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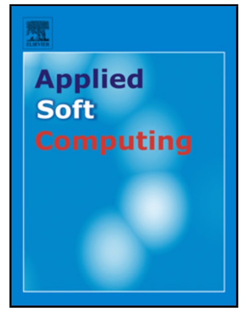

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Hybrid Adaptive Evolutionary Algorithm Based on Decomposition

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Abstract

The performance of search operators varies across the different stages of the search/optimisation process of Evolutionary Algorithms (EA). In general, a single search operator may not do well in all these stages when dealing with different optimization and search problems. To mitigate this, adaptive search operator schemes have been introduced. The idea is that when a search operator hits a difficult patch (under-performs) in the search space, the EA scheme "reacts" to that by potentially calling upon a different search operator. Hence, several multiple-search operator schemes have been proposed and employed within EA. In this paper, a Hybrid Adaptive Evolutionary Algorithm Based on Decomposition (HAEA/D) that employs four different crossover operators is suggested. Its performance has been evaluated on the well-known IEEE CEC'09 test instances. HAEA/D has generated promising results which compare well against several well-known algorithms including MOEA/D, on a number of metrics such as the Inverted Generational Distance (IGD), the hyper-volume, the Gamma and Delta functions. These results are included and discussed in this paper.

Keywords: Multi-Objective Optimization, Adaptive Operator Selection, MOEA, MOEA/D

1. Introduction

The performance of search operators varies across the different stages of the search/optimisation process of Evolutionary Algorithms (EA). In general, it is difficult for a single search operator to do well in all stages of EAs when dealing with various optimization and search problems. To mitigate this, adaptive search operator schemes have been introduced. The idea is that when a search operator hits a difficult patch (under-performs) in the search space, the EA scheme "reacts" to it by potentially calling upon a different search operator. Hence, several multiple-search operator schemes have been proposed and employed within EAs. Note that this approach is different from the Multiple Algorithms Single Formulation (MASF) approach advocated in [54]. In [54], algorithms which do not perform well may eventually die out completely when the resources allocated to them are exhausted and not replenished. Here, operators remain alive throughout the search process. Although the approach put forward here is innovative, it is not entirely new. We already know, for instance, that the performance of EA is greatly affected by search operators with self-adaptive capabilities, [4, 64, 58, 66, 71, 67]. Adaptive operators selection procedures including probability matching [15, 20] and adaptive pursuit methods [10, 18] have recently been investigated on exploitation and exploration issues that arise during the evolutionary process. There is no doubt that different operators are suitable at different stages of evolutionary optimization. The use of many crossovers, is therefore, a good idea to cope with new and complex optimization and search problems.

This paper suggests a hybrid adaptive EA based on decomposition (HAEA/D) in which multiple crossover operators, namely, Differential Evolution (ADE), [53], the Center of Mass Crossover (CMX) [61, 62], Parent Centric Crossover (PCX), [12], and Trigonometric Mutation (TM) [17], Simplex Crossover (SPX), [63], Simulation Based Crossover (SBX), [13, 11], are employed. The extent of their deployment in the search process is based on their recent individual performances and the way they generate new populations of individuals. The more successful they are in terms of the quality of the solutions they find, the more often then are used in the search. Poor performance of an operator means that it will barely feature in the search. However, it will not be dropped completely; leaving it in the frame means that it can potentially be called upon at some point when the others have hit difficult patches in the search space, which is always a possibility. If it does well, then it can be called upon more often to contribute to the overall search/optimisation process.

HAEA/D uses MOEA/D [73] as a global search technique. It is implemented, applied to the CEC'09 test instances [74] and then compared to a number of recently developed algorithms. Note that the decomposition nature of HAEA/D, being based on MOEA/D, means that meaningful comparisons can only be with similar type algorithms and schemes. These include multiobjective memetic algorithm based on decomposition [44], multiobjective cloud particle optimization algorithm based on decomposition (MOEA/D-CPDE) [34] Multiple Trajectory Search (MTS) [59, 60], Differential Evolution with self-adaptation and local search for constrained multiobjective optimization algorithm (DECMOSA-SQP) [70], generalized DE3 (GDE3) [32] and MOEA/D [33]. Different metrics such as the Inverted Generational Distance (IGD) [74], Relative Hyper-volume indicator (HYP) [65], the Gamma function (Γ) [13] and the Delta function (Δ) [13] have been applied to conduct a performance assessment of the proposed algorithm. The experimental results returned by HAEA/D are competitive in terms of proximity and diversity when dealing with most benchmark functions.

The rest of this paper is organized as follows. Section 1 gives the generic form of the Multi-objective Optimisation Problem (MOP) and gives a brief review of the background literature on the topic. Section 1 offers the template of the proposed algorithm. Section 4 presents the test problems and indicator functions used in our experiments. Section 5 discuses the experimental results produced by HAEA/D and its competitors. Section 6 concludes the paper and suggests future research directions.

2. Problem Definition and Background Literature

Multi-objective optimization (MOO) is concerned with problems involving more than one objective function that need to be optimized simultaneously subject to a set of constraints or bounds. MOO problems can be discrete, continuous or both. They arise in various applications including in air traffic routing, the design of telephone networks, electrical and hydraulic applications, cable TV and computer systems, road networks and other. Continuous optimization is widely used in mechanical design, chemical engineering, economics, finance, agriculture and the food industry, to name a few, [57, 35, 8, 1, 6].

A generic minimization multi-objective optimization problem (MOP) can be formally defined as follows:

$$\begin{aligned} &\text{minimize } F(x) = (f_1(x), f_2(x) \ldots, f_m(x)) \\ &\text{such that } x \in \Omega, \end{aligned} \tag{1}$$

where Ω is the decision variable space, $x = (x_1, x_2, \ldots, x_n)^T$ is a decision variable vector with x_i, $i = 1, \ldots, n$, their decision variables, $F(x) : \Omega \rightarrow R^m$ involves $m \geq 2$ real valued conflicting objective functions and R^m is the objective space.

If Ω is a closed and connected region in R^n and all the objective functions are continuous in x then problem (1) will be continuous. Furthermore, if $m = 1$, then problem (1) is a single objective problem (SOP).

A solution $u = (u_1, u_2, \ldots, u_n) \in \Omega$ is said to be Pareto optimal if there does not exist any other solution $v = (v_1, v_2, \ldots, v_n) \in \Omega$ for which $f_j(u) \leq f_j(v)$, $\forall j = 1, \ldots, m$ and for at least index k, $f_k(u) < f_k(v)$. An objective vector is said to be Pareto optimal if the corresponding decision vector is also Pareto optimal. All Pareto optimal solutions in the decision space of a MOP form a Pareto set (PS) and their corresponding image in the objective space is called a Pareto Front (PF). This idea of Pareto optimality was first proposed by Francis Ysidro Edgeworth in 1881 and has later been generalized by Vilfredo Pareto, [11, 7].

Multiobjective Evolutionary Algorithms (MOEAs) are well-established stochastic techniques for solving various MOP test suites and MOPs arising in real-world applications. They use a number of intrinsic evolutionary operators (variation and selection operators) to evolve their populations and do not rely on derivative information related to the objective functions of the MOPs.

The first MOEA known as "Vector Evaluated Genetic Algorithm (VEGA)" was developed in a seminal work of David Schaffer, [56, 55]. VEGA divides the population into m sub-populations, each of which evolves toward a single objective. The main advantages of VEGA is low time complexity because it does not calculate the dominance level of individuals in its populations. After the appearance of VEGA, a wide range of MOEAs have been developed that mostly follow the mechanisms introduced by David Goldberg such as the non-dominance concept and diversity-preserving techniques, [19]. These algorithms, most of the time, provide Pareto optimal solutions in a single simulation run for a variety of problems including those to be found in test suites of MOPs. Of course, the no free lunch (NFL) theorem, [68, 31], holds here.

In general, MOEAs can be divided into three main categories based on fitness assignment strategies; they are the Pareto dominance based MOEAs (e.g., [9, 13, 78, 77, 51, 21]), the Decomposition based MOEAs (e.g., [22, 33, 73, 38, 42, 44, 24, 25, 26]), and the Indicator based algorithms (e.g., [80, 5, 23, 3, 2, 14]). Pareto dominance MOEAs use explicitly the Pareto dominance concept in order to determine the reproduction probability of each individual of its population. Unfortunately, the time complexity of most existing Pareto dominance based MOEAs is not attractive. Because of that they are not suitable for dealing with many objective optimization problems (MOPs) and especially real-world problems [30, 36, 69, 52]. Indicator based MOEAs often incorporate hyper-volume in their selection process in order to evolve their population during the course of optimization. This is computationally very expensive when solving practical problems and problems in test suites with many conflicting objective functions. Both aforementioned categories [47] do not associate their solution populations with any particular scalar optimization problem and solve the given problem directly unlike the MOEAs based on decomposition (MOEA/D), [72].

In the simple MOEA/D, [72], two different paradigms, namely calassical mathematical programming and evolutionary computing have been coupled to address fitness assignment and diversity maintenance issues that cause difficulties for non-decomposition based MOEAs. It decomposes the problem of approximating the PF into N different single objective optimization subproblems and then optimizes all of them at the same time with the help of a generic EA. A neighborhood relationship among these subproblems is one of its key features which is defined using the distances between their weight vectors. This neighbouring procedure among the subproblems can speed up the search process of MOEA/D, [72] by exchanging information between problems. It keeps one solution in memory that cannot be the best solution found so far for its subproblems and updates it if the new solution produced is better.

In [33], an enhanced version of MOEA/D [72] is developed in which the Simulated Binary Crossover (SBX) [27] has been replaced with Differential Evolution (DE) [53]. This gave MOEA/D-DE, [33]. The purpose of this replacement is to produce a solution while inducing two different neighbourhoods, one with each child solution. One of these solutions is then allowed to replace a very small number of old solutions. In [73], resources are allocated dynamically to each sub-problem as used in the MOEA/D paradigm. In [29, 42, 28, 46, 73], the impact of multiple search operators coupled to a self-adaptive scheme has been studied. It has then been tested on instances designed for the special session on MOEA competition at the Congress of Evolutionary Computing of 2009 (CEC'09), [74]. In [40, 44], DE and PSO [16] have been used simultaneously within the framework of MOEA/D, [72]. This variant was then applied to five standard ZDT test problems [79] as well as the CEC'09 test instances [74]. In [39, 37, 43, 45], MOEA/D [72] and NSGA-II [13], two different MOEA approaches have been used synergetically at population and generation levels. These two algorithms have also been used in [48] to solve hard multiobjective optimization problems. Fuzzy Dominance (FD) concepts have been introduced in [50] to further improve the algorithmic behavior of the MOEA/D paradigm. The effect of the combined use of neighbourhood sizes with a self-adaptive strategy has been investigated in [75]. For more details please refer to [38, 41, 76].

3. A Hybrid Adaptive Evolutionary Algorithm Based on Decomposition: HAEA/D

The proposed HAEA/D as outlined in the Algorithm 1 is an improved version of MOEA/D that uses the Tchebycheff Aggregation Function (TAF) [49], to transform the given MOP into N scalar optimization sub-problems with fixed N weight vectors and then optimizes all N sub-problems simultaneously. The suggested algorithm incorporates multiple search operators based on an adaptive operator selection (AOS) method and decides which operator should be applied to evolve their population of solutions; for further details, please refer to Algorithm 3. The suggested AOS performs mainly two tasks: the selection of operators and the allocation of awards to them based on their solutions' fitness improvement. Furthermore, HAEA/D defines the neighbouring relationships among the N sub-problems using minimum Euclidean distances between the N weighted vectors/coefficients of the TAF.

Let $\lambda^1, \ldots, \lambda^N$ be a set of N weight vectors and $z_j^* = min\{f_j(x)|x \in \Omega\}$ be the reference point. We use the Tchebycheff Aggregation Function [49] to transform the approximated PF of problem (1) into N scalar optimization subproblems whose j^{th} subproblem is as follows:

$$\begin{aligned} \text{minimize } g(x|\lambda, z) = max_{1\le j\le n}\{\lambda_j|f_j(x) - z_j^*|\} \\ z_j = min\{f_j(x)|x \in \Omega\} \\ z^* = \{z_1, z_2, \ldots, z_j\}. \end{aligned} \tag{2}$$

Algorithm 3, the main part of the HAEA/D framework, allocates resources to $q = 4$ crossover operators at population generation level. The first part, $\zeta \times p_t^k$, in the above suggested adaptive model ensures that all crossovers are active in the process of population evolution. This is because the best crossover is not necessarily going to perform best at all stages of the optimization process. Therefore, the proposed adaptive methodology does not allow any weak crossover to be inactive due to the concept of no free lunch (NFL) theorem [31]. In other words, no single operator can always perform better within any MOEA framework while dealing with complicated problems like CEC'09 test instances [74], or weakness may be only temporary. The proposed AOS method mainly makes use of valuable information found in both previous and current populations of solutions when allocating resources to the crossovers involved.

The search process uses π^i, the utility of subproblem i, to measure the improvement that has been due to x^i in reducing the objective of this subproblem i; this is defined as

$$\pi^i = \begin{cases} 1 & \text{if } \Lambda^i > 0.001; \\ (0.95 + 0.05\frac{\Lambda^i}{0.001})\pi^i & \text{otherwise.} \end{cases}$$

If gen is a multiple of 50, then compute Λ^i, the relative decrease of the objective for each subproblem i. In each generation, HAEA/D selects a set of solutions from the current population based on their utilities as outlined in Algorithm 2. As in MOEA/D [33, 73], each i^{th} offspring solution of HAEA/D is restricted to replace at most n_r solutions in its T-neighbouring solutions based their scalar objective function values. Further, we have employed the polynomial mutation as defined in Equation (3) after the use of each crossover in our HAEA/D to mutate the resulting new solution with the rate of probability p_m.

$$y_k^* = \begin{cases} y_k + \sigma_k(u_k - l_k) & \text{with probability } p_m, \\ y_k & \text{with probability } 1 - p_m, \end{cases} \tag{3}$$

where l_k and u_k are the lower and upper bound of the the k^{th} decision variable, respectively.

$$\sigma_k = \begin{cases} (2 \times rand)^{\frac{1}{\eta+1}} - 1 & \text{if rand } < 0.5, \\ 1 - (2 - 2 \times rand)^{\frac{1}{\eta+1}} & \text{otherwise.} \end{cases}$$

Algorithm 1 HAEA/D: Hybrid Adaptive EA Based on Decomposition.

$[PS, PF] \leftarrow$ **HAEA-D**(N, T, n_r, MOP) ▷ $PS = \{x^1, \ldots, x^N\}, PF = \{F(x^1), \ldots, F(x^N)\}$
$\{x^1, x^2, \ldots, x^N\} \leftarrow a + b - a \times rand(N, n)$ ▷ Generate initial population P uniformly and randomly. Here a is the lower and b is the upper limit of the decision space of the given MOP.
$\{f_j(x^1), f_j(x^2), \ldots, f_j(x^N)\} \leftarrow$ **Eval-Function**(P) ▷ Evaluate the initial population P of size N;
Initialize the weight vector $\{\lambda^1, \ldots, \lambda^N\}$ uniformly and randomly.
Initialize $z^* = min\{f_j(x^1), f_j(x^2), \ldots, f_j(x^N)\}$;
Set the utility function $\pi^i = 1$;
for $i \leftarrow 1 : N$ **do**
  $B(i) = \{\lambda^{i_1}, \lambda^{i_2}, \ldots, \lambda^{i_T}\}$, where $\lambda^{i_1}, \lambda^{i_2}, \ldots, \lambda^{i_T}$ are T closest weight vectors to each i^{th} weight vector;
end for
Assign equal probabilities to each k^{th} operator, $p_t^k = \{\frac{1}{q}\}$;
while $Stopping\ criteria\ not\ Satisfied$ **do**
  Let $I = 1, .., i, ..q$ be the set of the indices of subproblems each having objective $f_i, i \in I$. Using 10-tournament selection based on π^i, select $\frac{N}{5} - m$ other indices and add them to I;
  if $uniform(0, 1) < \delta$ **then**
    $P \leftarrow B(i)$;
  else
    $P \leftarrow \{1, 2, \ldots, N\}$;
  end if
  Divide population P into $P_1, P_2, \ldots, P_q$ based on P^t;
  for $i \in I = \{P_1, P_2, \ldots, P_q\}$ **do**
    if $i \in P_1$ **then**
      $x^l, x^m, x^i \leftarrow P_1$ such that $x^i \neq x^l \neq x^m$;
      $\overline{y} \leftarrow XOR_1(x^i, x^l, x^m)$;
    else
      if $i \in P_2$ **then**
        $x^l, x^m, x^i \leftarrow P_2$ such that $x^i \neq x^l \neq x^m$;
        $\overline{y} \leftarrow XOR_2(x^i, x^l, x^m)$;
      end if
    end if
    if $i \in P_m$ **then**
      $x^l, x^m, x^i \leftarrow P_q$ such that $x^i \neq x^l \neq x^m$;
      $\overline{y} \leftarrow XOR_q(x^i, x^l, x^m)$;
    end if
    $y \leftarrow$ **Mutate**$(\overline{y}, p_m)$ and evaluate their fitness;
    Update the current reference point $z^* = (z_1, z_2, \ldots, z_m)^T$;
    To update Population call Algorithm 2;
  end for
  To update p_t^k call Algorithm 3.
end while

Algorithm 2 Population Update and Resource Allocation.

Set $c = 0$; $n_r = 0.001 \times N$; $T = 0.1 \times N$;
while $c < n_r$ or $P = \emptyset$ **do**
  Pick a solution x^j from P;
  if $g(y|\lambda^j, z^*) \leq g(x^j|\lambda^j, z^*)$; **then**
    $x^j \leftarrow y$, $F(x^j) = F(y)$;
    Remove x^j from P;
    $\Lambda = \frac{g(x^j|\lambda^j,z^*)-g(y|\lambda^j,z^*)}{g(x^j|\lambda^j,z^*);}$
    $c = c + 1$;
    $\Phi(c) \leftarrow \Lambda$;
  end if
end while
$\nabla(i, 2) \leftarrow \sum \Phi$ and save it with tag number assigned to each crossover.
$C_G = C_G + 1$;
if $mod(C_G, 50) == 0$; **then**
  Update utility π^i of each subproblem i.
end if

Algorithm 3 Adaptive Multiple Crossover Selection.

Initially, HAEA/D uses $q = 4$ different crossovers with equal selection ratio of $\frac{1}{q}$ in their framework to generate an offspring for the new population.
After the first generation, their selection ratios are updated according to the difference between fitness values of each parent x and offspring solution y as below.
for $k \leftarrow 1 : q$ **do**
  Define $0 < \zeta < 1$;
  $\nabla_k \leftarrow \sum \frac{g(x^j|\lambda^j,z^*)-g(y|\lambda^j,z^*)}{g(x^j|\lambda^j,z^*)}$;
  $p^k_{t+1} = \zeta \times p^k_t + (1 - \zeta) \times \frac{\nabla_k}{\sum_{k=1}^{q} \nabla_k}$;
end for
$p^k_t \leftarrow p^k_{t+1}$, return to Algorithm 1.

4. Test Problems and Indicator Functions

Due to the flurry of MOEAs recently developed, their performances are measured on different MOP test suites, some related to real-world applications, while most were generated for testing purposes. Several such test suites comprising unconstrained (but bound constrained) as well as constrained problems have been presented in special sessions at events such as the CEC 09. This particular one provides performance assessment guidelines and code in web-sites such as http://dces.essex.ac.uk/staff/qzhang/moeacompetition09.htm. Table 1 records the statistics of the ten unconstrained CEC'09 test instances [74] used in our experiments.

4.1. Parameter Settings

The following parameter values have been used in our experiments.

- $N = 600$ for 2-objective test instances;
- $N = 1000$ for 3-objective test instances;
- $T = 0.1N$ are closest weight vectors;
- $n_r = 0.01N$ is the maximum number of solutions replaced by each new solution;

Table 1. IEEE CEC'09 Benchmark Functions Characteristics.

CEC'09	Objectives	Search Domain	Characteristics of PF
UF1	2	$[0,1] \times [-1,1]^{n-1}$	Concave
UF2	2	$[0,1] \times [-1,1]^{n-1}$	Concave
UF3	2	$[0,1]^n$	Concave
UF4	2	$[0,1] \times [-2,2]^{n-1}$	Convex
UF5	2	$[0,1] \times [-1,1]^{n-1}$	21 point front
UF6	2	$[0,1] \times [-1,1]^{n-1}$	One isolated point and two disconnected parts
UF7	2	$[0,1] \times [-1,1]^{n-1}$	Continuous straight line
UF8	3	$[0,1]^2 \times [-2,2]^{n-2}$	Parabolic
UF9	3	$[0,1]^2 \times [-2,2]^{n-2}$	Planar
UF10	3	$[0,1]^2 \times [-2,2]^{n-2}$	Parabolic

- $\delta = 0.9$ is the probability with regard to selecting P;
- $\eta = 20$ and $p_m = 1/n$ in the polynomial mutation operator;
- ADE parameter $CR = F = 0.5 * (1 + rand)$, where $rand() \in [0, 1]$;
- The maximum number of function evaluations is $300,000$;

4.2. *Weight Vector Selection*

A set of N weight vectors, W, is generated according to the following procedure [73]:

1. Uniformly randomly generate $5,000$ weight vectors to form set W_1. Set W is initialised with weight vectors $(1, 0, \ldots, 0, 0), (0, 1, \ldots, 0, 0), \ldots, (0, 0, \ldots, 0, 1)$;
2. Find the weight vector in set W_1 with the largest distance to set W; include it in set W and remove it from set W_1.
3. If $|W| = N$, stop and return it. Else, go to 2.

4.3. *Performance Indicators*

Two main goals must be kept in sight when dealing with MOP. They are:

1. the convergence towards the Pareto-optimal front,
2. the uniformity and good distribution of the set of multiple solutions that cover the true PF of the problem in hand [11].

Several performance metrics found in the specialized literature on evolutionary computing (EC) [65, 13, 11, 81] are used to rank algorithms in terms of performance. They are the inverted generational distance (IGD), [81, 74], the relative HYPer-volume (HYP), [65, 11], the Gamma (Γ) and Delta (Δ) indicators, [11, 13]; they are commonly used in several comparative analyses of a variety of algorithms. These performance indicators can only be used if the reference set of the problem at hand is known in advance or is available with the test suites. In this paper, we have used the following performance indicators.

4.3.1. The Inverted Generational Distance (IGD)

Let P^* be a set of uniformly distributed points along the PF. Let A be an approximate set of the PF, the average distance from P^* to A is defined as [74]:

$$D(A, P) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|},$$

where $d(v, A)$ is the minimum Euclidean distance between v and the points in A. If P^* is large enough to represent the PF very well, then $D(A, P)$ could measure both the diversity and convergence of A in a sense. The closer the IGD metric values, the better the approximation set is. We have used $P^* = 500$ in our experiments to tackle 2-objective test instances and $P^* = 1000$ to solve 3-objective problems.

4.3.2. The Relative Hyper-volume Indicator (HYP)

HYP is mathematically expressed as

$$HYP(A) = \frac{HV(P^*) - HV(A)}{HV(P^*)},$$

where HV denotes the hyper-volume of the approximate set A of P^* and it is calculated as follows, [65, 66]:

$$HV(A) = \text{volume} \cup_{i=1}^{|A|} z_i$$

where $i \in A$ and z_i is the i^{th} hypercube constructed with respect to reference point W and the solution i as the diagonal corners of the hypercube. The closer the value of HYP to zero, the closer the approximate set of solutions to the true Pareto-optimal set.

4.3.3. The Gamma (Γ) Performance Indicator

To use the Γ metric [13], we generate $P^* = 500$ uniformly spaced solutions from the true Pareto optimal front in the objective space of the problem at hand to calculate the Γ metric values. Then, we compute the minimum Euclidean distance of each individual solution belonging to the approximated set of solutions denoted by A between P^* the Pareto-optimal solutions. The average of these distances represents the Γ metric values. In practice, if the Γ metric value is close to zero, the approximate set will converge well to the true Pareto front. This metric measures the extent of convergence to a known set of Pareto-optimal solutions. However, it fails to provide complete information about the spread in the obtained solutions. For this reason, we use another metric denoted Δ which is explained below.

4.3.4. The Delta (Δ) Performance Indicator

Δ is a metric function calculated as follows, [13].

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} d_i - \overline{d}}{d_f + d_l + (N-1)\overline{d}},$$

where d_f and d_l are the Euclidean distances of the extreme and the boundary solutions belonging to the approximate set of the optimal solutions set and $\overline{d}$ denotes the average of all Euclidean distances d_i between consecutive solutions in the final approximate set of optimal solutions provided by a particular algorithm.

5. Experimental Results: Discussion

The experiments have been carried out on the following computing platform and parameter values.

- Operating system: Windows XP Professional;
- Programming language: Matlab;
- CPU: Core 2 Quad 2.4 GHz;
- RAM: 4 GB DDR2 1066 MHz;

- Execution: 30 times each algorithm with different random seeds;

In our experimental investigation, first we have embedded some crossover operators, namely, CMX [61, 62], ADE [53], PCX [12], and TM [17], one by one in MOEA/D, [73] without any modification of the original framework. As a result MOEA/D-CMX, MOEA/D-ADE, MOEA/D-PCX and MOEA/D-TM have been developed and form the core matter of this paper.

Secondly, we developed HAEA/D by enhancing MOEA/D [73] with the four cross-overs mentioned above. Each used crossover receives resources based on the performance of its generated individuals/solutions within the HAEA/D framework. This performance is in terms of the fitness values of the parents allocated to each crossover and the offspring produced by a given crossover. HAEA/D and the other considered algorithms have been executed with the same parameter settings as explained in Section 4.1 and their algorithmic behaviors have been investigated thoroughly using four different indicators as explained in Section 4.3.1.

The IGD-metric values obtained by HAEA/D and four other versions of MOEA/D [73] on each of UF1 through to UF10 problems, are recorded in Table 2 and Table 3. The 1^{st} column in each of these tables shows the minimum (Min), the 2^{nd} the maximum (Max), the 3^{rd} the mean, and the 4^{th} column shows the standard deviation (Std) of IGD-metric values. From these tables one can conclude that HAEA/D has found a better approximate set of solutions with minimum average IGD-values compared to the other algorithms on most CEC'09 test instances, [74].

The last columns of Tables 2 and 3, indicate that the experimental results obtained with all algorithms for instances UF5 and UF6 are not good due to the fact that the objective function profiles of these problems are very complicated; small perturbations in the data have a big effect on the populations of solutions generated by these algorithms and cause them to be dominated and/or get stuck in the local basins of attraction of some solutions. Note also that HAEA/D does not allow evolved solutions to replace all T neighbouring solutions as in the original MOEA/D, [72]. The mating restriction in HAEA/D, a type of elitism, which prevents promising solutions from taking part in the process of evolution, may have a drawback. However, mating restriction strategies are quite useful in that they improve the time complexity of algorithms.

Table 4 shows the IGD-metric values produced by A) MTS, [60], B) GDE3, [32], C) DECMOSA-SQP, [70], algorithms to cope with UF1-UF10 test instances. Among all these algorithms, MTS has handled both UF1 and UF6 with minimum IGD-values over 30 independent runs as compared to GDE3, DECMOSA-SQP, HAEA/D and four other different versions of MOEA/D [73] considered in this paper.

Table 5 presents the values of the relative hypervolume (HYP) in the 1^{st} row, Γ in the 2^{nd} and Δ in the 3^{rd}, for each of UF1 to UF10, respectively. Columns 1^{st}, through to 5^{th} of this table lists the best, median, average, standard deviation (std) and maximum values of the relative hypervolume, Γ and Δ indicators, respectively. The average variation accrued in the values of these indicators are displayed in Figures 8, 9 and 10 which clearly demonstrate that HAEA/D has performed well on most CEC'09 test instances as compared to the four different versions of MOEA/D [73] based on a single crossover with average indicator values.

The best PFs of some CEC'09 test instances, [74] with respect to a) HAEA/D, b) MOEA/D-CMX, c) MOEA/D-ADE, d) MOEA/D-PCX, e) MOEA/D-TM are shown in Figures 1, 2 and 3. One can see from these figures that the proposed algorithm has found better a PF for most test instances with good convergence and diversity as compared to the MOEA/D versions. Note that we did not include the figures corresponding to MOEA/D-PCX and MOEA/D-TM to keep down the volume of the paper.

In Figures 4, 5 and 6, we have plotted all 30 PFs together to show the distribution ranges of the final populations approximated by the above mentioned algorithms in 30 independent runs on CEC'09 test instances. The figures clearly demonstrate that HAEA/D has found better solutions in terms of the distribution ranges compared to the ones generated by the algorithms used in the comparative investigation.

The average IGD-metric values are plotted against the number of generations in Figure 7. This figure shows that HAEA/D has solved most problems with minimum average IGD-metric value as compared to most of the algorithms considered here.

5.1. Statistical Significance Analysis of the HAEA/D

In order to have statistically sound conclusions, we conducted the Wilcoxons rank sum tests at 0.05 significance level aim at establishing significance differences between the suggested algorithm and the rest of the state-of-the-art MOEAs considered. In this regard, the IGD-metric values of the suggested algorithm are used along with the other used MOEAs to assign them ranks according to their performance, where the last columns of Tables 2, 3 and 4 provide details regarding the ranks of each MOEA. We have highlighted the best ranking MOEA in bold in the mentioned tables.

Table 2. IGD-metric values of A) HAEA/D, B) MOEA/D-CMX, C) MOEA/D-ADE, D) MOEA/D-PCX), E) MOEA/D-TM, F) MOEA/D-DE+PSO [44], G) MOEA/D-CPDE [34] when applied to UF1-UF5 of the IEEE CEC09 test instances [74].

CEC'09	Min	Median	Mean	Std	Max	EAs	RK
UF1	**0.003943**	**0.004253**	**0.004275**	**0.000222**	**0.00494**	**A**	1
	0.004178	0.004589	0.005397	0.003487	0.022876	B	4
	0.004847	0.010675	0.010995	0.003732	0.019072	C	6
	0.031234	0.063827	0.074934	0.037351	0.188678	D	9
	0.044052	0.077233	0.072754	0.017260	0.113685	E	8
	0.004466	0.0182737	0.0285723	0.02360902	0.0711827	F	7
	0.004594	-	0.005203	0.000248	0.006144	G	2
UF2	**0.004135**	**0.004125**	**0.004138**	**0.000715**	**0.009050**	**A**	1
	0.005287	0.006875	0.006958	0.001154	0.009084	B	4
	0.004580	0.005584	0.00653	0.001551	0.010123	C	3
	0.010176	0.018078	0.020656	0.010734	0.047623	D	8
	0.024549	0.029689	0.031660	0.006182	0.053112	E	10
	0.010218	0.011671	0.011737	0.000756	0.013261	F	5
	0.008998	-	0.011877	0.001714	0.015334	G	6
UF3	0.004043	0.021355	0.021138	0.020926	0.059050	A	2
	0.004454	0.022377	0.032645	0.027109	0.091392	B	3
	0.006955	0.0468554	0.044923	0.02754	0.09362	C	4
	0.114871	0.206634	0.204245	0.038934	0.269345	D	8
	0.031612	0.694083	1.665783	4.869497	24.661914	E	10
	0.0039404	**0.0058284**	**0.006039**	**0.001826**	**0.010494**	**F**	1
	0.008435	-	0.044706	0.029862	0.114730	G	5
UF4	0.039089	0.045792	0.047761	0.007650	0.062431	A	6
	0.053578	0.060667	0.061493	0.004765	0.073944	B	9
	0.041028	0.048083	0.049150	0.005221	0.062834	C	7
	0.049639	0.056073	0.058145	0.007545	0.078592	D	8
	0.047843	**0.054604**	**0.0040458**	**0.004053**	**0.071241**	**E**	1
	0.043632	-	0.045344	0.001319	0.049677	G	5
UF5	0.188700	0.242179	0.295313	0.118853	0.707107	A	6
	0.222475	0.383753	0.425495	0.154682	0.712491	B	8
	0.127310	0.270832	0.281138	0.087448	0.480059	C	5
	0.170921	0.379723	0.378716	0.0131373	0.643197	D	7
	0.200144	0.417523	0.458011	0.139898	0.822995	E	9
	0.282111	0.454364	0.490882	0.118561	0.708999	F	10
	0.115859	-	0.203445	0.045309	0.258254	G	4

Table 3. IGD-metric values of A) HAEA/D, B) MOEA/D-CMX, C) MOEA/D-ADE, D) MOEA/D-PCX), E) MOEA/D-TM, F) MOEA/D-DE+PSO [44], G) MOEA/D-CPDE [34] when applied to UF6-UF10 of IEEE CEC09 test instances [74].

CEC'09	Min	Median	Mean	Std	Max	EAs	RK
UF6	0.051234	0.173364	0.196282	0.101955	0.474144	A	5
	0.150081	0.443699	0.398187	0.153898	0.761923	B	9
	0.041672	0.0902887	0.0903197	0.030015	0.250136	C	2
	0.077549	0.204678	0.284849	0.205672	0.832798	D	7
	0.193984	0.399496	0.390574	0.128275	0.727768	E	8
	0.186140	0.823351	0.778214	0.157339	0.843885	F	10
	0.073819	-	0.091573	0.030068	0.210588	G	3
UF7	**0.004549**	**0.005769**	**0.005988**	**0.001489**	**0.012287**	**A**	1
	0.004884	0.006387	0.008573	0.005581	0.031779	B	2
	0.004898	0.008514	0.009190	0.003596	0.023703	C	3
	0.012956	0.114638	0.190943	0.195067	0.619975	D	8
	0.017341	0.024890	0.144043	0.154892	0.495634	E	9
	0.006726	0.008776	0.243517	0.237355	0.674953	F	10
	0.005112	-	0.006067	0.000640	0.007461	G	7
UF8	0.058455	0.074751	0.082124	0.018202	0.125892	A	3
	0.058878	**0.068588**	**0.075276**	**0.016682**	**0.101398**	**B**	1
	0.086072	0.088657	0.087818	0.006782	0.102548	C	4
	0.891378	1.016922	1.009308	0.066704	1.117835	D	9
	1.187237	1.370320	1.368381	0.074510	1.516075	E	10
	0.057600	0.076222	0.078455	0.006532	0.1019020	F	2
	0.112641	-	0.123347	0.006231	0.139016	G	5
UF9	0.039255	0.122295	0.107791	0.052809	0.175837	A	5
	0.049189	0.151583	0.128087	0.060854	0.325666	B	8
	0.043128	0.069057	0.087238	0.042770	0.177565	C	4
	0.096845	0.18768	0.170278	0.036245	0.219371	D	9
	0.207515	0.287547	0.280701	0.046702	0.343331	E	10
	0.035499	**0.038980**	**0.071131**	**0.035008**	**0.149478**	**F**	1
	0.074763	-	0.080028	0.002555	0.083058	G	2
UF10	0.282175	0.413275	0.421193	0.059551	0.545143	A	6
	0.428067	0.478093	0.484467	0.035198	0.553108	B	8
	0.215141	0.344499	0.372537	0.107854	0.627219	C	4
	0.195734	0.402962	0.382064	0.147843	0.761573	D	5
	0.417532	0.489873	0.494567	0.046058	0.599181	E	9
	0.184050	0.187033	0.187158	0.001552	0.190097	F	2
	0.369133	-	0.499921	0.078278	0.688156	G	10

Table 4. IGD-metric values over 30 independent runs of (H) MTS [60],(I) GDE3 [32]and (J) DECMOSA-SQP [70] when applied to UF1-UF10 CEC'09 test instances [74].

H) MTS, I) GDE3, J) DECMOSA-SQP						
CEC'09	Min	Mean	Std	Max	EAs	RK
UF1	0.005782	0.006467	0.0003485	0.007221	H	5
	0.004815	0.005342	0.000342	0.006242	I	3
	0.055126	0.0770281	0.039379	0.0880129	J	10
UF2	0.005188	0.006157	0.000508	0.007499	H	2
	0.009020	0.011953	0.001541	0.014972	I	7
	0.017336	0.028342	0.031318	0.04022	J	9
UF3	0.037394	0.053107	0.0117366	0.077863	H	9
	0.085759	0.106395	0.012900	0.138381	I	7
	0.030545	0.093500	0.19795	0.16816	J	6
UF4	0.022486	0.023561	0.0006641	0.024950	H	3
	0.025857	0.026506	0.000372	0.027550	I	2
	0.031624	0.033926	0.005370	0.035643	J	4
UF5	**0.009743**	**0.003277**	**0.0032771**	**0.022036**	**H**	1
	0.031791	0.039281	0.003947	0.045880	I	2
	0.133012	0.167139	0.089508	0.237081	J	3
UF6	**0.041631**	**0.059178**	**0.0106224**	**0.090268**	**H**	1
	0.204163	0.250913	0.019573	0.282468	I	6
	0.057917	0.126042	0.561753	0.589904	J	4
UF7	0.015951	0.040794	0.0144456	0.081103	H	5
	0.014125	0.025228	0.008891	0.042002	I	4
	0.0198913	0.024163	0.022349	0.0427502	J	6
UF8	0.090927	0.112517	0.0129335	0.138652	H	6
	0.194990	0.248556	0.035521	0.365385	I	8
	0.098938	0.215834	0.121475	0.228895	J	7
UF9	0.062463	0.114423	0.0254955	0.182694	H	6
	0.045261	0.082482	0.022485	0.133812	I	3
	0.062463	0.114423	0.0254955	0.182694	J	7
UF10	**0.124504**	**0.153065**	**0.0158331**	**0.198014**	**H**	1
	0.393773	0.433261	0.012323	0.445574	I	7
	0.238279	0.369857	0.65322	0.580852	J	3

Table 5. Relative hypervolume, Γ and Δ function values found by HAEA/D in 30 independent runs on UF1-UF10 CEC'09 test instances [74].

HYP: Relative Hypervolume, Γ: Gamma function, Δ: Delta function						
CEC'09	Min	Median	Mean	Std	Max	Metrics
UF1	0.00220045	0.01255353	0.01202675	0.00535225	0.02275685	HYP
	0.00121840	0.00174985	0.00180623	0.00022546	0.00238502	Γ
	0.07801665	0.11353670	0.11433680	0.01288763	0.15680761	Δ
UF2	0.00291273	0.00465720	0.00843102	0.01004413	0.02768225	HYP
	0.00203678	0.00218194	0.00180623	0.00265505	0.00715213	Γ
	0.11293602	0.13404632	0.14648345	0.04293143	0.23655212	Δ
UF3	0.00004254	0.00014643	0.00053125	0.00264534	0.01403214	HYP
	0.00121481	0.00242053	0.00430134	0.00656321	0.03284612	Γ
	0.07029556	0.10969542	0.10161512	0.08430107	0.50246075	Δ
UF4	0.00598240	0.55587351	0.60157204	0.32040712	0.83525984	HYP
	0.04764373	0.05537687	0.05456813	0.00326493	0.06052927	Γ
	0.12937862	0.23535025	0.23170045	0.04069459	0.30314138	Δ
UF5	0.00598240	0.65587451	0.60247204	0.35000214	0.78654364	HYP
	0.79072623	0.81263565	0.81472453	0.91047034	0.98745035	Γ
	0.98730817	0.82635653	0.83724536	0.86547032	0.98748051	Δ
UF6	0.02205605	0.37265342	0.40516564	0.20103465	0.85639675	HYP
	0.00010862	0.08427560	0.07814091	0.05140101	0.19555661	Γ
	0.87963604	0.90446772	0.90248338	0.08398419	0.93354324	Δ
UF7	0.00043681	0.00516514	0.00475982	0.00763563	0.03453752	HYP
	0.00105175	0.00170317	0.00172401	0.00047320	0.00323692	Γ
	0.06703412	0.09153683	0.08672308	0.04332503	0.25342028	Δ
UF8	0.87601574	0.85057461	0.84050834	0.00060703	0.89179615	HYP
	0.01510523	0.03068203	0.03202178	0.03741104	0.17045402	Γ
	0.01716533	0.04068203	0.04310127	0.04230103	0.18079422	Δ
UF9	0.86172901	0.87114726	0.87139222	0.00356474	0.98670624	HYP
	0.08674953	0.20654221	0.21256451	0.17921201	0.09908802	Γ
	0.45456348	0.66454237	0.57154904	0.08541573	0.83435762	Δ
UF10	0.87004740	0.87389041	0.86918374	0.00210681	0.91342709	HYP
	0.04122707	0.50831050	0.51040308	0.41692692	0.09303015	Γ
	0.5487735	0.71304252	0.71140378	0.13701056	0.61245170	Δ

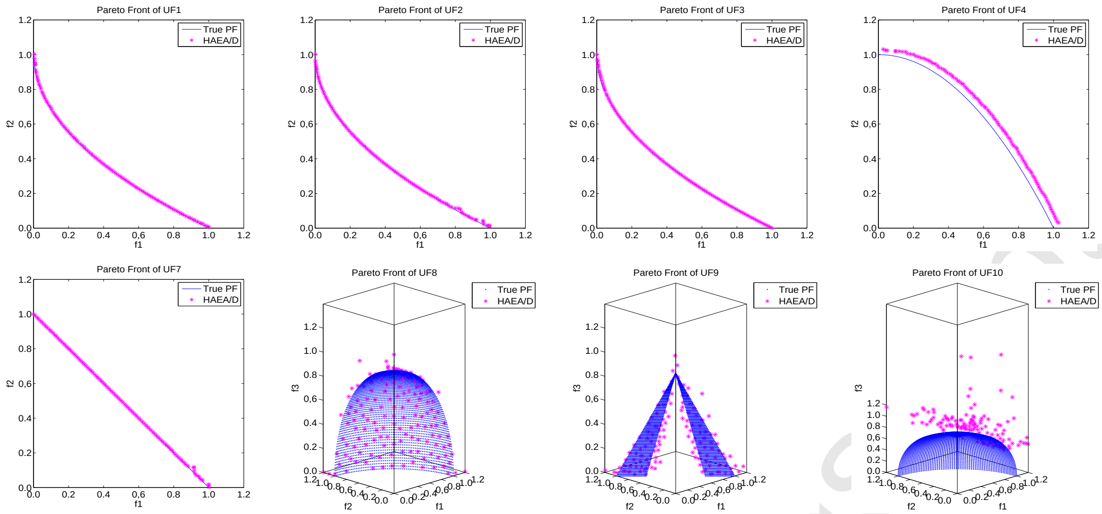

Figure 1. Plots of the approximated Pareto Fronts of the best run among 30 independent runs of HAEA/D on CEC'09 test instances [74].

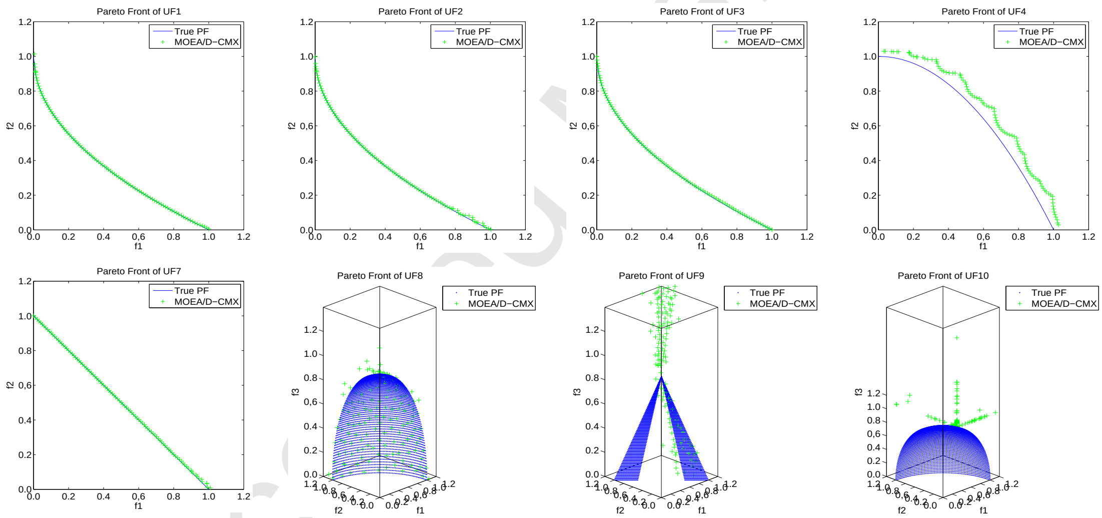

Figure 2. Plots of the approximated Pareto Fronts of the best run among 30 independent runs of MOEA/D-CMX on CEC'09 test instances [74].

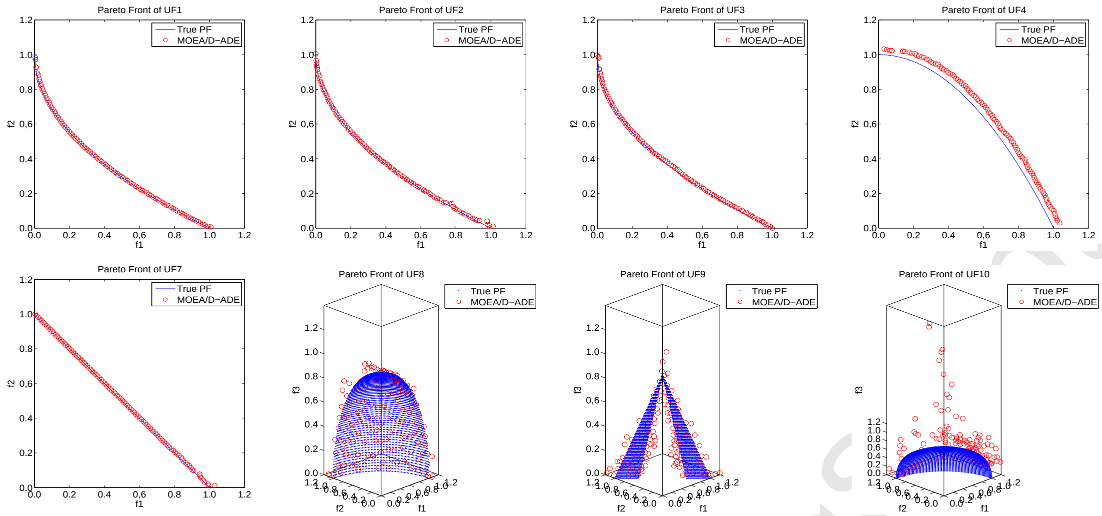

Figure 3. Plots of the approximated Pareto Fronts of the best run among 30 independent runs of MOEA/D-ADE on CEC'09 test instances [74].

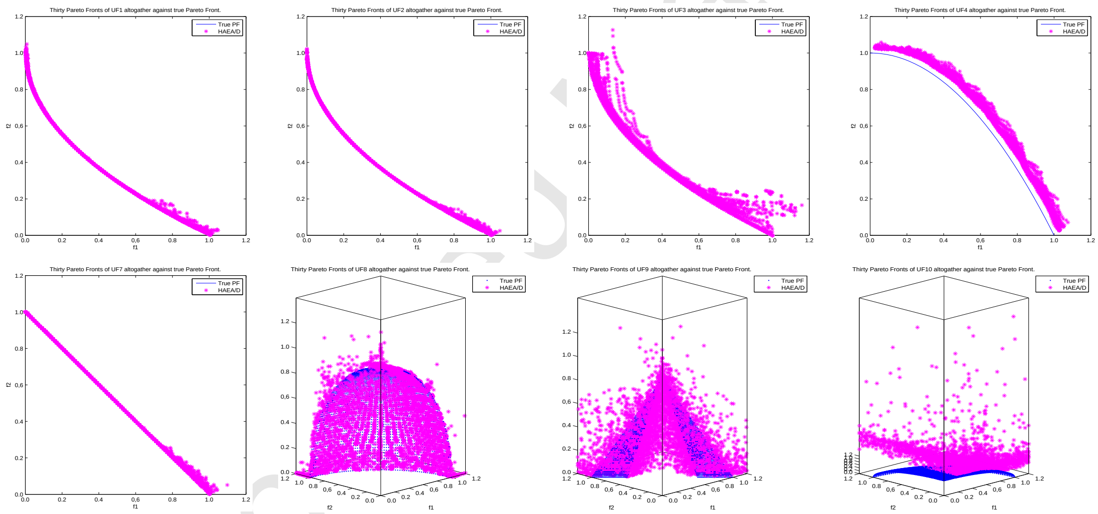

Figure 4. Plots of the 30 approximated Pareto Fronts found by HAEA/D on CEC'09 test instances, [74].

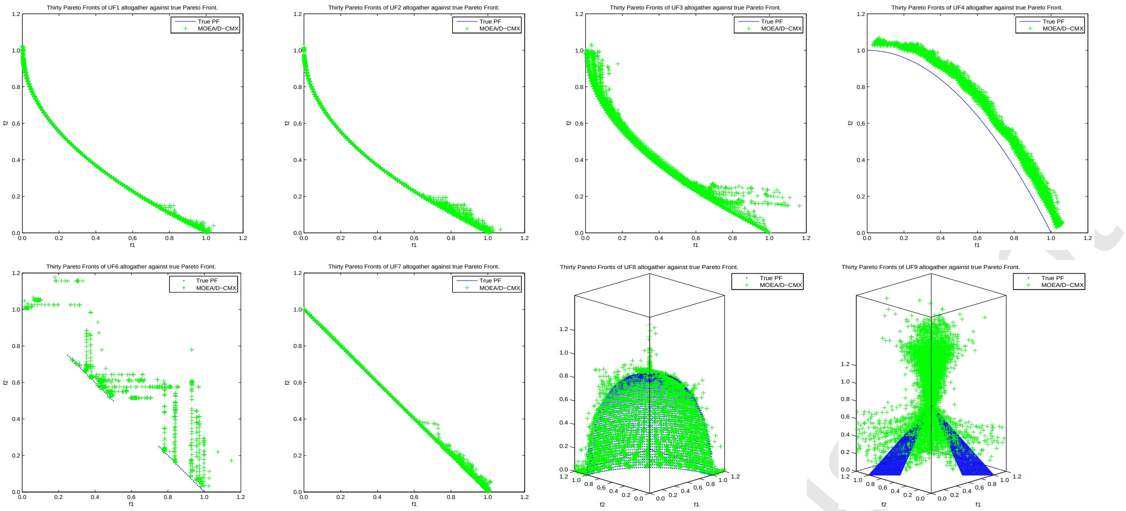

Figure 5. Plots of the 30 approximated Pareto Fronts found by MOEA/D-CMX for CEC'09 test instances, [74].

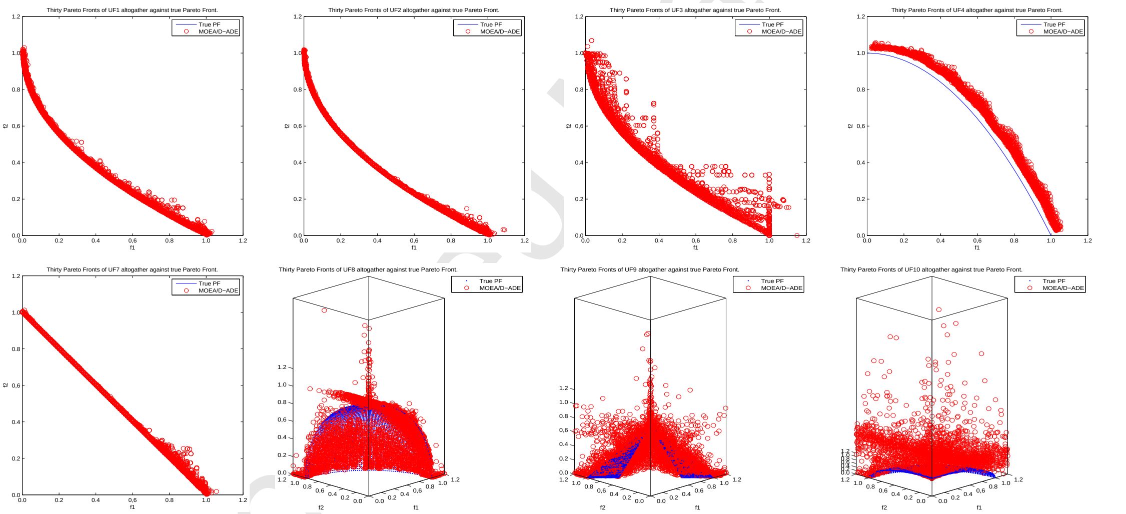

Figure 6. Plots of 30 approximate Pareto Fronts found by MOEA/D-ADE for CEC'09 test instances, [74].

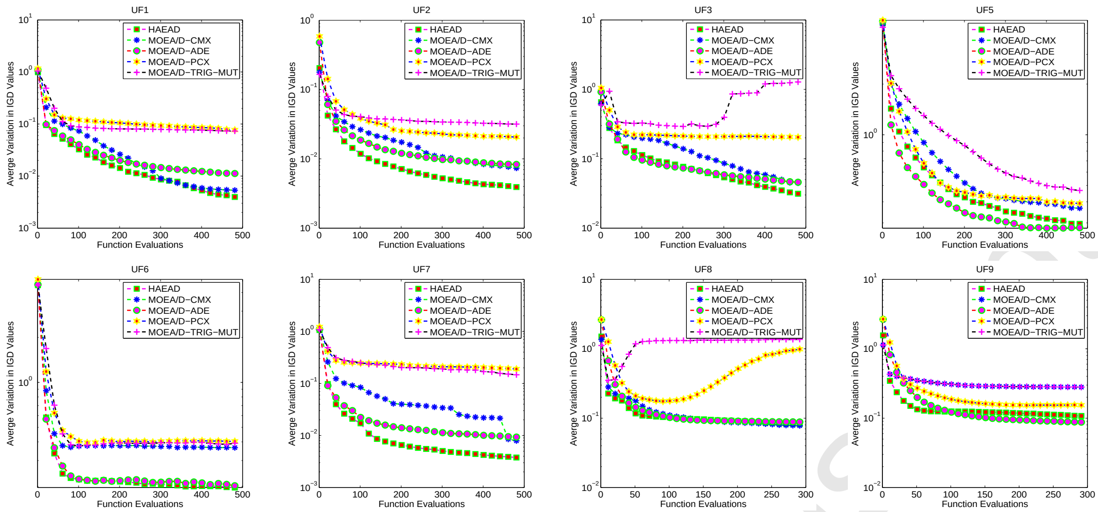

Figure 7. Plots of the final solutions with lowest IGD values found by HAEA/D, MOEA/D-CMX, MOEA/D-ADE,MOEA/D-PCX and MOEA/D-TM in 30 independent runs on CEC'09 test instances, [74].

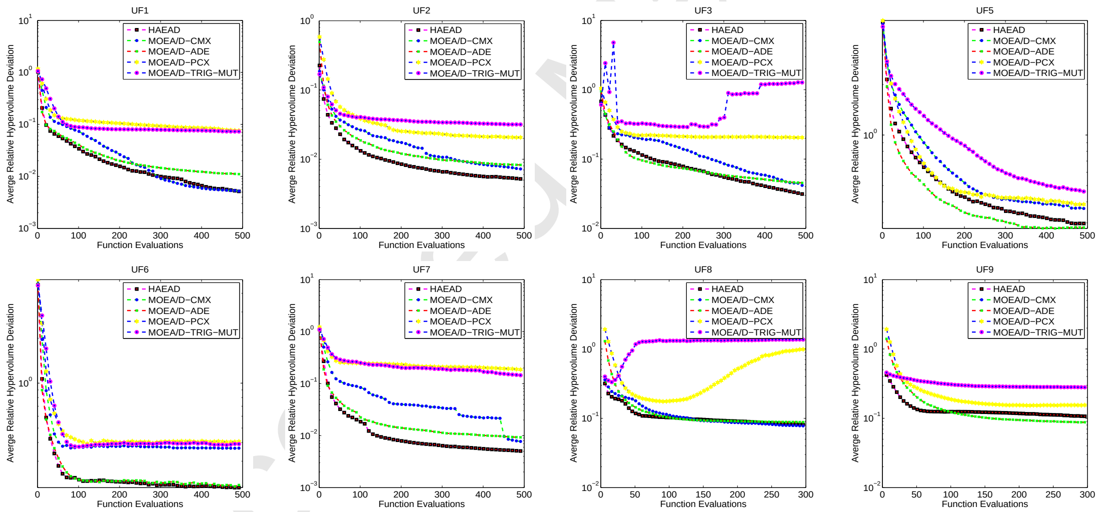

Figure 8. Plots of the final solutions with the lowest Relative Hypervolume values found by HAEA/D, MOEA/D-CMX, MOEA/D-ADE, MOEA/D-PCX and MOEA/D-TM in 30 independent runs on CEC'09 test instances, [74].

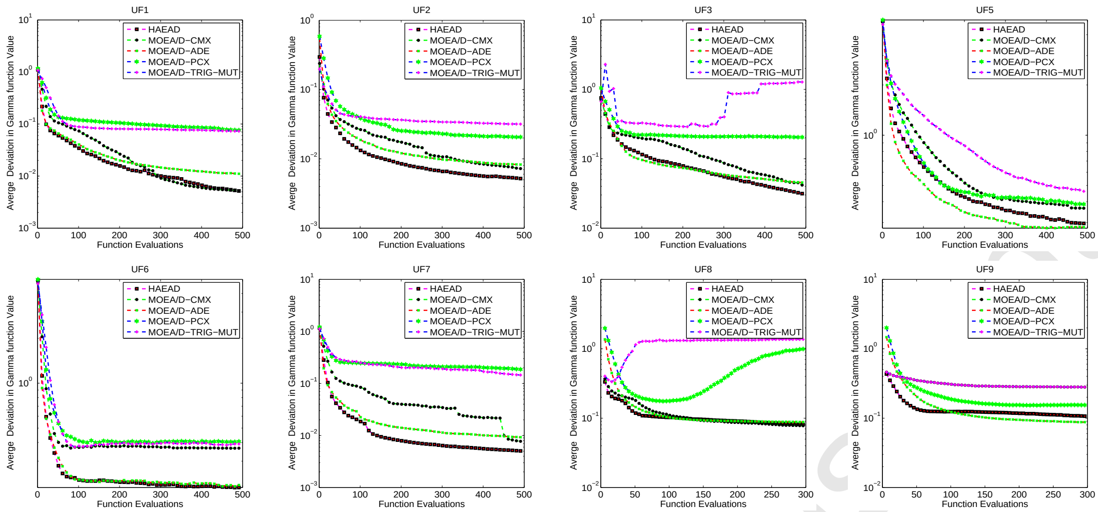

Figure 9. Plots of the final solutions with the lowestΓ function values found by HAEA/D, MOEA/D-CMX, MOEA/D-ADE, MOEA/D-PCX and MOEA/D-TM in 30 independent runs on CEC'09 test instances [74].

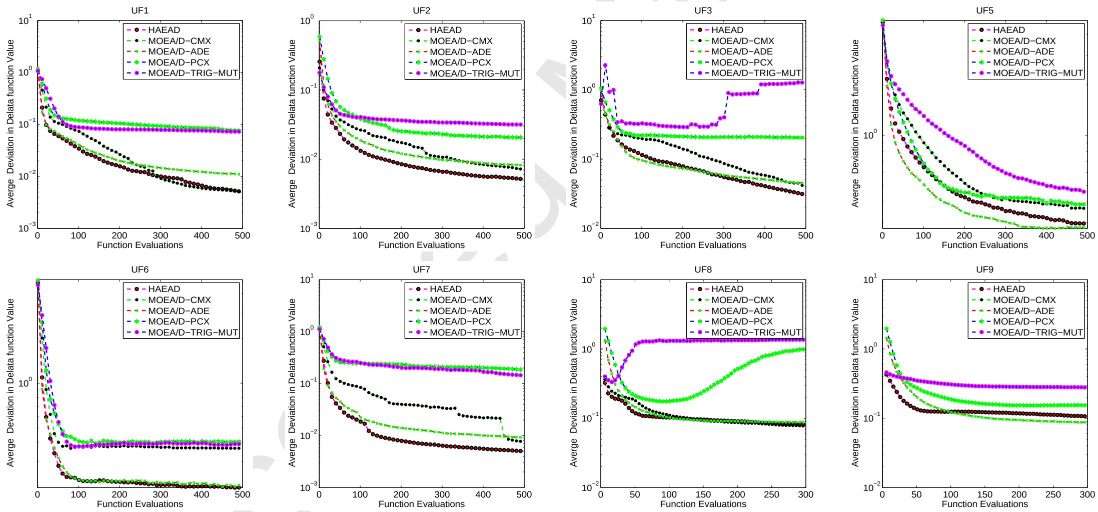

Figure 10. Plots of the final solutions with the lowest Δ function values found by HAEA/D, MOEA/D-CMX, MOEA/D-ADE, MOEA/D-PCX and MOEA/D-TM in 30 independent runs on CEC'09 test instances, [74].

5.2. Sensitivity of Population Size N and Neighbourhood Size T in HAEA/D

The neighbourhood size T is one of the most important parameters in HAEA/D. Its setting is based on population size N as in MOEA/D [72, 73, 33]. It is, therefore, important to study its impact when different population sizes N are used in the HAEA/D framework. The last columns of Tables 6 and 7 provide the different values of N and T used to obtain the recorded IGD-metric values corresponding to the solutions returned by HAEA/D on CEC'09 test instances. Note that all other parameters are as explained in Section 4.1 when the algorithm is run 30 times independently over each CEC'09 test problem [74]. In general, as clearly shown in Figure 11, the performance of the suggested algorithm gets better with large size populations and high neighbourhood sizes.

Table 6. IGD-metric values for solutions found by HAEA/D with different values of N applied to CEC'09 test instance [74].

CEC'09	Min	Median	Mean	Std	Max	N	T
UF1	0.0154861	0.043135	0.043793	0.0364123	0.150717	100	10
	0.011752	0.0097635	0.009703	0.034175	0.137849	200	20
	0.004129	0.005037	0.006873	0.007852	0.046767	300	30
	0.003803	0.006011	0.0060303	0.0075932	0.046752	400	40
	0.004017	0.005767	0.005690	0.002031	0.010453	500	50
UF2	0.0109156	0.013032	0.013201	0.001016	0.017311	100	10
	0.007832	0.010328	0.012010	0.001002	0.015842	200	20
	0.006134	0.004357	0.005265	0.000897	0.006245	300	30
	0.006223	0.004343	0.004427	0.000619	0.010179	400	40
	0.005304	0.004378	0.004231	0.000602	0.008324	500	50
UF3	0.047194	0.05124	0.051419	0.061489	0.125030	100	10
	0.045174	0.0472456	0.050420	0.06035	0.122030	200	20
	0.004188	0.02404	0.0245372	0.025863	0.086023	300	30
	0.004075	0.023814	0.024143	0.020288	0.0.71245	400	40
	0.004654	0.022030	0.022021	0.006756	0.034507	500	50
UF4	0.066031	0.080604	0.080407	0.008061	0.105614	100	10
	0.050347	0.053500	0.053462	0.004838	0.071733	200	20
	0.05036	0.013207	0.013301	0.015603	0.071123	300	30
	0.057589	0.060794	0.01120	0.005536	0.083088	400	40
	0.050508	0.015137	0.010154	0.004771	0.071678	500	50
UF5	0.3547931	0.526634	0.523743	0.127420	0.770570	100	10
	0.324541	0.400737	0.410643	0.116410	0.702550	200	20
	0.304162	0.361650	0.369325	0.070253	0.640103	300	30
	0.26791	0.377591	0.369084	0.070084	0.630531	400	40
	0.213475	0.353221	0.361207	0.069768	0.608106	500	50

Table 7. IGD-metric values for HAEA/D with different values of N when applied to CEC'09 test instance [74].

CEC'09	Min	Median	Mean	Std	Max	N	T
UF6	0.163743	0.430572	0.430632	0.132023	0.703027	100	10
	0.153843	0.423102	0.423134	0.132012	0.702076	200	20
	0.154252	0.420657	0.42101	0.130392	0.702004	300	30
	0.150471	0.407079	0.408064	0.130553	0.700130	400	40
	0.143015	0.412482	0.308267	0.123729	0.704633	500	50
UF7	0.008045	0.018031	0.181943	0.170417	0.534720	100	10
	0.007012	0.07022	0.071020	0.163422	0.546720	200	20
	0.005247	0.005635	0.006501	0.002456	0.016846	300	30
	0.005969	0.005531	0.0055401	0.0024201	0.0142367	400	40
	0.005032	0.005104	0.005201	0.001652	0.0140052	500	50
UF8	0.113042	0.132746	0.147027	0.034742	0.232342	100	10
	0.102062	0.131764	0.132125	0.034664	0.230363	200	20
	0.060263	0.088290	0.086985	0.034453	0.215247	500	50
	0.051623	0.065814	0.066302	0.016072	0.130021	600	60
	0.051145	0.065743	0.0655787	0.016023	0.130014	800	80
UF9	0.114720	0.201512	0.202102	0.023442	0.250684	100	10
	0.114617	0.201302	0.202113	0.023488	0.260696	200	20
	0.057576	0.152300	0.154230	0.0230373	0.189787	500	50
	0.057067	0.152041	0.151974	0.032441	0.118181	600	60
	0.053023	0.076842	0.07720	0.020167	0.10234	800	80
UF10	0.427416	0.510471	0.512021	0.056342	0.511059	100	10
	0.426425	0.511302	0.512011	0.056472	0.511057	200	20
	0.354046	0.456048	0.456202	0.045423	0.526323	500	50
	0.353065	0.440197	0.445363	0.037592	0.520965	600	60
	0.314561	0.436521	0.436701	0.044025	0.530024	800	80

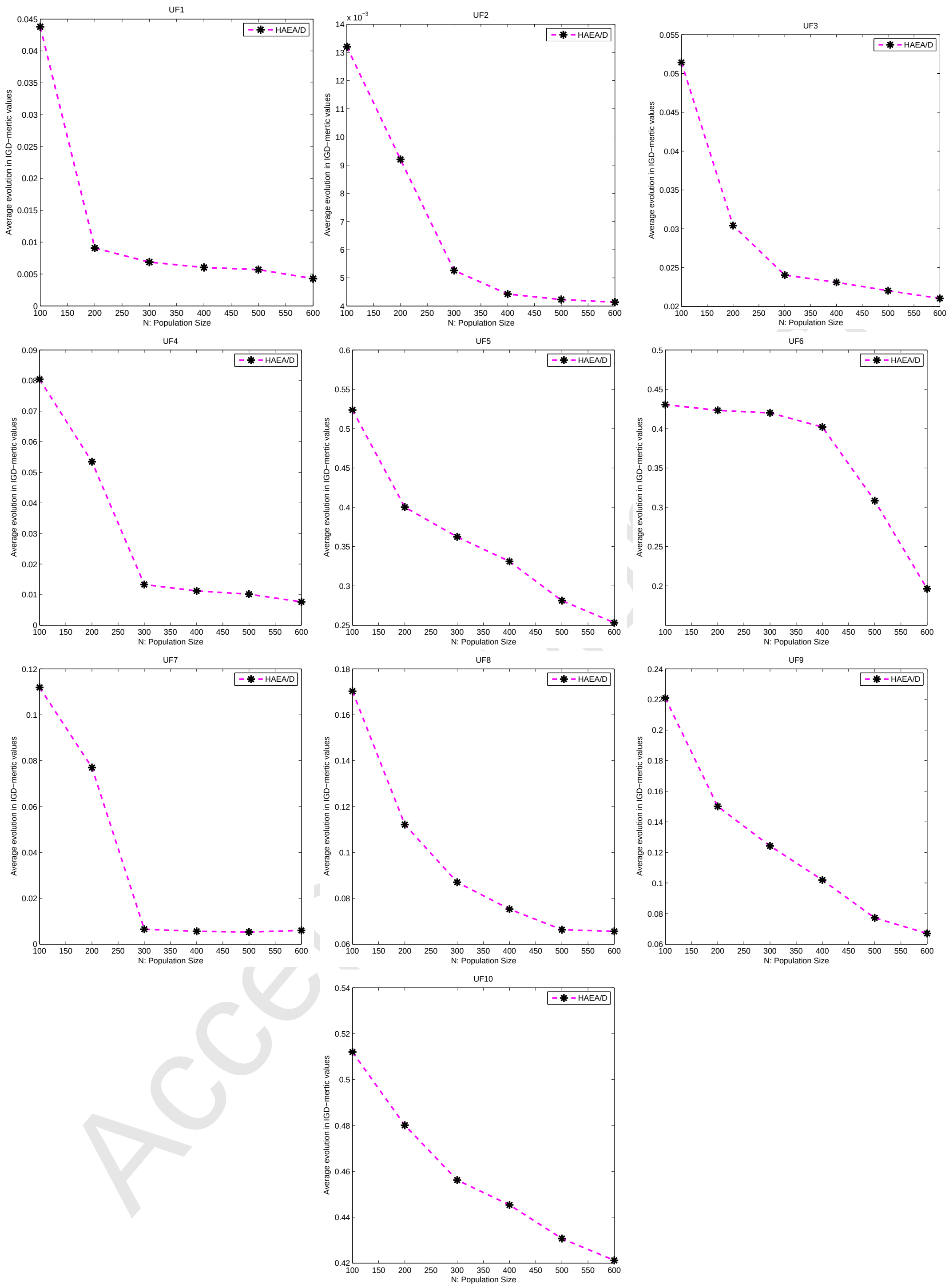

Figure 11. The average IGD-metric value versus the value of population size in HAEA/D for CEC'09 test instances [74].

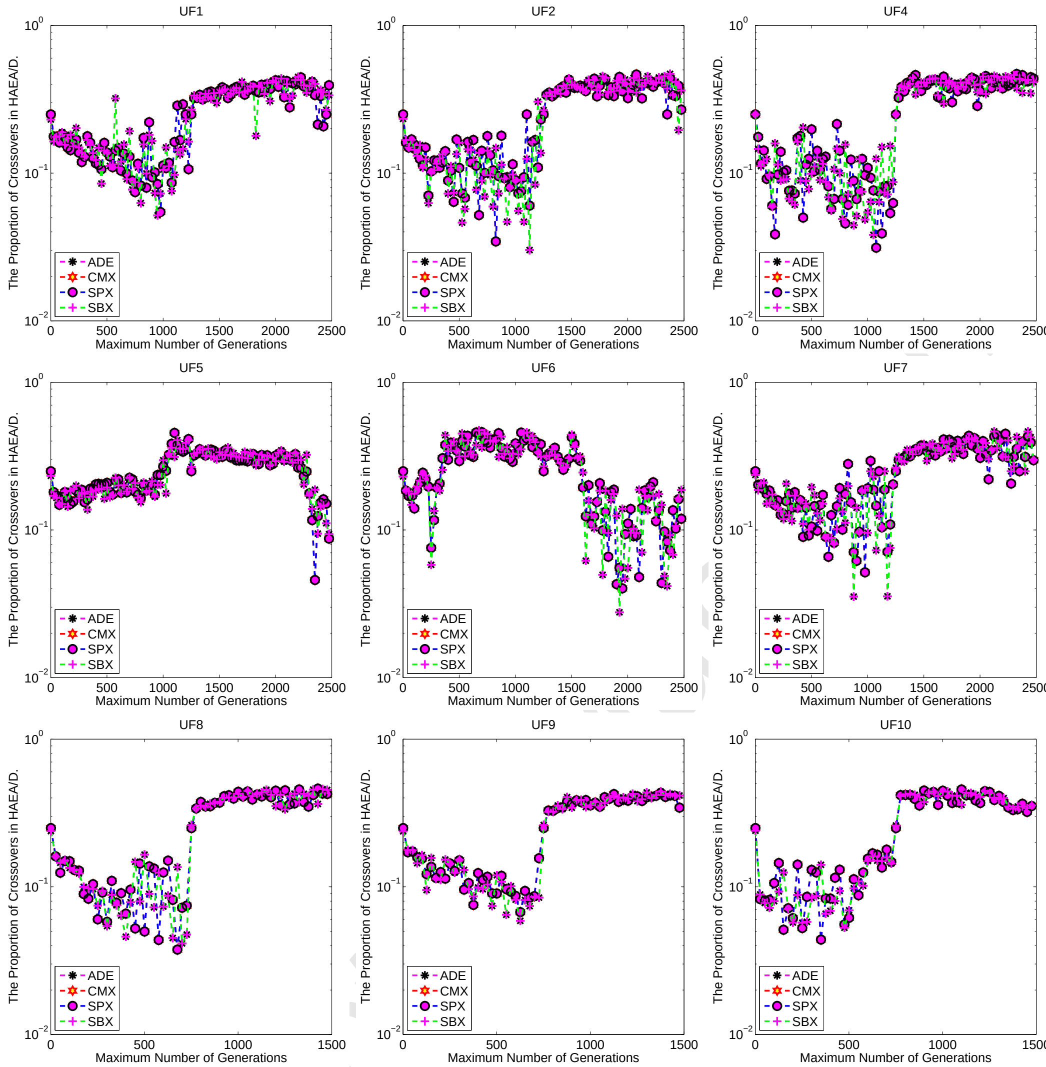

Figure 12. The proportion of crossover operators selected during the evolution process of HAEA/D applied to CEC'09 test problems.

The implementation of the Adaptive Operators Selection (AOS) within Pareto dominance-based MOEAs is tedious and complex compared to implementing decomposition-based MOEA. The latter are more flexible and suitable as a framework in which to deploy multiple search operators. This is because improvements in the search are easier to measure by virtue of the basic concept of decomposability which allows to convert the given MOP into N scalar subproblems. In this paper, therefore, we have studied the effect of the use of multiple search operators in adaptive and ensemble manner. In carried out experiment, we found that no single operator dominates the whole search process of the HAEA/D when applied to CEC'09 test instance [74].

Figure 12 demonstrates the effect of the use of multiple search operators in adaptive and ensemble manner. It can be seen in these figures that no single operator dominates the whole search process of the HAEA/D when applied to CEC'09 test instance [74].

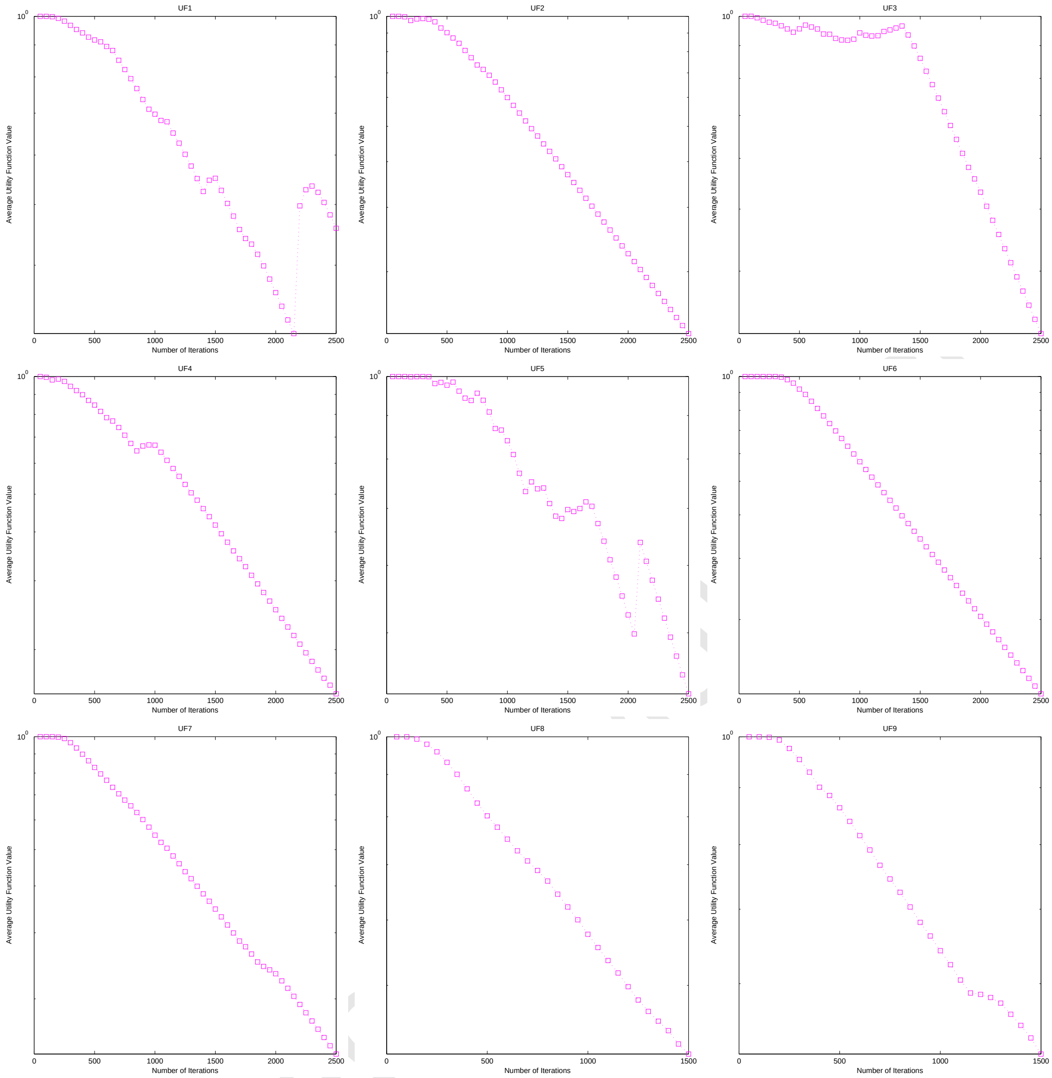

Figure 13. Dynamic evolution of the utility function values in HAEA/D when solving CEC'09 test problems.

Decomposition-based approaches convert the problem of approximating the PF into *N* scalar optimization problems (SOPs). These SOPs require different amounts of computational resources. Figure 13 shows how the suggested HAEA/D algorithm allocates resources to each of the *N* subproblems based on the measured improvement made by each solution in reducing the single objective function values.

6. Conclusion

Adaptive operator selection procedures employ multiple genetic operators and local search optimizers within an evolutionary algorithm framework to find the most suitable search operator for the given problems. A trial-and-error approach for MOEAs is unlikely to work. Engaging simultaneously various genetic operators not only improves the performance of the base line algorithm but also saves on the time that is necessary to find the operators that perform best in the different stages of the optimization process. This paper proposes a hybrid adaptive evolutionary algorithm based on decomposition which employs multiple search operators in an MOEA/D framework based on a self-adaptive scheme. The proposed methodology allocates a population of solutions dynamically to each crossover operator based on their respective performances, to create new solutions. The overall performance of HAEA/D has been evaluated on the CEC'09 test instances. The results have been compared to those of four different versions of MOEA/D: MOEA/D with MTS, MOEA/D with GDE3, DECMOSA-SQP and MOEA/D-CPDE. Four different performance indicators have been used in the comparative analysis.

HAEA/D has performed better on most CEC09 test instances in terms of proximity and diversity. These results suggest, therefore, that using adaptive genetic operators may be a good idea in other contexts. And using multiple solution approaches simultaneously as in the Multiple Algorithms Single Formulation paradigm or MAFS of [54] is worthwhile particularly when problem instances are new and the choice of appropriate algorithms for the given problems is not straightforward. Further investigation and testing is therefore warranted. In future, we also intend to test the performance of our suggested algorithm on dynamic multi-objective benchmarks developed for the sessions of the IEEE CEC'14 and IEEE CEC'15.

We also intend to investigative the algorithmic performance of our proposal on single objective constrained optimization problems. The basic idea is to convert a single objective constrained problem into a MOPs by treating the violation of constraints as an extra objective function.

7. Acknowledgment

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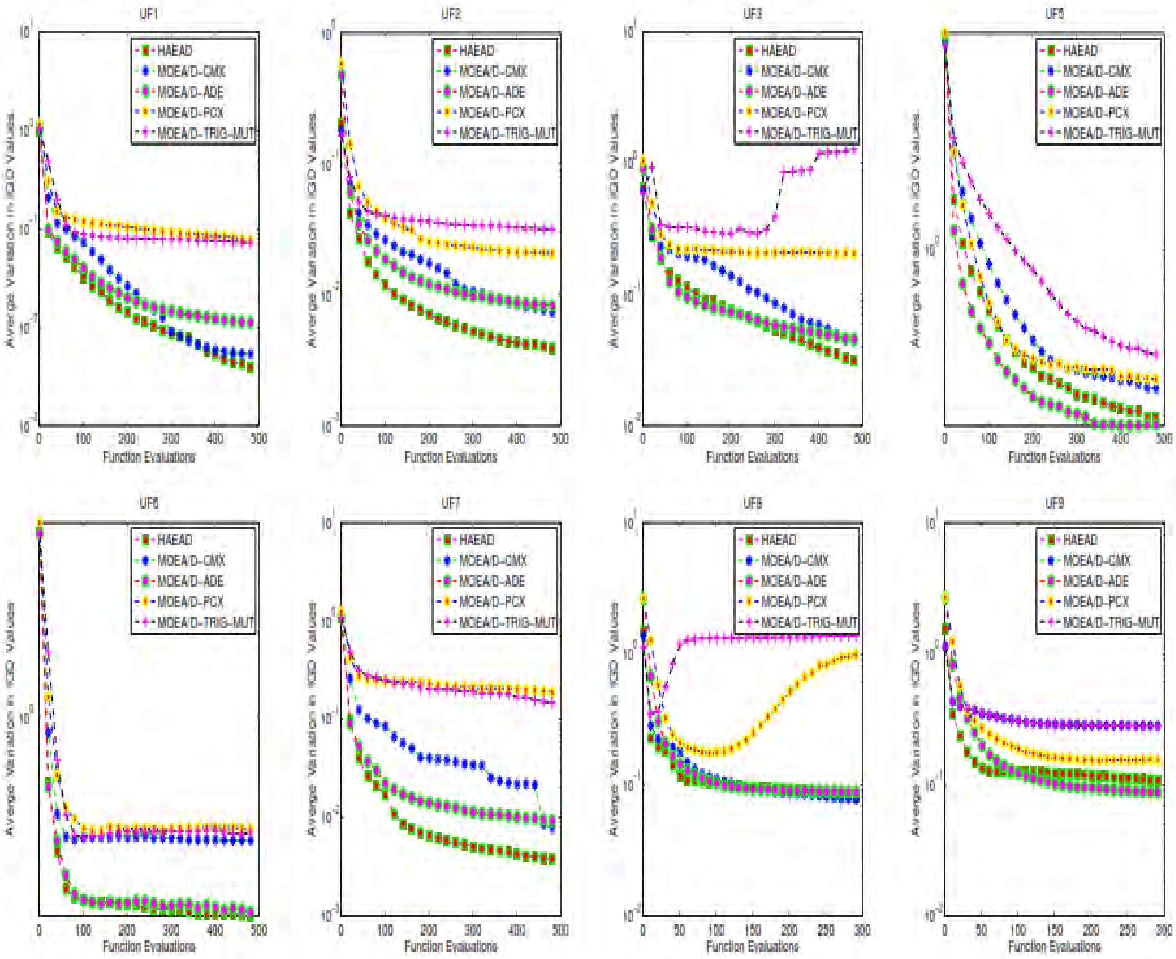
- We suggest the hybrid adaptive evolutionary algorithm based on decomposition (HAEA/D) which akin to the classical MOEA/D enhanced with the use of multiple search operators and an adaptive selection procedure at the population level to choose which genetic operator to use at a given point in the search/optimisation process.

- HAEA/D splits its population of candidate designs into subpopulations according to the number of operators; it then allocates resources to different search operators according to their respective performances, in adaptive manner.

- Experimental results obtained with HAEA/D on standard test suite of multi-objective optimization problems are compared with those of a number of state-of-the-art-MOEAs based on the decomposition concept.

- The experimental analysis of the HAEA/D include multi-objective memetic algorithm based on decomposition, multi-objective cloud particle optimization algorithm based on decomposition (MOEA/D-CPDE), Multiple Trajectory Search (MTS), Differential Evolution with self-adaptation and local search for constrained multi-objective optimization algorithm (DECMOSA-SQP), generalized DE3 (GDE3) and different variants of the MOEA/D paradigm.

- Performance indicators including the inverted generational distance (IGD), the relative hypervolme (RHV), Γ and Δ functions are used to compare the proposed algorithm against established algorithms as mentioned above. Note that the IGD metric has the ability to measure both the convergence and spread of the obtained optimal solutions.


UF1
UF2
UF3
UF5
UF6
UF7
UF8
UF9
HAEAD
MOEA/D-CMX
MOEA/D-ADE
MOEA/D-PCX
MOEA/D-TRIG-MUT
Averge Variation in IGD Values
Function Evaluations