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Global attractors for von Karman equations with nonlinear interior dissipation

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Abstract

In this paper we study the asymptotic behavior of weak solutions for von Karman equations with nonlinear interior dissipation. We prove the existence of a global attractor in the space $\mathring{W}_2^2(\Omega) \times L_2(\Omega)$.

Keywords: Global attractors; von Karman equations

1. Introduction

Let Ω be a bounded smooth domain in R^2 with boundary $\partial\Omega$. We consider the following von Karman system with the homogeneous boundary conditions:

$$w_{tt} + \Delta^2 w + g(w_t) = \left[\mathcal{F}(w), w\right] + h \quad \text{in } (0, +\infty) \times \Omega, \tag{1.1}$$

$$\Delta^2 \mathcal{F}(w) = -[w, w] \quad \text{in } (0, +\infty) \times \Omega, \tag{1.2}$$

$$w = \frac{\partial w}{\partial \nu} = \mathcal{F} = \frac{\partial \mathcal{F}}{\partial \nu} = 0 \quad \text{on } (0, +\infty) \times \partial\Omega, \tag{1.3}$$

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$$w(0,\cdot)=w_0,\quad w_t(0,\cdot)=w_1\quad \text{in } \Omega, \tag{1.4}$$

where $h \in L_2(\Omega)$, the vector ν denotes an outward normal and von Karman bracket is given by

$$[u,v]\equiv \frac{\partial^2 u}{\partial x_1^2}\frac{\partial^2 v}{\partial x_2^2}+\frac{\partial^2 u}{\partial x_2^2}\frac{\partial^2 v}{\partial x_1^2}-2\frac{\partial^2 u}{\partial x_1 \partial x_2}\frac{\partial^2 v}{\partial x_1 \partial x_2}.$$

The damping function $g \in C^1(R)$ satisfies the condition

$$g(0)=0,\qquad g \text{ strictly increasing},\quad \text{and}\quad \liminf_{|s|\to\infty} g'(s)>0. \tag{1.5}$$

The long-time behavior of solutions for von Karman equations with interior dissipations were studied in [1–7] and references therein. The wellposedness of weak solutions of problem (1.1)–(1.4) has been established in [3] (see also [6]). The problem of existence of weak attractors for (1.1)–(1.4) in the case when $g(\cdot)$ is linear, was studied in [2]. In the case of nonlinear dissipation, the most general treatment for the problem (1.1)–(1.4) to our knowledge is given in [7]. In that article the authors have proved the existence of a global attractor in $\mathring{W}_2^2(\Omega)\times L_2(\Omega)$ for large values of the damping parameter.

Our main goal in this paper is to prove the existence of a global attractor for the problem (1.1)–(1.4) without assuming large values for the damping parameter. The sharp regularity of Airy's stress function obtained in [8] plays a key role in our result.

2. Preliminaries

Denote the spaces $\mathring{W}_2^s(\Omega)$, $W_2^s(\Omega)$ and $L_2(\Omega)$, by H_0^s, H^s, and H, respectively. The norm and scalar product in H are denoted by $\|\cdot\|$ and $\langle,\rangle$, respectively. It is known that under condition (1.5) the solution operator $S(t)(w_0,w_1)=(w(t),w_t(t))$, $t\geqslant 0$, of problem (1.1)–(1.4) generates a C^0-semigroup on the energy space $H_0^2\times H$ (see [3,6]) in which

$$E\big(w(t)\big)+\frac{1}{4}\big\|\Delta\mathcal{F}\big(w(t)\big)\big\|^2+\int_s^t\int_\Omega g\big(w_t(\tau,x)\big)w_t(\tau,x)\,dx\,d\tau-\big\langle h,w(t)\big\rangle$$
$$\leqslant E\big(w(s)\big)+\frac{1}{4}\big\|\Delta\mathcal{F}\big(w(s)\big)\big\|^2-\big\langle h,w(s)\big\rangle \tag{2.1}$$

and

$$E\big(w(t)-u(t)\big)+\int_s^t\int_\Omega\big(g\big(w_t(\tau,x)\big)-g\big(u_t(\tau,x)\big)\big)\big(w_t(\tau,x)-u_t(\tau,x)\big)\,dx\,d\tau$$
$$\leqslant E\big(w(s)-u(s)\big)+\int_s^t\big\langle\big[\mathcal{F}\big(w(\tau)\big),w(\tau)\big]-\big[\mathcal{F}\big(u(\tau)\big),u(\tau)\big],w_t(\tau)-u_t(\tau)\big\rangle\,d\tau, \tag{2.2}$$

hold for $(w(t),w_t(t))=S(t)(w_0,w_1)$ and $(u(t),u_t(t))=S(t)(u_0,u_1)$, where $E(v(t))=\frac{1}{2}(\|\Delta v(t)\|^2+\|v_t(t)\|^2)$ and $t\geqslant s\geqslant 0$.

Denote by $G(u,v)$ a solution to a biharmonic problem:

$$z \equiv G(u,v) \quad \text{iff} \quad \Delta^2 z = [u,v] \quad \text{in } \Omega \quad \text{and} \quad z = \frac{\partial}{\partial \nu} z = 0 \quad \text{on } \partial\Omega.$$

We will use the following theorem on sharp regularity of Airy's stress function from [8], and prove some lemmas in order to show asymptotic compactness of $S(t)$.

Theorem 1. [8] *The map $(u, v) \to G(u,v)$ is bounded from $H^2 \times H^2 \to H^3 \cap W^2_\infty(\Omega)$.*

Lemma 1. *Let $g(\cdot)$ satisfy condition* (1.5)*. Then for any $\delta > 0$ there exists $c(\delta) > 0$, such that*

$$|u-v|^2 \leqslant \delta + c(\delta)\big(g(u)-g(v)\big)(u-v) \quad \textit{for } u, v \in R. \tag{2.3}$$

Proof. Assume (2.3) does not hold. Then there exist $\delta_0 > 0$, $c_n \to +\infty$, and $u_n \in R$, $v_n \in R$ such that

$$|u_n - v_n|^2 > \delta_0 + c_n\big(g(u_n)-g(v_n)\big)(u_n - v_n)$$

from which we obtain

$$|u_n - v_n|^2 > \delta_0 \quad \text{and} \quad \frac{1}{u_n - v_n}\int_{v_n}^{u_n} g'(s)\,ds \to 0,$$

which contradicts (1.5). □

Lemma 2. *Assume that $w \in L_\infty(0,T;H_0^2)$ and $w_t \in L_\infty(0,T;H)$. Then $\mathcal{F}(w) \in C(0,T;H_0^2)$ and*

$$\frac{1}{4}\big\|\Delta\mathcal{F}\big(w(t)\big)\big\|^2 = -\int_s^t \big\langle\big[\mathcal{F}\big(w(\tau)\big), w(\tau)\big], w_t(\tau)\big\rangle\,d\tau + \frac{1}{4}\big\|\Delta\mathcal{F}\big(w(s)\big)\big\|^2, \tag{2.4}$$

for every t, $s \in [0,T]$.

Proof. Since $w \in L_\infty(0,T;H_0^2)$ and $w_t \in L_\infty(0,T;H)$, we have $w \in C(0,T;H_0^1)$ and consequently $w \in C_s(0,T;H_0^2)$ (see [9, Lemma 8.1, p. 275]). It means that if $t_n \to t_0$, then $w(t_n) \to w(t_0)$ weakly in H_0^2. So by Theorem 1 and the compact embedding theorems we obtain

$$\mathcal{F}\big(w(t_n)\big) \to \mathcal{F}\big(w(t_0)\big) \quad \text{strongly in } H_0^2.$$

Hence $\mathcal{F}(w) \in C(0,T;H_0^2)$.

Let the sequence $w^n \in C_0^\infty((0,T)\times\Omega)$ be such that

$$w^n \to w \quad \text{strongly in } L_4\big(0,T;H_0^2\big)$$

and

$$w_t^n \to w_t \quad \text{strongly in } L_4(0,T;H)$$

as n tends to infinity. Then by Theorem 1 we have

$$\mathcal{F}(w^n) \to \mathcal{F}(w) \quad \text{strongly in } L_2\big(0,T;H^3 \cap W_\infty^2(\Omega)\big) \tag{2.5}$$

and

$$\big\langle \big[\mathcal{F}(w^n), w^n\big], w_t^n\big\rangle \to \big\langle \big[\mathcal{F}(w), w\big], w_t\big\rangle \quad \text{strongly in } L_1(0,T). \tag{2.6}$$

Taking into account $\frac{\partial}{\partial t}\|\Delta\mathcal{F}(w^n(t))\|^2 = -4\langle[\mathcal{F}(w^n(t)), w^n(t)], w_t^n(t)\rangle$ from (2.5)–(2.6) we find that

$$\frac{\partial}{\partial t}\big\|\Delta\mathcal{F}\big(w(t)\big)\big\|^2 = -4\big\langle\big[\mathcal{F}\big(w(t)\big), w(t)\big], w_t(t)\big\rangle \in L_\infty(0,T),$$

which implies (2.4). □

Lemma 3. *Assume* $\{w^n(t)\}$ *and* $\{w_t^n(t)\}$ *are weakly star convergent in* $L_\infty(0,T;H_0^2)$ *and* $L_\infty(0,T;H)$, *respectively. Then*

$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T \big\langle\big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t) - w_t^m(t)\big\rangle\, dt = 0. \tag{2.7}$$

Proof. Let

$$\begin{cases} w^n \to w & \text{weakly star in } L_\infty(0,T;H_0^2),\\ w_t^n \to w_t & \text{weakly star in } L_\infty(0,T;H). \end{cases} \tag{2.8}$$

By the compact embedding theorem (see [10, Theorem 5.1, p. 58]) from (2.8) we have

$$w^n \to w \quad \text{strongly in } L_p\big(0,T;H_0^{2-\varepsilon}\big) \tag{2.9}$$

for $1 \leqslant p < \infty$ and $\varepsilon > 0$.

Using $(2.8)_1$, (2.9) and the property of the von Karman bracket, we obtain

$$\big[w^n, w^n\big] \to [w,w] \quad \text{weakly star in } L_\infty\big(0,T;H^{-2}\big)$$

and consequently

$$\mathcal{F}(w^n) \to \mathcal{F}(w) \quad \text{weakly star in } L_\infty\big(0,T;H_0^2\big). \tag{2.10}$$

From $(2.8)_1$, (2.9) and (2.10) we have

$$\big[\mathcal{F}(w^n), w^n\big] \to \big[\mathcal{F}(w), w\big] \quad \text{weakly star in } L_\infty\big(0,T;H^{-2}\big). \tag{2.11}$$

On the other hand, by $(2.8)_1$ and Theorem 1 we find that $\{[\mathcal{F}(w^n), w^n]\}$ is bounded in $L_\infty(0,T;H)$, which together with (2.11) gives

$$\big[\mathcal{F}(w^n), w^n\big] \to \big[\mathcal{F}(w), w\big] \quad \text{weakly star in } L_\infty(0,T;H). \tag{2.12}$$

From (2.8), also follows that

$$w^n \to w \quad \text{weakly in } C\big(0,T;H_0^1\big) \tag{2.13}$$

which according to [9, Lemma 8.1, p. 275], together with $(2.8)_1$ yields $w^n \in C_s(0,T;H_0^2)$. So $\langle w^n(\cdot),\varphi\rangle \in C[0,T]$ and

$$\left|\langle w^n(t),\varphi\rangle\right| \leqslant \left\|\langle w^n(\cdot),\varphi\rangle\right\|_{C[0,T]} \leqslant \left\|w^n\right\|_{L_\infty(0,T;H_0^2)}\|\varphi\|_{H^{-2}}, \tag{2.14}$$

for every $t \in [0,T]$ and $\varphi \in H^{-2}$.

From (2.13) and (2.14) we obtain

$$w^n(t) \to w(t) \quad \text{weakly in } H_0^2$$

for every $t \in [0,T]$. Thus by Theorem 1 we find that

$$\mathcal{F}\big(w^n(t)\big) \to \mathcal{F}\big(w(t)\big) \quad \text{weakly in } H^3 \tag{2.15}$$

for every $t \in [0,T]$.

By Lemma 2, we have

$$\int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t) - w_t^m(t)\big\rangle\,dt$$
$$= \frac{1}{4}\Big[\big\|\Delta\mathcal{F}\big(w^n(0)\big)\big\|^2 + \big\|\Delta\mathcal{F}\big(w^m(0)\big)\big\|^2 - \big\|\Delta\mathcal{F}\big(w^n(T)\big)\big\|^2 - \big\|\Delta\mathcal{F}\big(w^m(T)\big)\big\|^2\Big]$$
$$- \int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big], w_t^m(t)\big\rangle\,dt - \int_0^T \big\langle \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t)\big\rangle\,dt.$$

Taking into account $(2.8)_2$, (2.12), (2.15) and passing to limit in the last equality, we obtain

$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t) - w_t^m(t)\big\rangle\,dt$$
$$= \frac{1}{2}\Big[\big\|\Delta\mathcal{F}\big(w(0)\big)\big\|^2 - \big\|\Delta\mathcal{F}\big(w(T)\big)\big\|^2\Big] - 2\int_0^T \big\langle \big[\mathcal{F}\big(w(t)\big), w(t)\big], w_t(t)\big\rangle\,dt,$$

which together with Lemma 2 imply (2.7). □

Lemma 4. *Assume the condition* (1.5) *is satisfied, and B is a bounded subset of $H_0^2 \times H$. Then for any $\varepsilon > 0$ there exists $T = T(\varepsilon, B)$ such that*

$$\limsup_{n\to\infty}\sup_{p\in\mathbb{N}}\big\|S(T)\theta_{n+p} - S(T)\theta_n\big\|_{H_0^2\times H} \leqslant \varepsilon, \tag{2.16}$$

where $\{\theta_n\}$ is a sequence in B and $\{S(t)\theta_n\}$ weakly star converges in $L_\infty(0,\infty;H_0^2\times H)$.

Proof. We will use techniques used in [6, Proof of Lemma 2.5] for similar estimates for von Karman equations (see also [7]). Let $(w^n(t), w_t^n(t)) = S(t)\theta_n$. From (2.2) we have

$$\int_0^T \int_\Omega \big(g\big(w_t^n(t,x)\big) - g\big(w_t^m(t,x)\big)\big)\big(w_t^n(t,x) - w_t^m(t,x)\big)\,dx\,dt$$
$$\leqslant \tilde{c}\big(\|B\|_{H_0^2\times H}\big) + \int_0^T \Big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big],$$
$$w_t^n(t) - w_t^m(t)\Big\rangle\,dt, \quad \text{for } T \geqslant 0,$$

where $\|B\|_{H_0^2\times H} = \sup_{v\in B}\|v\|_{H_0^2\times H}$. Taking into account (2.3) in the last, inequality we obtain

$$\int_0^T \big\|w_t^n(t) - w_t^m(t)\big\|^2\,dt$$
$$\leqslant \delta T \operatorname{mes}\Omega + c(\delta)\tilde{c}\big(\|B\|_{H_0^2\times H}\big)$$
$$+ c(\delta)\int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t) - w_t^m(t)\big\rangle\,dt, \tag{2.17}$$

for every $\delta > 0$. On the other hand, multiplying both sides of

$$\big(w^n - w^m\big)_{tt} + \Delta^2\big(w^n - w^m\big) + g\big(w_t^n\big) - g\big(w_t^m\big) = \big[\mathcal{F}\big(w^n\big), w^n\big] - \big[\mathcal{F}\big(w^m\big), w^m\big]$$

by $(w^n - w^m)$, integrating over $[0, T] \times \Omega$ and taking into account (2.1), we find that

$$\int_0^T \big\|\Delta\big(w^n(t) - w^m(t)\big)\big\|^2\,dt$$
$$\leqslant \tilde{c}\big(\|B\|_{H_0^2\times H}\big) + \int_0^T \big\|w_t^n(t) - w_t^m(t)\big\|^2\,dt$$
$$+ \int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w^n(t) - w^m(t)\big\rangle\,dt$$
$$+ \int_0^T \int_\Omega \big(g\big(w_t^m(t,x)\big) - g\big(w_t^n(t,x)\big)\big)\big(w^n(t,x) - w^m(t,x)\big)\,dx\,dt,$$
$$\text{for } T \geqslant 0. \tag{2.18}$$

Thus by (2.17) and (2.18) we have

$$\int_0^T E\big(w^n(t) - w^m(t)\big)\,dt$$

$$
\begin{aligned}
&\leqslant \delta T \operatorname{mes} \Omega + \tilde{c}\big(\|B\|_{H_0^2\times H}\big)\left(c(\delta)+\frac{1}{2}\right) \\
&\quad + c(\delta)\int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t) - w_t^m(t)\big\rangle\, dt \\
&\quad + \frac{1}{2}\int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w^n(t) - w^m(t)\big\rangle\, dt \\
&\quad + \frac{1}{2}\int_0^T\int_\Omega \big(g\big(w_t^m(t,x)\big) - g\big(w_t^n(t,x)\big)\big)\big(w^n(t,x) - w^m(t,x)\big)\, dx\, dt, \\
&\text{for } T \geqslant 0,
\end{aligned}
$$

which together with (2.2) implies

$$
\begin{aligned}
&E\big(w^n(T) - w^m(T)\big) \\
&\quad\leqslant \delta \operatorname{mes} \Omega + \frac{1}{T}\tilde{c}\big(\|B\|_{H_0^2\times H}\big)\left(c(\delta)+\frac{1}{2}\right) \\
&\qquad + \frac{1}{T}c(\delta)\int_0^T \big\langle \big[\mathcal{F}\big(w^n(t)\big), w^n(t)\big] - \big[\mathcal{F}\big(w^m(t)\big), w^m(t)\big], w_t^n(t) - w_t^m(t)\big\rangle\, dt \\
&\qquad + \frac{1}{T}\int_0^T\int_t^T \big\langle \big[\mathcal{F}\big(w^n(s)\big), w^n(s)\big] - \big[\mathcal{F}\big(w^m(s)\big), w^m(s)\big], w_t^n(s) - w_t^m(s)\big\rangle\, ds\, dt \\
&\qquad + \frac{1}{2T}\int_0^T\int_\Omega \big(g\big(w_t^m(t,x)\big) - g\big(w_t^n(t,x)\big)\big)\big(w^n(t,x) - w^m(t,x)\big)\, dx\, dt \\
&\qquad + \frac{1}{2T}\int_0^T \big\langle \big[\mathcal{F}\big(w^n(\tau)\big), w^n(\tau)\big] - \big[\mathcal{F}\big(w^m(\tau)\big), w^m(\tau)\big], w^n(\tau) - w^m(\tau)\big\rangle\, d\tau \\
&\quad\equiv \delta \operatorname{mes} \Omega + \frac{1}{T}\tilde{c}\big(\|B\|_{H_0^2\times H}\big)\left(c(\delta)+\frac{1}{2}\right) + K_1 + K_2 + K_3 + K_4. \qquad (2.19)
\end{aligned}
$$

By Lemma 3 we have

$$
\lim_{n\to\infty}\lim_{m\to\infty} K_1 = 0 \quad \text{and} \quad \lim_{n\to\infty}\lim_{m\to\infty} K_2 = 0. \qquad (2.20)
$$

Since $\{(w^n, w_t^n)\}_{n=1}^\infty$ is bounded in $C(0,T; H_0^2 \times H)$ and the embedding $H_0^2 \subset C(\bar{\Omega})$ is compact, by Arzela theorem $\{w^n\}_{n=1}^\infty$ is compact in $C(0,T; C(\bar{\Omega}))$. On the other hand, $\{w^n\}_{n=1}^\infty$ converges weakly star in $L_\infty(0,T; H_0^2)$. Thus $\{w^n\}_{n=1}^\infty$ strongly converges in $C(0,T; C(\bar{\Omega}))$.

Since by (1.5) and (2.1)

$$\int_0^T \int_\Omega \left|g\left(w_t^n(t,x)\right)\right| dx\,dt = \int_0^T \left[\int_{\{x:\ x\in\Omega,\ |w_t^n(t,x)|\geqslant 1\}} \left|g\left(w_t^n(t,x)\right)\right| dx + \int_{\{x:\ x\in\Omega,\ |w_t^n(t,x)|<1\}} \left|g\left(w_t^n(t,x)\right)\right| dx \right] dt$$
$$\leqslant \int_0^T \int_\Omega g\left(w_t^n(t,x)\right) w_t^n(t,x)\,dx\,dt + T \operatorname{mes}\Omega\left(g(1)+\left|g(-1)\right|\right)$$
$$\leqslant T \operatorname{mes}\Omega\left(g(1)+\left|g(-1)\right|\right) + \tilde{c}\left(\|B\|_{H_0^2\times H}\right),$$

we have

$$|K_3| \leqslant \frac{1}{T}\left\|w^n - w^m\right\|_{C(0,T;C(\bar{\Omega}))}\left(T \operatorname{mes}\Omega\left(g(1)+\left|g(-1)\right|\right) + \tilde{c}\left(\|B\|_{H_0^2\times H}\right)\right). \tag{2.21}$$

On the other hand, for K_4 we find that

$$|K_4| \leqslant \tilde{c}\left(\|B\|_{H_0^2\times H}\right)\left\|w^n - w^m\right\|_{C(0,T;C(\bar{\Omega}))}. \tag{2.22}$$

From (2.21) and (2.22) we obtain

$$\lim_{n\to\infty}\lim_{m\to\infty} K_3 = 0 \quad \text{and} \quad \lim_{n\to\infty}\lim_{m\to\infty} K_4 = 0. \tag{2.23}$$

Thus by (2.19), (2.20) and (2.23) we get

$$\limsup_{n\to\infty}\limsup_{m\to\infty} E\left(w^n(T) - w^m(T)\right) \leqslant \delta \operatorname{mes}\Omega + \frac{1}{T}\tilde{c}\left(\|B\|_{H_0^2\times H}\right)\left(c(\delta)+\frac{1}{2}\right),$$

consequently

$$\limsup_{n\to\infty}\sup_{p\in\mathbb{N}} E\left(w^{n+p}(T) - w^n(T)\right)$$
$$\leqslant 2\limsup_{n\to\infty}\sup_{p\in\mathbb{N}}\limsup_{m\to\infty} E\left(w^{n+p}(T) - w^m(T)\right) + 2\limsup_{n\to\infty}\limsup_{m\to\infty} E\left(w^m(T) - w^n(T)\right)$$
$$\leqslant 4\left(\delta \operatorname{mes}\Omega + \frac{1}{t}\tilde{c}\left(\|B\|_{H_0^2\times H}\right)\left(c(\delta)+\frac{1}{2}\right)\right),$$

which yields (2.16). □

3. Global attractors

In this section, we shall show the existence of the global attractor. To this end, we first prove the asymptotic compactness of $S(t)$ in $H_0^2 \times H$, which is given in the following theorem:

Theorem 2. *Assume the condition* (1.5) *holds. Then for any bounded subset B of $H_0^2 \times H$, the set $\{S(t_n)\theta_n\}_{n=1}^{\infty}$ is relatively compact in $H_0^2 \times H$, where $t_n \to \infty$ and $\{\theta_n\}_{n=1}^{\infty} \subset B$.*

Proof. Since B is bounded, by (2.1) we have $\sup_{t \geqslant 0} \sup_{\theta \in B} \|S(t)\theta\|_{H_0^2 \times H} < \infty$. Therefore there exists a bounded subset B_0 of $H_0^2 \times H$ such that $S(t)\theta \in B_0$, for every $t \geqslant 0$ and $\theta \in B$. Let $\varepsilon_m > 0$ and $\varepsilon_m \to 0$. By Lemma 4, for every ε_m there exists $T_m = T_m(B_0) > 0$ such that

$$\limsup_{k\to\infty} \sup_{p\in\mathbb{N}} \big\| S(T_m)\varphi_{k+p} - S(T_m)\varphi_k \big\|_{H_0^2 \times H} \leqslant \varepsilon_m, \tag{3.1}$$

where $\{\varphi_k\}_{n=1}^{\infty}$ is a sequence in B_0 and $\{S(t)\varphi_k\}_{n=1}^{\infty}$ weakly star converges in $L_\infty(0,\infty; H_0^2 \times H)$.

Now for ε_1, choose a subsequence $\{n_k^{(1)}\} \subset \{n\}$ such that $t_{n_k^{(1)}} \geqslant T_1$ and $\{S(t)S(t_{n_k^{(1)}} - T_1)\theta_{n_k^{(1)}}\}_{k=1}^{\infty}$ weakly star converges in $L_\infty(0,\infty; H_0^2 \times H)$. For ε_2, choose a subsequence $\{n_k^{(2)}\} \subset \{n_k^{(1)}\}$ such that $t_{n_k^{(2)}} \geqslant T_2$ and $\{S(t)S(t_{n_k^{(2)}} - T_2)\theta_{n_k^{(2)}}\}_{k=1}^{\infty}$ weakly star converges in $L_\infty(0,\infty; H_0^2 \times H)$. Continuing this procedure we have $\{n_k^{(1)}\} \supset \{n_k^{(2)}\} \supset \cdots \supset \{n_k^{(m)}\} \supset \cdots$, such that $t_{n_k^{(m)}} \geqslant T_m$ and $\{S(t)S(t_{n_k^{(m)}} - T_m)\theta_{n_k^{(m)}}\}_{k=1}^{\infty}$ weakly star converges in $L_\infty(0,\infty; H_0^2 \times H)$. Taking $\varphi_k = S(t_{n_k^{(m)}} - T_m)\theta_{n_k^{(m)}}$ in (3.1), we obtain

$$\limsup_{k\to\infty} \sup_{p\in\mathbb{N}} \big\| S\big(t_{n_{k+p}^{(m)}}\big)\theta_{n_{k+p}^{(m)}} - S\big(t_{n_k^{(m)}}\big)\theta_{n_k^{(m)}} \big\|_{H_0^2 \times H} \leqslant \varepsilon_m, \tag{3.2}$$

for every $m \in \mathbb{N}$.

Now we construct the diagonal subsequence $\{S(t_{n_k^{(k)}})\theta_{n_k^{(k)}}\}$. Since for every $m \in \mathbb{N}$, the sequence $\{S(t_{n_k^{(k)}})\theta_{n_k^{(k)}}\}_{k=m}^{\infty}$ is a subsequence of $\{S(t_{n_k^{(m)}})\theta_{n_k^{(m)}}\}_{k=1}^{\infty}$, by (3.2) we have

$$\limsup_{k\to\infty} \sup_{p\in\mathbb{N}} \big\| S\big(t_{n_{k+p}^{(k+p)}}\big)\theta_{n_{k+p}^{(k+p)}} - S\big(t_{n_k^{(k)}}\big)\theta_{n_k^{(k)}} \big\|_{H_0^2 \times H} \leqslant \varepsilon_m.$$

Since $\varepsilon_m \to 0$, the last inequality means that the sequence $\{S(t_{n_k^{(k)}})\theta_{n_k^{(k)}}\}_{k=1}^{\infty}$ is a Cauchy sequence in $H_0^2 \times H$ and consequently this sequence strongly converges in $H_0^2 \times H$. In other words, the sequence $\{S(t_n)\theta_n\}_{n=1}^{\infty}$ has a subsequence which is strongly convergent in $H_0^2 \times H$. It can be seen in a similar way that every subsequence of $\{S(t_n)\theta_n\}_{n=1}^{\infty}$ has a subsequence strongly convergent in $H_0^2 \times H$. Thus the set $\{S(t_n)\theta_n\}_{n=1}^{\infty}$ is relatively compact in $H_0^2 \times H$. □

Since by (2.1) the problem (1.1)–(1.4) admits a "good" Lyapunov function (see [11, p. 41]) $L(w(t)) = E(w(t)) + \frac{1}{4}\|\Delta\mathcal{F}(w(t))\|^2 - \langle h, w(t)\rangle$ and since the set of stationary solutions is bounded in H_0^2, using the results of [11, pp. 49–50], we can formulate our main result.

Theorem 3. *Assume that* (1.5) *holds. Then problem* (1.1)–(1.4) *has a global attractor in $H_0^2 \times H$, which is invariant and compact.*

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