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Joint transceiver FIR filter design for multiuser MIMO channel shortening equalization and full equalization using channel duality

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Abstract: In this work, joint transceiver filter design for channel shortening equalization is studied for a multiuser multiple input multiple output (MIMO) frequency selective channel. We develop algorithms for the multiuser MIMO, broadcast, multiple access, and interference channel models, where the link between these models is established via channel duality. We also provide an algorithm for full equalization of these channel models as a special case of the proposed channel shortening equalizer. The main contributions of this paper are investigating the multiuser scenario for the joint transceiver design for MIMO channel shortening equalization and extending the channel duality theorem to shortening equalization and full equalization problems.

Key words: Channel shortening, equalization, frequency selective, joint transmitter–receiver design, multiple input multiple output, multiuser, channel duality

1. Introduction

The interest in multiple input multiple output (MIMO) communications [1] continues increasing due to their superior performance and effective resource utilizing properties. The use of multiple antennas increases data rate (via multiplexing) or improves diversity according to quality of service requirements. MIMO has become one of the most frequently used methods of wireless communication standards including IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi), HSPA+ (3G), and Long-Term Evolution (4G).

In spite of the advantages of MIMO, such as increased channel capacity and transmission reliability, intersymbol interference (ISI) continues to be a problem. There are several techniques to manage ISI but their performance is limited or they have rather complex structures. For instance, orthogonal frequency division multiplexing (OFDM) [2] and the Viterbi algorithm [3] achieve good performance mitigating ISI but have increased complexity. In OFDM, the use of a cyclic prefix (CP) causes a loss in data rate. Similarly, the computational complexity of the Viterbi algorithm increases exponentially with increasing channel taps length. To overcome these problems one could employ channel shortening equalization to obtain an effective channel that is shorter than the actual channel. This would either increase the throughput of OFDM or decrease the computational complexity of the Viterbi algorithm.

Channel shortening equalization developed for different objectives has been described in the literature for more than two decades [4–6]. However, almost all proposals are for receiver-only processing. We have eliminated the constraint of using only a receive filter and tried to form a joint transceiver in order to reduce

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the complexity of the receiver. It is known that the joint transceiver design [7–9] has superior performance compared to the receiver-only design. The transceiver in our work differs from the ones in [7,8] because of the channel shortening procedure. Instead of full channel equalization, we perform shortening equalization in order to achieve improved data rate with negligible trade-off on the ISI mitigation. In [9] the authors considered a frequency flat channel scenario; we have improved this assumption and considered frequency selective fading channels.

In [10], an algorithm for joint transceiver design for MIMO channel shortening was developed and shown to perform better than receiver-only channel shortening for a single user scenario. In our work, we improve [10] and investigate a multiuser scenario. To the best of our knowledge, joint transceiver design has not been investigated in the literature for MIMO channel shortening for a multiuser (MU) scenario except [11]. However, in [11], the problem is handled only for the transmitter that performs power allocation. In this paper we form finite impulse response (FIR) filters not only at the receiver but also at the transmitter.

Minimum mean square error (MMSE) [2] channel shortening equalization is considered in this paper and it is extended to the four different multiuser MIMO channel scenarios: MU-MIMO, broadcast (BC), multiple access (MAC), and interference (IC) channels [4]. For the BC scenario, channel duality [12,13] is used for the optimization process.

We also examined full equalization (filtering into a single tap) as a special case of the developed channel shortening equalization method, where the channel is fully equalized to a single tap. The work in this paper upgrades our previous works [14,15] as we investigate both channel shortening equalization and full channel equalization. Moreover, as an addition, optimization of the MIMO interference channel scenario is established. The proposed transceiver design is a general framework that is applicable to both multiple antenna and single antenna systems and MAC, BC, and IC scenarios where the channel is frequency selective.

2. System model

2.1. Multiuser multiple access channel model

Consider a frequency selective multiuser MAC scenario with K noncooperating transmitters and a receiver with K cooperating receive filters, where the transmitter and the receiver of the k^{th} user have N_{T_k} and N_{R_k} antennas, respectively, and the base-station has a total of N_R antennas as demonstrated in Figure 1.

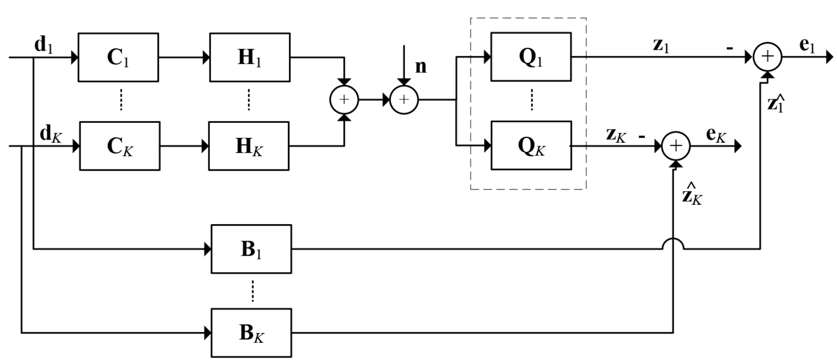

Figure 1. Multiple access channel model. The box with the dashed lines indicates cooperation among filters.

Here $\mathbf{C}_k, \mathbf{Q}_k$, and $\mathbf{H}_k$ are the transmit filter, receive filter, and the channel matrix for the k^{th} user, respectively. As stated earlier, the channel is assumed to be frequency selective, i.e. the signal bandwidth is larger than the coherence bandwidth of the channel. In frequency selective channels the received signal experiences significant intersymbol interference (ISI). In order to reduce the ISI, a multicarrier approach is performed in this study where the channel is divided into small subchannels so that each subchannel experiences frequency flat fading. Each channel is generated from zero-mean circularly symmetric complex Gaussian random variables where the average energy of all channels is assumed to be equal and is set to unity. The lower branches shown in Figure 1, with matrices $\mathbf{B}_k, k = 1, \ldots, K$, do not exist in reality. These branches correspond to the virtual target impulse response (TIR) giving the equalized channel impulse response for the k^{th} user.

The optimization in this work is conducted in the frequency domain; hence filters and channels in Figure 1 will be demonstrated by their frequency response representations as a function of frequency, ω.

The transmit filter $\mathbf{C}_k(\omega)$, the receive filter $\mathbf{Q}_k(\omega)$, the multiple access channel $\mathbf{H}_k(\omega)$, and the target impulse response $\mathbf{B}_k(\omega)$ for the k^{th} user have dimensions $N_{T_k} \times n_{i_k}$, $n_{i_k} \times N_{R_k}$, $N_{R_k} \times N_{T_k}$, and $n_{i_k} \times n_{i_k}$, respectively. Here the k^{th} user's data stream is n_{i_k}.

The aim of this work is to design the joint transceiver filter pairs $(\mathbf{C}_k(\omega), \mathbf{Q}_k(\omega))$ in order to shorten the multiuser MAC of length N_b taps to a shortened channel of length $n_b(< N_b)$ taps according to the MMSE criterion, as demonstrated in [10] for a single user scenario. Here n_b is a predefined and fixed system parameter.

The signal at the output of the k^{th} receiver at frequency ω is

$$\mathbf{z}_k(\omega) = \mathbf{Q}_k(\omega)\mathbf{H}_k(\omega)\mathbf{C}_k(\omega)\mathbf{d}_k(\omega) + \sum_{j=1, j\neq k}^{K} \mathbf{Q}_k(\omega)\mathbf{H}_j(\omega)\mathbf{C}_j(\omega)\mathbf{d}_j(\omega) + \mathbf{Q}_k(\omega)\mathbf{n}_k(\omega), \tag{1}$$

where $\mathbf{d}_k(\omega)$ is frequency domain representation of the k^{th} user data sequence $(\mathbf{d}_k[n])$, which is an $n_{i_k} \times 1$ vector with independent and identically distributed (i.i.d.) zero mean elements chosen within a unit energy constellation. Zero mean circularly symmetric additive white Gaussian noise (AWGN) vector with dimensions $N_{R_k} \times 1$ and variance σ_n^2 at the receiver is given by $\mathbf{n}_k(\omega)$. It is accepted that the data and AWGN are mutually statistically independent. The first term in the received signal is the desired signal for the k^{th} user. The second and third terms correspond to the interference from other users and the channel noise, respectively.

The optimization criterion for channel shortening equalization is chosen as MMSE in this work. Therefore the error has to be calculated. The error for the k^{th} user is defined as the difference of the receive filter and the TIR output,

$$\mathbf{e}_k(\omega) = \mathbf{z}_k(\omega) - \hat{\mathbf{z}}_k(\omega) = \mathbf{B}_k(\omega)\mathbf{d}_k(\omega) - \left(\sum_{j=1}^{K} \mathbf{Q}_k(\omega)\mathbf{H}_j(\omega)\mathbf{C}_j(\omega)\mathbf{d}_j(\omega) + \mathbf{Q}_k(\omega)\mathbf{n}_k(\omega)\right). \tag{2}$$

The MSE for the k^{th} user at frequency ω can be shown as

$$\begin{aligned} &MSE_k\left(\mathbf{Q}_k(\omega), \{\mathbf{C}_j(\omega), \mathbf{H}_j(\omega), \mathbf{B}_j(\omega)\}_{j=1}^{K}\right) = tr\left\{E\left\{\mathbf{e}_k(\omega)\mathbf{e}_k^H(\omega)\right\}\right\} \\ &= tr\left\{\mathbf{B}_k(\omega)\mathbf{B}_k^H(\omega) - \mathbf{B}_k(\omega)\mathbf{C}_k^H(\omega)\mathbf{H}_k^H(\omega)\mathbf{Q}_k^H(\omega) - \mathbf{Q}_k(\omega)\mathbf{H}_k(\omega)\mathbf{C}_k(\omega)\mathbf{B}_k^H(\omega)\right. \\ &\left. + \mathbf{Q}_k(\omega)\left(\sum_{j=1}^{K}\mathbf{H}_j(\omega)\mathbf{C}_j(\omega)\mathbf{C}_j^H(\omega)\mathbf{H}_j^H(\omega)\right)\mathbf{Q}_k^H(\omega) + \sigma_n^2\mathbf{Q}_k(\omega)\mathbf{Q}_k^H(\omega)\right\}. \end{aligned} \tag{3}$$

The minimization is performed over sum MSE, which is calculated over all users and all frequencies. A more precise notation would be integration over ω rather than summation. However, for implementation purposes we opt for the summation and assume that the fast Fourier transform (FFT) size is large enough to prevent aliasing.

Without any constraints, minimizing this term will result in a trivial solution; therefore, we impose two constraints on the problem. A per user power constraint is considered as the first constraint. The second constraint forces the TIR to be orthonormal in order to avoid interference among users. The optimization problem for the MAC becomes

$$\begin{aligned} &\min_{\mathbf{Q}_k(\omega),\mathbf{C}_k(\omega),\mathbf{B}_k(\omega)} && \sum_{k=1}^{K}\sum_{\omega} MSE_k\left(\mathbf{Q}_k(\omega),\{\mathbf{C}_j(\omega),\mathbf{H}_j(\omega),\mathbf{B}_j(\omega)\}_{j=1}^{K}\right) \\ &\text{s.t.} && tr\left\{\sum_{\omega}\mathbf{C}_k(\omega)\mathbf{C}_k^H(\omega)\right\}\le P_k,\forall k \\ & && \mathbf{B}_k(\omega)\mathbf{B}_l^H(\omega)=\Sigma_{kl},\forall k,l,\omega, \end{aligned} \tag{4}$$

where P_k is the power constraint of the k^{th} user, $\mathbf{I}$ is the identity matrix, and $\boldsymbol{\Sigma}_{kl}=\begin{cases}\mathbf{I}, & k=l\\ 0\mathbf{I}, & k\neq l\end{cases}$.

For the rest of the analysis, the frequency index (ω) will be dropped as a subscript for the clarity of notation where convenient (e.g., $\mathbf{Q}_k(\omega)\rightarrow\mathbf{Q}_{k,\omega}$).

2.2. Multiuser broadcast channel model

Multiuser BC with K cooperating transmitters and K noncooperating receivers is considered in Figure 2, where the cooperating transmitters have a total of N_T antennas.

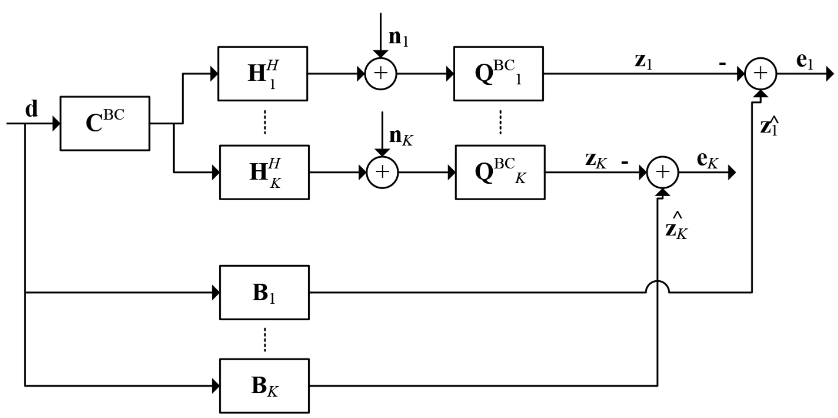

Figure 2. Broadcast channel model.

The aim is again to design a joint transceiver filter pair ($\mathbf{C}_{\omega}^{BC}=\left[\begin{array}{ccc}\mathbf{C}_{1,\omega}^{BC} & \cdots & \mathbf{C}_{K,\omega}^{BC}\end{array}\right],\mathbf{Q}_{k,\omega}^{BC}$) in order to shorten the multiuser broadcast channel according to the MMSE criterion.

The error at the output of the k^{th} receiver at frequency ω and MSE are

$$\mathbf{e}_{k,\omega}=\mathbf{z}_{k,\omega}-\hat{\mathbf{z}}_{k,\omega}=\mathbf{B}_{k,\omega}\mathbf{d}_{k,\omega}-\left(\sum_{j=1}^{K}\mathbf{Q}_{k,\omega}^{BC}\mathbf{H}_{j,\omega}^{H}\mathbf{C}_{j,\omega}^{BC}\mathbf{d}_{j,\omega}+\mathbf{Q}_{k,\omega}^{BC}\mathbf{n}_{k,\omega}\right), \tag{5}$$

$$MSE_{k,\omega} = tr\left\{E\left\{\mathbf{e}_{k,\omega}\mathbf{e}_{k,\omega}^{H}\right\}\right\} = tr\left\{\mathbf{B}_{k,\omega}\mathbf{B}_{k,\omega}^{H} - \mathbf{B}_{k,\omega}\mathbf{C}_{k,\omega}^{BC,H}\mathbf{H}_{k,\omega}\mathbf{Q}_{k,\omega}^{BC,H} - \mathbf{Q}_{k,\omega}^{BC}\mathbf{H}_{k,\omega}^{H}\mathbf{C}_{k,\omega}^{BC}\mathbf{B}_{k}^{H}(\omega) + \mathbf{Q}_{k,\omega}^{BC}\left(\sum_{j=1}^{K}\mathbf{H}_{j,\omega}^{H}\mathbf{C}_{j,\omega}^{BC}\mathbf{C}_{j,\omega}^{BC,H}\mathbf{H}_{j,\omega}\right)\mathbf{Q}_{k,\omega}^{BC,H} + \sigma_n^2\mathbf{Q}_{k,\omega}^{BC}\mathbf{Q}_{k,\omega}^{BC,H}\right\}. \quad (6)$$

Similar to the MAC scenario, the optimization problem for the BC scenario becomes

$$\begin{aligned} \min_{\mathbf{Q}_{k,\omega}^{BC},\mathbf{C}_{k,\omega}^{BC},\mathbf{B}_{k,\omega}} \quad & \sum_{k=1}^{K}\sum_{\omega} MSE_{k,\omega} \\ \text{s.t.} \quad & tr\left\{\sum_{\omega}\mathbf{C}_{k,\omega}^{BC}\mathbf{C}_{k,\omega}^{BC,H}\right\} \le P_k, \forall k \\ & \mathbf{B}_{k,\omega}\mathbf{B}_{l,\omega}^{H} = \Sigma_{kl}, \forall k,l,\omega. \end{aligned} \quad (7)$$

Due to the diagonal structure of the receiver in the BC scenario, the problem in (7) is difficult to optimize. One method to solve this problem is to utilize the BC-MAC duality [13], stating that under certain conditions both the broadcast and the multiple access channels achieve the same sum MSE region. In order to find the optimum solution of the problem in (7), one can obtain equivalent MAC transformation, solve the MAC problem, and, using this transformation, obtain optimal filters of the BC scenario.

The transformation between original BC transmit (receive) and equivalent MAC receive (transmit) filters depends on the scalar and the transformation given below [16]

$$\alpha_k = \sqrt{\frac{P_k}{tr\left\{\sum_{\omega}\mathbf{Q}_{k,\omega}^{MAC}\mathbf{Q}_{k,\omega}^{MAC,H}\right\}}}, \mathbf{C}_{k,\omega}^{BC} = \alpha_k\mathbf{Q}_{k,\omega}^{MAC,H}, \text{ and } \mathbf{Q}_{k,\omega}^{BC} = \frac{1}{\alpha_k}\mathbf{C}_{k,\omega}^{MAC,H}. \quad (8)$$

The resulting equivalent MAC is shown in Figure 3.

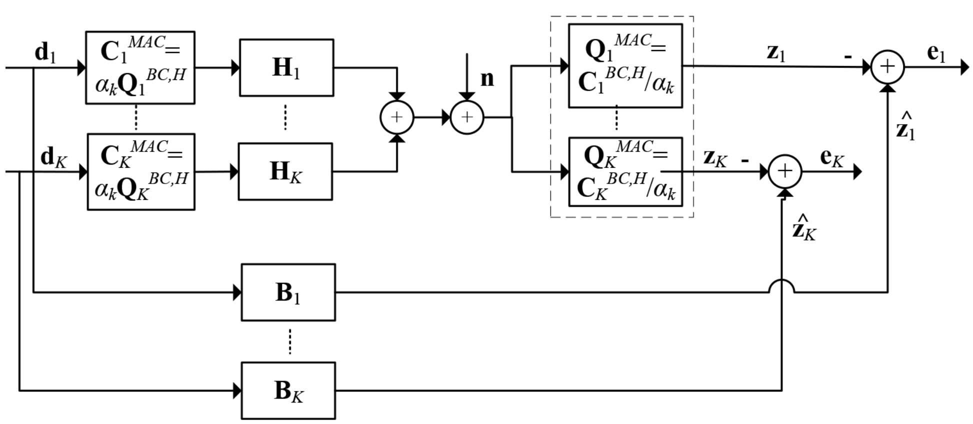

Figure 3. Equivalent MAC channel model, transformed from BC.

For the equivalent MAC scenario, the optimization problem takes the following form:

$$\begin{aligned} \min_{\mathbf{Q}_{k,\omega}^{MAC},\mathbf{C}_{k,\omega}^{MAC},\mathbf{B}_{k,\omega}} \quad & \sum_{k=1}^{K}\sum_{\omega} MSE_{k,\omega}^{MAC} \\ \text{s.t.} \quad & tr\left\{\sum_{\omega}\mathbf{C}_{k,\omega}^{MAC}\mathbf{C}_{k,\omega}^{MAC,H}\right\} \leq P_k, \forall k \\ & \mathbf{B}_{k,\omega}\mathbf{B}_{l,\omega}^{H} = \Sigma_{kl}, \forall k, l, \omega, \end{aligned} \tag{9}$$

$$\begin{aligned} MSE_{k,\omega}^{MAC} = \; & tr\left\{\mathbf{B}_{k,\omega}\mathbf{B}_{k,\omega}^{H} - \mathbf{B}_{k,\omega}\mathbf{C}_{k,\omega}^{MAC,H}\mathbf{H}_{k,\omega}^{H}\mathbf{Q}_{k,\omega}^{MAC,H} - \mathbf{Q}_{k,\omega}^{MAC}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}^{MAC}\mathbf{B}_{k}^{H}(\omega)\right. \\ & \left. + \mathbf{Q}_{k,\omega}^{MAC}\left(\sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}^{MAC}\mathbf{C}_{j,\omega}^{MAC,H}\mathbf{H}_{j,\omega}^{H}\right)\mathbf{Q}_{k,\omega}^{MAC,H} + \sigma_n^2\mathbf{Q}_{k,\omega}^{MAC}\mathbf{Q}_{k,\omega}^{MAC,H}\right\}. \end{aligned} \tag{10}$$

2.3. Multiuser MIMO channel and multiuser interference channel model

In addition to the multiuser MAC and BC scenarios, the MU-MIMO and IC scenarios are also investigated in this work. In the MIMO case both the transmitters and receivers are in cooperation and in the IC case neither the transmitters nor the receivers cooperate.

In the multiuser MIMO scenario shown in Figure 4, the transmitters and the receivers are in cooperation. This scenario is very similar to the author's previous work in [10]. In [10] a single user (SU) scenario is handled, but increasing the layers allocated to the single user is equal to multiple users with one layer for the MIMO channel. Therefore, the MU-MIMO scenario in this work is obtained by increasing the layers for a SU-MIMO scenario. For the MU-MIMO scenario the MSE at frequency ω is

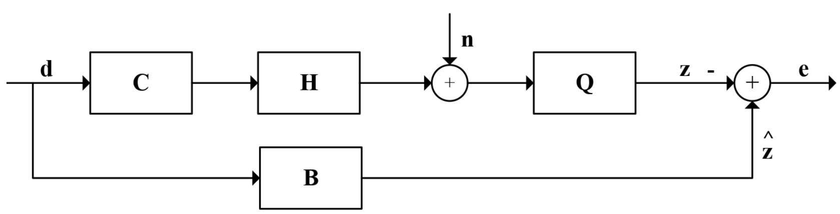

Figure 4. MU-MIMO channel model.

$$\mathbf{e}_k(\omega) = \mathbf{z}_k(\omega) - \hat{\mathbf{z}}_k(\omega) = \mathbf{B}_k(\omega)\mathbf{d}_k(\omega) - \left(\sum_{j=1}^{K}\mathbf{Q}_k(\omega)\mathbf{H}_j(\omega)\mathbf{C}_j(\omega)\mathbf{d}_j(\omega) + \mathbf{Q}_k(\omega)\mathbf{n}_k(\omega)\right). \tag{11}$$

and the optimization problem with power constraint P_{total} is

$$\begin{aligned} \min_{\mathbf{Q}_{\omega},\mathbf{C}_{\omega},\mathbf{B}_{\omega}} \quad & \sum_{\omega} MSE_{\omega} \\ \text{s.t.} \quad & tr\left\{\sum_{\omega}\mathbf{C}_{\omega}\mathbf{C}_{\omega}^{H}\right\} \leq P_{total} \\ & \mathbf{B}_{\omega}\mathbf{B}_{\omega}^{H} = \mathbf{I}, \forall \omega, \end{aligned} \tag{12}$$

In the multiuser IC scenario shown in Figure 5, receivers consider the messages from other sources as interference. It is assumed that the interference is taken as noise. For the MU-IC scenario the MSE (k^{th} user, ω frequency) and the optimization problem are

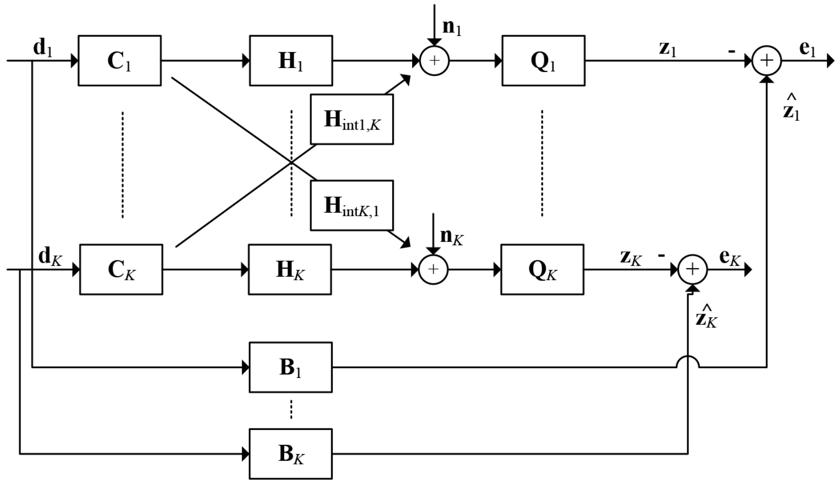

Figure 5. Interference channel model.

$$\begin{aligned} MSE_{k,\omega} &= tr\left\{E\left\{\mathbf{e}_{k,\omega}\mathbf{e}_{k,\omega}^H\right\}\right\} = tr\left\{\mathbf{B}_{k,\omega}\mathbf{B}_{k,\omega}^H - \mathbf{B}_{k,\omega}\mathbf{C}_{k,\omega}^H\mathbf{H}_{k,\omega}^H\mathbf{Q}_{k,\omega}^H - \mathbf{Q}_{k,\omega}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}\mathbf{B}_{k,\omega}^H \right. \\ &\left. + \sum_{j=1}^{K}\mathbf{Q}_{j,\omega}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}\mathbf{C}_{j,\omega}^H\mathbf{H}_{j,\omega}^H\mathbf{Q}_{j,\omega}^H + \sigma_n^2\mathbf{Q}_{k,\omega}\mathbf{Q}_{k,\omega}^H\right\}, \end{aligned} \tag{13}$$

$$\begin{aligned} \min_{Q_{k,\omega},,\mathbf{C}_{k,\omega},\mathbf{B}_{k,\omega}} \quad & \sum_{k=1}^{K}\sum_{\omega} MSE_{k,\omega} \\ \text{s.t.} \quad & tr\left\{\sum_{\omega}\mathbf{C}_{k,\omega}\mathbf{C}_{k,\omega}^H\right\} \le P_k, \forall k \\ & \mathbf{B}_{k,\omega}\mathbf{B}_{l,\omega}^H = \Sigma_{kl}, \forall k,l,\omega. \end{aligned} \tag{14}$$

as given above.

3. Optimum filter design

3.1. Filter design for MAC

First, the Lagrange multipliers method is applied to the problem in (4),

$$L = \sum_{k=1}^{K}\sum_{\omega} MSE_{k,\omega}^{\mathbf{Q},\mathbf{C},\mathbf{B}} - \lambda_k\left(tr\left\{\sum_{\omega}\mathbf{C}_{k,\omega}\mathbf{C}_{k,\omega}^H\right\} - P_k\right) + \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{\omega}\mu_{k,l,\omega}tr\left\{\mathbf{B}_{k,\omega}\mathbf{B}_{l,\omega}^H - \mathbf{\Sigma}_{kl}\right\}, \tag{15}$$

where L is the Lagrangian and the MSE in Eq. (3) is shown as $MSE_{k,\omega}^{\mathbf{Q},\mathbf{C},\mathbf{B}}$. Taking the derivative of the Lagrangian w. r. t. the receive filter and equating to zero yields

$$\frac{\partial L}{\partial \mathbf{Q}_{k,\omega}^H} = -\mathbf{B}_{k,\omega}\mathbf{C}_{k,\omega}^H\mathbf{H}_{k,\omega}^H + \mathbf{Q}_{k,\omega}\left(\sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}\mathbf{C}_{j,\omega}^H\mathbf{H}_{j,\omega}^H\right) + \sigma_n^2\mathbf{Q}_{k,\omega} = \mathbf{0}. \tag{16}$$

Therefore the optimum receive filter ($\mathbf{Q}_{k,\omega,\text{opt}}$) as a function of $\mathbf{C}_{k,\omega}$ and $\mathbf{B}_{k,\omega}$ becomes

$$\mathbf{Q}_{k,\omega,\text{opt}} = \mathbf{B}_{k,\omega}\mathbf{C}_{k,\omega}^H\mathbf{H}_{k,\omega}^H \left[\sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}\mathbf{C}_{j,\omega}^H\mathbf{H}_{j,\omega}^H + \sigma_n^2\mathbf{I}\right]^{-1}. \tag{17}$$

Substituting this solution back into the MSE in (3) reduces the problem to

$$\begin{aligned} \min_{\mathbf{C}_{k,\omega},\mathbf{B}_{k,\omega}} \quad & \sum_{k=1}^{K}\sum_{\omega} MSE_{k,\omega}^{\mathbf{C},\mathbf{B}} = \sum_{k=1}^{K}\sum_{\omega} tr\left\{\mathbf{B}_{k,\omega}\left[\mathbf{I}-\mathbf{C}_{k,\omega}^H\mathbf{H}_{k,\omega}^H\mathbf{T}^{-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}\right]\mathbf{B}_{k,\omega}^H\right\} \\ \text{s.t.} \quad & tr\left\{\sum_{\omega}\mathbf{C}_{k,\omega}\mathbf{C}_{k,\omega}^H\right\} \leq P_k, \forall k \\ & \mathbf{B}_{k,\omega}\mathbf{B}_{l,\omega}^H = \Sigma_{kl}, \forall k,l,\omega, \end{aligned} \tag{18}$$

where $\mathbf{T} = \sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}\mathbf{C}_{j,\omega}^H\mathbf{H}_{j,\omega}^H + \sigma_n^2\mathbf{I}$, and $MSE_{k,\omega}^{\mathbf{C},\mathbf{B}}$ is the MSE found by substituting the optimum receive filter. So far, a closed form solution for the problem in (18) cannot be found. Therefore, iterative algorithms will be used and in each iteration TIR and transmit filters will be updated one by one. For transmit filter update, the projected gradient algorithm [17] is used. An iteration of this algorithm is as follows:

$$\mathbf{C}_{k,\omega}^{(l+1)} = \left[\mathbf{C}_{k,\omega}^{(l)} - \eta\frac{\partial MSE_{k,\omega}^{\mathbf{C},\mathbf{B}}}{\partial\mathbf{C}_{k,\omega}^{(l)}}\right]_{\perp}, \tag{19}$$

where l shows the iteration number, η shows step size, $\perp$ operator is a projection to the hyperball defined in the first constraint of the problem in (18) with radius P_k, and the derivative of the MSE can be calculated as

$$\frac{\partial MSE_{k,\omega}^{\mathbf{C},\mathbf{B}}}{\partial\mathbf{C}_{k,\omega}} = -\mathbf{H}_{k,\omega}^H\mathbf{T}^{-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}\mathbf{B}_{k,\omega}^H\mathbf{B}_{k,\omega}\left[\mathbf{I}+\mathbf{C}_{k,\omega}^H\mathbf{H}_{k,\omega}^H\mathbf{T}^{-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}\right]. \tag{20}$$

The sum MSE is decreased and transmit and TIR filters are updated at each step. The convergence of this algorithm is proved in [17]. The new transmit filter ($\mathbf{C}_{k,\omega}^{(l+1)}$) is substituted into the optimization problem in (18) and the TIR filter is as

$$\begin{aligned} \min_{\mathbf{B}_{k,\omega}} \quad & \sum_{k=1}^{K}\sum_{\omega} tr\left\{\mathbf{B}_{k,\omega}\left[\mathbf{I}-\mathbf{C}_{k,\omega}^{(l+1),H}\mathbf{H}_{k,\omega}^H\mathbf{T}^{(l+1),-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}^{(l+1)}\right]\mathbf{B}_{k,\omega}^H\right\} \\ \text{s.t.} \quad & \mathbf{B}_{k,\omega}\mathbf{B}_{l,\omega}^H = \mathbf{\Sigma}_{kl}, \forall k,l,\omega. \end{aligned} \tag{21}$$

Because the problem in (21) is decoupled in terms of the TIR, the MSE expression for user k and frequency k,l,ω can be rewritten as in [10]

$$\begin{aligned} MSE_{k,\omega} &= tr\left\{\mathbf{B}_{k,\omega}\left[\mathbf{I}-\mathbf{C}_{k,\omega}^{(l+1),H}\mathbf{H}_{k,\omega}^H\mathbf{T}^{(l+1),-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}^{(l+1)}\right]\mathbf{B}_{k,\omega}^H\right\} \\ &= tr\left\{\hat{\mathbf{B}}_{k,\omega}\mathbf{R}_{k,\omega}\hat{\mathbf{B}}_{k,\omega}^H\right\}, \end{aligned} \tag{22}$$

where

$$\mathbf{B}_{k,\omega} = \hat{\mathbf{B}}_k\mathbf{W} = \left[\begin{array}{cccc} \hat{\mathbf{B}}_{k,0} & \hat{\mathbf{B}}_{k,1} & \cdots & \hat{\mathbf{B}}_{k,(n_b-1)} \end{array}\right]\left[\begin{array}{cccc} \mathbf{I} & e^{-j\omega}\mathbf{I} & \cdots & e^{-j\omega(n_b-1)}\mathbf{I} \end{array}\right]^T. \tag{23}$$

Here $\mathbf{R}_{k,\omega} = \mathbf{W}\left[\mathbf{I} - \mathbf{C}_{k,\omega}^{(l+1),H}\mathbf{H}_{k,\omega}^{H}\mathbf{T}^{(l+1),-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}^{(l+1)}\right]\mathbf{W}^{H}$, n_b is the shortened channel length, $j = \sqrt{-1}$, and $\hat{\mathbf{B}}_{k,j}, j = 0,\ldots,(n_b - 1)$ are the channel matrix taps of the TIR filter. It can also be shown that the orthonormality constraint of the problem in (21) is equivalent to $\hat{\mathbf{B}}_{k,\omega}\hat{\mathbf{B}}_{l,\omega}^{H} = \mathbf{\Sigma}_{kl}, \forall k,l$. Hence, the solution $\hat{\mathbf{B}}_{k,\omega}^{H}$ minimizing the problem in (21) becomes the eigenvector set corresponding to the smallest n_{i_k} eigenvalues of the matrix $\mathbf{R}_{k,\omega}$. The optimization procedure is summarised in Algorithm 1 in Table 1.

Table 1. Algorithm 1 – Sum MSE optimization for multiuser MIMO-MAC.

- Initialize: $\mathbf{C}_{k,\omega}^{(0)} = \mathbf{I}, \mathbf{B}_{k,\omega}^{(0)} = \mathbf{I}, l = 1$.
- Repeat:
  - $\mathbf{T}^{(l)} = \sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}^{(l)}\mathbf{C}_{j,\omega}^{(l),H}\mathbf{H}_{j,\omega}^{H} + \sigma_n^2\mathbf{I}$
  - Update Transmit Filter: Projected Gradient Algorithm
  - Return $\mathbf{C}_{k,\omega}^{(l+1)} = \sqrt{\frac{P_k}{\left\|\mathbf{C}_{k,\omega}^{(l)} - \eta\frac{\partial MSE_{k,\omega}}{\partial\mathbf{C}_{k,\omega}^{(l)}}\right\|_F^2}}\left[\mathbf{C}_{k,\omega}^{(l)} - \eta\frac{\partial MSE_{k,\omega}}{\partial\mathbf{C}_{k,\omega}^{(l)}}\right]$
  - Update Target Impulse Response:
  - $\mathbf{T}^{(l+1)} = \sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}^{(l+1)}\mathbf{C}_{j,\omega}^{(l+1),H}\mathbf{H}_{j,\omega}^{H} + \sigma_n^2\mathbf{I}$
  - $\mathbf{R}_{k,\omega} = \mathbf{W}\left[\mathbf{I} - \mathbf{C}_{k,\omega}^{(l+1),H}\mathbf{H}_{k,\omega}^{H}\mathbf{T}^{(l+1),-1}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}^{(l+1)}\right]\mathbf{W}^{H}$
  - Return $\hat{\mathbf{B}}_{k}^{(l),H}$ as the last n_{i_k} eigenvectors of $\mathbf{R}_{k,\omega}$.
- Until Stopping Criterion. Return $\mathbf{C}_{k,\omega,\text{opt}} = \mathbf{C}_{k,\omega}^{(end)}$ and $\mathbf{B}_{k,\omega,\text{opt}} = \hat{\mathbf{B}}_{k}^{(end)}\mathbf{W}$.
- Return $\mathbf{Q}_{k,\omega,\text{opt}} = \mathbf{B}_{k,\omega,\text{opt}}\mathbf{C}_{k,\omega,\text{opt}}^{H}\mathbf{H}_{k,\omega}^{H}\left[\sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega,\text{opt}}\mathbf{C}_{j,\omega,\text{opt}}^{H}\mathbf{H}_{j,\omega}^{H} + \sigma_n^2\mathbf{I}\right]^{-1}$.

3.2. Filter design for BC

As mentioned before, the optimization of the BC is done using its dual equivalent MAC. If the problem in (9) is observed and compared with the problem in (4), it can be obviously seen that the problems are exactly the same. Hence one can use Algorithm 1 given in Section 3.1 to find the optimum filter of the equivalent MAC.

After the optimal filters for the equivalent MAC have been found, the transformation has to be performed to calculate the filters of the original BC. The algorithm for the optimization of broadcast channel filters is shown in Algorithm 2 in Table 2.

3.3. Filter design for MU-MIMO and MU-IC

The procedures that find optimal filters of MU-MIMO and MU-IC scenarios are almost the same as the procedure in Algorithm 1. For the MU-MIMO scenario the matrix $\mathbf{T}_{k,\omega}$

Table 2. Algorithm 2 – Sum MSE optimization for multiuser MIMO-BC.

- Formulate the equivalent MAC and run Algorithm 1.
- Calculate the transformation scalar using the equation in (8).
- Transform the equivalent MAC filters to the original BC filters using the equations in (8).

in the problem in (18) should be replaced with

$$\mathbf{T}_{\omega} = \mathbf{H}_{\omega}\mathbf{C}_{\omega}\mathbf{C}_{\omega}^{H}\mathbf{H}_{\omega}^{H} - \sigma_n^2\mathbf{I}, \tag{24}$$

and the derivative of MSE in (20) should be replaced with

$$\frac{\partial MSE_{k,\omega}^{\mathbf{C},\mathbf{B}}}{\partial \mathbf{C}_{k,\omega}} = -\mathbf{H}_{\omega}^{H}T\mathbf{H}_{\omega}\mathbf{C}_{\omega}\mathbf{B}_{\omega}^{H}\mathbf{B}_{\omega}\left[\mathbf{I} + \mathbf{C}_{\omega}^{H}\mathbf{H}_{\omega}^{H}\mathbf{T}^{-1}\mathbf{H}_{\omega}\mathbf{C}_{\omega}\right]. \tag{25}$$

For the MU-IC, changing the matrix $\mathbf{T}_{k,\omega}$ in (18) with the following is sufficient

$$\mathbf{T}_{k,\omega} = \mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega}\mathbf{C}_{k,\omega}^{H}\mathbf{H}_{k,\omega}^{H} - \sigma_n^2\mathbf{I}. \tag{26}$$

3.4. Special case – Full equalization

Up to this point, the main focus was to shorten the effective length of the channel. However, alternatively the channel could be fully equalized to a single tap where a symbol-by-symbol detector is used afterwards. Again, a joint transceiver design is adopted. To achieve full equalization, the TIR is replaced with an identity matrix.

Therefore, MSEs for MAC, BC, IC, and MU-MIMO scenarios for the k^{th} user at ω become

$$\begin{aligned} MSE_{k,\omega}^{MAC} &= tr\Big\{\mathbf{I} - \mathbf{C}_{k,\omega}^{H}\mathbf{H}_{k,\omega}^{H}\mathbf{Q}_{k,\omega}^{H} - \mathbf{Q}_{k,\omega}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega} \\ &\quad + \mathbf{Q}_{k,\omega}\left(\sum_{j=1}^{K}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}\mathbf{C}_{j,\omega}^{H}\mathbf{H}_{j,\omega}^{H}\right)\mathbf{Q}_{k,\omega}^{H} + \sigma_n^2\mathbf{Q}_{k,\omega}\mathbf{Q}_{k,\omega}^{H}\Big\}, \end{aligned} \tag{27}$$

$$\begin{aligned} MSE_{k,\omega}^{BC} &= tr\Big\{\mathbf{I} - \mathbf{C}_{k,\omega}^{BC,H}\mathbf{H}_{k,\omega}\mathbf{Q}_{k,\omega}^{BC,H} - \mathbf{Q}_{k,\omega}^{BC}\mathbf{H}_{k,\omega}^{H}\mathbf{C}_{k,\omega}^{BC} \\ &\quad + \mathbf{Q}_{k,\omega}^{BC}\left(\sum_{j=1}^{K}\mathbf{H}_{j,\omega}^{H}\mathbf{C}_{j,\omega}^{BC}\mathbf{C}_{j,\omega}^{BC,H}\mathbf{H}_{j,\omega}\right)\mathbf{Q}_{k,\omega}^{BC,H} + \sigma_n^2\mathbf{Q}_{k,\omega}^{BC}\mathbf{Q}_{k,\omega}^{BC,H}\Big\}, \end{aligned} \tag{28}$$

$$\begin{aligned} MSE_{k,\omega}^{IC} &= tr\Big\{\mathbf{I} - \mathbf{C}_{k,\omega}^{H}\mathbf{H}_{k,\omega}^{H}\mathbf{Q}_{k,\omega}^{H} - \mathbf{Q}_{k,\omega}\mathbf{H}_{k,\omega}\mathbf{C}_{k,\omega} \\ &\quad + \sum_{j=1}^{K}\mathbf{Q}_{j,\omega}\mathbf{H}_{j,\omega}\mathbf{C}_{j,\omega}\mathbf{C}_{j,\omega}^{H}\mathbf{H}_{j,\omega}^{H}\mathbf{Q}_{j,\omega}^{H} + \sigma_n^2\mathbf{Q}_{k,\omega}\mathbf{Q}_{k,\omega}^{H}\Big\}, a \end{aligned} \tag{29}$$

$$MSE_{\omega}^{MU-MIMO} = tr\left\{\mathbf{I} - \mathbf{C}_{\omega}^{H}\mathbf{H}_{\omega}^{H}\mathbf{Q}_{\omega}^{H} - \mathbf{Q}_{\omega}\mathbf{H}_{\omega}\mathbf{C}_{\omega} + \mathbf{Q}_{\omega}\mathbf{H}_{\omega}\mathbf{C}_{\omega}\mathbf{C}_{\omega}^{H}\mathbf{H}_{\omega}^{H}\mathbf{Q}_{\omega}^{H} + \sigma_n^2\mathbf{Q}_{\omega}\mathbf{Q}_{\omega}^{H}\right\}, \tag{30}$$

respectively. Similar to channel shortening, in order to find the optimum receive filter, Algorithm 1 and Algorithm 2 can be used by replacing the TIR matrices with an identity matrix. The rest are the same for the optimization process and hence will not be repeated.

4. Simulation results

In the simulations, two-user MIMO and IC (each user pair has two antennas at the transmitter and two antennas at the receiver) and BC and MAC (each user has two antennas and the base stations have 4 antennas) are compared. The original channel has a *10* tapped impulse response where average energy of all channels is set to unity. The channel of each user is shortened to a predefined number of taps with the proposed algorithm and as a special case the channel is also fully equalized. The frequency domain design has an FFT size of $N = 128$. The transmit power constraint for MIMO is taken as 1 W and for MAC/BC and IC cases it is taken as $1/K$ W. Throughout the simulations, solid lines in the figures denote channel shortening results and dashed lines denote full equalization results.

In Figure 6a for 18 dB input signal to noise ratio (SNR, in dB) sum MSE values in each iteration for channel shortening and full equalization algorithms are demonstrated. The input SNR is the ratio of the energy of the signal, at the front-end of the receiver, to the noise energy. In Figure 6b sum MSE vs. input SNR results for channel shortening and full equalization scenarios are provided. In both simulations, the TIR length is set to 5 taps. It can be seen that for both simulations, the fully coordinated MIMO scheme achieves minimum MSE values and the interference channel has the maximum. This is an expected result since MIMO transmitters and receivers are in full cooperation; however, in other scenarios cooperation is limited. Moreover, channel shortening achieves better performance than full equalization does as a result of extended channel tap length. However, reduced channel length of full equalization allows its optimization algorithm to converge faster.

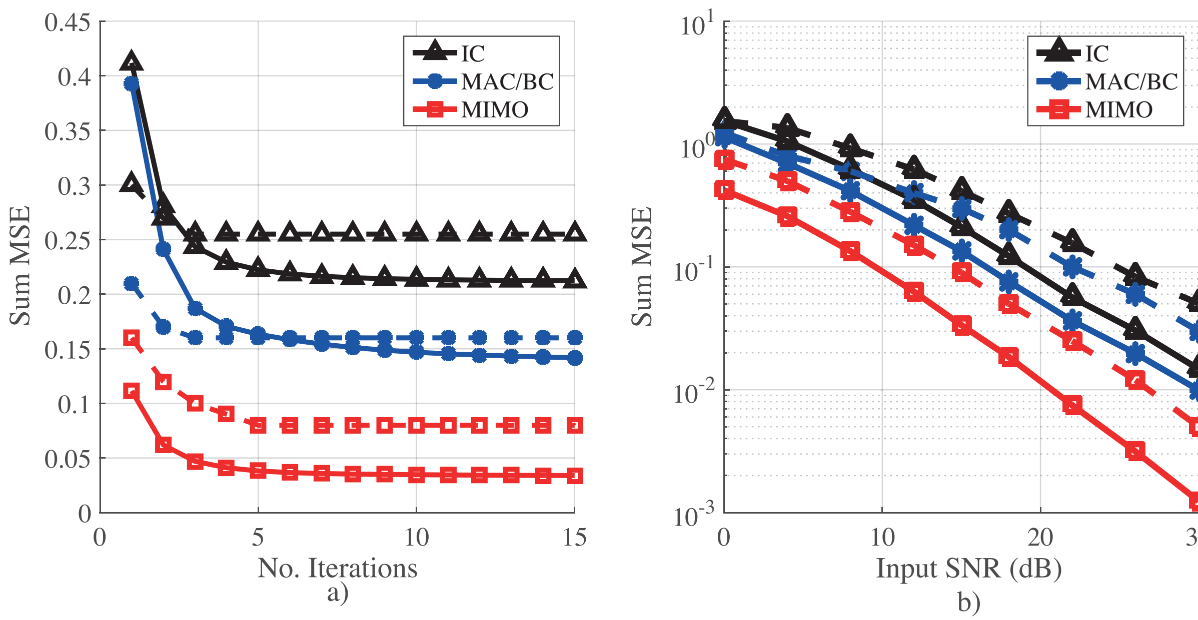

Figure 6. a) Sum MSE vs. no. iterations for channel shortening (solid lines) and full equalization (dashed lines), b) Sum MSE vs. input SNR for channel shortening (solid lines) and full equalization (dashed lines).

To assess the performance of channel shortening and full equalization, output shortening signal to interference plus noise ratio (SSINR, in dB) and output signal to interference plus noise ratio (SINR, in dB) metrics are used, which are defined as

$$SSINR = \frac{E_{\text{Shortened Channel Output Signal}}}{E_{\text{Interference}} + E_{\text{Noise}}}, \text{ and } SINR = \frac{E_{\text{Equalized Channel Output Signal}}}{E_{\text{Interference}} + E_{\text{Noise}}}. \tag{31}$$

The SSINR (solid lines) and SINR (dashed lines) performances vs. input SNR are demonstrated in Figure 7a and the sum capacity (in bits/channel use) vs. input SNR simulations are shown in Figure 7b. For channel shortening, TIR length is set to 5 taps. The sum capacity calculation is based on $\log_2(1+SSINR)$ for channel shortening and $\log_2(1+SINR)$ for full equalization. It can be deduced that shortening has a better SSINR than the SINR of full equalization for both cases, and the superiority of shortening can also be seen in the capacity simulations. Moreover, both SSINR (SINR for full equalization) and capacity of MU-MIMO are the highest whereas IC has the lowest.

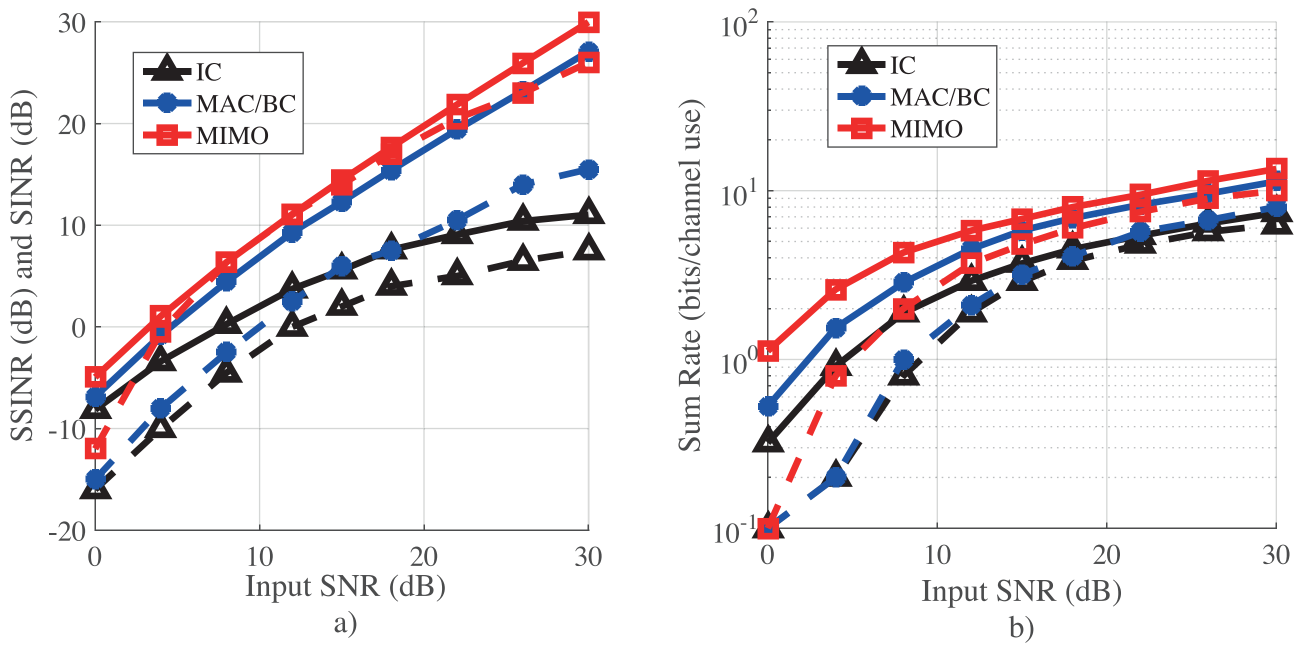

Figure 7. a) SSINR vs. input SNR for channel shortening (solid lines) and SINR vs. input SNR for full equalization (dashed lines), b) Capacity vs. input SNR for channel shortening (solid lines) and full equalization (dashed lines).

BER comparisons of four channel scenarios for full equalization and channel shortening (to 3 taps) are given in Figure 8. In the figure, MIMO achieved the best BER and the IC has the worst. An interesting point to emphasize here is the BER performance of channel shortening together with Viterbi algorithm is superior to that of full equalization for all coordination schemes. This is the result of channel shortening having extra degrees of freedom via the extended TIR coefficients to further decrease BER in contrast to full equalization where the equalized channel is forced to be approximately a single tap with unity gain.

5. Conclusions

In this paper we investigated the problem of designing the joint transceiver filters for channel shortening equalization in a *multiuser* MIMO frequency selective environment. Four different channel scenarios are considered, namely the MU-MIMO, MAC, BC, and IC. It is assumed that for BC the transmit filters are in cooperation, for MAC the receive filters are in cooperation, for MU-MIMO both the transmit and receive filters are in cooperation, and for IC there is no cooperation. We also investigated a full equalization scheme as a special case of the proposed method. For the optimization of BC, channel duality theorem is used to find an equivalent MAC.

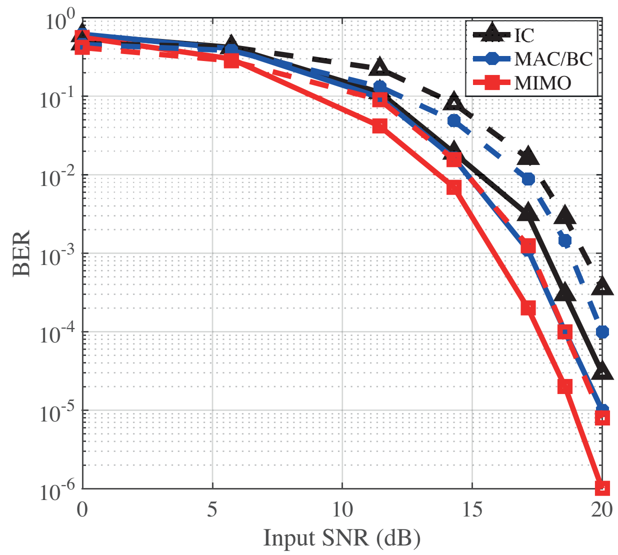

Figure 8. BER vs. input SNR for channel shortening (solid lines) and SINR vs. input SNR for full equalization (dashed lines).

In the optimization problem, the transceiver pairs are designed in order to minimize total MSE over all users and all frequency bands subject to per user power constraints and interference constraints. The solution to the optimization problem is an iterative combination of a projected gradient algorithm and an eigenvector problem in which at each iteration the transceiver and TIR filters are updated.

We assessed the algorithms developed in terms of attainable SINR, MSE, BER, and capacity. The simulations showed that the MSE decreases at each iteration of the iterative optimization algorithm proposed in Algorithm 1. It has also been seen that channel shortening achieved better performance than full equalization did as a result of having a longer channel impulse response. Moreover, the superiority of multiuser MIMO to BC, MAC, and IC scenarios, as a result of the cooperation in both the transmitters and the receivers, is verified.

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