of their lifetime.

Mortality data and life tables, originate from observations concerning a whole national population or a specific part of a population (e.g. retired workers, disabled people, etc.) or an insurer's portfolio, and so on.

The past life table data does not assure future outcome. Hence in order to price insurance products, and obtain the solvency of insurance companies, actuaries must develop projections of future insured events. To do this actuaries develop mathematical models for mortality, next life tables are generated. The life tables are called mortality tables if they show death rates and morbidity tables if they show various types of sickness or disability rates.

The standard notations required for a life table includes:

Number of Survivors

Radix is the number of people alive at the age of 0. It is shown as l_0 and this is the starting point for l_x. l_x is the number of survivors at age x. It is the number of those original radix who are still alive on their x^{th} birthday. As age increases the number of people alive decreases to a finite limit of 0. Also, ω is the limiting age of the mortality table and l_n is zero for all $n \geq \omega$.

Also, l_0 can be calculated as;

$$l_0 = \sum_{x=0}^{w} {}_1d_x$$

Number of Deaths

Number of people who die between age x and $x+n$ is defined as ${}_nd_x$. It can be calculated using the formula;

$${}_nd_x = d_x + d_{x+1} + ... + d_{x+n-1}.$$

$$= l_x - l_{x+n}$$

Probability of Living and Dying

Probability of dying between age x and $x+n$ is defined as ${}_nq_x$.

$${}_nq_x = \frac{{}_nd_x}{l_x}$$

Also, when mortality is following a mathematical model ${}_xq_0$ can be calculated as;

$$ {}_{x}q_0 = 1 - \exp[-\int_0^x \mu_a da] $$

Probability of surviving between age x and $x+n$ is defined as ${}_{n}p_x$.

$$ {}_{n}p_x = \frac{l_{x+n}}{l_x} $$

Also, when mortality is following a mathematical model ${}_{x}p_0$ can be calculated as;

$$ {}_{x}p_0 = \exp[-\int_0^x \mu_a da] $$

Since the only possible alternatives from one age x to the next $x+n$ are living and dying, the relationship between these two probabilities;

$$ {}_{n}q_x + {}_{n}p_x = 1. $$

Life Expectancy

Another statistic that can be obtained from a life table is life expectancy. Life expectancy is the average number of years of life expected to remain for a person at age x. It is defined as e_x.

$$ e_x = \sum_{t=1}^{\infty} {}_{t}p_x $$

Also, when mortality is following a mathematical model, e_x can be calculated as;

$$ e_x = \int_0^{\infty} {}_{t}p_x dt $$

The expected lifetime is often used to compare mortality in various populations. The definition of e_x is based on the probability distribution of the lifetime conditional on being alive at age x. Thus when $x=0$ the mortality at all ages contributes to the value of e_x.

Within populations, differences in life expectancy exist with regard to gender. Females tend to outlive males in all populations, and have lower mortality rates at all ages, starting from infancy.

Force of Mortality

Force of mortality refers to an instantaneous rate of mortality at a certain age measured on an annualized basis. In a life table, we consider the probability of a person dying between age x and age $x+t$; as ${}_tq_x$. In the continuous case, we could also consider the conditional probability that a person who has attained age x will die between age x and age $x+t$. This is called the force of mortality and defined as follows:

$$\mu_x = \lim_{t \to 0} \frac{\mathrm{P}[T_x \le t]}{t} = \lim_{t \to 0} \frac{{}_tq_x}{t}$$

Where T_x is the remaining lifetime of a person at age x. And hence it represents the instantaneous rate of mortality at a given age x.

From

$$\mathrm{P}[T_x \le t] = F_x(t) = \frac{F_0(x+t) - F_0(x)}{1 - F_0(x)} = \frac{F_0(x+t) - F_0(x)}{S(x)}$$

we obtain

$$\mu_x = \lim_{t \to 0} \frac{F_0(x+t) - F_0(x)}{tS(x)} = \frac{f_0(x)}{S(x)} = \frac{-\frac{d}{dx}S(x)}{S(x)} = -\frac{d}{dx}\ln S(x)$$

Hence, once the survival function $S(x)$ has been assigned, the force of mortality can be derived.

Life tables can be constructed using projections of future mortality rates, but more often they are snapshots of age-specific mortality rates in the recent past. For these reasons, the older ages in the life table may have greater chance of not being representative of what lives at these ages may experience in future, as it is predicated on current advances in medicine, public health, and safety standards that did not exist in the early years of this cohort.

Annuity Cost

People generally feel a necessity to make savings to help them when they retire. An annuity specifically provides that. Everyone at the same age group makes equal payments to a life insurance company to be a beneficiary of the accumulated payments in the future under the condition of reaching a certain age. In this manner the survivors from this group gets more than they pay initially. No one knows when they will die and the payment is conditional on staying alive therefore, life annuities are conditional annuities. Making regular savings, rental payments, coupon and bond interest payments, life insurance premiums are examples of annuity problems. If not recognized, the changing mortality rates can and will cause companies to fail to meet their liabilities.

A life annuity consists of a series of payments which are made while the annuitant (of initial age x) lives. To ensure the future payments insured needs to make contributions to the fund, this quantity is called net premium. The net single premium of a life annuity is its expected present value, $\bar{a}_x$. It is defined as

$$\bar{a}_x = \int_0^\infty {}_t p_x \exp[-\delta t] dt$$

Where δ is the force of interest rate.

Survival Function

Survival function gives the proportion of individuals reaching the age x. Assume that the function $S(t)$, called the survival function and defined for $t \geq 0$ as follows:

$$S(t) = \mathrm{P}[T_0 > t]$$

T_0 denotes the random lifetime for a newborn. Where w is the limiting age it is usual to assume that the possible outcomes of T_x lie in $(0, w)$ and the probability measure outside this interval is zero.

The survival curve moves towards a rectangular shape with time due to improvements in mortality rates and increases in life expectancy, and hence the term rectangularization is used to describe this feature.

Rectangularization of survival curve is also associated with a reduction in the variability of age at death. As deaths become concentrated in a narrow age range, the slope of the survival curve becomes steeper, and the curve itself begins to appear more rectangular.

The point of maximum downwards slope of the survival curve progressively moves towards the very old ages; this feature is called the expansion of the survival function.

2.2. Methodology

2.2.1. Entropy and Life Expectancy

A measure of uncertainty associated with a random variable comes from the field of information theory and is called entropy. In actuarial science entropy is a single figure index that quantifies the effect of proportional change in the force of mortality over the whole range of age on life expectancy. It gives the percentage change in life expectancy produced by a reduction of one percent in the force of mortality if equal progress is achieved against mortality at all ages.

Entropy determines the expected uncertainty on the result of an experiment and presents information about the foreseeability of the result of a random variable. The larger the entropy the less compressed the distribution and thus an observation about random variable provides less information. Measure of entropy may be regarded as a descriptive quantity of the corresponding probability density function as it belongs to the class of measures of variability like the median, mode, variance or the standard deviation [14].

The most known measure of entropy, Shannon's information entropy equation, is defined by;

$$H(X) = -\sum_{x} p(x) \ln p(x)$$

or

$$H(X) = -\int f(x)\ln f(x)dx$$

Where, $p(x)$ is the probability mass function of X and $f(x)$ is the probability density function of X depending on whether the random variable X is discrete or continuous, respectively [9].

The Shannon's entropy, measures the information content of a random variable and represents a lower bound on how much that random variable can be compressed without losing its content. Also, Shannon's entropy measure initiated the mathematical study of information theory, and his results are the bases of communication over networks today.

The formula for the entropy measure for actuarial work frame differs from that associated with information theory.

The expectation of life at age x e_x^0, calculated from life tables is a commonly used summary measure for population's level of mortality. Variations in life expectancies also can be used to compare between countries. The role of single-figure indices such as e_x^0, summarizing the lifetime probability distribution, should not be underestimated. It does suffer from some of the usual disadvantages of single-figure indices but at the same time there are good reasons for adopting such a summary measure. Apart from the fact that a single index such as e_x^0 condense the information of a full mortality schedule considerably, life expectancies also have the advantage of easing the interpretation.

Assume that the overall force of mortality at age x is μ_x, and the force of mortality changes by 100φ percent where φ is a constant change in the force of mortality at all ages, and could be positive or negative depending on whether the mortality rates are increasing or decreasing respectively. Then, the new force of mortality at age x is, $\mu_x^* = \mu_x(1+\varphi)$.

This new force of mortality leads to a new survivorship curve, and the new probability of surviving from age 0 to x becomes

$$ {}_{x}p_0^* = \exp[-\int_0^x \mu_a^* da] $$

$$ = \exp[-\int_0^x \mu_a (1+\varphi) da] $$

$$ = \exp[-\int_0^x \mu_a da]^{1+\varphi} $$

$$ = ({}_{x}p_0)^{1+\varphi} \qquad (2.1) $$

The new expectation of life is

$$ e_0^{o*} = \int_0^w ({}_{a}p_0)^{1+\varphi} da \, , \qquad (2.2) $$

where w is the maximum limit age of a population in the life table.

Keyfitz [11] has shown how the logarithm of the survivorship curve, $[\ln {}_{n}p_x]$, contains useful information about the effects of changes in force of mortality on life expectancies. So as to determine the effect of changes in force of mortality, μ_x, on the expectation of life, e_x^0, we consider the derivative of equation (2.2) with respect to φ;

$$ \frac{de_0^{o*}}{d\varphi} = \int_0^w \ln({}_{a}p_0)({}_{a}p_0)^{1+\varphi} da \qquad (2.3) $$

In the neighborhood of $\varphi = 0$ based on a Taylor expansion, we have the following approximation

$$ \frac{\Delta e_0^0}{e_0^0} \cong \left[\frac{\int_0^w [\ln {}_{a}p_0]\, {}_{a}p_0 da}{\int_0^w {}_{a}p_0 da} \right] \varphi \qquad (2.4) $$

To make H a positive quantity we define it as minus the expression in brackets.

$$\frac{\Delta e_0^0}{e_0^0} = -H\varphi \tag{2.5}$$

Equation (2.5) means that the proportional change in life expectancy can be approximated in simple terms as H times φ, where φ means that a small proportional change in the force of mortality if equal progress is achieved against mortality at all ages.

The quantity H as the entropy measure is shown as

$$H = -\frac{\int_0^w [\ln {}_a p_0]\, {}_a p_0 da}{\int_0^w {}_a p_0 da} \tag{2.6}$$

Entropy is a positive quantity and the ratio of the integrals is always negative because ${}_a p_0$ is always between zero and one and $\ln({}_a p_0)$ is always negative. Eventually entropy is a weighted average of values of $\ln {}_a p_0$ as age ranges from 0 to w. As ${}_t p_x$ declines from one to zero over the age span $-\ln {}_t p_x$ increases from zero to infinitely large values. Therefore, there is no apparent mathematical explanation of why $H_x(\delta)$ should be bounded from above [12]. Hence, it has no upper bound and life tables may have entropy values above one. Those life tables exhibit extremely high death rates in the young ages.

The effect of a mortality change on the life expectancy depends directly on the concavity of the survival curve (same as ${}_t p_x$, or μ_x, which ${}_t p_x$ was derived) [15]. As mortality improve over time, in other words the curve of ${}_t p_x$ turns down more sharply which is the same as saying that μ_x turns up more sharply, more people die in a narrow range of old ages before w. This leads H closer to zero. Hence, a change in μ_x has almost no effect on life expectancy. The lower H, the greater the tendency for all of us to die at about the same age, a tendency that apparently accompanies mortality decline. Indeed, H would be 0 if all mortality were concentrated at a single age (everyone dies at the same age). Hence, a change in

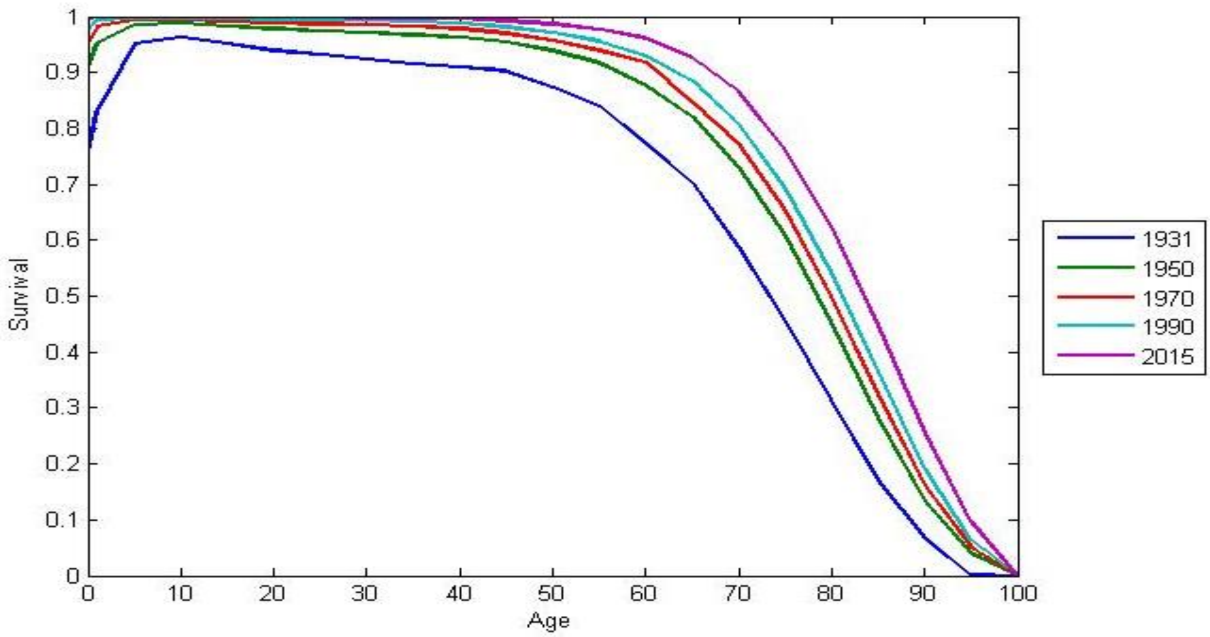
μ_x is reflected by an equal proportionate change in e_0. Also, in the case of complete rectangularization of the survival curve, entropy is zero regardless of the age at which all members of the population die.

Entropy is roughly the percentage change in life expectancy originated from a reduction of 1% in the force of mortality at all ages. Such as if entropy is 1, a decrease in mortality rates by 1% results in an increase in life expectancy at birth by 1%. Similarly, if entropy is 0.2, a uniform decrease in the force mortality by 1% results in an increase in life expectancy at birth by 0.2%.

Analyses point out that there is an inverse relationship between the period expectation of life and the entropy measure H. This decline in H is, in large measure, an outcome of the progress of reducing deaths in infancy- the age at which the highest number of years of life expectancy are lost. Another result of the progress in reducing deaths in infancy is a shift in the ages where further improvements of mortality would be most effective in increasing life expectancy. About a century ago, most of the potential for saving years of life was concentrated in the first years of childhood; today, in developed countries most is in old age. In the course of mortality declines and life expectancy increases, there is an upward shift in the ages where further declines in mortality would be most effective in increasing life expectancy. This gives rise to life expectancy being less sensitive to further declines in the force of mortality, and so we would expect the entropy measure to decrease over time as mortality improves. As a consequence, future movement towards rectangularization and a reduction in the force of mortality by one percent at all ages would produce a much lower increase in life expectancy today than would have been the case fifty years or a century ago [16].

Rectangularization occurs when the age-at-death distribution shows an increased building up around a certain old age, as the survival function takes more and more a rectangular shape. It is often being seen as a sign that the life expectancy is approaching to its biological limit. Approaching the average life span (biological limit of the life expectancy) implies that increases in total life expectancy must decelerate [16].

Great developments were made in the 20th century towards minimizing hazards to survival at the young ages. Survival rates at the older ages are also continuing to improve. As a result of these developments; Figure 2.1 presents the rectangularization of the Turkish female and male survival curves respectively for selected calendar years 1931, 1950, 1970, 1990 and 2015.



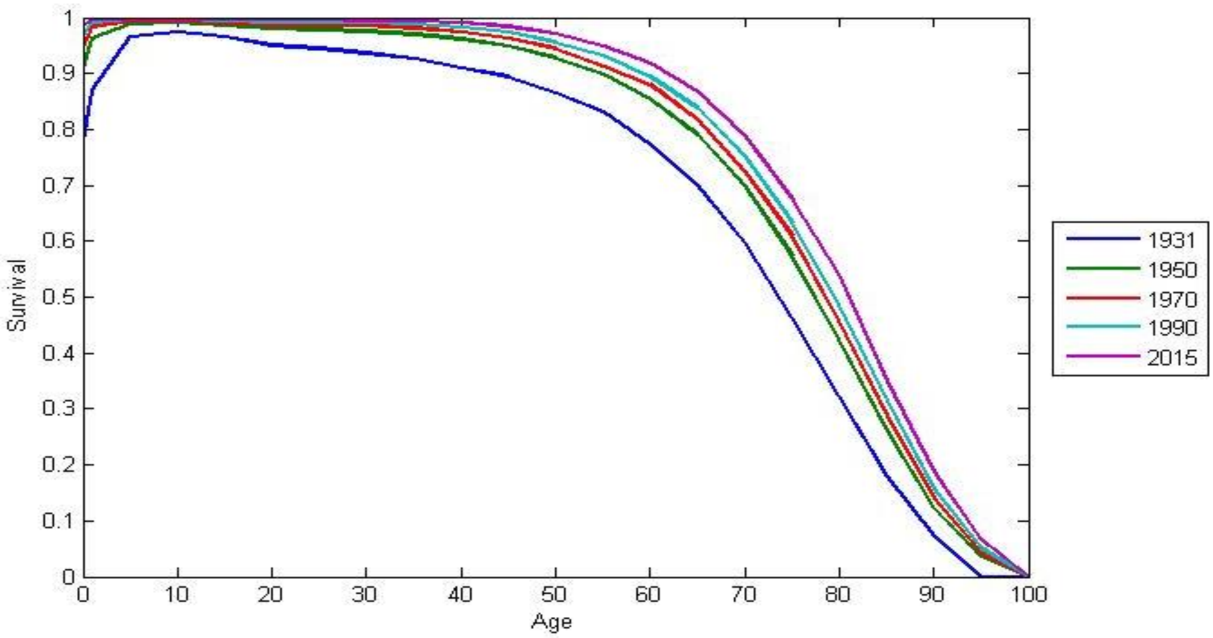


Fig. 2.1. Turkish female and male survival curves for selected calendar years.

Rectangularization is gradually evolving over the last 8 decades. The differences between men and women is more pronounced at the older ages in 2015 than in 1931. Women made greater gains than man and remarkable improvements have been registered in the probabilities of survival to older age.

Therefore, entropy measure can be seen as an indicator of rectangularization and it can also be interpreted as a measure of the heterogeneity of the population with respect to mortality at different ages [11], [16].

The theory presented in this section has considered the concept of entropy in relation with the expectation of life at birth. We now investigate how to extend this approach in order to find the corresponding entropy value for annuity costs.

2.2.2. Entropy and Cost of Annuity

This section provides a method to find the effect of a uniform proportional change in the force of mortality over the whole age range on the cost of an annuity.

Using the same assumption as considered in Section 2.1: Assume that the overall force of mortality at age x , μ_x , changes by 100φ percent where φ is a constant change in the force of mortality at all ages, and could be positive or negative depending on whether the mortality rates are increasing or decreasing respectively. This new force of mortality leads to a new value of a life annuity.

The new value of a life annuity at age x becomes

$$\bar{a}_x^* = \int_0^\infty ({}_t p_x)^{1+\varphi} \exp[-\delta t]dt \tag{2.7}$$

To determine the effect of changes in force of mortality, φ , on the value of a life annuity, $\bar{a}_x$, we consider the derivative of equation (2.7) with respect to φ .

$$\frac{d}{d\varphi}\bar{a}_x^* = \int_0^\infty (\ln {}_t p_x)({}_t p_x)^{1+\varphi} \exp[-\delta t]dt \tag{2.8}$$

And using a Taylor expansion for the neighborhood of $\varphi = 0$, it concludes that,

$$\frac{\Delta \bar{a}_x}{\bar{a}_x} \cong \left[\frac{\int_0^\infty (\ln {}_t p_x)({}_t p_x) \exp[-\delta t] dt}{\int_0^\infty {}_t p_x \exp[-\delta t] dt} \right] \varphi \qquad (2.9)$$

$H_x(\delta)$ can be thought of as minus the weighted average value of $\ln {}_t p_x$, weighted by ${}_t p_x \exp[-\delta t]$. Hence;

$$\frac{\Delta \bar{a}_x}{\bar{a}_x} = -H_x(\delta)\varphi \qquad (2.10)$$

Where $H_x(\delta)$ is the entropy measure of an annuity value for a person at age x with the force of interest, δ.

$$H_x(\delta) = \frac{-\int_0^\infty (\ln {}_t p_x)({}_t p_x) \exp[-\delta t] dt}{\int_0^\infty {}_t p_x \exp[-\delta t] dt} \qquad (2.11)$$

Here, $H_x(\delta)$ is a positive quantity because the numerator of the integrals in (2.11) is always negative.

When mortality rates improve over time, we would expect a larger percent of deaths to occur at older ages because of the decrease in volatility of age-at-death distribution. Also, the value of $H_x(\delta)$ decreases and becomes closer to 0. As stated previously, the pace at which this happens depends on the concavity of the survivorship curve. Additionally, as the interest rates increases, $H_x(\delta)$ is expected to decrease, because any change of the force of mortality is expected to have a lower impact on the cost of a life annuity at higher interest rates.

Equation (2.11) can be written as weighted average of $\bar{a}_{x+s}$ with using integration by parts;

$$H_x(\delta) = \frac{-\int_0^\infty \mu_{x+s}\ {}_s p_x e^{-\delta s} \bar{a}_{x+s} ds}{\bar{a}_x} \tag{2.12}$$

Hereby, $H_x(\delta)$ can be seen as a measure of heterogeneity easily, as ${}_s p_x \mu_{x+s}$ is the probability density function for the time of death of a person aged x.

To compute the values of entropy measure, we first need to arrange the ratio of integrals in equation (2.11), by using a numerical approach designed by Pollard [10].

Let

$${}_t Q_x = \int_0^t \mu_{x+u} du = -\ln\left(\frac{l_{x+t}}{l_x}\right) = -\ln {}_t p_x ,$$

and

$${}_t E_x = {}_t p_x \exp(-\delta t) .$$

Using these two equations and the mean value theorem for integrals; the integrals in (2.11) are replaced by sums of the one-year integrals. As a result of this, it leads the entropy measure to the following approximation:

$$H_x(\delta) \approx \frac{\sum_{t=0}^{w} {}_{t+\frac{1}{2}} Q_x \ {}_{t+\frac{1}{2}} E_x}{\sum_{t=0}^{w} {}_{t+\frac{1}{2}} E_x} \tag{2.13}$$

Where w is the maximum limit age in the mortality tables. Hence, equation (2.13) can be used directly to calculate the values of entropy measure for different values of x and δ.

3. ENTROPY VALUES USING TURKISH LIFE TABLES

İn this section of the thesis, entropy measure $H_x(\delta)$ is calculated using the Turkish Life Mortality Tables over the time interval between 1931 and 2015. These mortality tables for each year was derived from the computed regression levels of the Construction of Turkish Life and Annuity Tables Project [17]. The project assumes that Turkish Life Tables and the Cole-Demeny West Model Life Tables [18] are alike. And the project data covering all age groups of the Turkish population had originally been derived from the Turkish population registers.

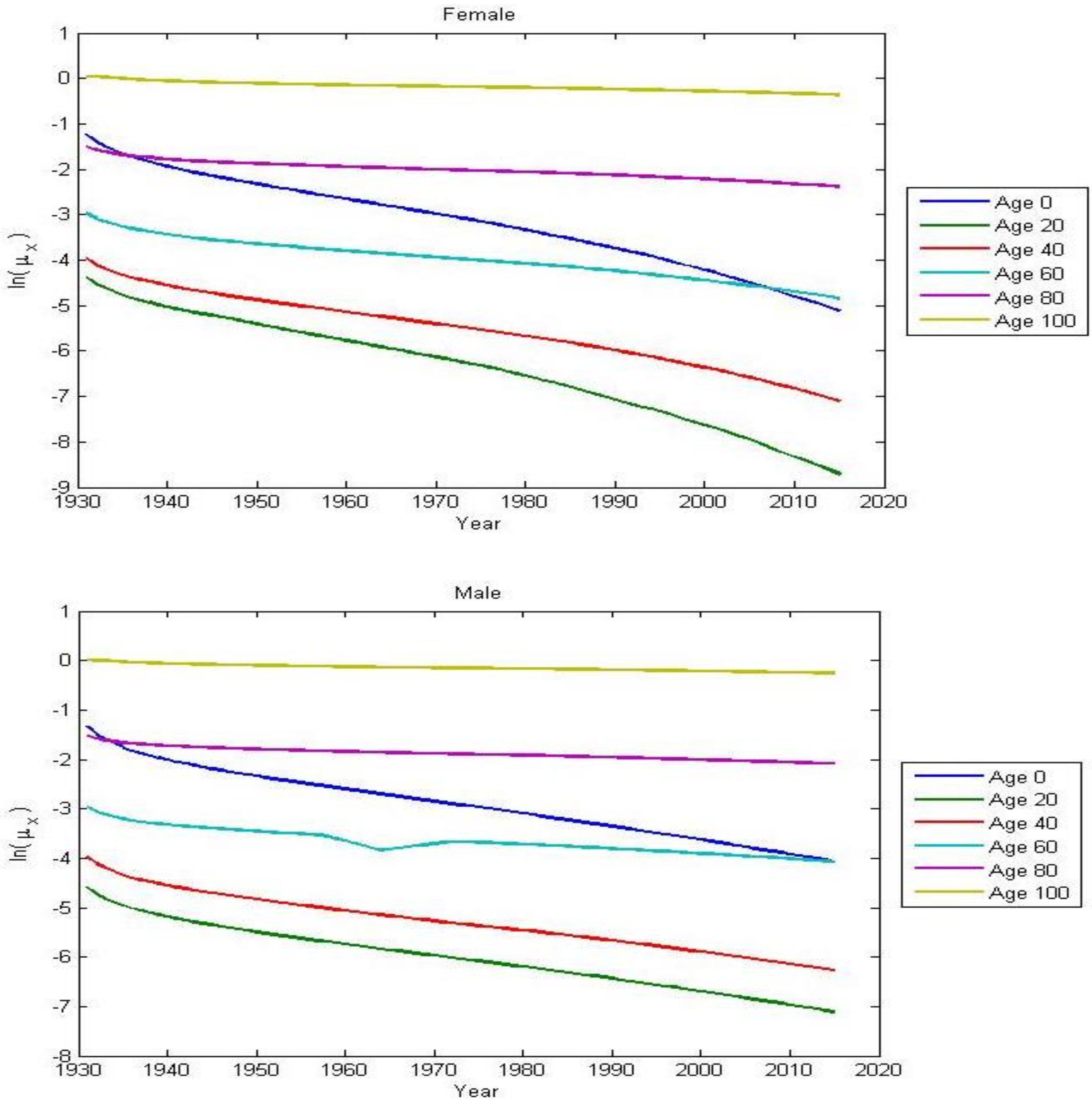


Fig. 3.1. Mortality rates for females and males in log scale over the period 1931-2015.

Firstly the plots of mortality rates were considered to determine the properties of population data. From the following Figure 3.1, it can be seen that mortality rates have been decreasing throughout the years. To facilitate understanding these rates have been plotted on a logarithmic scale. These life tables will allow us to investigate the entropy measure changes over an almost 85 year period.

The entropy measure is calculated separately for males and females to see and examine the importance of gender in mortality. Furthermore, it is calculated at different rates of interest (e.g. 0%, 2%, 4%, 10%, 8%, 10%).

Currently, until the year of 2036 the retirement age in Turkey is 60 for males and 58 for females as stated in the law number 4447 [19]. Beginning from 2048 the age of retiring from Social Security Institution is expected to be 65 for both females and males [20]. As the primary area of interest for the application of the concept of entropy for annuities is for people who retired, the age ranges 60-100 and 70-100 are considered, as well as the whole age range 0-100 to have a general outlook. Hence in the next subtitles $H_0(\delta)$, $H_{60}(\delta)$ and $H_{70}(\delta)$ are computed and investigated respectively.

3.1. Entropy Values at Age 0

Firstly, the values of the entropy measure for the whole age range, $H_0(\delta)$, is calculated. The change in the mortality rates at all ages shows its overall impact on $H_0(\delta)$. Also, different interest rates are taken into consideration when using the generated Turkish Life Tables between the years 1931-2015. The results are shown in Figures 3.2. and 3.3. for females and males respectively. Also, tables of these values are contained in the Appendix 2.

From the figures it can be clearly seen that, with the decreased mortality rates over the years, the value of $H_0(\delta)$ also decreased. This decrease in $H_0(\delta)$ means that any improvement in the force of mortality in a population will have a much smaller increasing effect on the expectation of life and the cost of the life annuity. Also as expected, the decreasing of entropy measure over the years has lesser impact at higher rates of interest. Because, the value of $H_0(\delta)$ decreases when interest rate increases.

As the number of survivors to older ages has increased and life expectancy at birth has improved over the years, the value of $H_0(\delta)$ has decreased. Consequently, the effect of proportional improvement of the age specific mortality rates on the cost of life annuity has diminished [13]. For recent mortality experiences in developed countries, $H_0(\delta)$ values appear to be below 0.2 [12].

As seen on the Figures 3.2. and 3.3., after the year of 1940 for all interest rate levels and each year, entropy measure is lower for females than males, due to the higher expectations of life for females than males in each year because females tend to outlive males and have lower mortality rates, at all ages. Also the female regression levels of the Construction of Turkish Life and Annuity Tables Project are lesser than men.

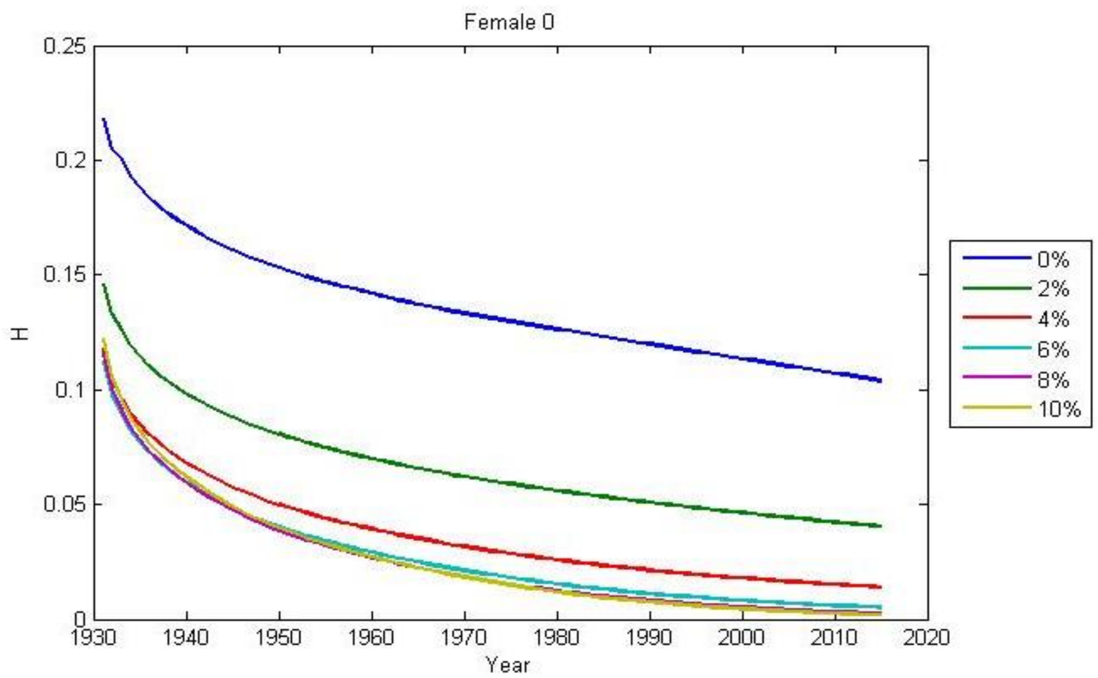


Fig. 3.2. Entropy values for Turkish life tables at different rates of interest for females at age 0.

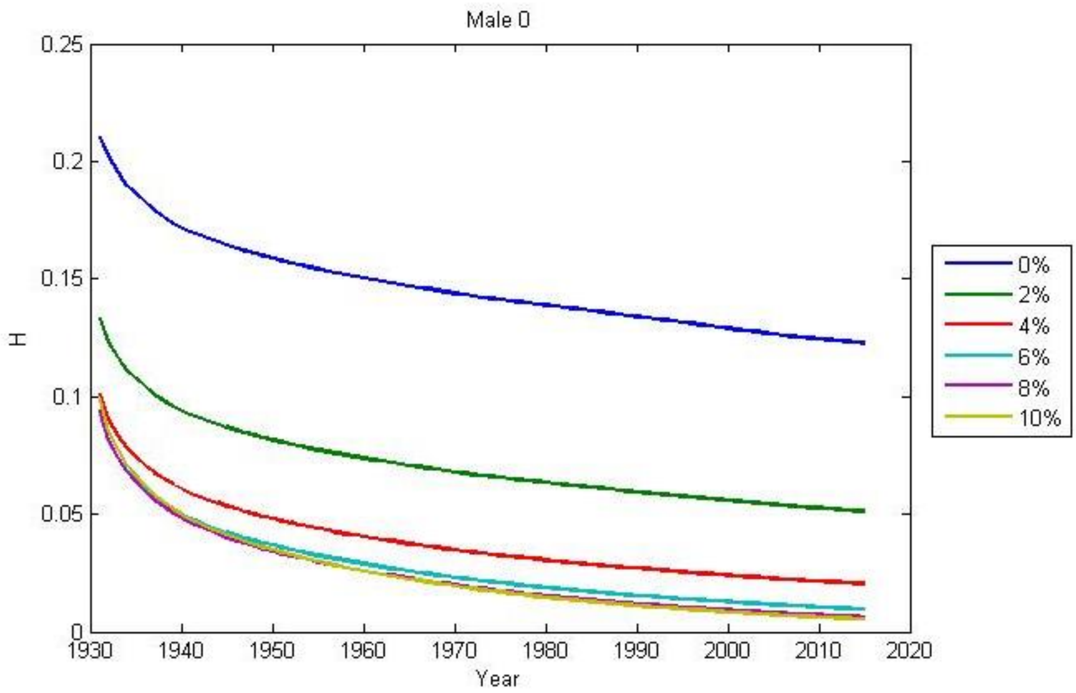


Fig. 3.3. Entropy values for Turkish life tables at different rates of interest for males at age 0.

It can also be seen from the tables 3.2. and 3.3. that the changes in the values of $H_0(\delta)$ fall into two distinct periods, for both males and females. Approximately before 1950s, when the levels of mortality were high, any change in the force of mortality has a stronger decreasing impact on the value of the entropy measure. However, this effect lessens with the increase of the interest rate. Hence, at high interest rates, the curve is more horizontal than the lower interest rates. The same applies in later years, when mortality is at a lower level. However in later years, any change in the mortality does not distinctively affects the entropy measure.

It can be explained by approximately dividing the curve into two sections. One on the left is where the mortality level is high and one on the right is where the mortality level is low. In high mortality part, the steeper slope reflects the fact that the value of $H_0(\delta)$ is more susceptible, and changes in the force of mortality will have a strong effect on the value of the entropy measure. However, for the low mortality part of the table, the more horizontal slope of the curve shows that the $H_0(\delta)$ is insusceptible and changes in the force of mortality do not have a great effect on the value of $H_0(\delta)$.

3.2. Entropy Values at Age 60 and 70

The primary area of interest is the effect of any change of the force of mortality on the cost of a life annuity at retiree ages with different rates of interest. In this section the values for $H_x(\delta)$ at ages 60 and 70 are computed for both genders. As in Section 3.1, the same data set and interest rates are used. Figures 3.4. and 3.5. show the values of the entropy measure at age 60, $H_{60}(\delta)$, for females and males respectively. Also, tables of these values are contained in the Appendix 3 and 4.

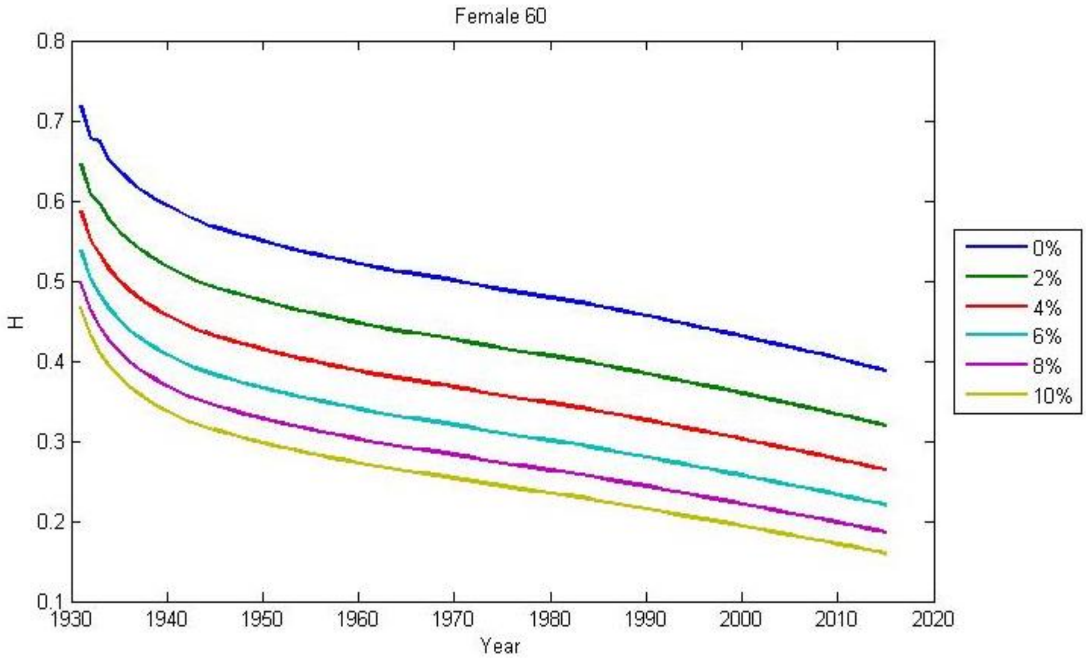


Fig. 3.4. Entropy values for Turkish life tables
at different rates of interest for females at age 60.

Because the mortality improvements (e.g. medical advances) are mostly in the older ages, for females it can be seen from the Tables 3.2. and 3.4. that the values of $H_{60}(\delta)$ are higher than the values of $H_0(\delta)$, in Section 3.1. This again shows the fact that greater effect of changes in the force of mortality on the expectation of life and the cost of a life annuity is in the higher age group (where mortality rates are much higher compared to younger ages).

At any year, the higher the interest rate the lower is the value of $H_{60}(\delta)$ and the effect of changes in the force of mortality on the cost of life annuity. As expected, at any interest rate the value of $H_{60}(\delta)$ is decreasing while levels of mortality get lower through 1931 to 2015. Hence, for the 2015 life table and $\delta = 0.1$ the values of $H_{60}(\delta)$ are lowest.

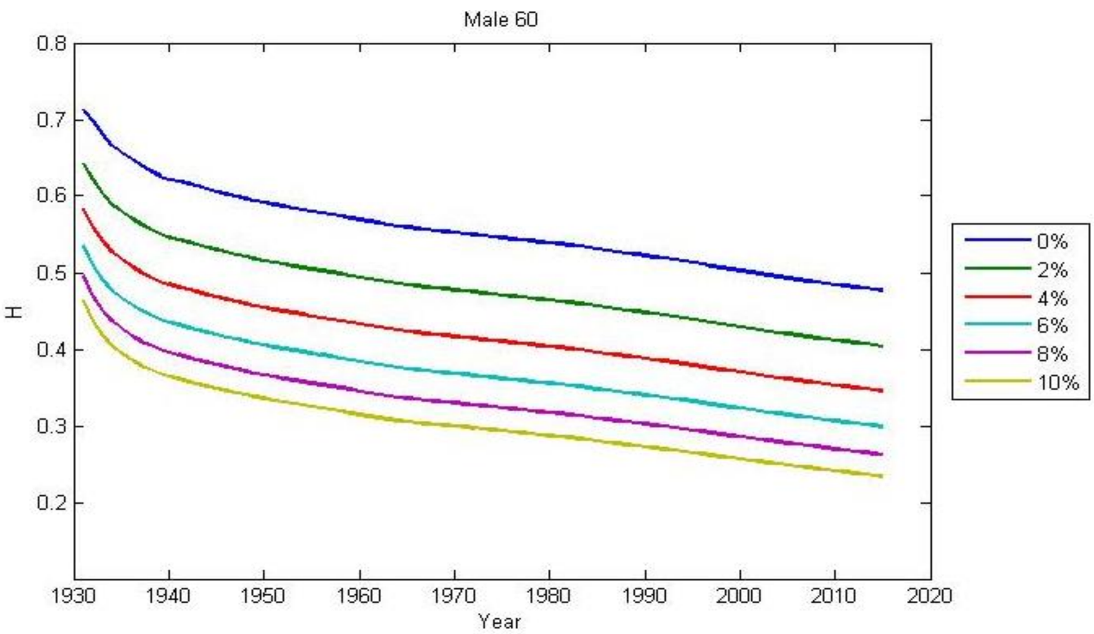


Fig. 3.5. Entropy values for Turkish life tables at different rates of interest for males at age 60.

Similar characteristics are valid in the entropy values for males. Expectedly, the values of $H_{60}(\delta)$ for males are higher than the values of $H_0(\delta)$ for males, and after the year 1931 they are also higher than the values of $H_{60}(\delta)$ for females. Also, the higher the interest rates the lower is the effect of changes in the force of mortality on the cost of life annuity, thus the lower are the values of $H_{60}(\delta)$. We expect the value of $H_{60}(\delta)$ to decrease when levels of mortality are lower so that, through 1931 to 2015 the values of $H_{60}(\delta)$ are on the decrease at all rates of interest.

Figures 3.6. and 3.7 show the values of $H_{70}(\delta)$ for both females and males respectively. As expected, the values of $H_{70}(\delta)$ are higher than the values of $H_{60}(\delta)$ and $H_0(\delta)$ at all interest rates. Again after the year 1931, for the all interest rate levels and each year, the value of entropy measure is lower for females than males due to higher expectations of life for females than males.

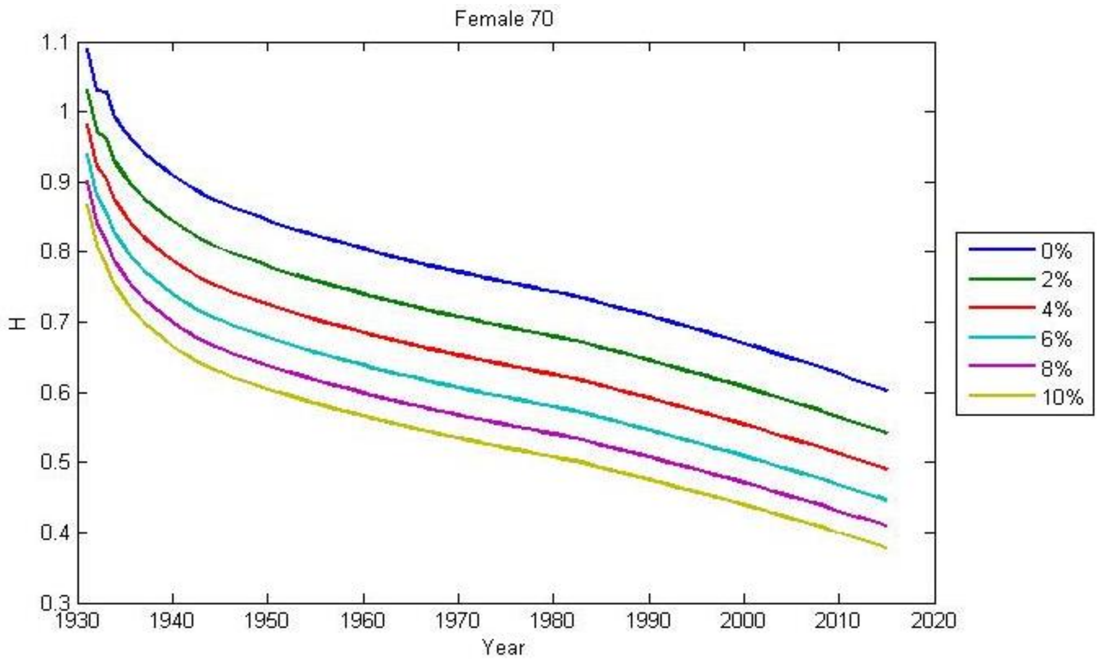


Fig. 3.6. Entropy values for Turkish life tables at different rates of interest for females at age 70.

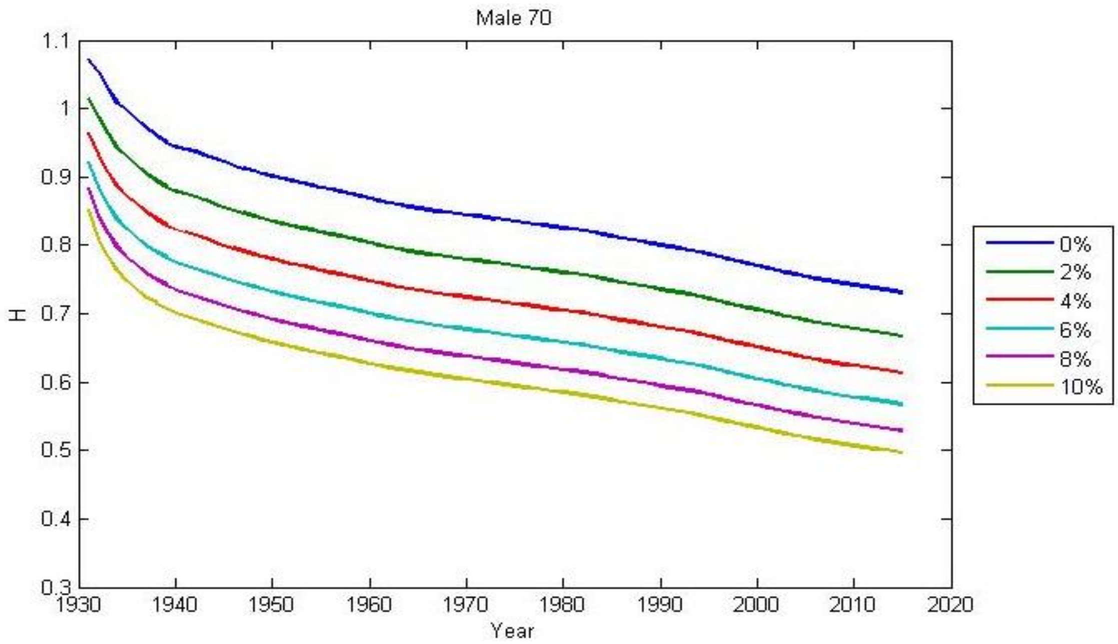


Fig. 3.7. Entropy values for Turkish life tables at different rates of interest for males at age 70.

4. ENTROPY VALUES AND ITS BEHAVIOR USING A MATHEMATICAL MODEL

If the progress against mortality will continue, what will happen to entropy measure and annuity costs? Some insight into these questions can be gained by a simple mathematical mortality model.

In this section of the thesis, the value of entropy measure, $H_x(\delta)$, is calculated with an assumption that mortality is following a chosen mathematical model considering the data from Construction of Turkish Life and Annuity Tables Project as the base mortality tables for 2000 and 2015. In this manner, mortality does not depend completely on observed data and hereby a cohort outlook is adopted rather than a period life table outlook.

By using the mathematical model assumption, it allows for a better examination of the different characteristics of entropy and how the value changes with different factors of a mathematical model.

Section 4.2 presents the entropy values under the assumption that force of mortality is following the Gompertz model. Also in this model, improvements in mortality rates over the years are allowed with a reduction factor. For the ease of calculation, the reduction factor is assumed to depend only on time.

4.1. Gompertz Law

Gompertz [21] suggested that a geometric progression pervades in mortality after a certain age. The mortality rates, particularly for ages over 30, suitably fits the Gompertz model. The model he proposed is often used to explain the distribution of adult lifespans by demographers and actuaries.

There are other and newer models for modelling particular features of mortality rates, but the Gompertz model has advantages over them for users, such as implementation simplicities and familiarity with formula.

The model suggests that

$$\mu_x = \exp(b + cx)$$

Where b denotes the level of the force of mortality at age 0 (location parameter), c is the rate of aging (shape parameter).

Considering the decreasing rates of mortality, a simple exponential reduction factor for the force of mortality, dependent only on the time past after the base year, ($RF(t) = \exp[-\alpha t]$) is used, so that

$$\mu_{x+u}^{t} = \mu_{x+u}^{o} RF(t) \tag{4.1}$$

Where μ_{x+u}^{o} is the mortality rate in the chosen base year at age $x+u$ and μ_{x+u}^{t} is the mortality rate in the tth year after the chosen base year at age $x+u$.

For these reasons we use the Gompertz model to demonstrate the application of entropy measure to cost of annuities. Therefore, annuity costs for different ages and time can be studied.

The behavior of the expectation of life, $e_x^0(t)$, and life annuity values as functions of time can be explored. As mentioned in Vaupel [16], it is straightforward to demonstrate that

$$\frac{\partial}{\partial t} e_x^0(t) \approx \frac{\alpha}{c}\left[1 - \mu_x^t e_x^0(t)\right] \tag{4.2}$$

So that, in the long run,

$$\frac{\partial}{\partial t} e_x^0(t) \approx \frac{\alpha}{c}$$

and is roughly constant. Thus, the rate of mortality change over time in the expectation of life at the end reaches a stable level (the same applies to annuity values). This is one of the driving forces behind the changes that have been demonstrated in $H_x(\delta)$ from the historical perspective of the Turkish life table calculations depicted in Figures 3.4., 3.5., 3.6. and 3.7.

4.2. Entropy Values Using Gompertz Law

The Gompertz law [21] with parameters b and c with a familiar exponential function in the form,

$$\mu_x = \exp(b + cx)$$

is used.

Hereby, the force of mortality of the base table (time 0) at age $x+u$ can be expressed as

$$\mu_{x+u}^0 = \exp[b + c(x+u)].$$

As in section 3.3, a reduction factor is used which depends only on time t for simplicity, in the form $\exp[-\alpha t]$, $\mu_{x+u}^t = \mu_{x+u}^0 \exp[-\alpha t]$. Hence, the probability of a person at age x lives to be $x+t$ is

$$
\begin{aligned}
{}_t p_x^* &= \exp\left[-\int_0^t \mu_{x+u}^u du\right] \\
&= \exp\left[-\int_0^t \mu_{x+u}^0 RF(u) du\right] \\
&= \exp\left[-\int_0^t \exp[b + c(x+u)] \exp[-\alpha u] du\right] \\
&= \exp\left[-\int_0^t \exp[b + c(x+u) - \alpha u] du\right] \\
&= \exp\left[-\mu_x^0 \left(\frac{e^{(c-\alpha)t} - 1}{c - \alpha}\right)\right] \qquad (4.3)
\end{aligned}
$$

Here, α is a bounded to be less than c, so the probability of survival in Equation (4.3) will be less than 1.

Entropy is now calculated on a cohort basis with the use of a reduction factor and it is a function of the parameters b and c, α and δ. Cohort life tables are preferable to period life tables at projecting a population into the future when mortality rate is expected to change over time, and for analyzing the general trends in mortality.

Hereafter, for the notation of the entropy measure, $H_x(b,c,\alpha,\delta)$ is used.

$$H_x(b,c,\alpha,\delta)=\frac{-\int_0^{\infty}\ln({}_tp_x^*)\,{}_tp_x^*e^{-\delta t}dt}{\int_0^{\infty}{}_tp_x^*e^{-\delta t}dt}$$

$$=\frac{\mu_x^0}{c-\alpha}\left[\frac{\int_0^t \exp\left(-\frac{\mu_x^0}{c-\alpha}\left(e^{(c-\alpha)t}-1\right)\right)e^{(c-\alpha-\delta)t}dt}{\int_0^t \exp\left(-\frac{\mu_x^0}{c-\alpha}\left(e^{(c-\alpha)t}-1\right)\right)e^{-\delta t}dt}-1\right] \tag{4.4}$$

One beneficial side of using a mathematical model for the force of mortality is that it enables to see the effects of its parameters in order to test their effects on $H_x(b,c,\alpha,\delta)$. For example; the effects of the base table used in computation (c), the assumed level of mortality improvement (α) and the interest rate (δ), can be seen on $H_x(b,c,\alpha,\delta)$. Hence, it helps to understand the behavior of the entropy measure.

Integrals in Equation (4.4.) can be written as the sums of one-year integrals using the same numerical approach and the mean value theorem for integrals in Chapter 3.

$H_x(b,c,\alpha,\delta)$ value has been calculated using data from the Construction of Turkish Life and Annuity Tables Project [17] for females and males at ages 0, 60 and 70. The base mortality tables used in calculations are the 2000 and 2015 mortality tables.

The values of μ_x^0, c, α and δ are needed to calculate $H_x(b,c,\alpha,\delta)$ values from the Equation (4.4). The values for μ_0^0, μ_{60}^0 and μ_{70}^0 are taken directly from the 2000 and 2015 base mortality tables. After that, Gompertz mortality model has been fitted to these data using MATLAB (R2013a) in order to obtain estimated c values by a simple regression approach. Note that assuming a lower c means that mortality level of the base table is at lower level. Table 4.1 and 4.2 shows the mortality rates and estimated c values for ages 0, 60 and 70, for the base tables of 2000 and 2015 respectively.

Table 4.1. 2000 base table mortality rates and estimated c values.

	Female		Male	
Age	μ_x^o	c	μ_x^o	c
0	0,014789	0,0741	0,026659	0,0664
60	0,011753	0,1037	0,020131	0,0924
70	0,037185	0,0996	0,052360	0,0910

Table 4.2. 2015 base table mortality rates and estimated c values.

	Female		Male	
Age	μ_x^o	c	μ_x^o	c
0	0,006029	0,0869	0,017054	0.0722
60	0,007890	0,1112	0,017071	0,0953
70	0,028844	0,1050	0,046953	0,0932

Mortality rates of the base year 2000 and Gompertz model's mortality rates are compared in Figure 4.1. After that mortality rates of the base year 2015 and Gompertz model's mortality rates are compared in Figure 4.2, which gives the similar results as in mortality rates of the year 2000. As explained in Section 4.1., Gompertz model is not a good fit for ages smaller than 30. This can be seen on the Figure 4.1 and 4.2 that Gompertz model is not a good fit for age 0, but for ages 60 and 70 mortality rate estimations are consistent with the data. For this reason age 0 is not taken into consideration in next sections.

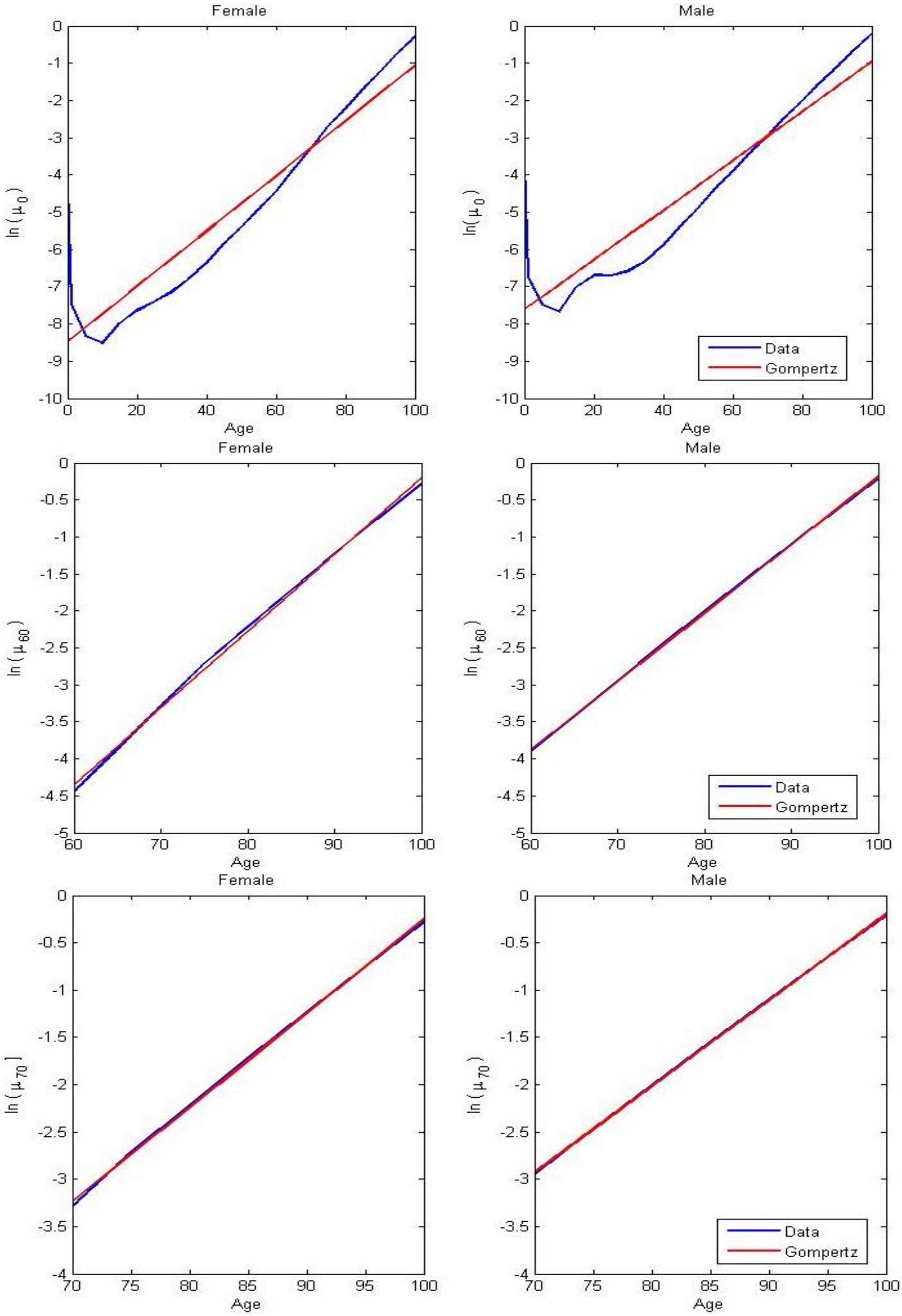


Fig. 4.1. Data and Gompertz model comparisons
of mortality rates of 2000, in log scale at ages 0, 60 and 70.

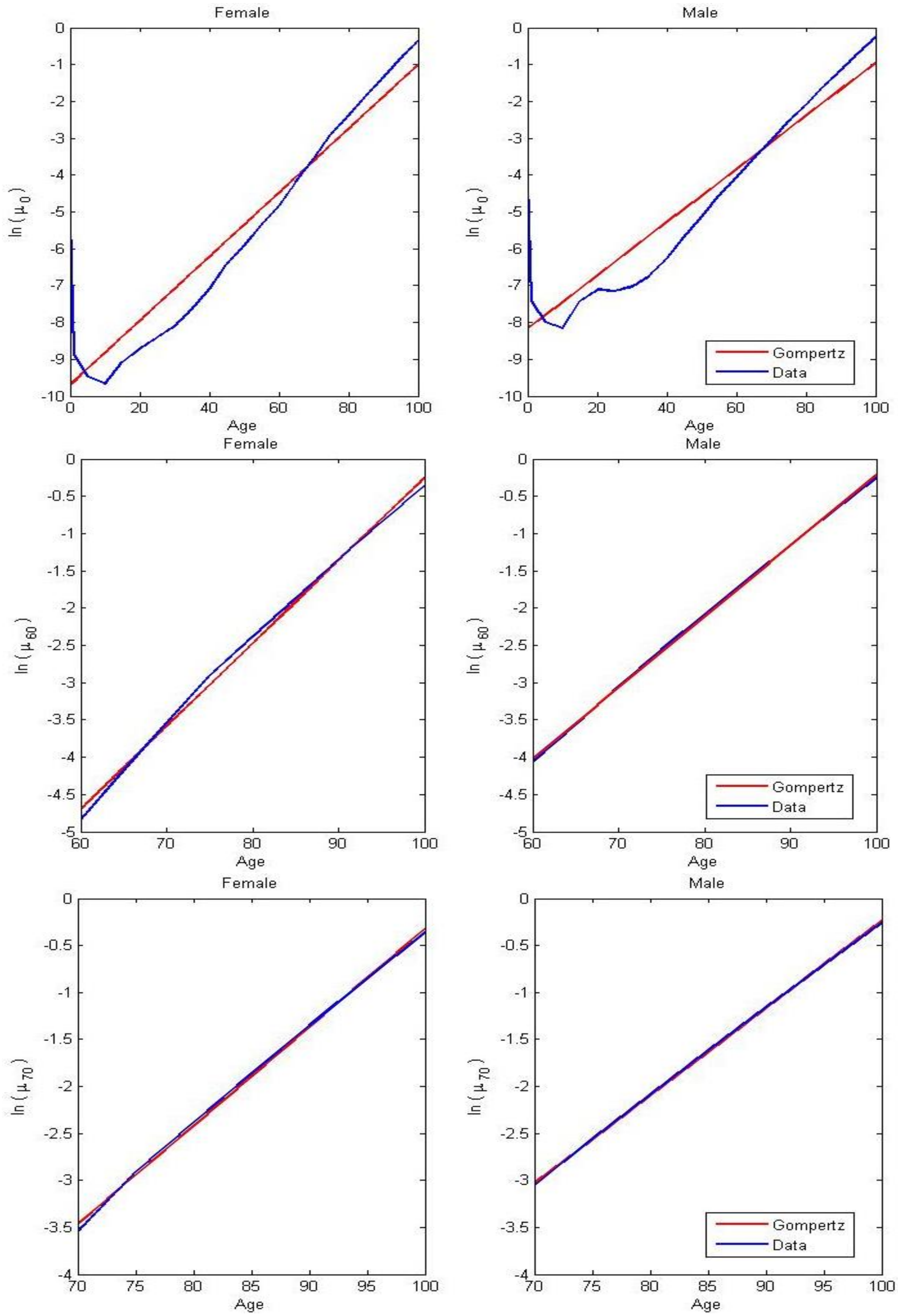


Fig. 4.2. Data and Gompertz model comparisons of mortality rates of 2015, in log scale at ages 0, 60 and 70.

Afterwards, the sensitivity of $H_x(b,c,\alpha,\delta)$ to changes in the values of α and δ are tested. Different levels of mortality improvement are considered by allowing α to vary between -0.1 and 0.1. Note that, positive values of α stands for improving mortality rates and negative values of α stands for deteriorating mortality rates over time. Also, different rates of interest are considered from 0% to 10%.

As can be seen from Figures 4.3 and 4.4 the value of entropy, $H_x(b,c,\alpha,\delta)$, decreases in all four cases as the interest rate increases. This effect arises from the decreasing effect of mortality improvement on the cost of annuities when the interest rate is higher. As Figures 4.3 and 4.4 show when the interest rate is lower the value of entropy is at its highest and directly proportional with the mortality rate improvement level. 2.19% increases with the α in direct proportion up until a level (breaking point of H) which thereafter, anymore improvement in mortality rate decreases H. Hence mortality rate improvements start to lose their effect on the cost of a life annuity. Despite that, when the rate of interest is high, $H_x(b,c,\alpha,\delta)$ decreases almost continuously and there is no peak point as mortality improves.

H values and the breaking points are consistently smaller for women than man. This reflects the relative difference in favor of women in the survival probabilities, annuity costs and life expectations at various ages. Within each gender, with the age increase from 60 to 70, H values and the breaking point for H increase too.

As a summary with $\alpha = 0.05$ and $\delta = 0.04$ changes in the annuity values are found with the Equation (2.10) by simply multiplying minus the entropy measure, H, with the α. For the base year 2000 the change in the cost of annuity for males; at age 60 is 1.68% and at age 70 is 2.59%. For the base year 2015 these values change to 1.54% at age 60 and 2.46% at age 70. For the base year 2000 the change in the cost of annuity for females; at age 60 is 1.26% and at age 70 is 2.19%. For the base year 2015 these values change to 1.01% at age 60 and 1.93% at age 70.

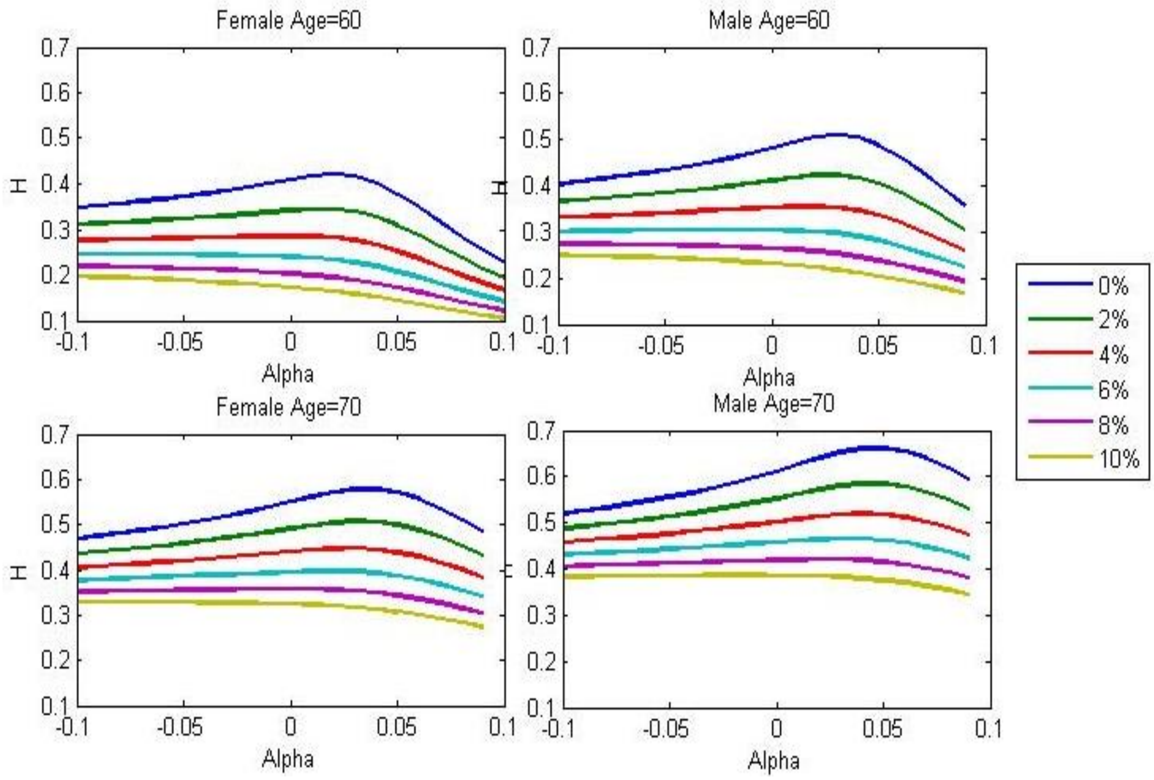


Fig. 4.3. Calculated values of $H_x(b,c,\alpha,\delta)$ in year 2000 for different values of α and δ for ages 60 and 70.

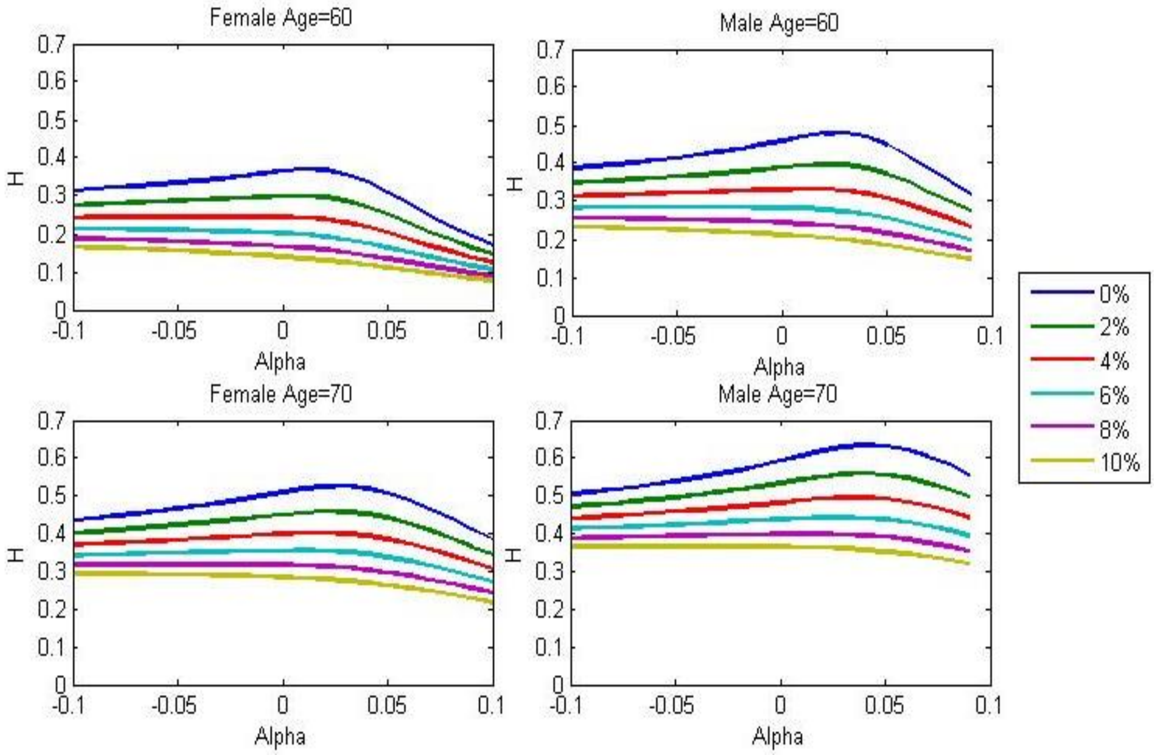


Fig. 4.4. Calculated values of $H_x(b,c,\alpha,\delta)$ in year 2015 for different values of α and δ for ages 60 and 70.

Additionally, effect of the base mortality table on $H_x(b,c,\alpha,\delta)$ can be tested by different values of c. To exemplify, the values of c between 0.09 and 0.115 is taken into consideration. Figures 4.5 and 4.6 shows results for $\delta = 0\%$ and Figure 4.7 and 4.8 shows results for $\delta = 4\%$.

Figures 4.5 and 4.6 shows entropy values for the base life tables 2000 and 2015 with the interest rate equal to zero. $H_x(b,c,\alpha,\delta)$, as a function of α, is peaked at one point (breaking point of H). When the reduction factor is greater than one (when α is negative), the values of $H_x(b,c,\alpha,\delta)$ are higher in lower mortality level ($c = 0.09$) than the ones in higher mortality levels ($c = 0.115$). On the contrary, when the reduction factor is less than one (when α is positive), this relationship is reversed with improving mortality rates.

Initially this verifies the conclusion that annuity cost with a high c value (high mortality group) is less sensitive to changes in the mortality rate than with a low c value, when the mortality rate is high. At low levels of mortality this is reversed that a high c value is more sensitive to changes in the mortality rate than a low c value.

As expected, when the interest rate increased to 4%, shown in Figure 4.7 for base year 2000 and Figure 4.8 for base year 2015, the effect of changes in the force of mortality on cost of life annuity is less distinctive and it results to lower values of $H_x(b,c,\alpha,\delta)$. This is based on the inverse proportion between interest rate and entropy measure, as in previous figures.

Again, H values and the breaking points are consistently smaller for women than man. This reflects the relative difference in favor of women in the survival probabilities, annuity costs and life expectations at various ages. Within each gender, with the age increase from 60 to 70, H values and the breaking point for H increase too.

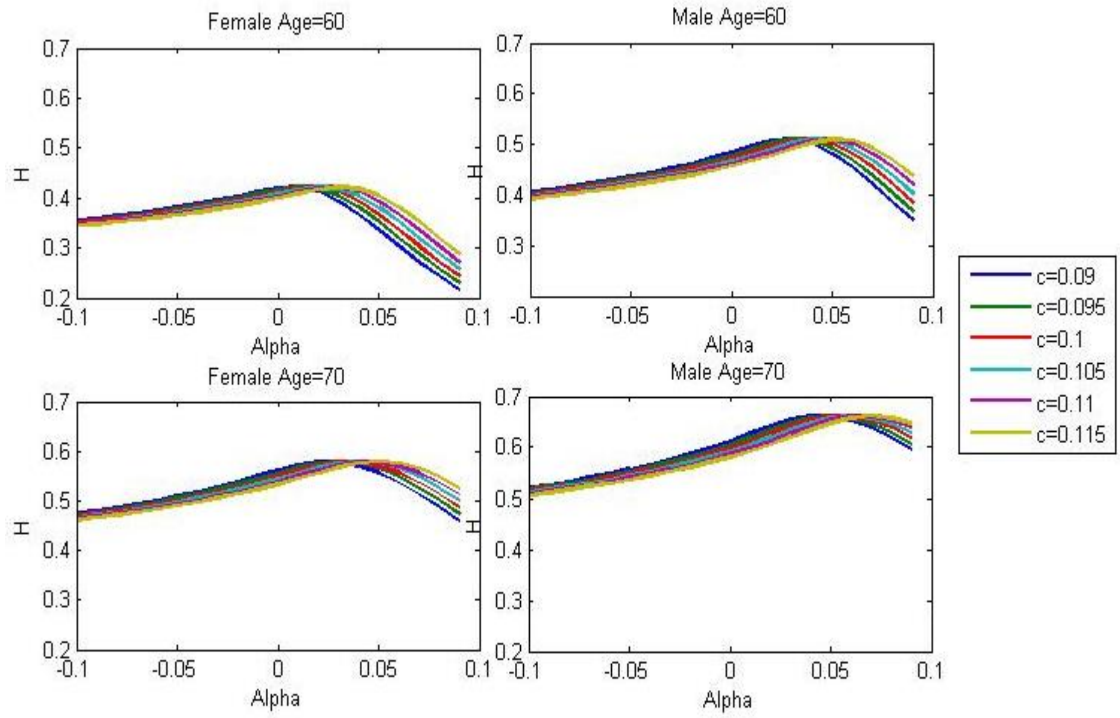


Fig. 4.5. H values for Females and Males at ages 60 and 70 in base year 2000 with different c values and $\delta = 0\%$.

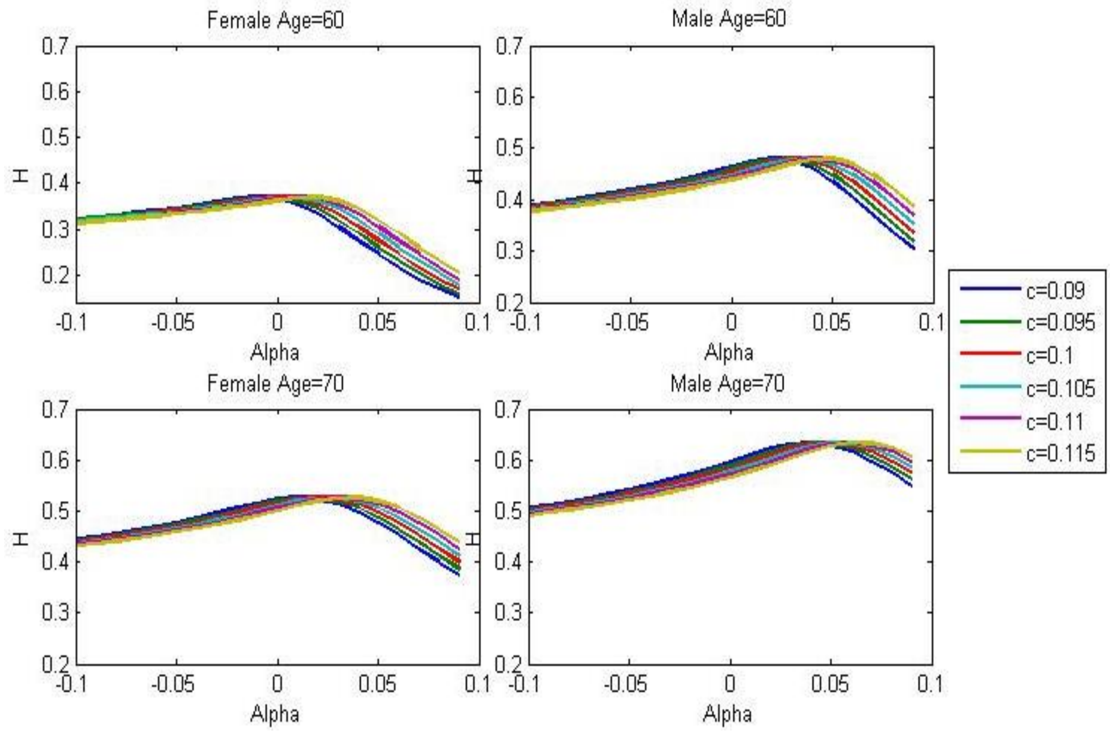


Fig. 4.6. H values for Females and Males at ages 60 and 70 in base year 2015 with different c values and $\delta = 0\%$.

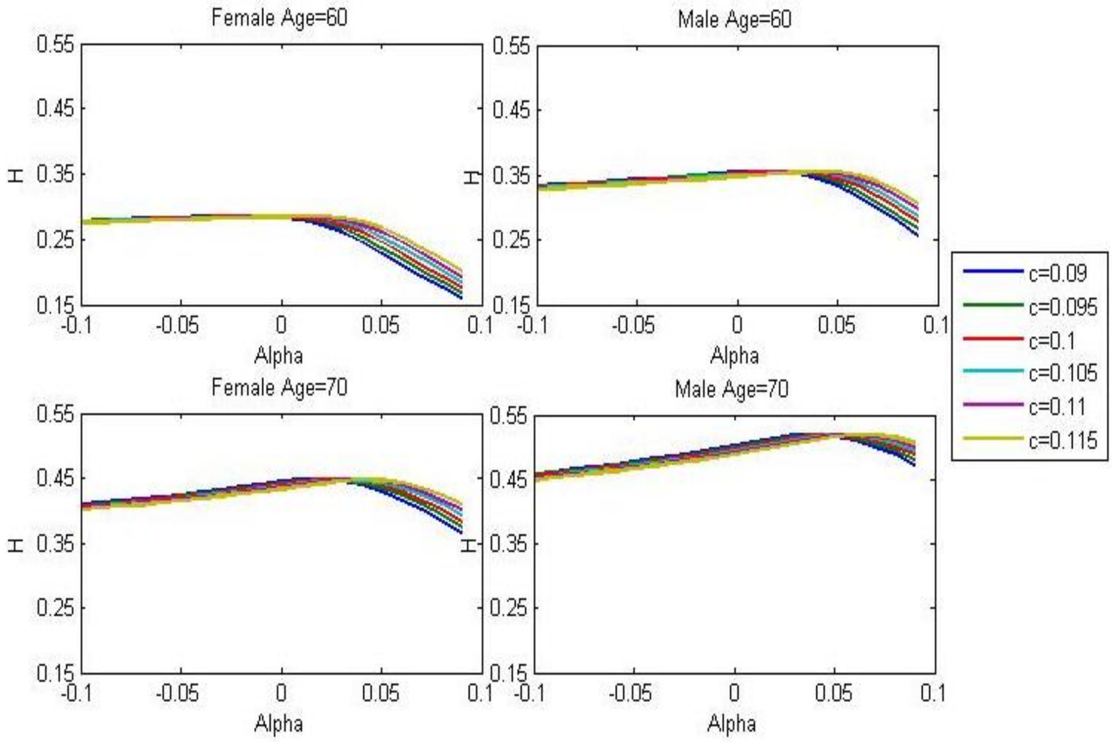


Fig. 4.7. H values for Females and Males at ages 60 and 70 in base year 2000 with different c values and $\delta = 4\%$.

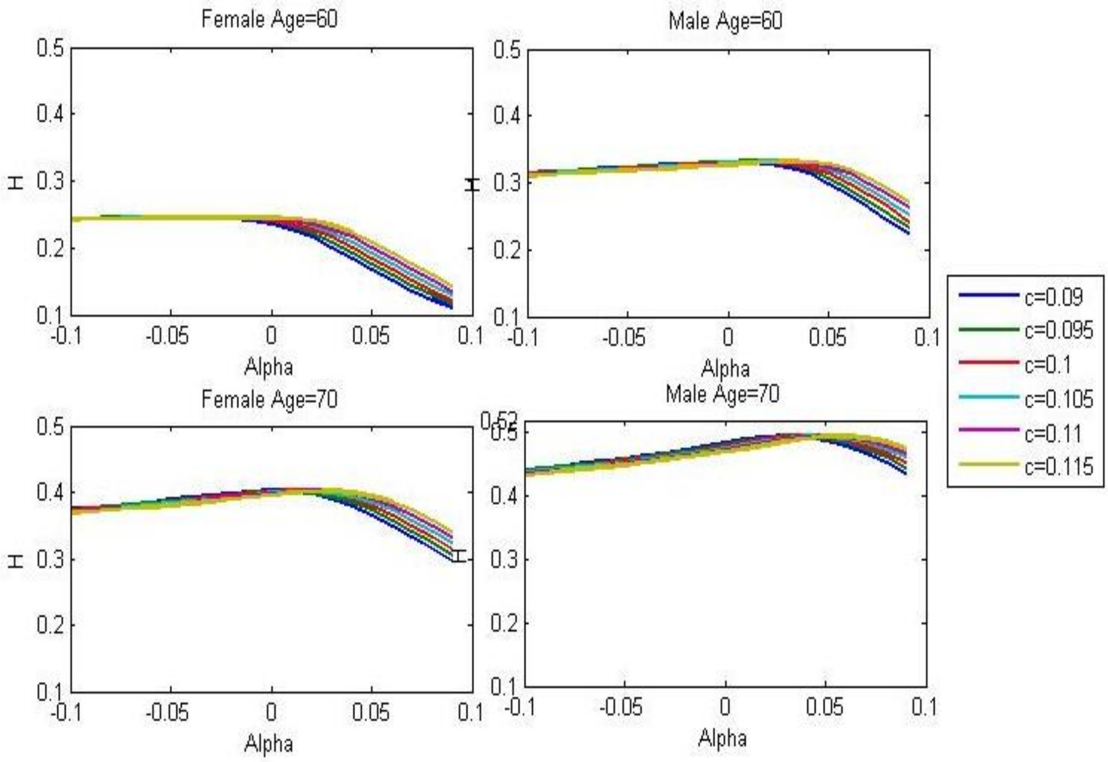


Fig. 4.8. H values for Females and Males at ages 60 and 70 in base year 2015 with different c values and $\delta = 4\%$.

5. RESULTS AND CONCLUSIONS

This persistent decrease in mortality rates was unexpected and it is producing a number of new issues and challenges at multiple levels. As a result of this unexpected decrease they have often been underestimated by annuity and pension providers. Mortality improvements and rising longevity for post-retirement ages have a significant financial effect on the annuity and pension plans. Because of the tax-paid structure, the deficiencies of public finance are aggravated by mortality improvements.

In several countries, defined benefit pension plans have continued to be replaced with defined contribution plans, which will most likely lead to the same result.

Furthermore a high number of governments including Turkey are planning to increase retirement ages by at least 2 to 5 years to be able to take into account the changing dynamics of mortality improvements, and the impacts of ageing populations upon the financing of pensions.

In this paper, the entropy measure has been examined in different scenarios to measure the effect of any changes in the force of mortality on the cost of life annuity. The entropy measure has been calculated with different mortality and interest rate assumptions. Also, it has been calculated over the whole age range (from age 0 to 100) and at older ages (from ages 60 and 70 to 100).

Computation results show that entropy is almost insensitive to gender of the person but it is highly sensitive to age, base year, interest rate and mortality changes. We see that cost of annuity and value of entropy measure have inverse proportion with interest rate and direct proportion with age.

There is a special case with the use of entropy measure to measure the effect of any changes in the force of mortality on the cost of life annuity: If the survival curve is already very rectangular shaped (when entropy is very close to zero), an expansion of the survival curve will have a great impact on the cost of life annuities but this would not be caught by the entropy measure.

For all examined cases, the lower the force of interest, the higher is the value of entropy. This means that there is a higher effect of mortality risk on the present value of annuity payments. This reflects the importance of mortality risk in the context of life annuity portfolios in particular with a low interest environment.

At very high and low levels of mortality, the numerical results suggest that the effect of mortality changes on the cost of life annuity is very small. This reflects the fact that if the mortality is already very high or very low, any positive or negative change in the force of mortality is less likely to have a marked effect on the cost of life annuity. This implies that even if mortality continues to improve in the future, it will reach a level beyond which any more improvements would not markedly affect the annuity costs.

For the future work, it would be useful to investigate the entropy measure with different models of mortality such as Lee Carter mortality forecasting model and Cairns-Blake-Dowd (CBD) stochastic mortality model. In addition, mortality model could be tested with distributional assumptions for its parameters. Also a stochastic model for the interest rate could be incorporated to see its effect on cost of life annuity.

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APPENDIX

Appendix 1: Construction of Turkish Life and Annuity Tables Project's female and male regression levels.

Appendix 2: Entropy values for Turkish life tables at different rates of interest for females and males at age 0.

Appendix 3: Entropy values for Turkish life tables at different rates of interest for females and males at age 60.

Appendix 4: Entropy values for Turkish life tables at different rates of interest for females and males at age 70.

Appendix 5: Entropy values for different mortality levels and rate of interests for the base years 2000 and 2015 at ages 60 and 70 for females and males.

Appendix 1: Construction of Turkish Life and Annuity Tables Project's female and male regression levels.

Life Table	Female Regression Level	Male Regression Level	Life Table	Female Regression Level	Male Regression Level	Life Table	Female Regression Level	Male Regression Level
1930	0,00	0,00	**1969**	19,00	19,60	**2008**	23,97	23,12
1931	5,57	8,20	**1970**	19,16	19,72	**2009**	24,07	23,19
1932	7,03	9,67	**1971**	19,32	19,84	**2010**	24,17	23,26
1933	8,05	10,65	**1972**	19,48	19,95	**2011**	24,27	23,33
1934	8,86	11,40	**1973**	19,63	20,06	**2012**	24,37	23,39
1935	9,55	12,02	**1974**	19,79	20,17	**2013**	24,47	23,46
1936	10,15	12,56	**1975**	19,93	20,28	**2014**	24,57	23,53
1937	10,69	13,03	**1976**	20,08	20,39	**2015**	24,67	23,59
1938	11,18	13,45	**1977**	20,23	20,49			
1939	11,63	13,83	**1978**	20,37	20,59			
1940	12,05	14,18	**1979**	20,51	20,70			
1941	12,44	14,51	**1980**	20,65	20,80			
1942	12,81	14,81	**1981**	20,79	20,89			
1943	13,15	15,09	**1982**	20,92	20,99			
1944	13,48	15,36	**1983**	21,06	21,09			
1945	13,80	15,62	**1984**	21,19	21,18			
1946	14,10	15,86	**1985**	21,32	21,27			
1947	14,39	16,09	**1986**	21,45	21,36			
1948	14,67	16,31	**1987**	21,58	21,45			
1949	14,94	16,52	**1988**	21,70	21,54			
1950	15,19	16,72	**1989**	21,83	21,63			
1951	15,44	16,92	**1990**	21,95	21,72			
1952	15,69	17,11	**1991**	22,07	21,80			
1953	15,92	17,29	**1992**	22,19	21,89			
1954	16,15	17,46	**1993**	22,31	21,97			
1955	16,37	17,63	**1994**	22,43	22,05			
1956	16,59	17,80	**1995**	22,55	22,14			
1957	16,80	17,96	**1996**	22,66	22,22			
1958	17,01	18,12	**1997**	22,78	22,30			
1959	17,21	18,27	**1998**	22,89	22,37			
1960	17,40	18,42	**1999**	23,00	22,45			
1961	17,60	18,56	**2000**	23,11	22,53			
1962	17,78	18,70	**2001**	23,22	22,61			
1963	17,97	18,84	**2002**	23,33	22,68			
1964	18,15	18,97	**2003**	23,44	22,76			
1965	18,33	19,10	**2004**	23,55	22,83			
1966	18,50	19,23	**2005**	23,65	22,90			
1967	18,67	19,36	**2006**	23,76	22,97			
1968	18,84	19,48	**2007**	23,86	23,05			

Appendix 2: Entropy values for different Turkish life tables at different rates of interest for females and males at age 0.

Female Age=0	**Interest Rate**					
Life Table	**%0**	**%2**	**%4**	**%6**	**%8**	**%10**
1931	0,218407	0,146166	0,11769	0,112047	0,115556	0,122012
1932	0,204658	0,132944	0,104239	0,097862	0,100369	0,105737
1933	0,200603	0,125872	0,096036	0,088963	0,090793	0,095473
1934	0,192701	0,119239	0,089613	0,082281	0,083669	0,087856
1935	0,187732	0,114317	0,084573	0,076965	0,077987	0,081781
1936	0,183656	0,110185	0,080312	0,072465	0,073176	0,076638
1937	0,179878	0,106582	0,076666	0,068631	0,069082	0,072263
1938	0,176623	0,10341	0,073435	0,065229	0,065449	0,068383
1939	0,173916	0,100654	0,070591	0,062226	0,062241	0,064955
1940	0,17137	0,098099	0,067965	0,059456	0,059283	0,061796
1941	0,168911	0,095772	0,065614	0,056981	0,056638	0,05897
1942	0,166588	0,093581	0,063402	0,054654	0,054152	0,056313
1943	0,164482	0,091568	0,061338	0,052456	0,051784	0,053772
1944	0,162593	0,089728	0,05941	0,05037	0,049514	0,05132
1945	0,16079	0,087974	0,057574	0,048382	0,047353	0,048987
1946	0,159104	0,086327	0,055858	0,046537	0,045357	0,04684
1947	0,157562	0,084807	0,054289	0,044874	0,043578	0,044943
1948	0,156077	0,083345	0,052781	0,043276	0,041869	0,04312
1949	0,154644	0,081936	0,051329	0,041737	0,040224	0,041366
1950	0,153225	0,080604	0,049985	0,040324	0,038719	0,039763
1951	0,151841	0,079325	0,048703	0,038981	0,037289	0,038242
1952	0,150499	0,078086	0,047462	0,03768	0,035906	0,03677
1953	0,149196	0,076884	0,046258	0,03642	0,034565	0,035344
1954	0,148042	0,075776	0,045139	0,035249	0,033321	0,034021
1955	0,146974	0,074728	0,044076	0,034137	0,032141	0,032767
1956	0,145931	0,073707	0,043041	0,033054	0,030992	0,031547
1957	0,144911	0,072712	0,042033	0,032	0,029873	0,030359
1958	0,143912	0,07174	0,041051	0,030973	0,028785	0,029203
1959	0,142883	0,070809	0,040138	0,03003	0,027788	0,028146
1960	0,141876	0,069899	0,039246	0,029109	0,026815	0,027114
1961	0,14089	0,069009	0,038374	0,028208	0,025864	0,026106
1962	0,139923	0,068137	0,03752	0,027327	0,024933	0,025119
1963	0,138975	0,067283	0,036685	0,026465	0,024022	0,024153
1964	0,138084	0,066471	0,035896	0,025657	0,023174	0,023256
1965	0,137218	0,06568	0,035128	0,024873	0,022351	0,022386
1966	0,136365	0,064902	0,034374	0,024103	0,021544	0,021533
1967	0,135527	0,064139	0,033635	0,023349	0,020752	0,020696
1968	0,134702	0,063389	0,032909	0,022608	0,019975	0,019874
1969	0,133891	0,062652	0,032196	0,021881	0,019212	0,019068
1970	0,133162	0,061992	0,031551	0,021215	0,018509	0,018323

1971	0,132443	0,061343	0,030916	0,020561	0,017818	0,017591
1972	0,131733	0,060703	0,030292	0,019917	0,017139	0,016871
1973	0,131033	0,060073	0,029677	0,019283	0,016471	0,016163
1974	0,130343	0,059452	0,029072	0,01866	0,015813	0,015466
1975	0,129661	0,05884	0,028476	0,018046	0,015165	0,01478
1976	0,128986	0,058245	0,027907	0,017469	0,014562	0,014143
1977	0,128317	0,057664	0,027362	0,016922	0,013994	0,013547
1978	0,127657	0,05709	0,026824	0,016384	0,013435	0,012959
1979	0,127004	0,056524	0,026293	0,015853	0,012883	0,012379
1980	0,126358	0,055966	0,02577	0,015329	0,012339	0,011808
1981	0,12572	0,055414	0,025254	0,014812	0,011803	0,011244
1982	0,125089	0,054869	0,024744	0,014302	0,011273	0,010688
1983	0,124439	0,054336	0,024267	0,013838	0,0108	0,010194
1984	0,123761	0,053813	0,023829	0,013433	0,010397	0,00978
1985	0,123087	0,053296	0,023397	0,013033	0,009999	0,00937
1986	0,122418	0,052784	0,02297	0,012638	0,009606	0,008966
1987	0,121754	0,052277	0,022547	0,012248	0,009218	0,008566
1988	0,121094	0,051775	0,022129	0,011861	0,008834	0,008171
1989	0,12044	0,051277	0,021715	0,01148	0,008455	0,007781
1990	0,11979	0,050784	0,021306	0,011102	0,00808	0,007395
1991	0,119142	0,05031	0,020927	0,010762	0,007746	0,007053
1992	0,118496	0,049848	0,020568	0,010447	0,007441	0,006742
1993	0,117853	0,04939	0,020213	0,010135	0,007138	0,006433
1994	0,117213	0,048935	0,019861	0,009826	0,00684	0,006129
1995	0,116575	0,048483	0,019512	0,009521	0,006544	0,005827
1996	0,11594	0,048035	0,019167	0,009219	0,006251	0,005529
1997	0,115308	0,047589	0,018824	0,008919	0,005961	0,005233
1998	0,114678	0,047147	0,018485	0,008623	0,005674	0,004941
1999	0,114051	0,046708	0,018149	0,00833	0,005391	0,004652
2000	0,113422	0,046295	0,017857	0,008094	0,005173	0,004434
2001	0,112793	0,045883	0,017568	0,00786	0,004956	0,004217
2002	0,112164	0,045473	0,017281	0,007628	0,004741	0,004003
2003	0,111536	0,045064	0,016996	0,007398	0,004529	0,00379
2004	0,110908	0,044658	0,016713	0,00717	0,004318	0,00358
2005	0,11028	0,044253	0,016432	0,006944	0,004109	0,003371
2006	0,109653	0,04385	0,016152	0,00672	0,003902	0,003164
2007	0,109027	0,043448	0,015875	0,006498	0,003697	0,002959
2008	0,108402	0,043048	0,015599	0,006277	0,003494	0,002756
2009	0,107775	0,042667	0,015354	0,006094	0,003332	0,002598
2010	0,107146	0,042294	0,015125	0,00593	0,003192	0,002463
2011	0,106513	0,041921	0,014896	0,005768	0,003053	0,002329
2012	0,105879	0,041549	0,014669	0,005606	0,002916	0,002196
2013	0,105242	0,041177	0,014442	0,005446	0,002779	0,002064
2014	0,104604	0,040805	0,014217	0,005286	0,002643	0,001933
2015	0,103963	0,040433	0,013992	0,005128	0,002508	0,001803

Male Age=0	Interest Rate					
Life Table	%0	%2	%4	%6	%8	%10
1931	0,210125	0,133419	0,101218	0,092709	0,094034	0,098783
1932	0,202081	0,1232	0,09016	0,080959	0,081464	0,085313
1933	0,19517	0,116381	0,083248	0,073723	0,073749	0,077056
1934	0,189706	0,111291	0,078184	0,068448	0,068133	0,071048
1935	0,186084	0,107427	0,074179	0,064235	0,063637	0,066239
1936	0,182547	0,10396	0,070626	0,060468	0,059587	0,061884
1937	0,179403	0,100915	0,067517	0,057176	0,056048	0,058081
1938	0,176356	0,098318	0,064972	0,054503	0,05318	0,055001
1939	0,173579	0,095957	0,06266	0,052076	0,050577	0,052206
1940	0,171577	0,094036	0,060705	0,050011	0,048363	0,049832
1941	0,170198	0,092472	0,059041	0,048241	0,046467	0,047802
1942	0,168897	0,09101	0,057489	0,046591	0,044699	0,045909
1943	0,167539	0,089611	0,056042	0,045064	0,043067	0,044164
1944	0,166011	0,088242	0,054696	0,043664	0,041579	0,042574
1945	0,164563	0,086947	0,053422	0,04234	0,04017	0,04107
1946	0,163186	0,085715	0,052212	0,041081	0,038832	0,039642
1947	0,161925	0,084567	0,051079	0,039903	0,03758	0,038307
1948	0,160799	0,083508	0,050025	0,03881	0,03642	0,037072
1949	0,159719	0,082493	0,049016	0,037762	0,035309	0,035888
1950	0,15868	0,081517	0,048047	0,036756	0,034243	0,034751
1951	0,157678	0,080578	0,047114	0,035788	0,033216	0,033658
1952	0,156739	0,079692	0,046236	0,034879	0,032253	0,032634
1953	0,155851	0,078852	0,045403	0,034019	0,031344	0,031667
1954	0,154991	0,078038	0,044598	0,033187	0,030465	0,030732
1955	0,154156	0,07725	0,043818	0,032381	0,029613	0,029826
1956	0,153345	0,076485	0,043062	0,0316	0,028787	0,028948
1957	0,152556	0,075742	0,042327	0,030842	0,027986	0,028097
1958	0,151793	0,075049	0,041645	0,030135	0,027238	0,0273
1959	0,151051	0,074383	0,040992	0,029459	0,02652	0,026537
1960	0,150327	0,073734	0,040356	0,028799	0,025822	0,025793
1961	0,149621	0,073102	0,039735	0,028157	0,025141	0,025068
1962	0,14893	0,072484	0,03913	0,02753	0,024477	0,024361
1963	0,148255	0,07188	0,038539	0,026919	0,023828	0,023672
1964	0,147594	0,07129	0,037962	0,026321	0,023195	0,022998
1965	0,14694	0,0707	0,037396	0,025747	0,022595	0,022362
1966	0,146298	0,070118	0,036842	0,025189	0,022012	0,021747
1967	0,145668	0,069549	0,036299	0,024642	0,021442	0,021144
1968	0,14505	0,06899	0,035768	0,024107	0,020883	0,020554
1969	0,144444	0,068443	0,035247	0,023583	0,020336	0,019976
1970	0,143848	0,067906	0,034736	0,023068	0,0198	0,01941
1971	0,143263	0,067379	0,034235	0,022564	0,019273	0,018854
1972	0,142688	0,066861	0,033744	0,022069	0,018757	0,018308
1973	0,142157	0,066382	0,033287	0,021608	0,018276	0,0178
1974	0,141661	0,065935	0,032859	0,021175	0,017823	0,017322

1975	0,141172	0,065495	0,032439	0,02075	0,017379	0,016851
1976	0,140691	0,065062	0,032025	0,020332	0,016942	0,016389
1977	0,140217	0,064636	0,031619	0,019921	0,016512	0,015935
1978	0,13975	0,064216	0,031219	0,019516	0,016089	0,015488
1979	0,139289	0,063804	0,030825	0,019118	0,015673	0,015048
1980	0,138835	0,063397	0,030437	0,018726	0,015263	0,014615
1981	0,138387	0,062996	0,030055	0,018341	0,01486	0,014189
1982	0,137945	0,062601	0,029679	0,01796	0,014463	0,013769
1983	0,137431	0,06219	0,029322	0,017621	0,01412	0,013413
1984	0,136913	0,061781	0,028971	0,01729	0,013787	0,013068
1985	0,136402	0,061378	0,028625	0,016964	0,013459	0,012728
1986	0,135897	0,060981	0,028284	0,016642	0,013136	0,012392
1987	0,135397	0,060588	0,027947	0,016325	0,012816	0,012061
1988	0,134902	0,060199	0,027614	0,016012	0,012502	0,011735
1989	0,134413	0,059816	0,027286	0,015703	0,012191	0,011413
1990	0,13393	0,059437	0,026962	0,015398	0,011884	0,011095
1991	0,133451	0,059063	0,026642	0,015097	0,011581	0,010781
1992	0,132977	0,058692	0,026326	0,014799	0,011283	0,010471
1993	0,132508	0,058326	0,026013	0,014505	0,010987	0,010165
1994	0,132004	0,057956	0,025713	0,014232	0,010717	0,009888
1995	0,131484	0,057584	0,02542	0,013971	0,010462	0,009627
1996	0,130968	0,057217	0,025131	0,013713	0,01021	0,009369
1997	0,130456	0,056852	0,024844	0,013458	0,009961	0,009114
1998	0,129948	0,056491	0,024561	0,013206	0,009714	0,008862
1999	0,129444	0,056134	0,02428	0,012956	0,009471	0,008613
2000	0,128943	0,055779	0,024003	0,01271	0,00923	0,008366
2001	0,128446	0,055428	0,023728	0,012466	0,008991	0,008123
2002	0,127953	0,05508	0,023456	0,012224	0,008756	0,007881
2003	0,127463	0,054735	0,023187	0,011985	0,008522	0,007643
2004	0,126977	0,054393	0,02292	0,011748	0,008292	0,007407
2005	0,126495	0,054054	0,022656	0,011514	0,008063	0,007173
2006	0,126016	0,053717	0,022394	0,011282	0,007837	0,006942
2007	0,125597	0,053412	0,022156	0,011074	0,007637	0,006739
2008	0,125212	0,053124	0,021931	0,01088	0,007453	0,006554
2009	0,124828	0,052838	0,021708	0,010688	0,00727	0,00637
2010	0,124446	0,052554	0,021487	0,010498	0,007089	0,006187
2011	0,124065	0,052272	0,021268	0,010309	0,00691	0,006007
2012	0,123686	0,051991	0,021051	0,010123	0,006732	0,005828
2013	0,123307	0,051713	0,020835	0,009937	0,006556	0,005651
2014	0,122931	0,051436	0,020621	0,009754	0,006382	0,005475
2015	0,122555	0,051162	0,020409	0,009572	0,006209	0,005301

Appendix 3: Entropy values for different Turkish life tables at different rates of interest for females and males at age 60.

Female Age=60	**Interest Rate**					
Life Table	**%0**	**%2**	**%4**	**%6**	**%8**	**%10**
1931	0,717799	0,646583	0,587351	0,538665	0,498954	0,466693
1932	0,678189	0,607655	0,549094	0,501074	0,462021	0,4304
1933	0,673751	0,59599	0,532906	0,482219	0,441706	0,409376
1934	0,649908	0,574644	0,513293	0,463809	0,424147	0,392439
1935	0,636652	0,561249	0,499887	0,450487	0,410971	0,379441
1936	0,626087	0,550354	0,488843	0,439423	0,399973	0,36856
1937	0,615894	0,540364	0,479038	0,429796	0,39052	0,359275
1938	0,607316	0,531809	0,470543	0,421393	0,38223	0,35111
1939	0,600429	0,524701	0,463327	0,414155	0,375028	0,343979
1940	0,593931	0,51807	0,456642	0,407478	0,368401	0,337429
1941	0,587435	0,511745	0,450464	0,401432	0,362478	0,33162
1942	0,581333	0,505809	0,444669	0,395763	0,356925	0,326176
1943	0,57601	0,500652	0,439649	0,390861	0,35213	0,321476
1944	0,571484	0,496296	0,435425	0,386748	0,348111	0,317541
1945	0,567178	0,492151	0,431409	0,382837	0,344291	0,3138
1946	0,563216	0,488307	0,427662	0,379175	0,340705	0,310283
1947	0,559702	0,484837	0,424238	0,375801	0,337384	0,307016
1948	0,556325	0,481504	0,420951	0,372563	0,334198	0,303882
1949	0,553072	0,478296	0,417789	0,36945	0,331134	0,300869
1950	0,549629	0,474987	0,414585	0,366333	0,328093	0,297893
1951	0,546208	0,471723	0,411443	0,363289	0,325128	0,294996
1952	0,5429	0,468568	0,408406	0,360345	0,322263	0,292197
1953	0,539696	0,465512	0,405464	0,357495	0,319488	0,289487
1954	0,53687	0,462717	0,402704	0,354774	0,316811	0,286855
1955	0,534259	0,460081	0,400066	0,352152	0,314217	0,284296
1956	0,531712	0,457514	0,397499	0,349602	0,311695	0,281808
1957	0,529225	0,455012	0,394999	0,347119	0,30924	0,279388
1958	0,526784	0,452561	0,392555	0,344696	0,306846	0,277029
1959	0,524062	0,449942	0,390018	0,342227	0,304438	0,274675
1960	0,521406	0,447385	0,387541	0,339819	0,302089	0,272379
1961	0,518809	0,444886	0,385122	0,337466	0,299795	0,270137
1962	0,516271	0,442444	0,382758	0,335168	0,297553	0,267946
1963	0,513787	0,440055	0,380445	0,332919	0,295361	0,265804
1964	0,51175	0,43808	0,378519	0,331036	0,293515	0,263989
1965	0,509832	0,436217	0,376701	0,329257	0,291768	0,26227
1966	0,507948	0,434388	0,374917	0,327511	0,290055	0,260584
1967	0,506095	0,432591	0,373166	0,325797	0,288374	0,25893
1968	0,504273	0,430825	0,371445	0,324114	0,286723	0,257307
1969	0,502473	0,429081	0,369746	0,322453	0,285095	0,255706
1970	0,500081	0,426726	0,367431	0,320184	0,282875	0,253538

1971	0,49773	0,424411	0,365157	0,317955	0,280694	0,251409
1972	0,495418	0,422137	0,362923	0,315765	0,278553	0,249317
1973	0,493143	0,4199	0,360726	0,313612	0,276447	0,247261
1974	0,490905	0,417699	0,358566	0,311495	0,274378	0,24524
1975	0,488702	0,415534	0,35644	0,309413	0,272342	0,243252
1976	0,486648	0,413526	0,354475	0,307491	0,270463	0,241417
1977	0,484713	0,411643	0,352639	0,305699	0,268712	0,239705
1978	0,482805	0,409788	0,35083	0,303933	0,266988	0,23802
1979	0,480924	0,407959	0,349047	0,302193	0,265289	0,236359
1980	0,479067	0,406154	0,347289	0,300478	0,263614	0,234723
1981	0,477236	0,404375	0,345556	0,298787	0,261964	0,23311
1982	0,475427	0,402619	0,343846	0,29712	0,260336	0,23152
1983	0,473324	0,400581	0,341866	0,295194	0,25846	0,229692
1984	0,470826	0,398167	0,339525	0,292922	0,256253	0,227546
1985	0,468356	0,395782	0,337215	0,29068	0,254076	0,225429
1986	0,465915	0,393425	0,334933	0,288466	0,251927	0,22334
1987	0,463501	0,391097	0,332679	0,286281	0,249805	0,221278
1988	0,461114	0,388795	0,330452	0,284123	0,247711	0,219243
1989	0,458754	0,38652	0,328252	0,281991	0,245643	0,217233
1990	0,456419	0,384271	0,326078	0,279885	0,2436	0,215249
1991	0,453912	0,381857	0,323745	0,277627	0,241411	0,213124
1992	0,451295	0,379339	0,321313	0,275274	0,239132	0,210914
1993	0,448703	0,376846	0,318907	0,272947	0,236879	0,20873
1994	0,446136	0,374379	0,316527	0,270647	0,234652	0,206572
1995	0,443592	0,371936	0,314172	0,268371	0,23245	0,204438
1996	0,441073	0,369518	0,311842	0,266121	0,230273	0,202329
1997	0,438577	0,367124	0,309536	0,263894	0,22812	0,200243
1998	0,436103	0,364753	0,307253	0,261691	0,22599	0,19818
1999	0,433646	0,362398	0,304987	0,259506	0,223878	0,196135
2000	0,430825	0,359702	0,302399	0,257015	0,221478	0,193818
2001	0,428026	0,357028	0,299834	0,254549	0,219102	0,191526
2002	0,425248	0,354377	0,297293	0,252106	0,216751	0,189258
2003	0,422491	0,351748	0,294774	0,249687	0,214422	0,187012
2004	0,419755	0,349141	0,292278	0,247291	0,212117	0,18479
2005	0,417039	0,346555	0,289805	0,244917	0,209834	0,182589
2006	0,414344	0,343991	0,287353	0,242565	0,207572	0,18041
2007	0,41167	0,341448	0,284922	0,240235	0,205333	0,178253
2008	0,409016	0,338926	0,282513	0,237926	0,203114	0,176116
2009	0,40609	0,336157	0,279878	0,235409	0,200705	0,173804
2010	0,403037	0,333274	0,277139	0,232799	0,198211	0,171415
2011	0,400001	0,330409	0,274422	0,230211	0,195738	0,169047
2012	0,396982	0,327564	0,271724	0,227643	0,193287	0,166701
2013	0,393981	0,324738	0,269046	0,225096	0,190857	0,164376
2014	0,390998	0,321931	0,266389	0,222569	0,188447	0,16207
2015	0,388033	0,319143	0,26375	0,220062	0,186057	0,159785

Male Age=60	Interest Rate					
Life Table	%0	%2	%4	%6	%8	%10
1931	0,712237	0,641729	0,583085	0,534913	0,495672	0,463848
1932	0,697509	0,620888	0,558463	0,508108	0,467723	0,435399
1933	0,679877	0,602909	0,54034	0,489995	0,449723	0,417575
1934	0,665508	0,588884	0,526613	0,476536	0,43651	0,404589
1935	0,657177	0,579704	0,516925	0,466586	0,426463	0,394548
1936	0,648432	0,57098	0,508245	0,457977	0,417945	0,386131
1937	0,64069	0,563322	0,500673	0,450496	0,410561	0,378848
1938	0,632982	0,556477	0,49443	0,444668	0,405021	0,373513
1939	0,62597	0,55026	0,488767	0,439386	0,400004	0,368682
1940	0,621402	0,545801	0,484399	0,435101	0,395794	0,364542
1941	0,618776	0,542744	0,48107	0,431618	0,392237	0,360962
1942	0,616269	0,539861	0,477951	0,428365	0,388922	0,357629
1943	0,613323	0,53677	0,474776	0,425155	0,385708	0,354433
1944	0,609479	0,533153	0,471327	0,421831	0,382481	0,351284
1945	0,605848	0,529736	0,468068	0,418691	0,379434	0,34831
1946	0,602404	0,526496	0,464978	0,415714	0,376544	0,34549
1947	0,599263	0,52349	0,462076	0,412894	0,373791	0,342795
1948	0,596471	0,52074	0,459365	0,410223	0,371161	0,340204
1949	0,593795	0,518106	0,45677	0,407667	0,368644	0,337726
1950	0,591227	0,515579	0,454281	0,405215	0,36623	0,33535
1951	0,588756	0,513148	0,451888	0,402859	0,363911	0,333067
1952	0,586413	0,510821	0,449582	0,40058	0,361663	0,330852
1953	0,584179	0,508586	0,447357	0,398375	0,359484	0,328702
1954	0,582017	0,506425	0,445207	0,396244	0,357378	0,326626
1955	0,579921	0,504332	0,443125	0,394182	0,355342	0,324618
1956	0,577888	0,502303	0,441108	0,392185	0,353369	0,322673
1957	0,575914	0,500333	0,439151	0,390247	0,351456	0,320787
1958	0,573651	0,498126	0,43698	0,388099	0,349321	0,318656
1959	0,571336	0,495882	0,43478	0,385922	0,347152	0,316485
1960	0,569083	0,493698	0,432639	0,383804	0,345042	0,314373
1961	0,566888	0,491571	0,430554	0,381742	0,342988	0,312316
1962	0,564748	0,489498	0,428522	0,379732	0,340987	0,310313
1963	0,562661	0,487475	0,426539	0,377771	0,339035	0,308359
1964	0,560623	0,485501	0,424604	0,375857	0,337129	0,306452
1965	0,559049	0,483958	0,423097	0,374391	0,335708	0,305079
1966	0,557616	0,482547	0,421721	0,373061	0,334432	0,303862
1967	0,556211	0,481166	0,420373	0,371758	0,333182	0,30267
1968	0,554834	0,479811	0,419053	0,370481	0,331957	0,301503
1969	0,553482	0,478483	0,417758	0,36923	0,330757	0,300358
1970	0,552155	0,477179	0,416488	0,368002	0,329579	0,299236
1971	0,550852	0,4759	0,415241	0,366798	0,328424	0,298135
1972	0,549572	0,474643	0,414018	0,365616	0,32729	0,297054
1973	0,548207	0,473295	0,412692	0,364319	0,326027	0,295826
1974	0,546783	0,471883	0,411296	0,362942	0,324671	0,294492

1975	0,545384	0,470496	0,409924	0,361589	0,323339	0,293183
1976	0,544007	0,469132	0,408576	0,36026	0,322031	0,291897
1977	0,542652	0,467791	0,407251	0,358953	0,320746	0,290634
1978	0,541319	0,466472	0,405948	0,357669	0,319482	0,289392
1979	0,540007	0,465173	0,404665	0,356405	0,318239	0,28817
1980	0,538715	0,463895	0,403404	0,355162	0,317017	0,286969
1981	0,537441	0,462637	0,402162	0,353939	0,315813	0,285786
1982	0,536187	0,461397	0,400938	0,352734	0,314629	0,284622
1983	0,534381	0,459647	0,399236	0,351075	0,31301	0,28304
1984	0,53254	0,457867	0,397507	0,349392	0,311368	0,281436
1985	0,530726	0,456113	0,395803	0,347733	0,30975	0,279857
1986	0,528937	0,454383	0,394124	0,346099	0,308156	0,278301
1987	0,527172	0,452678	0,392469	0,344488	0,306586	0,276767
1988	0,525432	0,450996	0,390837	0,3429	0,305038	0,275256
1989	0,523714	0,449337	0,389227	0,341334	0,303511	0,273766
1990	0,522019	0,4477	0,387639	0,339789	0,302006	0,272296
1991	0,520346	0,446085	0,386073	0,338265	0,300521	0,270847
1992	0,518694	0,44449	0,384526	0,336761	0,299056	0,269417
1993	0,517063	0,442916	0,383	0,335277	0,29761	0,268006
1994	0,51508	0,441019	0,381182	0,33353	0,295928	0,266382
1995	0,512926	0,438966	0,379222	0,331657	0,294133	0,264657
1996	0,510798	0,436937	0,377286	0,329807	0,292361	0,262954
1997	0,508694	0,434931	0,375373	0,327979	0,29061	0,261271
1998	0,506614	0,432949	0,373482	0,326173	0,28888	0,259609
1999	0,504557	0,43099	0,371614	0,324388	0,28717	0,257967
2000	0,502524	0,429053	0,369767	0,322623	0,285481	0,256344
2001	0,500513	0,427138	0,367941	0,320879	0,283811	0,254741
2002	0,498524	0,425243	0,366135	0,319155	0,28216	0,253155
2003	0,496556	0,42337	0,364349	0,317449	0,280528	0,251588
2004	0,494609	0,421517	0,362583	0,315763	0,278914	0,250038
2005	0,492683	0,419683	0,360835	0,314095	0,277317	0,248505
2006	0,490777	0,417869	0,359107	0,312445	0,275738	0,246989
2007	0,489065	0,416198	0,357472	0,310844	0,274171	0,245456
2008	0,487463	0,414612	0,355895	0,309278	0,272619	0,24392
2009	0,485874	0,413039	0,354332	0,307727	0,271082	0,242401
2010	0,484295	0,411478	0,352784	0,30619	0,269561	0,240897
2011	0,482729	0,409931	0,351249	0,304669	0,268054	0,239409
2012	0,481173	0,408396	0,349729	0,303162	0,266563	0,237936
2013	0,479628	0,406873	0,348221	0,301669	0,265086	0,236478
2014	0,478094	0,405363	0,346727	0,30019	0,263624	0,235034
2015	0,476571	0,403865	0,345246	0,298724	0,262176	0,233604

Appendix 4: Entropy values for different Turkish life tables at different rates of interest for females and males at age 70.

Female Age=70	Interest Rate					
Life Table	%0	%2	%4	%6	%8	%10
1931	1,089369	1,032354	0,982407	0,938935	0,901282	0,868776
1932	1,029689	0,972689	0,922807	0,879447	0,841944	0,809621
1933	1,027737	0,960685	0,90348	0,854903	0,813764	0,778959
1934	0,991192	0,926981	0,871916	0,82493	0,784966	0,751027
1935	0,971807	0,907168	0,851809	0,804638	0,764574	0,730601
1936	0,956524	0,891278	0,835483	0,788017	0,74777	0,713697
1937	0,941486	0,876352	0,82065	0,773271	0,733107	0,699117
1938	0,928934	0,863709	0,807948	0,76054	0,720375	0,686406
1939	0,918932	0,853346	0,79732	0,74973	0,70945	0,675418
1940	0,909503	0,843672	0,787465	0,739754	0,699402	0,665336
1941	0,899961	0,834258	0,778156	0,730535	0,690264	0,656272
1942	0,891039	0,825457	0,769456	0,721921	0,681727	0,647805
1943	0,883277	0,817832	0,761942	0,714499	0,674384	0,640533
1944	0,876694	0,811402	0,755633	0,70829	0,668258	0,634478
1945	0,870449	0,805301	0,749648	0,702399	0,662446	0,628734
1946	0,864736	0,799684	0,74411	0,696927	0,657032	0,623371
1947	0,859719	0,794681	0,739122	0,691959	0,652086	0,61845
1948	0,854908	0,789885	0,734343	0,687198	0,647348	0,613737
1949	0,850283	0,785277	0,729752	0,682627	0,642799	0,609213
1950	0,845326	0,780442	0,725014	0,677968	0,638206	0,604677
1951	0,840392	0,775657	0,720348	0,673397	0,633712	0,600247
1952	0,83563	0,771039	0,715844	0,668985	0,629375	0,595972
1953	0,831027	0,766575	0,711491	0,664721	0,625183	0,59184
1954	0,826968	0,762521	0,707445	0,660687	0,621166	0,587844
1955	0,823218	0,758716	0,703601	0,656821	0,617291	0,58397
1956	0,819565	0,755013	0,699864	0,653064	0,613526	0,580208
1957	0,816005	0,751407	0,696227	0,649409	0,609867	0,576552
1958	0,812512	0,747878	0,692673	0,645843	0,606298	0,572989
1959	0,808558	0,744014	0,688882	0,642112	0,602618	0,569354
1960	0,804706	0,740249	0,685187	0,638476	0,599033	0,565813
1961	0,800948	0,736576	0,681584	0,63493	0,595536	0,562359
1962	0,79728	0,732991	0,678066	0,63147	0,592124	0,558989
1963	0,793697	0,729489	0,674631	0,62809	0,588791	0,555697
1964	0,790294	0,726127	0,671303	0,624793	0,585524	0,552459
1965	0,786984	0,722848	0,668052	0,621568	0,582325	0,549286
1966	0,783741	0,719637	0,66487	0,618412	0,579195	0,546182
1967	0,780562	0,716491	0,661751	0,615321	0,576129	0,543141
1968	0,777444	0,713406	0,658695	0,612291	0,573125	0,540163
1969	0,774385	0,71038	0,655698	0,60932	0,570179	0,537242
1970	0,771373	0,707394	0,652736	0,606381	0,567265	0,534352

1971	0,768414	0,704462	0,649828	0,603497	0,564405	0,531515
1972	0,765506	0,701582	0,646973	0,600665	0,561596	0,528731
1973	0,762648	0,698752	0,644167	0,597884	0,558838	0,525996
1974	0,759837	0,695969	0,641409	0,59515	0,556128	0,523309
1975	0,757072	0,693232	0,638697	0,592462	0,553463	0,520669
1976	0,75426	0,690453	0,635949	0,58974	0,550768	0,517999
1977	0,75142	0,687652	0,63318	0,587001	0,548057	0,515315
1978	0,748624	0,684894	0,630455	0,584306	0,54539	0,512675
1979	0,74587	0,682178	0,627772	0,581653	0,542765	0,510076
1980	0,743157	0,679503	0,625129	0,57904	0,54018	0,507518
1981	0,740483	0,676867	0,622526	0,576466	0,537634	0,504999
1982	0,737847	0,674269	0,619961	0,573931	0,535126	0,502517
1983	0,734707	0,671176	0,616907	0,570913	0,532143	0,499566
1984	0,730897	0,667424	0,613205	0,567257	0,528528	0,495993
1985	0,72714	0,663725	0,609557	0,563653	0,524968	0,492473
1986	0,723434	0,660077	0,605959	0,560102	0,521459	0,489004
1987	0,719778	0,65648	0,602412	0,5566	0,518	0,485586
1988	0,71617	0,652931	0,598914	0,553148	0,51459	0,482216
1989	0,712609	0,649429	0,595463	0,549743	0,511227	0,478894
1990	0,709094	0,645973	0,592058	0,546383	0,507911	0,475617
1991	0,705258	0,642203	0,588344	0,54272	0,504295	0,472046
1992	0,701222	0,638237	0,584438	0,538869	0,500494	0,468293
1993	0,697234	0,63432	0,580582	0,535067	0,496744	0,464591
1994	0,693294	0,630451	0,576774	0,531315	0,493042	0,460937
1995	0,689401	0,626629	0,573014	0,52761	0,489388	0,457331
1996	0,685552	0,622853	0,5693	0,523951	0,48578	0,453772
1997	0,681749	0,619122	0,565631	0,520337	0,482218	0,450257
1998	0,677989	0,615434	0,562006	0,516768	0,4787	0,446787
1999	0,674259	0,611777	0,558411	0,513229	0,475212	0,443347
2000	0,669814	0,607422	0,554133	0,50902	0,471067	0,439262
2001	0,665417	0,603116	0,549905	0,504862	0,466973	0,435228
2002	0,661067	0,598857	0,545725	0,500751	0,462927	0,431243
2003	0,656763	0,594645	0,541592	0,496689	0,458929	0,427305
2004	0,652503	0,590479	0,537505	0,492672	0,454978	0,423414
2005	0,648288	0,586357	0,533463	0,488702	0,451072	0,419569
2006	0,644117	0,582279	0,529465	0,484776	0,447211	0,415768
2007	0,639989	0,578245	0,525511	0,480893	0,443394	0,412011
2008	0,635902	0,574253	0,5216	0,477053	0,439619	0,408296
2009	0,631254	0,569719	0,517165	0,472705	0,43535	0,4041
2010	0,626349	0,564941	0,512495	0,468131	0,430862	0,399693
2011	0,621491	0,560211	0,507874	0,463606	0,426424	0,395334
2012	0,616679	0,555527	0,503299	0,459128	0,422034	0,391024
2013	0,611912	0,550889	0,498771	0,454697	0,41769	0,386761
2014	0,60719	0,546296	0,494289	0,450312	0,413393	0,382544
2015	0,602513	0,541749	0,489852	0,445973	0,409142	0,378373

Male Age=70	Interest Rate					
Life Table	%0	%2	%4	%6	%8	%10
1931	1,07092	1,014335	0,964687	0,921429	0,883933	0,85155
1932	1,053957	0,988641	0,93267	0,884934	0,84434	0,809865
1933	1,02838	0,9624	0,905951	0,857895	0,817108	0,782539
1934	1,007421	0,941664	0,885407	0,837522	0,796891	0,762467
1935	0,995823	0,928885	0,871753	0,823247	0,782194	0,747498
1936	0,983159	0,916187	0,859033	0,810521	0,769481	0,734813
1937	0,971988	0,905062	0,847945	0,799472	0,758475	0,723854
1938	0,960671	0,894703	0,838325	0,79041	0,749832	0,715525
1939	0,950379	0,885302	0,829607	0,782209	0,742017	0,707997
1940	0,943856	0,878859	0,823233	0,775895	0,735756	0,701785
1941	0,940309	0,874762	0,818723	0,771086	0,73074	0,696631
1942	0,9369	0,870878	0,814481	0,766585	0,726058	0,69183
1943	0,932761	0,866539	0,809991	0,76199	0,721395	0,687126
1944	0,927185	0,861195	0,804831	0,756972	0,716489	0,682309
1945	0,921928	0,856157	0,799964	0,75224	0,711862	0,677766
1946	0,916953	0,851389	0,795358	0,747761	0,707483	0,673466
1947	0,912419	0,846987	0,791061	0,743547	0,703337	0,669377
1948	0,908393	0,842989	0,787087	0,739596	0,699409	0,665472
1949	0,904543	0,839166	0,783288	0,73582	0,695655	0,661742
1950	0,900852	0,835503	0,779648	0,732202	0,69206	0,658169
1951	0,897306	0,831984	0,776153	0,728729	0,688609	0,65474
1952	0,893942	0,828617	0,772787	0,725369	0,685259	0,651404
1953	0,890732	0,825384	0,76954	0,722116	0,682008	0,648161
1954	0,88763	0,822261	0,766404	0,718975	0,67887	0,645032
1955	0,884627	0,81924	0,763372	0,71594	0,675838	0,642008
1956	0,881717	0,816313	0,760436	0,713002	0,672904	0,639082
1957	0,878894	0,813476	0,75759	0,710154	0,67006	0,636248
1958	0,875536	0,810194	0,754376	0,707004	0,666969	0,633213
1959	0,872069	0,806831	0,751106	0,703817	0,663857	0,63017
1960	0,868702	0,803564	0,747928	0,700719	0,660834	0,627214
1961	0,865428	0,800387	0,744838	0,697707	0,657893	0,624338
1962	0,862242	0,797295	0,741831	0,694776	0,65503	0,621539
1963	0,859139	0,794284	0,738901	0,691919	0,652241	0,618812
1964	0,856114	0,791348	0,736044	0,689134	0,649521	0,616152
1965	0,853899	0,789132	0,733825	0,686911	0,647295	0,613924
1966	0,851922	0,787132	0,731802	0,684866	0,64523	0,611842
1967	0,849987	0,785175	0,729822	0,682866	0,643212	0,609807
1968	0,848091	0,783259	0,727885	0,680909	0,641236	0,607816
1969	0,846232	0,781381	0,725987	0,678992	0,639303	0,605868
1970	0,844409	0,77954	0,724128	0,677115	0,637409	0,60396
1971	0,84262	0,777735	0,722304	0,675274	0,635553	0,60209
1972	0,840864	0,775963	0,720516	0,673469	0,633733	0,600257
1973	0,838923	0,774015	0,718562	0,671511	0,631773	0,598297
1974	0,83685	0,771941	0,71649	0,669444	0,629715	0,596249

1975	0,834813	0,769905	0,714456	0,667416	0,627695	0,59424
1976	0,832812	0,767906	0,712459	0,665425	0,625711	0,592267
1977	0,830846	0,76594	0,710497	0,663469	0,623764	0,59033
1978	0,828912	0,764008	0,708569	0,661547	0,62185	0,588426
1979	0,82701	0,762109	0,706674	0,659657	0,619969	0,586556
1980	0,825138	0,76024	0,704809	0,657799	0,61812	0,584717
1981	0,823296	0,758401	0,702975	0,655972	0,616301	0,582909
1982	0,821482	0,756591	0,70117	0,654174	0,614512	0,58113
1983	0,818781	0,753934	0,698549	0,651585	0,611952	0,578597
1984	0,816025	0,751226	0,695881	0,648951	0,60935	0,576024
1985	0,813313	0,748561	0,693256	0,64636	0,606789	0,573491
1986	0,810643	0,745937	0,690671	0,643809	0,604269	0,570999
1987	0,808012	0,743353	0,688126	0,641298	0,601788	0,568546
1988	0,805421	0,740808	0,685619	0,638824	0,599344	0,566129
1989	0,802868	0,7383	0,683149	0,636388	0,596937	0,56375
1990	0,800351	0,735829	0,680715	0,633987	0,594566	0,561405
1991	0,79787	0,733392	0,678316	0,63162	0,592228	0,559094
1992	0,795424	0,73099	0,675951	0,629288	0,589924	0,556817
1993	0,793011	0,728621	0,673619	0,626987	0,587653	0,554571
1994	0,789892	0,725541	0,670583	0,623998	0,584712	0,551681
1995	0,786427	0,722115	0,667203	0,620672	0,581446	0,548478
1996	0,783011	0,718736	0,663871	0,617394	0,578226	0,545319
1997	0,779642	0,715404	0,660585	0,614161	0,575051	0,542204
1998	0,776318	0,712117	0,657343	0,610972	0,571919	0,539132
1999	0,773038	0,708874	0,654146	0,607826	0,568829	0,536101
2000	0,769802	0,705674	0,650991	0,604722	0,565781	0,53311
2001	0,766608	0,702517	0,647877	0,601658	0,562772	0,530159
2002	0,763455	0,6994	0,644804	0,598635	0,559803	0,527246
2003	0,760342	0,696322	0,64177	0,59565	0,556871	0,52437
2004	0,757268	0,693284	0,638774	0,592703	0,553977	0,52153
2005	0,754231	0,690283	0,635815	0,589792	0,551118	0,518726
2006	0,751232	0,687319	0,632893	0,586917	0,548295	0,515956
2007	0,748693	0,684816	0,630417	0,584464	0,54586	0,513538
2008	0,746412	0,682573	0,628194	0,582249	0,543648	0,511325
2009	0,74415	0,680348	0,625991	0,580056	0,541458	0,509134
2010	0,741904	0,678143	0,623808	0,577885	0,53929	0,506966
2011	0,739677	0,675956	0,621645	0,575734	0,537144	0,504821
2012	0,737467	0,673788	0,619501	0,573603	0,535019	0,502698
2013	0,735274	0,671638	0,617377	0,571492	0,532915	0,500596
2014	0,733098	0,669506	0,615271	0,569401	0,530831	0,498515
2015	0,730939	0,667391	0,613183	0,567329	0,528767	0,496454

Appendix 5: Entropy values for different mortality levels and rate of interests for the base years 2000 and 2015 at ages 60 and 70 for females and males.

Age=60 Female	**0,00**	**0.02**	**0.04**	**0.06**	**0.08**	**1,00**	**Age=70 Female**	**0,00**	**0.02**	**0.04**	**0.06**	**0.08**	**1,00**
-0,10	0,3484	0,3103	0,2765	0,2466	0,2203	0,1972	**-0,10**	0,4692	0,4356	0,4050	0,3772	0,3519	0,3291
-0,09	0,3523	0,3126	0,2774	0,2464	0,2193	0,1956	**-0,09**	0,4745	0,4395	0,4076	0,3787	0,3526	0,3290
-0,08	0,3569	0,3151	0,2784	0,2462	0,2182	0,1940	**-0,08**	0,4804	0,4437	0,4105	0,3804	0,3533	0,3289
-0,07	0,3618	0,3179	0,2794	0,2460	0,2171	0,1922	**-0,07**	0,4870	0,4484	0,4135	0,3822	0,3540	0,3288
-0,06	0,3669	0,3207	0,2804	0,2456	0,2157	0,1902	**-0,06**	0,4941	0,4534	0,4168	0,3840	0,3548	0,3286
-0,05	0,3725	0,3236	0,2813	0,2451	0,2142	0,1880	**-0,05**	0,5016	0,4586	0,4202	0,3859	0,3554	0,3284
-0,04	0,3785	0,3267	0,2822	0,2444	0,2125	0,1856	**-0,04**	0,5097	0,4642	0,4237	0,3878	0,3560	0,3280
-0,03	0,3852	0,3300	0,2831	0,2436	0,2105	0,1829	**-0,03**	0,5186	0,4702	0,4274	0,3897	0,3565	0,3274
-0,02	0,3924	0,3335	0,2839	0,2425	0,2082	0,1800	**-0,02**	0,5283	0,4767	0,4314	0,3916	0,3570	0,3267
-0,01	0,4005	0,3372	0,2846	0,2412	0,2056	0,1767	**-0,01**	0,5390	0,4837	0,4355	0,3936	0,3572	0,3258
0,00	0,4093	0,3411	0,2850	0,2395	0,2027	0,1731	**0,00**	0,5506	0,4912	0,4397	0,3954	0,3573	0,3246
0,01	0,4176	0,3443	0,2848	0,2371	0,1991	0,1690	**0,01**	0,5623	0,4985	0,4437	0,3969	0,3570	0,3230
0,02	0,4217	0,3447	0,2826	0,2333	0,1946	0,1642	**0,02**	0,5722	0,5045	0,4466	0,3975	0,3559	0,3208
0,03	0,4172	0,3396	0,2769	0,2273	0,1886	0,1585	**0,03**	0,5781	0,5075	0,4474	0,3965	0,3536	0,3176
0,04	0,4021	0,3275	0,2668	0,2186	0,1810	0,1519	**0,04**	0,5779	0,5062	0,4450	0,3932	0,3497	0,3133
0,05	0,3778	0,3093	0,2526	0,2073	0,1719	0,1444	**0,05**	0,5706	0,4997	0,4389	0,3872	0,3438	0,3075
0,06	0,3476	0,2868	0,2358	0,1944	0,1617	0,1364	**0,06**	0,5563	0,4881	0,4290	0,3786	0,3360	0,3004
0,07	0,3151	0,2626	0,2176	0,1807	0,1512	0,1281	**0,07**	0,5363	0,4721	0,4159	0,3675	0,3265	0,2921
0,08	0,2831	0,2384	0,1994	0,1669	0,1407	0,1200	**0,08**	0,5122	0,4528	0,4002	0,3546	0,3157	0,2829
0,09	0,2532	0,2155	0,1821	0,1538	0,1307	0,1123	**0,09**	0,4855	0,4315	0,3829	0,3405	0,3039	0,2729
0,10	0,2262	0,1946	0,1661	0,1416	0,1213	0,1050	**0,10**						
Age=60 Male	**0,00**	**0.02**	**0.04**	**0.06**	**0.08**	**1,00**	**Age=70 Male**	**0,00**	**0.02**	**0.04**	**0.06**	**0.08**	**1,00**
-0,10	0,4038	0,3657	0,3316	0,3011	0,2740	0,2499	**-0,10**	0,5197	0,4872	0,4575	0,4304	0,4056	0,3832
-0,09	0,4089	0,3691	0,3335	0,3018	0,2737	0,2490	**-0,09**	0,5259	0,4919	0,4609	0,4327	0,4070	0,3837
-0,08	0,4143	0,3725	0,3353	0,3024	0,2734	0,2478	**-0,08**	0,5324	0,4968	0,4644	0,4350	0,4084	0,3843
-0,07	0,4203	0,3762	0,3373	0,3029	0,2729	0,2466	**-0,07**	0,5396	0,5021	0,4682	0,4375	0,4097	0,3847
-0,06	0,4268	0,3803	0,3393	0,3035	0,2723	0,2451	**-0,06**	0,5474	0,5079	0,4722	0,4400	0,4111	0,3851
-0,05	0,4339	0,3845	0,3414	0,3039	0,2715	0,2435	**-0,05**	0,5558	0,5140	0,4764	0,4427	0,4125	0,3855
-0,04	0,4415	0,3891	0,3435	0,3043	0,2705	0,2417	**-0,04**	0,5650	0,5206	0,4809	0,4455	0,4139	0,3858
-0,03	0,4500	0,3939	0,3457	0,3045	0,2694	0,2396	**-0,03**	0,5750	0,5277	0,4857	0,4484	0,4153	0,3860
-0,02	0,4593	0,3992	0,3479	0,3046	0,2680	0,2372	**-0,02**	0,5860	0,5354	0,4907	0,4513	0,4166	0,3861
-0,01	0,4698	0,4049	0,3502	0,3044	0,2662	0,2345	**-0,01**	0,5981	0,5438	0,4961	0,4544	0,4179	0,3861
0,00	0,4815	0,4110	0,3524	0,3040	0,2641	0,2314	**0,00**	0,6115	0,5529	0,5019	0,4576	0,4191	0,3858
0,01	0,4941	0,4173	0,3544	0,3031	0,2615	0,2278	**0,01**	0,6260	0,5625	0,5078	0,4606	0,4201	0,3853
0,02	0,5052	0,4224	0,3553	0,3013	0,2581	0,2236	**0,02**	0,6404	0,5719	0,5133	0,4633	0,4206	0,3843
0,03	0,5102	0,4234	0,3534	0,2976	0,2534	0,2184	**0,03**	0,6527	0,5798	0,5176	0,4649	0,4203	0,3825
0,04	0,5047	0,4177	0,3472	0,2911	0,2469	0,2122	**0,04**	0,6604	0,5842	0,5195	0,4647	0,4186	0,3797
0,05	0,4876	0,4043	0,3361	0,2815	0,2384	0,2047	**0,05**	0,6613	0,5839	0,5179	0,4620	0,4150	0,3756
0,06	0,4608	0,3843	0,3206	0,2691	0,2282	0,1962	**0,06**	0,6547	0,5781	0,5123	0,4564	0,4094	0,3700
0,07	0,4280	0,3600	0,3021	0,2548	0,2169	0,1870	**0,07**	0,6407	0,5668	0,5027	0,4479	0,4016	0,3628
0,08	0,3929	0,3336	0,2822	0,2395	0,2050	0,1776	**0,08**	0,6207	0,5509	0,4896	0,4369	0,3920	0,3543
0,09	0,3581	0,3071	0,2622	0,2242	0,1931	0,1682	**0,09**	0,5962	0,5313	0,4738	0,4237	0,3809	0,3447
0,10							**0,10**						

Age=60 Female	0	0.02	0.04	0.06	0.08	1	Age=70 Female	0	0.02	0.04	0.06	0.08	1
-0,1	0,3137	0,2759	0,2426	0,2133	0,1878	0,1657	**-0,1**	0,4356	0,4015	0,3706	0,3425	0,3171	0,2942
-0,09	0,3175	0,2779	0,2431	0,2128	0,1866	0,1639	**-0,09**	0,4408	0,4051	0,3728	0,3437	0,3174	0,2938
-0,08	0,3215	0,2800	0,2437	0,2123	0,1852	0,1621	**-0,08**	0,4465	0,4091	0,3753	0,3450	0,3178	0,2934
-0,07	0,3255	0,2819	0,2441	0,2116	0,1837	0,1600	**-0,07**	0,4526	0,4133	0,3780	0,3464	0,3181	0,2929
-0,06	0,3297	0,2839	0,2444	0,2107	0,1820	0,1578	**-0,06**	0,4590	0,4177	0,3807	0,3477	0,3184	0,2923
-0,05	0,3344	0,2861	0,2447	0,2096	0,1801	0,1553	**-0,05**	0,4659	0,4222	0,3834	0,3490	0,3185	0,2916
-0,04	0,3395	0,2884	0,2450	0,2084	0,1779	0,1527	**-0,04**	0,4733	0,4271	0,3863	0,3502	0,3185	0,2907
-0,03	0,3450	0,2907	0,2451	0,2070	0,1756	0,1498	**-0,03**	0,4814	0,4324	0,3893	0,3515	0,3185	0,2897
-0,02	0,3511	0,2932	0,2450	0,2054	0,1729	0,1466	**-0,02**	0,4902	0,4381	0,3924	0,3527	0,3183	0,2884
-0,01	0,3577	0,2958	0,2449	0,2034	0,1700	0,1432	**-0,01**	0,4999	0,4441	0,3957	0,3539	0,3179	0,2870
0	0,3646	0,2982	0,2443	0,2010	0,1666	0,1394	**0**	0,5099	0,4502	0,3988	0,3548	0,3172	0,2852
0,01	0,3693	0,2991	0,2425	0,1977	0,1625	0,1351	**0,01**	0,5189	0,4555	0,4012	0,3550	0,3159	0,2828
0,02	0,3677	0,2958	0,2381	0,1927	0,1574	0,1302	**0,02**	0,5248	0,4585	0,4019	0,3540	0,3136	0,2798
0,03	0,3566	0,2865	0,2299	0,1853	0,1508	0,1244	**0,03**	0,5254	0,4576	0,3998	0,3509	0,3099	0,2756
0,04	0,3361	0,2709	0,2177	0,1755	0,1428	0,1178	**0,04**	0,5193	0,4520	0,3943	0,3454	0,3044	0,2702
0,05	0,3089	0,2508	0,2025	0,1639	0,1338	0,1106	**0,05**	0,5065	0,4414	0,3852	0,3373	0,2971	0,2635
0,06	0,2787	0,2283	0,1858	0,1513	0,1242	0,1033	**0,06**	0,4878	0,4264	0,3728	0,3269	0,2881	0,2557
0,07	0,2483	0,2056	0,1688	0,1387	0,1147	0,0960	**0,07**	0,4649	0,4081	0,3580	0,3147	0,2778	0,2469
0,08	0,2197	0,1839	0,1526	0,1265	0,1055	0,0890	**0,08**	0,4393	0,3876	0,3414	0,3011	0,2666	0,2375
0,09	0,1939	0,1642	0,1377	0,1153	0,0970	0,0825	**0,09**	0,4126	0,3660	0,3239	0,2869	0,2549	0,2278
0,1	0,1713	0,1466	0,1242	0,1051	0,0893	0,0766	**0,1**	0,3859	0,3443	0,3063	0,2725	0,2431	0,2179
Age=60 Male	**0**	**0.02**	**0.04**	**0.06**	**0.08**	**1**	**Age=70 Male**	**0**	**0.02**	**0.04**	**0.06**	**0.08**	**1**
-0,1	0,3861	0,3478	0,3136	0,2831	0,2561	0,2323	**-0,1**	0,5033	0,4703	0,4402	0,4127	0,3877	0,3650
-0,09	0,3908	0,3507	0,3151	0,2835	0,2556	0,2311	**-0,09**	0,5092	0,4747	0,4433	0,4147	0,3888	0,3653
-0,08	0,3959	0,3539	0,3166	0,2838	0,2550	0,2297	**-0,08**	0,5155	0,4794	0,4466	0,4168	0,3899	0,3656
-0,07	0,4015	0,3573	0,3183	0,2841	0,2542	0,2283	**-0,07**	0,5225	0,4845	0,4501	0,4191	0,3911	0,3659
-0,06	0,4077	0,3609	0,3200	0,2843	0,2534	0,2266	**-0,06**	0,5301	0,4900	0,4539	0,4214	0,3922	0,3661
-0,05	0,4142	0,3648	0,3217	0,2844	0,2523	0,2248	**-0,05**	0,5384	0,4959	0,4579	0,4238	0,3934	0,3663
-0,04	0,4214	0,3688	0,3234	0,2844	0,2511	0,2227	**-0,04**	0,5472	0,5022	0,4621	0,4263	0,3945	0,3663
-0,03	0,4293	0,3732	0,3251	0,2842	0,2496	0,2204	**-0,03**	0,5569	0,5090	0,4665	0,4289	0,3956	0,3663
-0,02	0,4381	0,3779	0,3269	0,2839	0,2479	0,2178	**-0,02**	0,5676	0,5164	0,4712	0,4315	0,3967	0,3661
-0,01	0,4478	0,3830	0,3286	0,2833	0,2458	0,2148	**-0,01**	0,5793	0,5244	0,4762	0,4342	0,3976	0,3657
0	0,4586	0,3884	0,3302	0,2824	0,2434	0,2115	**0**	0,5923	0,5330	0,4815	0,4370	0,3984	0,3651
0,01	0,4699	0,3937	0,3314	0,2810	0,2404	0,2076	**0,01**	0,6059	0,5419	0,4868	0,4395	0,3990	0,3642
0,02	0,4786	0,3971	0,3312	0,2784	0,2365	0,2032	**0,02**	0,6189	0,5502	0,4915	0,4415	0,3989	0,3628
0,03	0,4799	0,3957	0,3278	0,2737	0,2312	0,1977	**0,03**	0,6290	0,5564	0,4945	0,4421	0,3979	0,3605
0,04	0,4704	0,3873	0,3199	0,2662	0,2241	0,1912	**0,04**	0,6337	0,5587	0,4948	0,4408	0,3953	0,3571
0,05	0,4499	0,3717	0,3073	0,2558	0,2152	0,1836	**0,05**	0,6314	0,5560	0,4914	0,4368	0,3908	0,3523
0,06	0,4211	0,3502	0,2909	0,2429	0,2049	0,1751	**0,06**	0,6217	0,5478	0,4841	0,4299	0,3843	0,3461
0,07	0,3878	0,3254	0,2722	0,2286	0,1936	0,1661	**0,07**	0,6051	0,5345	0,4730	0,4203	0,3758	0,3384
0,08	0,3534	0,2995	0,2527	0,2137	0,1821	0,1570	**0,08**	0,5831	0,5170	0,4587	0,4084	0,3656	0,3295
0,09	0,3201	0,2741	0,2334	0,1989	0,1707	0,1481	**0,09**	0,5574	0,4965	0,4421	0,3947	0,3541	0,3197
0,1							**0,1**						

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