INTERACTION OF ELECTROMAGNETIC WAVE AND PLASMA SLAB WITH PARTIALLY LINEAR AND SINU-SOIDAL ELECTRON DENSITY PROFILE

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Abstract—Interaction of an electromagnetic wave and an inhomogeneous plasma slab with electron distribution in the form of partially linear and sinusoidal profiles is analysed to determine new reflection, absorbtion and transmission characteristics. Broadband and tunable absorbtion performance of the plasma layer accompanied with narrowband reflection characteristic is presented as the function of electron density profile parameters and external magnetic field excitation. According to the resulting performance characteristics, proposed plasma layer is found to be useful as a new absorbing layer for shielding and stealth applications.

1. INTRODUCTION

The subject of interaction of electromagnetic waves and tunable plasma mediums has been studied by various authors and attracted growing attention as an important topic in plasma applications [1–16]. By using plasmas as cover layers, optimal reflection, absorbtion and transmission performances required in stealth applications, radio communications, radio astronomy etc. can be obtained. In the study by Vidmar [5], it has been found that inhomogeneous plasma slab can be used as a broadband radar absorbing layer. Interaction between electromagnetic

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wave and magnetized plasma is numerically studied in [6]. Then in a following study, the results of interaction of microwaves and plasma generated at atmospheric pressure are presented [7]. Recursive matrix approach called as scattering matrix method is proposed for the analysis of electromagnetic wave propagation through plasmas [8]. Guo and Wang [11] have studied the characteristics of electromagnetic wave incidence onto unmagnetized uniform plasma layer covering a metal surface in atmosphere conditions as the function of plasma parameters. Interaction between electromagnetic wave and conductor coating plasma is also analyzed in literature [9, 14].

In addition to the general reflection, absorbtion and transmission behaviors, special plasma responses under certain circumstances are also presented in literature. Gradov and Stenflo [3] have discussed the possibility for the nonlinear transparency of a dense magnetized plasma and shown that according to the parametric excitation of the surface waves; significant part of the radiation can penetrate through the plasma. In the following study, they have suggested a mechanism to amplify the transmitted and reflected electromagnetic wave power from an immobile plasma having properly varying slab density in time. They pointed out that total radiation energy can be larger than the incident energy due to the release of the stored wave energy in the plasma during short periods [4].

In literature, various electron density functions of the plasma layer are considered such as exponential, parabolic and steepened profiles to obtain special characteristics [8, 10, 15]. In a very recent study, sinusoidal electron distribution is analyzed, and resulting frequency selective characteristics are discussed as the function of sinusoidal profile parameters and external magnetic field excitation [16]. Similar to the nonuniform plasmas, uniform plasmas are also analyzed in literature with constant electron density [12].

In the present study, electromagnetic wave propagation through a plasma layer linearly increasing electron density followed by sinusoidal profile is firstly studied as a new plasma medium to determine special reflection, absorbtion and transmission behaviors. In the analysis, plasma medium is taken as cold, weakly ionized, steady state and collisional. Background magnetic field is assumed to be uniform and parallel to the magnetized slab. The first part of the plasma with linearly varying electron density is used to provide matching of the electromagnetic wave and the following sinusoidally varying profile to obtain improved absorbtion and transmission performances. As shown in the study, these characteristics are tunable by means of external magnetic field excitation.

In the analysis, whole plasma layer is firstly divided into

sufficiently thin, adjacent subslabs with constant plasma parameters. Then starting with Maxwell's equations, reflected, absorbed and transmitted power expressions are derived. The effects of collision frequency, maximum electron number density value, the lengths of two parts of the density function with respect to each other and external magnetic field strength are discussed to determine appropriate conditions for broadband and tunable absorbtion performance accompanied with narrowband reflection behavior.

2. FORMULATION

Maxwell's equations can be generally expressed for a plane wave propagation in the magnetized plasma medium with $\exp(j\omega t)$ time dependence as

$$\nabla \times E = -j\omega\mu_0 H \tag{1}$$
$$\nabla \times H = j\omega\varepsilon_0\tilde{\varepsilon}_r E \tag{2}$$

where complex dielectric constant of the plasma with conductivity σ is given in the form

$$\tilde{\varepsilon}_r = \varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0} \tag{3}$$

Then, wave equation for electric field is obtained as

$$\nabla^2 E = \left(\frac{\tilde{\varepsilon}_r}{c^2}\right)\omega^2 E \tag{4}$$

where c is the speed of light.

For an incident electromagnetic wave $E_i = E_0\exp(j\omega t - \gamma z)\mathbf{a}_x$ through a plasma slab, the complex propagation constant is defined in the form

$$\gamma^2 = \tilde{\varepsilon}_r\frac{\omega^2}{c^2} \tag{5}$$

The complex dielectric constant of the magnetized plasma in the case of plane wave incidence is given by the Appleton's formula [1]. In this study, for the case of normally incident plane wave, magnetic field is parallel to the propagation direction, and complex dielectric constant is obtained as

$$\tilde{\varepsilon}_r = 1 - \frac{(\omega_p/\omega)^2}{1 - j\dfrac{(v_{en} - \omega_{ce})}{\omega}} \tag{6}$$

where v_{en} is the collision frequency; ω_p and ω_{ce} are plasma frequency and electron cyclotron frequency, respectively, which are given in [1, 2]

as

$$w_p^2 = e^2 \frac{N}{m\varepsilon_0} \tag{7}$$

$$w_{ce} = \frac{eB}{m} \tag{8}$$

where e, m are charge and mass of an electron, respectively; N is the electron number density; ε_0 is the permittivity of free space; B is the external magnetic field strength.

For the formulation of absorbtion, transmission and reflection characteristics of the plasma layer, it is divided into equal width, sufficiently thin, adjacent subslabs through which electromagnetic wave propagation will be modeled as shown in Fig. 1. In each sublayer, plasma is taken as uniform.

Electron density profile N along the plasma layer for a linearly increasing profile followed by sinusoidal part is defined as

$$N = \begin{cases} N_m z/L' & \text{for } z < L' \\ N_m(0.5 + 0.5\cos(\pi(z - L')/(L - L'))) & \text{for } z > L' \end{cases} \tag{9}$$

having maximum electron number density of N_m. L is the thickness of the total plasma slab, and L' is the length of the linear profile.
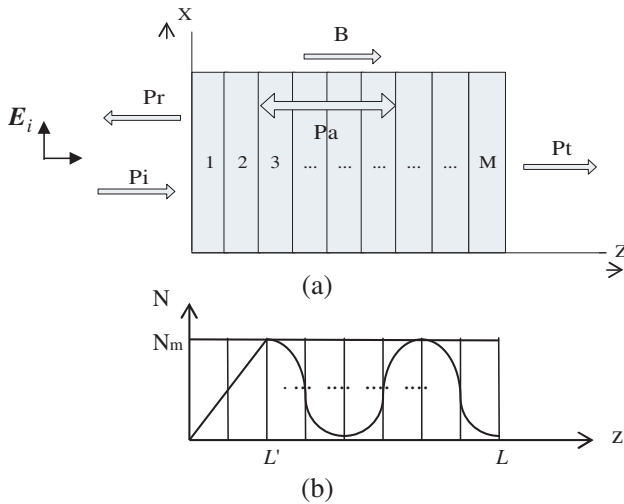

Figure 1. (a) Electromagnetic wave propagation through a plasma layer with equal width subslabs. (b) Electron density profile along the plasma layer having linear and sinusoidal parts.

In this study, normal incidence is assumed as a general case. The reflection coefficient on the $(i+1)$th interface with neglecting multiple reflections from the interfaces is obtained as

$$\Gamma(i+1) = \frac{\sqrt{\tilde{\varepsilon}_r(i)} - \sqrt{\tilde{\varepsilon}_r(i+1)}}{\sqrt{\tilde{\varepsilon}_r(i)} + \sqrt{\tilde{\varepsilon}_r(i+1)}} \tag{10}$$

Thus, reflected power is calculated by using attenuation constants $\alpha(i)$ of each subslab, and reflection coefficients is $\Gamma(j)$ evaluated at their interfaces.

The power through the plasma layer at the first interface is given as

$$P_i = \Gamma(1)^2 \times P_i + P_{t1} \Rightarrow P_{t1} = P_i \left(1 - \Gamma(1)^2\right) \tag{11}$$

Along the propagation towards the second interface, transmitted wave power exponentially reduces as

$$P_{t2} = e^{-2\alpha(1)d} P_i \left(1 - \Gamma(1)^2\right) \tag{12}$$

Similar procedure can be repeated for each interface as

$$\begin{aligned} P_r &= P_i \Gamma(1)^2 + P_i e^{-4\alpha(1)d}(1 - \Gamma(1)^2)\Gamma(2)^2 \\ &\quad + P_i e^{-4\alpha(1)d}(1 - \Gamma(1)^2)(1 - \Gamma(2)^2)e^{-4\alpha(2)d}\Gamma(3)^2 + ... \end{aligned} \tag{13}$$

Then, total reflected power can be expressed for totally M layered plasma in the form

$$P_r = P_i \left\{ \Gamma(1)|^2 + \sum_{j=2}^{M} \left(|\Gamma(j)|^2 \prod_{i=1}^{j-1} \left(\exp\left[-4\alpha(i)d\right] \left(1 - |\Gamma(i)|^2\right) \right) \right) \right\} \tag{14}$$

where d is the thickness of the subslabs. Then, total transmitted power is expressed as

$$P_t = P_i \prod_{i=1}^{M} \left\{ \exp\left[-2\alpha(i)d\right] \left(1 - |\Gamma(i)|^2\right) \right\} \tag{15}$$

Thus, absorbed power in the plasma slab can be simply calculated from

$$P_a = P_i - (P_r + P_t) \tag{16}$$

The results are presented in the next section.

3. NUMERICAL RESULTS AND DISCUSSION

In this part, absorbtion, transmission and reflection characteristics of an electromagnetic wave incidence onto a plasma slab having

partially linear and sinusoidal electron density profile are analyzed. In the numerical analysis, plasma layer is divided into sufficiently thin subslabs to provide convergence of the results in each cases. In the first section of the study, $L = 12\,\text{cm}$ plasma slab is studied with the first and second halves with linearly and sinusoidally varying electron density profiles, respectively. The normalized absorbed power as the function of collision frequency when maximum value of the electron density is $N_m = 1 \times 10^{18}\,\text{m}^{-3}$, and external magnetic field strength is $B = 0.4\,\text{T}$ is shown in Fig. 2.
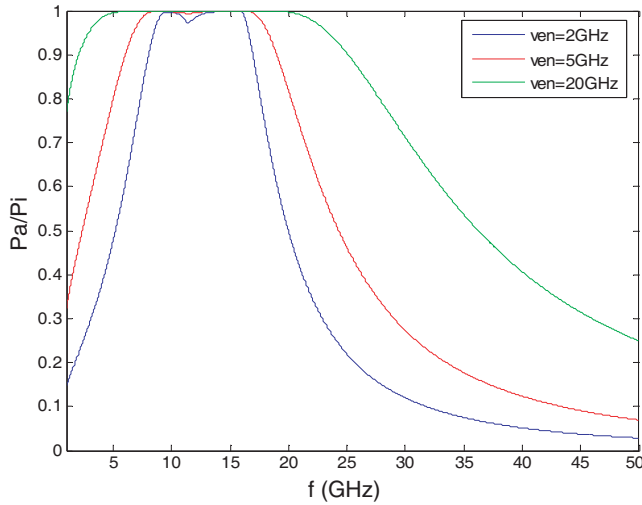

Figure 2. Normalized absorbed power with wave frequency for different collision frequencies. $N_m = 1 \times 10^{18}\,\text{m}^{-3}$, $B = 0.4\,\text{T}$, $L = 12\,\text{cm}$.

It is observed that absorbtion band becomes wider as the collision frequency of the plasma increases. The peak point of absorbtion is around electron cyclotron frequency which is $14\,\text{GHz}$ when $B = 0.4\,\text{T}$. Fig. 3 shows the transmission characteristics for different collision frequencies. As shown in the figure, transmitted power increases with decreasing collision frequency for wave frequencies higher than $15\,\text{GHz}$. Around electron cyclotron frequency, transmitted power decreases sharply to zero because of the increase in absorbed power as shown in Fig. 2.

From the last two figures it is seen that absorbtion and reflection behaviors of the plasma for different collison frequencies are dual of each other which means small amount of reflection form the plasma. It can also be noted that highly collisonal plasma absorbs considerable amount of electromagnetic wave power in a wide frequency band.
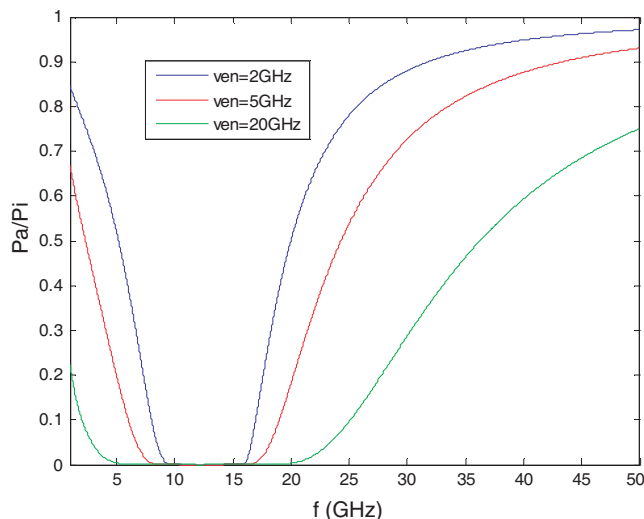
Figure 3. Normalized transmitted power versus wave frequency for different collision frequencies, $N_m = 1 \times 10^{18}\,\mathrm{m^{-3}}$, $B = 0.4\,\mathrm{T}$, $L = 12\,\mathrm{cm}$.

In this part of the study, in order to analyze the effect of plasma thickness, length of the plasma slab is doubled having linear and sinusoidal profiles of equal length. In Fig. 4, normalized absorbed power with respect to maximum electron number density for $v_{en} = 20\,\mathrm{GHz}$ is shown. When $N_m = 1 \times 10^{18}\,\mathrm{m^{-3}}$ the absorbtion band is wider with respect to other maximum electron number density values. The normalized peak value of absorbed power decreases in addition to decrease in the maximum electron number density value.

The comparison of Fig. 4 and Fig. 2 for $N_m = 1 \times 10^{18}\,\mathrm{m^{-3}}$, $v_{en} = 20\,\mathrm{GHz}$ case indicates that the magnitude and bandwidth of the absorbed power increase for the plasma having wider linear and sinusoidal profile regions.

In this part, the length ratio of the linear and sinusoidal regions is changed in order to observe the reflected power variation as shown in Fig. 5. The total thickness of the plasma is L, and the length of the linearly varying part is L'. The normalized reflected power for $L'/(L - L') = 1/7$ ratio is named as "case 1", and $L'/(L - L') = 2$ is named as "case 2" in this figure. It can be concluded from Fig. 5 that when the $L'/(L - L')$ ratio increases overall reflected power decreases which is very small in amount in both cases. The increased length of the linear region provides better matching of the incident wave resulting in wider absorbtion band.
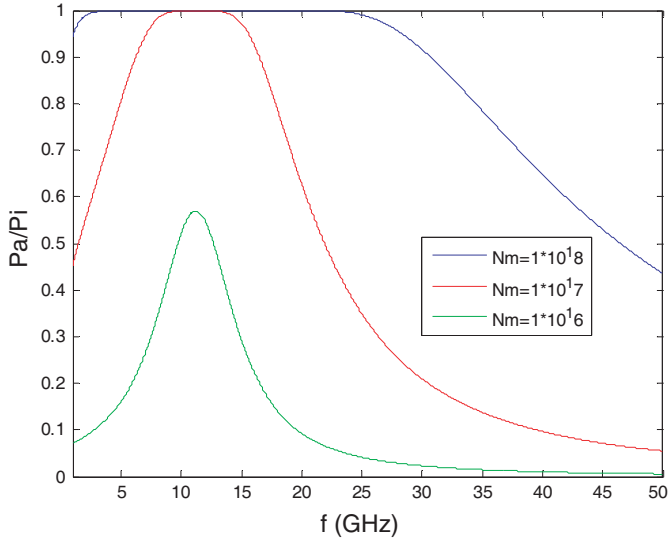
Figure 4. Normalized absorbed power versus wave frequency for different maximum plasma number density values, $v_{en} = 20\,\text{GHz}$, $B = 0.4\,\text{T}$, $L = 12\,\text{cm}$.
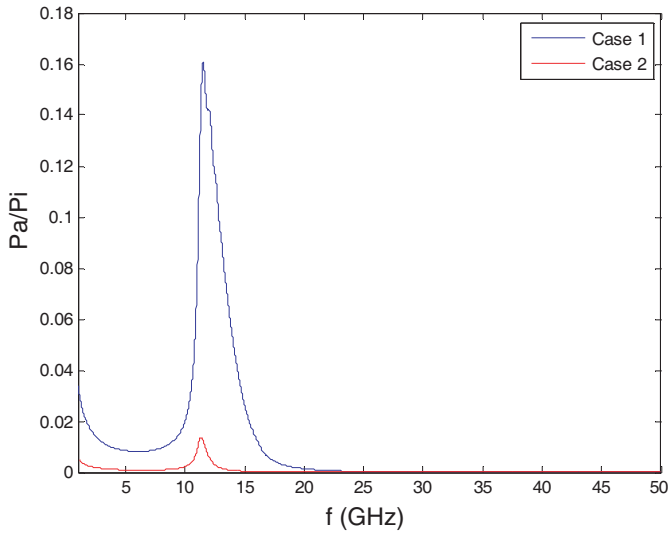

Figure 5. Normalized reflected power versus wave frequency for $L'/(L - L') = 1/7$ (case 1) and $L'/(L - L') = 2$ (case 2). $N_m = 1 \times 10^{18}\,\text{m}^{-3}$, $v_{en} = 2\,\text{GHz}$, $B = 0.4\,\text{T}$, $L = 24\,\text{cm}$.

The effect of external magnetic field tuning on the absorbtion characteristic of the plasma is shown in Fig. 6. It is seen that absorbtion band can be shifted by adjusting the magnetic field strength due to changing resonant frequency. This tuning property of the magnetic field can be similarly applied to the other characteristics of the plasma-wave interaction.
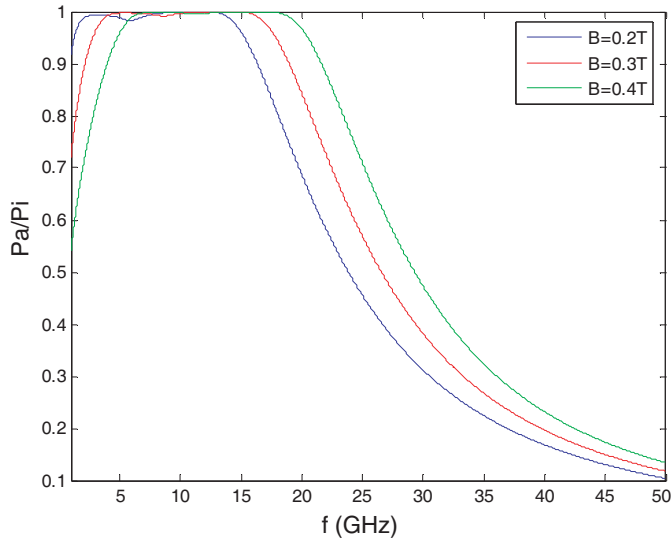

Figure 6. Normalized absorbed power versus wave frequency for different external magnetic field strengths. $N_m = 1 \times 10^{18} \, \text{m}^{-3}$, $v_{en} = 5 \, \text{GHz}$, $L = 24 \, \text{cm}$.

4. CONCLUSION

In this study, interaction of an electromagnetic wave and nonuniform plasma slab having new electron density profile is analyzed by varying the plasma parameters to determine absorbtion, transmission and reflection characteristics. It is shown that highly collisonal and high-density plasma greatly absorbs the electromagnetic wave power along the wide frequency band. The effects of total thickness and the ratio of two electron profile function lengths on the general characteristics are also analyzed. Increased length of the plasma is shown to improve wideband absorbtion characteristic. It is also shown that normalized reflected power decreases while absorbed power increases as the matching provided by the linearly varying part with the tuning

provided by the sinusoidal part. Thus, adjustment of parameters of two profile regions has an important role in decreasing the reflected power while increasing the absorbtion due to provided matching. It is shown that considerably improved new absorber characteristics can be obtained from the designed plasma layer due to proper adjustment of the other plasma parameters, such as collison frequency and maximum electron density value. Hence, plasma layer with new electron number density profile can be proposed as a broadband absorber for some special shielding applications.

ACKNOWLEDGMENT

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