Improved Exponential Type Estimators for Finite Population Mean in Stratified Random Sampling

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Abstract

In this paper new exponential type estimators are suggested for estimating the population mean in stratified random sampling. The biases and mean square errors of the suggested estimators are obtained. Also an empirical study is carried out to show the properties of the proposed estimators.

Keywords: Ratio estimator; Exponential estimator; Mean square error; Efficiency; Stratified random sampling.

1. Introduction

In sampling theory the role of auxiliary information has a great importance. It is well known that using suitable auxiliary information such as population total, mean, skewness, correlation, attribute etc. can improve precision of estimates. Many authors have used this information in their ratio, product and exponential type estimators to get more efficient estimator under different sampling design. In stratified random sampling scheme the authors including Diana (1993), Singh and Vishwakarma (2008), Koyuncu and Kadilar (2009, 2010), Tailor et al. (2012), Koyuncu (2013), Yadav et al. (2014) etc. have suggested estimators using auxiliary information. In this paper we have tried to generalize Singh et al. (2008) and Yadav et al. (2014) estimators and suggested two exponential type estimators in stratified random sampling.

Consider a finite population $U=(u_1,u_2,...,u_N)$ of size *N* and let *y* and *x*, respectively, be the study and auxiliary variable associated with each unit u_j $(j=1,2,....,N)$ of the population. Assume that the population of size, *N*, is stratified into *L* strata with *h*-th stratum containing N_h units, where $h=1,2,...,L$ such that $\sum_{h=1}^{L} N_h = N$. A simple random sample of size n_h is drawn without replacement from the *h*-th stratum such that $\sum_{h=1}^{L} n_h = n$. Let (y_{hi}, x_{hi}) denote the observed values of *y* and *x* on the *i*-th unit of the *h*-th stratum, where $i=1,2,...,N_h$. Moreover, let $\bar{y}_h = \sum_{i=1}^{n_h} \frac{y_{hi}}{n_h}$, $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$, and $\bar{Y}_h = \sum_{i=1}^{N_h} \frac{y_{hi}}{N_h}$, $\bar{Y} = \bar{Y}_{st} = \sum_{h=1}^{L} W_h \bar{Y}_h$ be the sample and population means of *y*, respectively, where $W_h = \frac{N_h}{N}$ is the stratum weight.

To obtain the bias and *MSE*, let us define $e_0 = (\bar{y}_{st} - \bar{Y})/\bar{Y}$ and $e_1 = (\bar{x}_{st} - \bar{X})/\bar{X}$. Using these notations, $E(e_0) = E(e_1) = 0$,

$$V_{rs} = \sum_{h=1}^{L} W_h^{r+s} \frac{E\left[(\bar{y}_h - \bar{Y}_h)^r (\bar{x}_h - \bar{X}_h)^s\right]}{\bar{Y}^r \bar{X}^s}. \tag{1}$$

Using (1), we can write

$$E(e_0^2) = \frac{\sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2}{\bar{Y}^2} = V_{20},\quad E(e_1^2) = \frac{\sum_{h=1}^{L} W_h^2 \lambda_h S_{xh}^2}{\bar{X}^2} = V_{02} \qquad E(e_0 e_1) = \frac{\sum_{h=1}^{L} W_h^2 \lambda_h S_{xyh}}{\bar{X}\bar{Y}} = V_{11}$$

where

$$S_{yh}^2 = \frac{\sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2}{N_h - 1},\quad S_{xh}^2 = \frac{\sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2}{N_h - 1},\quad S_{xyh} = \frac{\sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)(y_{hi} - \bar{Y}_h)}{N_h - 1},\quad \lambda_h = \frac{1 - f_h}{n_h},$$

and $f_h = \dfrac{n_h}{N_h}$.

Singh et al. (2008) have suggested estimators of population mean using auxiliary information, given by

$$t_1 = \bar{y}_{st} \exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right) \tag{2}$$

$$t_2 = \bar{y}_{st} \exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right) \tag{3}$$

where $\bar{x}_1 = \sum_{h=1}^{L} W_h (\bar{x}_h + C_{xh})$, $\bar{X}_1 = \sum_{h=1}^{L} W_h (\bar{X}_h + C_{xh})$, $\bar{x}_2 = \sum_{h=1}^{L} W_h (\bar{x}_h + \beta_{2h}(x))$ and $\bar{X}_2 = \sum_{h=1}^{L} W_h (\bar{X}_h + \beta_{2h}(x))$.

To generalize t_1 and t_2 estimators in a class we can define

$$k_1 = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st} + 2A_{st}}\right) \tag{4}$$

where $A_{st} = \sum_{h=1}^{L} W_h A_h$ and A_h may be some population information of the auxiliary variable for *h*-th stratum such as S_{xh}, coefficient of variation C_{xh}, skewness $\beta_{1h}(x)$, kurtosis $\beta_{2h}(x)$, correlation coefficient $\rho_{h(xy)}$. Note that t_1 and t_2 estimators are member of k_1 if we take A_h as C_{xh} and $\beta_{2h}(x)$ respectively as given in Table2.

Expressing (4) in terms of e's, we have

$$(k_1 - \bar{Y}) = \bar{Y}\left\{ -\frac{\bar{X}}{2(\bar{X} + A_{st})} e_1 + \frac{3\bar{X}^2}{8(\bar{X} + A_{st})^2} e_1^2 + e_0 - \frac{\bar{X}}{2(\bar{X} + A_{st})} e_0 e_1 \right\} \tag{5}$$

Squaring both sides of (5) and neglecting the terms of e's having greater than two, we can write

$$\left(k_1-\bar{Y}\right)^2=\bar{Y}^2\left\{e_0^2+\frac{\bar{X}^2}{4\left(\bar{X}+A_{st}\right)^2}e_1^2-\frac{\bar{X}}{\left(\bar{X}+A_{st}\right)}e_0e_1\right\} \tag{6}$$

Taking expectation of (6) we get the MSE of k_1, under the first order of approximation is given by

$$MSE(k_1)=\left\{\sum_{h=1}^{L}W_h^2\lambda_h S_{yh}^2+\frac{\bar{Y}^2}{4(\bar{X}+A_{st})^2}\sum_{h=1}^{L}W_h^2\lambda_h S_{xh}^2-\frac{\bar{Y}}{(\bar{X}+A_{st})}\sum_{h=1}^{L}W_h^2\lambda_h S_{xyh}\right\}$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left(S_{yh}^2+\frac{\bar{Y}^2}{4\left(\bar{X}+A_{st}\right)^2}S_{xh}^2-\frac{\bar{Y}}{\left(\bar{X}+A_{st}\right)}S_{xyh}\right)$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left(S_{yh}^2+\frac{1}{4}R_{k1}^2S_{xh}^2-R_{k1}S_{xyh}\right) \tag{7}$$

where $R_{k1}=\dfrac{\bar{Y}}{\left(\bar{X}+A_{st}\right)}$.

Secondly Singh et al. (2008) have suggested estimators using more auxiliary information

$$t_3=\bar{y}_{st}\exp\left(\frac{\bar{X}_3-\bar{x}_3}{\bar{X}_3+\bar{x}_3}\right) \tag{8}$$

$$t_4=\bar{y}_{st}\exp\left(\frac{\bar{X}_4-\bar{x}_4}{\bar{X}_4+\bar{x}_4}\right) \tag{9}$$

where $\bar{x}_3=\sum_{h=1}^{L}W_h\left(\bar{x}_h\beta_{2h}(x)+C_{xh}\right)$, $\quad \bar{X}_3=\sum_{h=1}^{L}W_h\left(\bar{X}_h\beta_{2h}(x)+C_{xh}\right)$,

$\bar{x}_4=\sum_{h=1}^{L}W_h\left(\bar{x}_hC_{xh}+\beta_{2h}(x)\right)$, $\quad \bar{X}_4=\sum_{h=1}^{L}W_h\left(\bar{X}_hC_{xh}+\beta_{2h}(x)\right)$.

To generalize t_3 and t_4 estimators let us define following class

$$k_2=\bar{y}_{st}\exp\left(\frac{A_{st}^*-a_{st}^*}{A_{st}^*+a_{st}^*+2B_{st}}\right) \tag{10}$$

where $A_{st}^*=\sum_{h=1}^{L}W_h\bar{X}_hA_h$, $a_{st}^*=\sum_{h=1}^{L}W_h\bar{x}_hA_h$, $B_{st}=\sum_{h=1}^{L}W_hB_h$, A_h and B_h may be some population information of the auxiliary variable for *h*-th stratum such as S_{xh}, coefficient of variation C_{xh}, skewness $\beta_{1h}(x)$, kurtosis $\beta_{2h}(x)$, correlation coefficient $\rho_{h(xy)}$.

To obtain the bias and *MSE*, let us define

$$e_1^*=\frac{\left(a_{st}^*-A_{st}^*\right)}{A_{st}^*}=\frac{\sum_{h=1}^{L}W_hA_h\left(\bar{x}_h-\bar{X}_h\right)}{A_{st}^*}, \qquad V_{r,s,t}^*=\sum_{h=1}^{L}W_h^{r+s}A_h^s\frac{E\left[\left(\bar{y}_h-\bar{Y}_h\right)^r\left(\bar{x}_h-\bar{X}_h\right)^{s+t}\right]}{\bar{Y}^rA_{st}^{*s}\bar{X}^t}$$

Using these notations we can write following notations

$$E(e_0^2)=V_{200}^*=V_{20} \qquad E\left(e_1^{*2}\right)=\frac{\sum_{h=1}^{L}W_h^2A_h^2\lambda_h S_{xh}^2}{A_{st}^{*2}}=V_{020}^*$$

$$E\left(e_0e_1^*\right)=\frac{\sum_{h=1}^{L}W_h^2A_h\lambda_h S_{xyh}}{\bar{Y}A_{st}^*}=V_{110}^*$$

$$E\left(e_1e_1^*\right)=\frac{\sum_{h=1}^{L}W_h^2A_h\lambda_h S_x^2}{\bar{X}A_{st}^*}=V_{011}^*, \qquad E\left(e_1^2\right)=\frac{\sum_{h=1}^{L}W_h^2\lambda_h S_{xh}^2}{\bar{X}^2}=V_{002}^*=V_{02},$$

$$E(e_0e_1)=\frac{\sum_{h=1}^{L}W_h^2\lambda_h S_{xyh}}{\bar{X}\bar{Y}}=V_{101}^*=V_{11}.$$

Expressing (10) in terms of e's, we have

$$k_2-\bar{Y}=\bar{Y}\left(-\frac{A_{st}^*}{2\left(A_{st}^*+B_{st}\right)}e_1^*+\frac{3}{8}\frac{\left(A_{st}^*\right)^2}{\left(A_{st}^*+B_{st}\right)^2}e_1^{*2}+e_0-\frac{1}{2}A_{st}^*e_0e_1^*+\frac{3}{8}\frac{\left(A_{st}^*\right)^2}{\left(A_{st}^*+B_{st}\right)^2}e_0e_1^{*2}\right) \quad (11)$$

Squaring both sides of (11) and neglecting the terms of e's having greater than two, we can write

$$\left(k_2-\bar{Y}\right)^2=\bar{Y}^2\left(e_0^2+\frac{1}{4\left(A_{st}^*+C_{st}\right)^2}A_{st}^{*2}e_1^{*2}-\frac{A_{st}^*}{\left(A_{st}^*+C_{st}\right)}e_1^*e_0\right) \quad (12)$$

Taking expectation both sides of (12) then we get *corrected MSE* of Singh et al. (2008) as follows:

$$MSE\left(k_2\right)=\bar{Y}^2\left(\frac{\sum_{h=1}^{L}W_h^2\lambda_h S_{yh}^2}{\bar{Y}^2}+\frac{A_{st}^{*2}}{4\left(A_{st}^*+C_{st}\right)^2}\frac{\sum_{h=1}^{L}W_h^2A_h^2\lambda_h S_{xh}^2}{A_{st}^{*2}}-\frac{A_{st}^*}{\left(A_{st}^*+C_{st}\right)}\frac{\sum_{h=1}^{L}W_h^2A_h\lambda_h S_{xyh}}{\bar{Y}A_{st}^*}\right)$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left\{S_{yh}^2-\frac{\bar{Y}}{\left(A_{st}^*+C_{st}\right)}A_hS_{xyh}+\frac{1}{4}\frac{\bar{Y}^2}{\left(A_{st}^*+C_{st}\right)^2}A_h^2S_{xh}^2\right\}$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left\{S_{yh}^2-R_{k2}S_{xyh}+\frac{1}{4}R_{k2}^2S_{xh}^2\right\} \quad (13)$$

where $R_{k2}=\dfrac{\bar{Y}A_h}{\left(A_{st}^*+B_{st}\right)}$.

Note that t_3 and t_4 estimators are member of k_2 for appropriate values of A_h and B_h as given in Table2. Also we have corrected the MSE of t_3 and t_4 estimators as given in Appendix with detail.

Yadav et al. (2014) have suggested ratio and product exponential estimators are given by

$$t_{RC}^{(a)} = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + (a-1)\bar{x}_{st}} \right) \tag{14}$$

$$t_{PC}^{(b)} = \bar{y}_{st} \exp\left(\frac{\bar{x}_{st} - \bar{X}}{\bar{X} + (b-1)\bar{x}_{st}} \right) \tag{15}$$

$$MSE_{\min}\left(t_{PC}^{(b)}\right) = MSE_{\min}\left(t_{RC}^{(a)}\right) = \bar{Y}^2 V_{20}\left[1 - \frac{V_{11}^2}{V_{02}V_{20}}\right] = \sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2 \left\{1 - \rho_c^2\right\} \tag{16}$$

where $\rho_c = \dfrac{\sum_{h=1}^{L} W_h^2 \lambda_h S_{xyh}}{\sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{xh}^2}\sqrt{\sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2}}$.

In sampling literature many authors have suggested estimators using auxiliary information. Koyuncu and Kadilar (2010) defined a general combined class in stratified random sampling to get a compact expression for the asymptotic variance and avoid tedious calculations, given by

$$t_c = g(\bar{y}_{st}, u_{st}) \tag{17}$$

where $u_{st} = \bar{x}_{st} / \bar{X}$ and $g(\bar{y}_{st}, u_{st})$ is a function of $\bar{y}_{st}$ and u_{st}. To study the properties of t_c we assume following regularity conditions:

1. The point $(\bar{y}_{st}, u_{st})$ assumes the value in a closed convex subset R_2 of two dimensional real space containing the point $(\bar{Y},1)$,
2. The function $g(\bar{y}_{st}, u_{st})$ is continuous and bounded in R_2,
3. $g(\bar{Y},1) = \bar{Y}$ and $g_0(\bar{Y},1) = 1$, where $g_0(\bar{Y},1)$ denotes the first order partial derivative of g with respect to $\bar{y}_{st}$,
4. The first and second order partial derivatives of $g(\bar{y}_{st}, u_{st})$ exist and are continuous and bounded in R_2.

Note that this class contains many estimator. We can say that Yadav et al. (2014) ratio and product exponential estimators are member of this class.
The bias and the *MSE* of t_c are respectively given by

$$B(t_c) = V_{02} g_2 + \bar{Y} V_{11} g_3 + \bar{Y}^2 V_{20} g_4, \tag{18}$$

$$MSE(t_c) = \bar{Y}^2 V_{20} + V_{02} g_1^2 + 2\bar{Y} V_{11} g_1. \tag{19}$$

where

$$g_1 = \left.\frac{\partial g}{\partial u_{st}}\right|_{\bar{y}_{st}=\bar{Y}, u_{st}=1}, \; g_2 = \left.\frac{1}{2}\frac{\partial^2 g}{\partial u_{st}^2}\right|_{\bar{y}_{st}=\bar{Y}, u_{st}=1}, \; g_3 = \left.\frac{1}{2}\frac{\partial^2 g}{\partial \bar{y}_{st}\partial u_{st}}\right|_{\bar{y}_{st}=\bar{Y}, u_{st}=1}, \; g_4 = \left.\frac{1}{2}\frac{\partial^2 g}{\partial \bar{y}_{st}^2}\right|_{\bar{y}_{st}=\bar{Y}, u_{st}=1}.$$

By using the optimal value of $g_1^* = -\frac{\bar{Y}V_{11}}{V_{02}}$, the minimum *MSE* of the estimators in the class t_c is found as

$$MSE(t_c)_{\min} = \bar{Y}^2 V_{20}\left[1 - \frac{V_{11}^2}{V_{02}V_{20}}\right] \tag{20}$$

which is also the *MSE* of combined regression type estimators and minimum *MSE* of Yadav et. al [8].

2. Proposed Exponential Type Estimators

Motivated by Koyuncu (2012) we propose the following estimator

$$\bar{y}_N = \left[w_1 \bar{y}_{st} + w_2\left(\frac{\bar{x}_{st}}{\bar{X}}\right)^{\gamma}\right]\exp\left[\frac{A_{st}(\bar{X} - \bar{x}_{st})}{A_{st}(\bar{X} + \bar{x}_{st}) + 2B_{st}}\right] \tag{21}$$

Some new estimators, which are generated from (21) for different combinations of A_h, B_h and γ are given in Table 2. To obtain the MSE, we are applying the same procedure as follows:

$$\bar{y}_N = \left[w_1\bar{Y}(1+e_0) + w_2(1+e_1)^{\gamma}\right]\left\{1 - \frac{\bar{X}A_{st}}{2(\bar{X}A_{st} + B_{st})}e_1 + \frac{3}{8}\frac{\bar{X}^2 A_{st}^2}{(\bar{X}A_{st} + B_{st})^2}e_1^2 + \ldots\right\} \tag{22}$$

$$\begin{aligned} \bar{y}_N - \bar{Y} = \Big\{&\bar{Y}(w_1 - 1) + w_1\left(\bar{Y}e_0 - \frac{1}{2}\psi\bar{Y}e_1 - \frac{1}{2}\psi\bar{Y}e_1e_0 + \frac{3}{8}\psi^2\bar{Y}e_1^2\right) \\ &+ w_2\left(1 + \gamma e_1 + \frac{\gamma(\gamma-1)}{2}e_1^2 - \frac{1}{2}\psi e_1 - \frac{1}{2}\psi\gamma e_1^2 + \frac{3}{8}\psi^2 e_1^2\right)\Big\} \end{aligned} \tag{23}$$

where $\psi = \frac{\bar{X}A_{st}}{(\bar{X}A_{st} + B_{st})}$.

Squaring both sides of (23) and taking expectation we have

$$\begin{aligned} MSE(\bar{y}_N) = E\Big\{&\bar{Y}^2 + \bar{Y}^2 w_1^2\left(1 + e_0^2 + \psi^2 e_1^2 - 2\psi e_0 e_1\right) + w_2^2\left(1 + \left(\gamma^2 + \psi^2 + \gamma(\gamma-1) - \psi\gamma - \gamma\psi\right)e_1^2\right) \\ &+ \bar{Y}^2 w_1\left(\psi e_0 e_1 - 2 - \frac{3}{4}\psi^2 e_1^2\right) + \bar{Y}w_2\left(\left(\psi\gamma - \frac{3}{4}\psi^2 - \gamma(\gamma-1)\right)e_1^2 - 2\right) \\ &+ \bar{Y}w_1w_2\left(2 + 2(\gamma - \psi)e_0e_1 + \left(2\psi^2 + \gamma(\gamma-1) - 2\gamma\psi\right)e_1^2\right)\Big\} \end{aligned} \tag{24}$$

The MSE of the proposed estimator is given by

$$MSE(\bar{y}_N) = \left[\bar{Y}^2 w_1^2 H + w_2^2 B + \bar{Y}^2 w_1 D + \bar{Y}w_2 G + \bar{Y}^2 + \bar{Y}w_1w_2 F\right] \tag{25}$$

where $H=\left(1+V_{20}+\psi^2V_{02}-2\psi V_{11}\right)$, $B=\left(1+\left(\gamma^2+\psi^2+\gamma(\gamma-1)-2\gamma\psi\right)V_{02}\right)$,

$D=\left(\psi V_{11}-2-\frac{3}{4}\psi^2V_{02}\right)$, $G=\left(\left(\psi\gamma-\frac{3}{4}\psi^2-\gamma(\gamma-1)\right)V_{02}-2\right)$

$F=\left(2+2(\gamma-\psi)V_{11}+\left(2\psi^2+\gamma(\gamma-1)-2\gamma\psi\right)V_{02}\right)$

The optimum values of w_1 and w_2, obtained by minimizing (25) respectively, are given by

$$w_{1(opt)}=\frac{GF-2DB}{\left(4BH-F^2\right)}\;w_{2(opt)}=\bar{Y}\frac{DF-2GH}{\left(4HB-F^2\right)} \tag{26}$$

Substituting these values in (25) we get the minimum MSE of $\bar{y}_N$ as follows:

$$MSE(\bar{y}_N)=\bar{Y}^2\left[1-\frac{BD^2-DFG+HG^2}{\left(4BH-F^2\right)}\right] \tag{27}$$

Note that the optimum choice of the constants w_1 and w_2 involve unknown parameters. These quantities can be guessed quite accurately through pilot sample survey or sample data or experience gathered in due course of time (Koyuncu and Kadilar (2009, 2010). Secondly we consider following estimator

$$\bar{y}_M=\left[w_1^*\bar{y}_{st}+w_2^*\left(\frac{\bar{x}_{st}}{\bar{X}}\right)^{\gamma}\right]\exp\left[\frac{A_{st}^*-a_{st}^*}{A_{st}^*+a_{st}^*+2B_{st}}\right] \tag{28}$$

Some new estimators, which are generated from (28) for different combinations of A_h, B_h and γ are given in Table 2. Expressing (28) in terms of e's, we have

$$\bar{y}_M=\left[w_1^*\bar{y}_{st}+w_2^*\left(1+e_1\right)^{\gamma}\right]\left\{1-\frac{A_{st}^*}{2\left(A_{st}^*+B_{st}\right)}e_1^*+\frac{3\left(A_{st}^*\right)^2}{8\left(A_{st}^*+B_{st}\right)^2}e_1^{*2}+...\right\} \tag{29}$$

$$\left(\bar{y}_M-\bar{Y}\right)=\left\{w_1^*\bar{Y}-\bar{Y}+w_2^*+w_2^*\frac{\gamma(\gamma-1)}{2}e_1^2-\frac{1}{2}\psi^*w_1^*\bar{Y}e_0e_1^*\right.$$
$$\left.-\frac{1}{2}w_2^*\psi^*\gamma e_1e_1^*+\frac{3}{8}w_1^*\psi^{*2}\bar{Y}e_1^{*2}+\frac{3}{8}w_2^*\psi^{*2}e_1^{*2}\right\} \tag{30}$$

where $\psi^*=\frac{A_{st}^*}{\left(A_{st}^*+B_{st}\right)}$.

Squaring both sides of (30) and taking expectation we have

$$MSE(\bar{y}_M)=E\left\{\bar{Y}^2+w_1\bar{Y}^2\left(-2+\psi^*e_0e_1^*-\frac{3}{4}\psi^{*2}e_1^{*2}\right)+w_1^2\bar{Y}^2\left(1+e_0^2+\psi^{*2}e_1^{*2}-2\psi^*e_0e_1^*\right)\right.$$
$$+\bar{Y}w_2\left(-2+\psi^*\gamma e_1e_1^*-\gamma(\gamma-1)e_1^2-\frac{3}{4}\psi^{*2}e_1^{*2}\right)+w_2^2\left(1+\left(\gamma^2+\gamma(\gamma-1)\right)e_1^2+\psi^{*2}e_1^{*2}-2\gamma\psi e_1e_1^*\right) \tag{31}$$
$$\left.+w_1w_2\bar{Y}\left(2-2\gamma\psi^*e_1e_1^*+\gamma(\gamma-1)e_1^2-2\psi^*e_0e_1^*+2\psi^{*2}e_1^{*2}+2\gamma e_0e_1\right)\right\}$$

The MSE of the proposed estimator is given by

$$MSE\left(\bar{y}_M\right)=\left[\bar{Y}^2+w_1^{*2}\bar{Y}^2H^*+w_2^{*2}B^*+w_1^*\bar{Y}^2D^*+\bar{Y}w_2^*G^*+w_1^*w_2^*\bar{Y}F^*\right] \tag{32}$$

where

$$H^*=\left(1+V_{200}^*+\psi^{*2}V_{020}^*-2\psi^*V_{110}^*\right)$$

$$B^*=\left(1+\left(\gamma^2+\gamma\left(\gamma-1\right)\right)V_{002}^*+\psi^{*2}V_{020}^*-2\gamma\psi^*V_{011}^*\right)$$

$$D^*=\left(-2+\psi^*V_{110}^*-\frac{3}{4}\psi^{*2}V_{020}^*\right)$$

$$G^*=\left(-2+\psi^*\gamma V_{011}^*-\gamma\left(\gamma-1\right)V_{002}^*-\frac{3}{4}\psi^{*2}V_{020}^*\right)$$

$$F^*=\left(2-2\gamma\psi^*V_{011}^*+\gamma\left(\gamma-1\right)V_{002}^*-2\psi^*V_{110}^*+2\psi^{*2}V_{020}^*+2\gamma V_{101}^*\right)$$

The optimum values of w_1 and w_2 are given

$$w_{1(opt)}^*=\frac{G^*F^*-2D^*B^*}{\left(4B^*A^*-F^{*2}\right)} \qquad w_{2(opt)}^*=\bar{Y}\frac{D^*F^*-2G^*H^*}{\left(4B^*A^*-F^{*2}\right)} \tag{33}$$

The minimum MSE of $\bar{y}_M$ at optimum values of w_1^* and w_2^*, is given by

$$MSE\left(\bar{y}_M\right)=\bar{Y}^2\left[1-\frac{B^*D^{*2}-D^*F^*G^*+H^*G^{*2}}{\left(4B^*H^*-F^{*2}\right)}\right] \tag{34}$$

3. Numerical Example

To analyze the performance of proposed estimator we use the data concerning the number of teachers as the study variable and the number of students as the auxiliary variable in both primary and secondary schools for 923 districts at 6 regions (as 1:Marmara 2:Agean 3:Mediterranean 4:Central Anatolia 5:Black Sea 6:East and Southeast Anatolia) in Turkey in 2007 (Source: Koyuncu and Kadilar (2009)). The summary statistics about the population is given in Table 1.

We have generated some new estimators using suitable values for A_h, B_h and γ. The *MSE* values of the k_1, k_2, $\bar{y}_N$ and $\bar{y}_M$ estimators have been obtained. These values are given in Table 2. When we examine Table 2, we observe that the proposed $\bar{y}_{M4}$ estimator has the smallest. From Table 2 we can conclude that using suitable auxiliary information is very important otherwise we can get worst estimates in each class.

4. Conclusion

In this paper we have tried to generalize Yadav et al. (2014) estimators and corrected mean square error formula of Singh et al. (2008) and suggested two exponential type estimators in stratified random sampling. To see the performance of estimators we used a real data. We investigate the usage of suitable information of auxiliary variable. We have found that suggested exponential class of estimators is highly efficient than Yadav et al. (2014) and combined regression estimator.

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Table 1: Data Statistics

N_1=127	N_2=117	N_3=103
N_4=170	N_5=205	N_6=201
n_1=31	n_2=21	n_3=29
n_4=38	n_5=22	n_6=39
S_{y1}=883.835	S_{y2}=644.922	S_{y3}=1033.467
S_{y4}=810.585	S_{y5}=403.654	S_{y6}=711.723
$\bar{Y}_1$=703.74	$\bar{Y}_2$=413	$\bar{Y}_3$=573.17
$\bar{Y}_4$=424.66	$\bar{Y}_5$=267.03	$\bar{Y}_6$=393.84
S_{x1}=30486.751	S_{x2}=15180.769	S_{x3}=27549.697
S_{x4}=18218.931	S_{x5}=8497.776	S_{x6}=23094.141
$\bar{X}_1$=20804.59	$\bar{X}_2$=9211.79	$\bar{X}_3$=14309.30
$\bar{X}_4$=9478.85	$\bar{X}_5$=5569.95	$\bar{X}_6$=12997.59
S_{xy1}=25237153.52	S_{xy2}=9747942.85	S_{xy3}=28294397.04
S_{xy4}=14523885.53	S_{xy5}=3393591.75	S_{xy6}=15864573.97
ρ_{xy1}=0.936	ρ_{xy2}=0.996	ρ_{xy3}=0.994
ρ_{xy4}=0.983	ρ_{xy5}=0.989	ρ_{xy6}=0.965
$\beta_2(x_1)$=4.593	$\beta_2(x_2)$=18.543	$\beta_2(x_3)$=15.446
$\beta_2(x_4)$=10.162	$\beta_2(x_5)$=21.947	$\beta_2(x_6)$=23.114

Table 2: Some members of k_1, k_2, $\bar{y}_N$ and $\bar{y}_M$ and MSE values

	A_h	B_h	γ	MSE
t_1	C_{xh}			602.5943
t_2	$\beta_{2h}(x)$			603.8937
t_3	$\beta_{2h}(x)$	C_{xh}		688.4160
t_4	C_{xh}	$\beta_{2h}(x)$		589.5579
$\bar{y}_{N1}$	$\beta_{2h}(x)$	C_{xh}	0	53.1396
$\bar{y}_{N2}$	$\beta_{2h}(x)$	C_{xh}	1	556.4491
$\bar{y}_{N3}$	$\beta_{2h}(x)$	C_{xh}	2	342.4030
$\bar{y}_{N4}$	C_{xh}	$\beta_{2h}(x)$	0	52.1180
$\bar{y}_{N5}$	C_{xh}	$\beta_{2h}(x)$	1	557.3786
$\bar{y}_{N6}$	C_{xh}	$\beta_{2h}(x)$	2	342.5917
$\bar{y}_{N7}$	$\rho_{h(xy)}$	C_{xh}	0	52.9641
$\bar{y}_{N8}$	$\rho_{h(xy)}$	C_{xh}	1	556.6087
$\bar{y}_{N9}$	$\rho_{h(xy)}$	C_{xh}	2	342.4354
$\bar{y}_{M1}$	C_{xh}	$\beta_{2h}(x)$	0	51.1939
$\bar{y}_{M2}$	C_{xh}	$\beta_{2h}(x)$	1	551.1354
$\bar{y}_{M3}$	C_{xh}	$\beta_{2h}(x)$	2	325.882
$\bar{y}_{M4}$	$\rho_{h(xy)}$	C_{xh}	0	**48.8562**
$\bar{y}_{M5}$	$\rho_{h(xy)}$	C_{xh}	1	505.5486
$\bar{y}_{M6}$	$\rho_{h(xy)}$	C_{xh}	2	331.6634
$MSE_{\min}\left(t_{PC}^{(b)}\right) = MSE_{\min}\left(t_{RC}^{(a)}\right)$				194.2832

Appendix

$$k_2 = \bar{y}_{st}\exp\left(\frac{A_{st}^* - a_{st}^*}{A_{st}^* + a_{st}^* + 2B_{st}}\right) \quad (1)$$

If we take A_h as $\beta_{2h}(x)$ and B_h as C_{xh} in A_{st}^*, a_{st}^* and B_{st} respectively we can get $A_{st}^* = \sum_{h=1}^{L} W_h \bar{X}_h \beta_{2h}(x)$, $a_{st}^* = \sum_{h=1}^{L} W_h \bar{x}_h \beta_{2h}(x)$, $B_{st} = \sum_{h=1}^{L} W_h C_{xh}$. Rewriting these values in (1) we get Singh et al (2008) estimator.

$$t_3 = \bar{y}_{st}\exp\left(\frac{\bar{X}_3 - \bar{x}_3}{\bar{X}_3 + \bar{x}_3}\right) \quad (2)$$

where $\bar{x}_3 = \sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)$ $\quad \bar{X}_3 = \sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)$

To find the MSE formula let us define $e_0 = \frac{\left(\bar{y}_{st} - \bar{Y}\right)}{\bar{Y}}$, $e_1^* = \frac{a_{st}^* - A_{st}^*}{A_{st}^*} = \frac{\sum_{h=1}^{L} W_h A_h\left(\bar{x}_h - \bar{X}_h\right)}{A_{st}^*}$

such that $E(e_0) = E(e_1^*) = 0$, $E(e_0^2) = \frac{\sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2}{\bar{Y}^2}$, $E(e_1^{*2}) = \frac{\sum_{h=1}^{L} W_h^2 A_h^2 \lambda_h S_{xh}^2}{A_{st}^{*2}}$,

$$E(e_0 e_1^*) = \frac{\sum_{h=1}^{L} W_h^2 A_h \lambda_h S_{xyh}}{\bar{Y} A_{st}^*}$$

Expressing k_2 with e term, we have

$$k_2 = \bar{Y}(1+e_0)\exp\left(\frac{A_{st}^* - A_{st}^*(1+e_1^*)}{A_{st}^* + A_{st}^*(1+e_1^*) + 2B_{st}}\right)$$

$$k_2 = \bar{Y}\left(1 - \frac{1}{2\left(A_{st}^* + B_{st}\right)} A_{st}^* e_1^* + \frac{3}{8}\frac{\left(A_{st}^*\right)^2}{\left(A_{st}^* + B_{st}\right)^2} e_1^{*2} + e_0 - \frac{1}{2} A_{st}^* e_0 e_1^* + \frac{3}{8}\frac{\left(A_{st}^*\right)^2}{\left(A_{st}^* + B_{st}\right)^2} e_0 e_1^{*2}\right)$$

$$k_2 - \bar{Y} = \bar{Y}\left(-\frac{A_{st}^*}{2\left(A_{st}^* + B_{st}\right)} e_1^* + \frac{3}{8}\frac{\left(A_{st}^*\right)^2}{\left(A_{st}^* + B_{st}\right)^2} e_1^{*2} + e_0 - \frac{1}{2} A_{st}^* e_0 e_1^* + \frac{3}{8}\frac{\left(A_{st}^*\right)^2}{\left(A_{st}^* + B_{st}\right)^2} e_0 e_1^{*2}\right) \quad (3)$$

Squaring both sides of (3) and neglecting the terms of e’s having power greater than two, we have

$$\left(k_2 - \bar{Y}\right)^2 = \bar{Y}^2\left(\frac{A_{st}^{*2}}{4\left(A_{st}^* + B_{st}\right)^2} e_1^{*2} + e_0^2 - \frac{A_{st}^*}{\left(A_{st}^* + B_{st}\right)} e_1^* e_0\right) \quad (4)$$

Taking expectation of both sides of (4) we get MSE of estimator k_2

$$MSE\left(k_2\right)=\bar{Y}^2\left(\frac{1}{4}\frac{A_{st}^{*2}}{\left(A_{st}^*+B_{st}\right)^2}\frac{\sum_{h=1}^{L}W_h^2A_h^2\lambda_hS_{xh}^2}{A_{st}^{*2}}+\frac{\sum_{h=1}^{L}W_h^2\lambda_hS_{yh}^2}{\bar{Y}^2}-\frac{A_{st}^*}{\left(A_{st}^*+B_{st}\right)}\frac{\sum_{h=1}^{L}W_h^2A_h\lambda_hS_{xyh}}{A_{st}^*\bar{Y}}\right)$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left\{S_{yh}^2-\frac{\bar{Y}A_h}{\left(A_{st}^*+B_{st}\right)}S_{xyh}+\frac{1}{4}\frac{\bar{Y}^2A_h^2}{\left(A_{st}^*+B_{st}\right)^2}S_{xh}^2\right\}$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left\{S_{yh}^2-R_{k2}S_{xyh}+\frac{1}{4}R_{k2}^2S_{xh}^2\right\}$$

where $R_{k2}=\dfrac{\bar{Y}A_h}{\left(A_{st}^*+B_{st}\right)}$.

If we take A_h as $\beta_{2h}\left(x\right)$ and B_h as C_{xh} in $MSE\left(k_2\right)$ we get corrected $MSE\left(t_3\right)$ as

$$MSE\left(t_3\right)=\sum_{h=1}^{L}W_h^2\lambda_h\left\{S_{yh}^2-\frac{\bar{Y}\beta_{2h}\left(x\right)}{\left(\sum_{h=1}^{L}W_h\bar{X}_h\beta_{2h}\left(x\right)+\sum_{h=1}^{L}W_hC_{xh}\right)}S_{xyh}+\frac{1}{4}\frac{\bar{Y}^2\beta_{2h}\left(x\right)^2}{\left(\sum_{h=1}^{L}W_h\bar{X}_h\beta_{2h}\left(x\right)+\sum_{h=1}^{L}W_hC_{xh}\right)^2}S_{xh}^2\right\}$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left\{S_{yh}^2-R_{t3}S_{xyh}+\frac{1}{4}R_{t3}^2S_{xh}^2\right\}$$

where $R_{t3}=\dfrac{\bar{Y}\beta_{2h}\left(x\right)}{\left(\sum_{h=1}^{L}W_h\left\{\bar{X}_h\beta_{2h}\left(x\right)+C_{xh}\right\}\right)}$.