

Regularized estimation of vertical total electron content from Global Positioning System data

F. Arikan

Department of Electrical and Electronics Engineering, Hacettepe University, Ankara, Turkey

C. B. Erol

National Electronics and Cryptology Research Institute (UEKAE), Scientific and Technical Research Council of Turkey (TUBITAK), Ankara, Turkey

O. Arikan

Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey

Received 24 July 2002; revised 25 September 2003; accepted 8 October 2003; published 31 December 2003.

[1] A novel regularization technique which can combine signals from all Global Positioning System (GPS) satellites for a given instant and a given receiver is developed to estimate the vertical total electron content (VTEC) values for the 24-hour period without missing any important features in the temporal domain. The algorithm is based on the minimization of a cost function which also includes a high pass penalty filter. Optional weighting function and sliding window median filter are added to enrich the processing and smoothing of the data. The developed regularized estimation algorithm is applied to GPS data for various locations for the solar maximum week of 23–28 April 2001. The parameter set that is required by the estimation algorithm is chosen optimally using appropriate error functions. This robust and optimum parameter set can be used for all latitudes and for both quiet and disturbed days. It is observed that the estimated TEC values are in general accordance with the TEC estimates from other global ionospheric maps, especially for quiet days and midlatitudes. Owing to its 30 s time resolution, the regularized VTEC estimates from the developed algorithm are very successful in representation and tracking of sudden temporal variations of the ionosphere, especially for high latitudes and during ionospheric disturbances. *INDEX TERMS:* 6979 Radio Science: Space and satellite communication; 2409 Ionosphere: Current systems (2708); 6974 Radio Science: Signal processing; 2494 Ionosphere: Instruments and techniques; 9820 General or Miscellaneous: Techniques applicable in three or more fields; *KEYWORDS:* total electron content, ionosphere, GPS, signal processing

Citation: Arikan, F., C. B. Erol, and O. Arikan, Regularized estimation of vertical total electron content from Global Positioning System data, *J. Geophys. Res.*, 108(A12), 1469, doi:10.1029/2002JA009605, 2003.

1. Introduction

[2] Total electron content (TEC) is defined as the number of free electrons along the ray path above 1 m². TEC is a direct way to investigate the structure of ionosphere and upper atmosphere. Both short-term and long-term variations, including irregularities and disturbances in the ionospheric structure, can be observed utilizing TEC values. In recent years, Global Positioning System (GPS) dual frequency signals are widely used to estimate both regional and global TEC values [Davies and Hartmann, 1997; Hocke and Pavelyev, 2001; Calais and Bernard Minster, 1998]. The advantages of GPS signals include the large number of satellites at an altitude of 20,000 km, global coverage, and commercially available receivers. Since the

frequencies that are used in GPS system are sufficiently high, the signals are minimally affected by the ionospheric absorption and Earth's magnetic field.

[3] Theoretically, the TEC values can be obtained by appropriate inversion of signal measurements that are recorded by the Earth-based GPS receivers. Since a complete forward physical signal model of the ionosphere is not available yet, TEC can be computed by various inversion techniques. These TEC computation methods can be grouped into three categories: computation of TEC using the pseudoranges, carrier phase delays, or a combination of pseudorange and phase delay measurements. The computation of TEC from the difference of pseudoranges (P2-P1) is simple and robust. The disadvantage is that the measurements are noisy and vulnerable to multipath corruption, especially for those satellite signals received at low elevation angles. The computed TEC values from the difference of carrier phase delays (L1-L2) is less noisy and more

immune to multipath corruption than those obtained from the pseudoranges but the computation procedure suffers from an initial phase ambiguity due to the hardware used in satellites and receivers. The third computation category aims to combine the advantages of the above methods by using both (L1-L2) and (P2-P1) data. Although the necessary instrumental biases and the initial phase ambiguities are corrected by this method, the problem of cycle slips requires preprocessing for the search of connected phase arcs. The (L1-L2) and (P2-P1) data can be combined in TEC estimation by many different algorithms. The above mentioned methods are widely discussed in the literature. A very brief partial list includes *Calais and Bernard Minster* [1998], *Jakowski* [1998], *Fraile* [1995], *Komjathy and Langley* [1996], *Lanyi and Roth* [1988], *Otsuka et al.* [2002], *Warnant* [1997], *Jakowski et al.* [1996], *Manucci et al.* [1998], *Ciraolo and Spalla* [1997].

[4] The above-listed, virtually simple computation procedures get complicated by various factors when the TEC values are estimated using active GPS satellite signals. The GPS receivers can receive signals from as many as 12 different satellites at a given instant. All of these satellites have different orbits and thus traverse the GPS receiver range with different paths in elevation and azimuth. Most of the estimation procedures provided in the literature assume both the spatial homogeneity of ionosphere for a wide range of elevation and azimuth angles and a temporal stationarity period of at least 5 to 15 min such as mentioned in the works of *Lanyi and Roth* [1988], *Wang et al.* [1998], and *Fraile* [1995]. This way some of the important spatial and temporal variations over the receiving station may be missed or not observed at all. In order to compensate such difficulties and reduce the corruption of signals due to multipath, most of the TEC computations are based on observations obtained only from the chosen satellites tracked while they are above a certain elevation angle for limited time periods [*Fraile*, 1995; *Wang et al.*, 1998]. Thus most of the measurement data which carries important information on the ionosphere is not used at all.

[5] In this paper, we devised a novel technique which can estimate the vertical TEC (VTEC) values by combining the received signals from all the satellites that are above GPS elevation angle limit of 10° with 30 s time resolution. The raw GPS signals that are used in the development of the estimation algorithm are downloaded from the internet site of the International GPS Service (IGS) for Geodynamics (available at <http://igs.ens.ign.fr>) and preprocessed to reduce the anomalies from the signals. The VTEC values for each data point can be obtained from those preprocessed signals by using any of the TEC computation methods mentioned in the above paragraph. These computed VTEC data for each satellite with 30 s period form the input of the proposed estimation algorithm. The satellite and receiver bias pairs obtained from the Center for Orbit Determination in Europe (CODE), University of Berne, Switzerland (available at ftp.unibe.ch/aiub/CODE/2001/P1P20104_ALL.DCB.Z) are also added to the computed VTEC values. The proposed method reduces the contamination due to multipath by applying an optional weighting function on the computed TEC data according to the satellite positions with respect to the local zenith. A two step regularization algorithm combines the computed and weighted vertical TEC and provides

smooth VTEC estimates for the 24-hour period with 30 s time resolution [*Erol et al.*, 2002a]. The first step of the regularization includes the minimization of error utilizing a high pass penalty function. This step requires the determination of two regularization parameters which are to be chosen from the error function formed by the L_2 norm of the difference between the estimated and actual VTEC values. The second step of regularization includes a sliding window median filter which further reduces the jagged features in the estimated VTEC values. The window length of the sliding median filter is another parameter to be determined. The estimation algorithm is applied to the computed VTEC data for four stations, namely Kiruna, Norway; Kiev, Ukraine; Ankara, Turkey; and M. Dragot, Israel for the solar maximum period of 23 April to 28 April 2001. Within this period, we observed both the typical quiet day TEC structure of the ionosphere and also the VTEC values of the most positively and the most negatively disturbed days of the month. The results of the proposed estimation algorithm are compared with other TEC estimation methods that are available on the internet. It is observed that the developed estimation procedure is highly accurate in construction of TEC values from various satellites [*Erol et al.*, 2002b]. The parameters that need to be determined for the estimation algorithm can be chosen robustly. Thus for the given data set, the same parameter set can be used both for the quiet and the disturbed days and all latitudes. The robustness of the parameters are an important indication of the strength of proposed estimation algorithm.

[6] In section 2, the preprocessing of GPS data is discussed. The VTEC values from various satellites are then further processed with the new algorithm to estimate the VTEC for 24 hour period for one GPS receiver. The developed algorithm is described in section 3. The procedure to choose the parameter set and the comparison of the estimated TEC values with other available data are provided in section 4.

2. Preprocessing of GPS Signals

[7] The receivers at GPS stations record signals transmitted at two L-band frequencies, namely, f_1 at 1575.42 MHz and f_2 at 1227.60 MHz. The time delay which occurs while these signals are propagating through the ionosphere are converted to pseudoranges and recorded as P1 and P2 signals. The carrier phase delay measurements on the f_1 and f_2 coherent frequencies are also recorded as L1 and L2, respectively. As mentioned in the first section, the TEC values can be calculated from the difference of P2 and P1 signals which is called the absolute TEC, the difference of L1 and L2 can be used to compute TEC which is called as relative TEC, and it is possible to compute TEC by using both (L1-L2) and (P2-P1) measurements and also solving for instrumental biases. Any of the above-mentioned methods can be used to compute TEC, and they can be used as inputs to the proposed algorithm in section 3. The TEC computation methods and their advantages and disadvantages are widely discussed in the literature. The computation of TEC from the difference of pseudoranges is very simple and unambiguous but usually corrupted by noise and multipath signals. Although low-noise, the computation of relative TEC is complicated due to the fact that the phase delay

measurements suffer from cycle ambiguities. There are various inversion procedures for fitting (L1-L2) to (P2-P1) and solving for instrumental biases. These unidentical methods try to combine the advantages of absolute and relative TEC and thus obtain an unambiguous and low-noise TEC. Yet all of these methods suffer from the problem of cycle slip, which occurs when the GPS receiver loses the lock with the satellite signals, especially at low elevation angles, causing discontinuity in the data set. Thus it is a necessity to use complicated preprocessing programs which will detect cycle slips, repair them, and adjust phase ambiguities by tracing continuous arcs of phase data. After considering the advantages and the disadvantages of all the above algorithms as discussed in the literature, we decided to use absolute TEC as input to our proposed algorithm. The absolute TEC computation is simple, standard, and does not require complicated preprocessing on data. The disadvantages of absolute TEC such as noise and multipath corruption for standard computation procedures can be overcome by the proposed algorithm and, as demonstrated in section 4, the estimated TEC from (P2-P1) with our algorithm is comparable those methods which use both (L1-L2) and (P2-P1).

[8] For any given time period, the GPS receiver records signals from various satellites at different elevation and azimuth angles with respect to the local zenith. An example of satellite paths is given in Figure 1 for Ankara, Turkey on 25 April 2001 between 0800 UT and 1000 UT. The circles indicate the direction of movement for the satellites. The numbers near the paths indicate the number of the corresponding satellite.

[9] The standard procedure to compute the absolute total electron content on the slant ray path (STEC) from the satellite to the receiver is provided in various studies in the literature, including *Jakowski* [1998], *Calais and Bernard Minster* [1998], *Jakowski et al.* [1996], and *Conkright et al.* [1997]. According to this procedure, STEC values are calculated from the difference of P1 and P2 recordings as follows:

$$\text{STEC}_m(n) = \frac{P2_m(n) - P1_m(n)}{A} \frac{f_1^2 f_2^2}{f_1^2 - f_2^2} \quad (1)$$

in el/m^2 , where the constant A is equal to $40.3 \text{ m}^3/\text{s}^2$, the subscript m in the upper equation denotes the satellite number, and n is the sample number. In the literature, the total electron content quantities are generally represented in TEC Units (TECU). Thus $\text{STEC}_m(n)$ in el/m^2 can be converted to TECU by multiplying STEC by 10^{-16} where $1 \text{ TECU} = 10^{16} \text{ el}/\text{m}^2$. The computed STEC values using equation (1) for the satellites paths over Ankara, between 0800 UT and 1000 UT on 25 April 2001 are provided in Figure 2a. The satellite numbers are again indicated on the STEC paths.

[10] Usually, the computed slant TEC is projected to the local zenith direction to obtain the vertical TEC through a mapping function, $M(\epsilon)$, assuming a thin shell model of the ionosphere, [*Jakowski*, 1998; *Fraile*, 1995; *Lanyi and Roth*, 1988; *Schaer*, 1999; *Feltens*, 1998; *Ciraolo and Spalla*, 1997]. The receiver and satellite biases are also added to compute the VTEC values as follows:

$$\text{VTEC}_m(n) = \text{STEC}_m(n)/M(\epsilon_m(n)) + s_m + r, \quad (2)$$

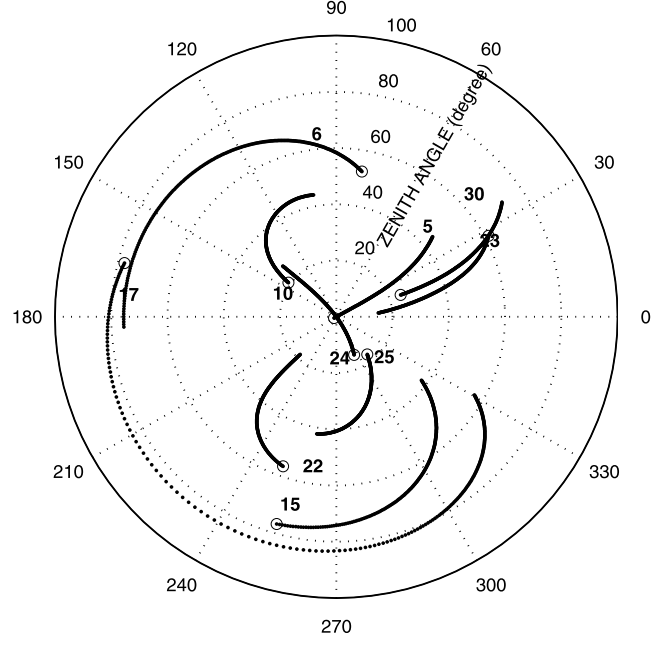


Figure 1. Satellite paths with respect to local zenith for Ankara, Turkey on 25 April 2001 between 0800 UT and 1000 UT; the circles indicate the direction of movement for the satellites; the satellite numbers are shown along with the corresponding paths.

where s_m is the bias for the m th satellite, r is the receiver bias, and

$$M(\epsilon_m(n)) = \left[1 - \left(\frac{R \cos \epsilon_m(n)}{R + h} \right)^2 \right]^{-1/2} \quad (3)$$

In the above formulas, $\epsilon_m(n)$ is the elevation angle in the local receiver coordinates for the m th satellite at the n th sample, R is the effective mean Earth radius, and h is the height of ionospheric pierce point. The mean height of the ionosphere is a time varying function, yet generally in the literature it is taken as constant [*Komjathy and Langley*, 1996; *Schaer*, 1999; *Lanyi and Roth*, 1988]. In *Schaer* [1999], four different mapping functions for single layer model (SLM) at various heights are examined according to their best fit to Chapman profile. For the SLM mapping function given in equation (3), it is found that best fit in the least squares sense to the Chapman profile is obtained when h is set to 428.8 km. Thus in this paper, we set $h = 428.8$ km after *Schaer* [1999]. The receiver and satellite bias pairs are obtained from the CODE internet site provided in section 1. Figure 2b contains the computed VTEC values for each satellite according to the equations (2) and (3).

[11] The standard procedure given in the above equations are used to obtain the STEC and VTEC values for four GPS receiver stations (Kiruna, Kiev, Ankara, and M. Dragot), whose geographic locations are provided in Table 1. These four stations are in the same time zone and almost equidistant from each other in terms of the latitude. The GPS receiver data for these stations are obtained from the internet site of the IGS for the period of 23 April 2001 to 28 April 2001, in Receiver INdependent EXchange (RINEX) format.

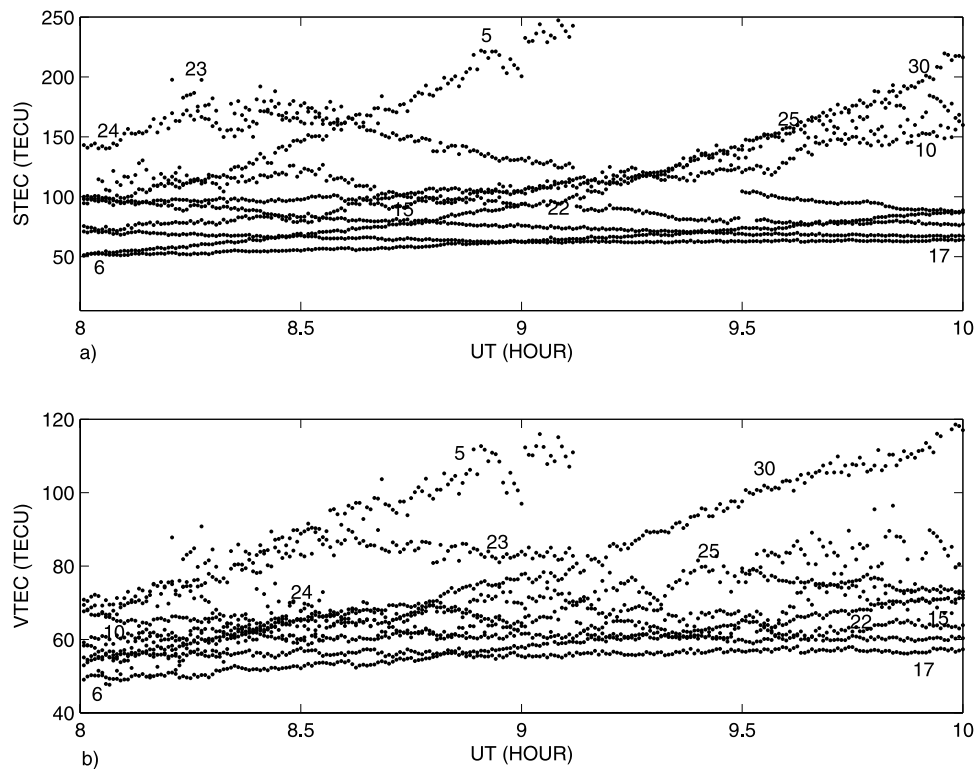


Figure 2. Computed total electron content in TECU for Ankara, Turkey on 25 April 2001 between 0800 UT and 1000 UT; some of the satellite numbers are indicated (a) STEC (b) VTEC.

The P1 and P2 pseudoranges are recorded for every 30 s for these stations. The chosen week corresponds to the solar maximum week of the year 2001, and the most disturbed days and one quite day can be observed within the same week as listed in the web site of the Ionospheric Dispatch Centre in Europe (IDCE) (available at <http://www.cbk.waw.pl/rwc/idce.html>). Both the satellite and receiver biases stayed constant for the week of interest [Sardon *et al.*, 1994].

[12] Some of the GPS recordings could be missing or inaccurate due to satellite and/or receiver failures, so all the data that have been downloaded are thoroughly examined and inaccurate or missing recordings are expelled from the data set. The STEC values are then computed according to equation (1) for all stations, all the days of the week, and all satellites.

[13] The daily positions of the satellites in Earth Centered and Earth Fixed (ECEF) coordinate system are downloaded from the internet site of National Imagery and Mapping Agency (available at <http://164.214.2.59>). In the downloaded files in ECEF coordinates, the satellite coordinates are given for every 15 min. Thus in order to obtain the satellite locations corresponding to P1 and P2 measurements with 30 s period, the 15 min coordinates are interpolated with 30 s separation. Then, the coordinates of the satellites are converted to the local coordinate system of each receiver location. The 10° elevation angle is the geometrical limit for the GPS receiver to lock to the passing satellite. The vertical TEC values are then computed using equation (2) for all receiver locations, all satellites over the 10° elevation angle limit with 30 s period. In the following section, we present the new method for estimation of VTEC

for the whole 24 hour period by using these computed VTEC data.

3. Proposed Method for Regularized TEC Estimation

[14] Our goal in this section is to obtain an accurate and robust estimate of the total electron content in the zenith direction of the GPS receiver combining the computed VTEC data from all available satellites for the 24 hour period. Since the ionosphere is spatially inhomogeneous and time varying, the computed STEC and VTEC values have different characteristics for each satellite path. This fact is illustrated in Figure 2. Within the 2 hour period, STEC and VTEC values demonstrate a wide variation. Missing data points cause discontinuities in computed VTEC such as those for PRN 23. Generally, in order to avoid missing and inaccurate data, the computed VTEC values are averaged for certain periods of time such as 5 min [Fraile, 1995] and 15 min [Wang *et al.*, 1998; Warnant, 1997]. Multipath corruption is another problem which affects the accuracy of the computed VTEC data. Thus in

Table 1. Geographic Locations and Receiver Biases of GPS Receivers of Interest (1 ns = 2.854 TECU)

GPS Receiver Location	Latitude	Longitude	Receiver Bias (CODE)
Kiruna, Norway	67.32°N	20.09°E	0.792 ns
Kiev, Ukraine	50.22°N	30.30°E	-12.075 ns
Ankara, Turkey	39.53°N	32.45°E	2.998 ns
M. Dragot, Israel	31.35°N	35.23°E	-13.041 ns

order to overcome multipath, generally in the literature, the satellite which has the highest elevation angle is tracked for limited time periods [Fraile, 1995; Wang et al., 1998]. In this study, the desired task is to develop an estimation algorithm for a single station which will combine all the VTEC values from all satellites without being affected by the multipath and the missing data points for the 24 hour period with 30 s time resolution. In our computations, we assume only the azimuthal homogeneity of the ionosphere with respect to the local zenith.

[15] In order to combine all the VTEC values from different satellites for the 24 hour period and obtain a smooth estimate, the following algorithm is proposed. Let

$$\mathbf{x}_m = [x_m(0) \dots x_m(n) \dots x_m(N-1)]_N^T \quad (4)$$

denote a set of computed VTEC values obtained from the m th satellite, where $1 \leq m \leq M$ and M is the total number of satellites; and also $0 \leq n \leq N-1$, where N is the total number of recordings in 24 hour period. Thus for a measurement of every 30 s, $N = 2 \times 60 \times 24 = 2880$ samples/day.

[16] In order to reduce the effect of multipath, we propose an optional weighting function, \mathbf{w}_m , to include all the contributions from satellites at an elevation angle higher than 60° and scale down the other VTEC values from satellites between 10° and 60° with a gaussian function. For the m th satellite, the weighting function, \mathbf{w}_m where

$$\mathbf{w}_m = [0 \dots 0 \ w_m(1) \dots w_m(N_m) \ 0 \dots 0]_N^T \quad (5)$$

can be defined as

$$w_m(n) = \begin{cases} 1, & \text{for } 60^\circ \leq \epsilon_m(n) \leq 90^\circ \\ \exp\left(-\frac{(90 - \epsilon_m(n))^2}{2\sigma^2}\right), & \text{for } 10^\circ \leq \epsilon_m(n) < 60^\circ \end{cases} \quad (6)$$

In the above equations, N_m denotes the number of nonzero measurements from the m th satellite within the 24 hour period; σ is the standard deviation which is chosen to reduce the contribution from the m th satellite by a factor of 0.001 when the elevation angle $\epsilon_m(n)$ is less than 10° . The effect of such a weighting function on the estimated VTEC values is demonstrated in the next section.

[17] Next step is to obtain estimates of TEC which minimize the error between the computed VTEC and the estimated VTEC in the least square sense. Therefore we will define a cost function which includes the \mathcal{L}_2 norm error between the estimated and computed VTEC values summed over all satellites plus a high pass penalty filter multiplied by a regularization parameter, μ , as follows:

$$J_{\mu, k_c}(\mathbf{x}) = \sum_{m=1}^M (\mathbf{x} - \mathbf{x}_m)^T \mathbf{W}_m (\mathbf{x} - \mathbf{x}_m) + \mu \mathbf{x}^T \mathbf{H}(k_c) \mathbf{x} \quad (7)$$

where $\mathbf{W}_m = \text{diag}(\mathbf{w}_m)$, and $\mathbf{H}(k_c)$ is the high pass penalty function. $\mathbf{H}(k_c)$ is designed to pass all signals up to a cutoff frequency, k_c . The purpose of adding a penalty function to the cost function is to control the smoothness of the data. For a given parameter set μ and k_c , we investigate the values

\mathbf{x} which will minimize J . Thus we search for \mathbf{x} which will satisfy $\nabla_{\mathbf{x}} J_{\mu, k_c}(\mathbf{x}) = \mathbf{0}$.

[18] The optimal solution to the minimization of the cost function in equation (7) reduces to the solution of the following linear system:

$$\mathbf{A}(\mu, k_c) \mathbf{x} = \mathbf{b} \quad (8)$$

where \mathbf{A} is an $N \times N$ matrix defined by

$$\mathbf{A}(\mu, k_c) = \sum_{m=1}^M \mathbf{W}_m + \mu \mathbf{H}(k_c) \quad (9)$$

and

$$\mathbf{b} = \sum_{m=1}^M \mathbf{W}_m \mathbf{x}_m. \quad (10)$$

Then, the estimate of \mathbf{x} , $\tilde{\mathbf{x}}$, is obtained from the above relations as

$$\tilde{\mathbf{x}}(\mu, k_c) = \mathbf{A}^{-1}(\mu, k_c) \mathbf{b}. \quad (11)$$

In order to represent the 24-hour cycle of the ionosphere, the high pass penalty function, $\mathbf{H}(k_c)$ can be designed as a Toeplitz matrix obtained from filters $h_n(k_c)$ in the following manner:

$$\mathbf{H}(k_c) = \begin{bmatrix} h_0(k_c) & h_1(k_c) & \dots & h_{N-1}(k_c) \\ h_{N-1}(k_c) & h_0(k_c) & \dots & h_{N-2}(k_c) \\ \vdots & \vdots & \ddots & \vdots \\ h_1(k_c) & h_2(k_c) & \dots & h_0(k_c) \end{bmatrix}_{N \times N} \quad (12)$$

where

$$h_n(k_c) = \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{H}_k(\omega_c) \exp\left(j \frac{2\pi}{N} kn\right) \quad (13)$$

where $\omega_c = 2\pi k_c / N$. For a possible choice of a high pass penalty function, $\mathbf{H}_k(\omega_c)$ can be chosen as follows:

$$H_k(\omega_c) = \begin{cases} 1, & \text{if } \pi - \omega_c \leq \frac{2\pi}{N} k \leq \pi + \omega_c \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

Then, the filter $h_n(k_c)$ becomes

$$h_n(k_c) = \begin{cases} 1 - \frac{1}{N} (2k_c + 1), & \text{for } n = 0 \\ -\sin\left(\frac{\pi n}{N} (2k_c + 1)\right) / \left(N \sin\left(\frac{\pi n}{N}\right)\right), & \text{for } n \neq 0 \end{cases} \quad (15)$$

[19] In order to determine the optimum choice for μ and k_c , an error function which will minimize the error between

the estimated VTEC, $\tilde{\mathbf{x}}$, and the data from each satellite, \mathbf{x}_m , can be defined as follows:

$$e(\mu, k_c) = \sum_{m=1}^M \|\mathbf{W}_m(\tilde{\mathbf{x}} - \mathbf{x}_m)\|^2, \quad (16)$$

where $\|\cdot\|$ denotes the \mathcal{L}_2 norm.

[20] After investigating for the optimum choice of μ and k_c by computing the \mathcal{L}_2 norm of the error between the computed and estimated VTEC values, an optional sliding window median filter can be applied to further reduce the jagged features and irregularities in the estimated TEC values. The window length of the median filter is another parameter that needs to be determined optimally. For the optimum choice of the median filter length, another error function which computes the square of the \mathcal{L}_2 norm between the estimated VTEC and the median filtered VTEC with the filter length N_f as follows:

$$e_f(N_f) = \|\tilde{\mathbf{x}} - \tilde{\mathbf{x}}_{N_f}\|^2 \quad (17)$$

where $\tilde{\mathbf{x}}_{N_f}$ denotes the median filtered $\tilde{\mathbf{x}}$ for the filter length N_f . In the following section, we will demonstrate the choice of the optimum filter length using the above error function on the estimated VTEC values.

[21] In the following section, we will apply the developed the regularized estimation algorithm of vertical total electron content for Kiruna, Kiev, Ankara and M. Dragot for the week of 23 April 2001 to 28 April 2001 and discuss various properties and novelties of the new estimation procedure. We will also demonstrate the choice of the parameters and their robustness.

4. Results

[22] The GPS signals, recorded by the receiving stations given in Table 1, are preprocessed as discussed in section 2 and then the regularized TEC estimation algorithm given in the previous section is applied for the week of 23 April to 28 April 2001. According to the information provided at the IDCE web site 23 April 2001 is listed as the most negatively disturbed day, 28 April 2001 is listed as the most positively disturbed day, and 25 April 2001 is the fifth most quiet day of the month of April 2001. On 23 April 2001, Kp index is over 3 for various hours and Dst index gets as low as -60 nT for those disturbed hours. On 28 April 2001, Kp index is over 4 and reaches 6 for many hours. For the disturbed hours on that day, Dst index peaks to 60 nT. Thus within 1 week, it is possible to observe the effects of the positive and negative disturbances and also an example of a quiet day. In order to estimate the 24 hour TEC values with the algorithm proposed in section 3, the parameters of the algorithm have to be determined optimally. In the first part of this section, we will demonstrate how to choose these parameters and discuss their robustness. In the second part, we will apply the regularization algorithm with the chosen parameters to the GPS receiver station signals recorded at the locations provided in Table 1. All of these stations are in the same time zone and they are roughly equidistant from each other in latitude. Thus we can observe the variation of TEC from north to south within one day, both for the quiet

and the disturbed days. In the last part of this section, we will compare the estimated TEC values with other TEC models available on the internet.

[23] According to the regularized estimation method in section 3, the algorithm has three main parameters that needs to be determined optimally: the regularization (or smoothness) parameter, μ , the cutoff frequency, k_c , and the sliding window filter length, N_f . The parameters μ and k_c are defined in equation (7), and the error function for the optimal choice of these parameters is provided in equation (16). We have applied the estimation algorithm with various values of the parameters to the GPS signals from the four stations and an example of the error function is provided in Figure 3. In Figure 3a, the error function for an high-latitude station on the most negatively disturbed day is provided. In Figure 3b, the error function for a midlatitude station on the fifth most quiet day of the month is given. For all other locations and on other days, the error function has similar behavior. The optimal choice of μ occurs at 0.1 where the error curve starts to increase with a significant slope for all values of the cutoff frequency k_c . Higher values of μ cause smoother TEC estimates and more regularization. Since the penalty function starts to dominate the cost function in equation (16), the smoother estimates with high values of μ may not follow the desired features of TEC data. For higher values of k_c , the estimates follow the local trends in the data very well, but since the cutoff frequency increases, more high-frequency components are included. Thus the estimates look more jagged and there may be undesired jumps and irregularities. In order to have a trade off, we choose the lowest possible smoothness parameter and a higher value of cutoff frequency. Thus for all stations and for all days, the optimum choice of μ is 0.1 and k_c is 8. The remedy for this choice of regularization parameters is to employ the sliding window median filter to improve the smoothness of the data as mentioned in the previous section.

[24] The sliding window median filter is a nonlinear filter. We expect that at the optimum value of the filter length, the output should follow all the local trends of the data closely, yet the filtered output should be free of the undesired irregularities. The optimum choice of the filter length is discussed in equation (17). The error function for Ankara on 25 April 2001 is plotted in Figure 4a, for various values of median filter length in samples. The optimum choice of median filter length can be obtained at a point where the error function settles to a plateau. The estimated TEC values for Ankara for the day of 25 April 2001 is given in Figure 4b. This TEC estimate is obtained with $\mu = 0.1$ and $k_c = 8$. As can be observed from this plot, the estimate is jagged. In order to further smooth it, the data in Figure 4b is median filtered with filter lengths $N_f = 85$ and $N_f = 105$. Both median filtered estimates are plotted in Figure 4c, where no apparent difference in the output for the choice of N_f for 85 or 105 samples is observed. We repeated this operation for all locations and for all days and the shorter filter length, $N_f = 85$, is appropriate for this data set.

[25] The optimum parameters for the regularized TEC estimation algorithm are then chosen as $\mu = 0.1$, $k_c = 8$ and $N_f = 85$. The same parameter set can be used for all the stations at various latitudes and both for the disturbed and quiet days. The robustness of these parameters denotes the power of the estimation algorithm.

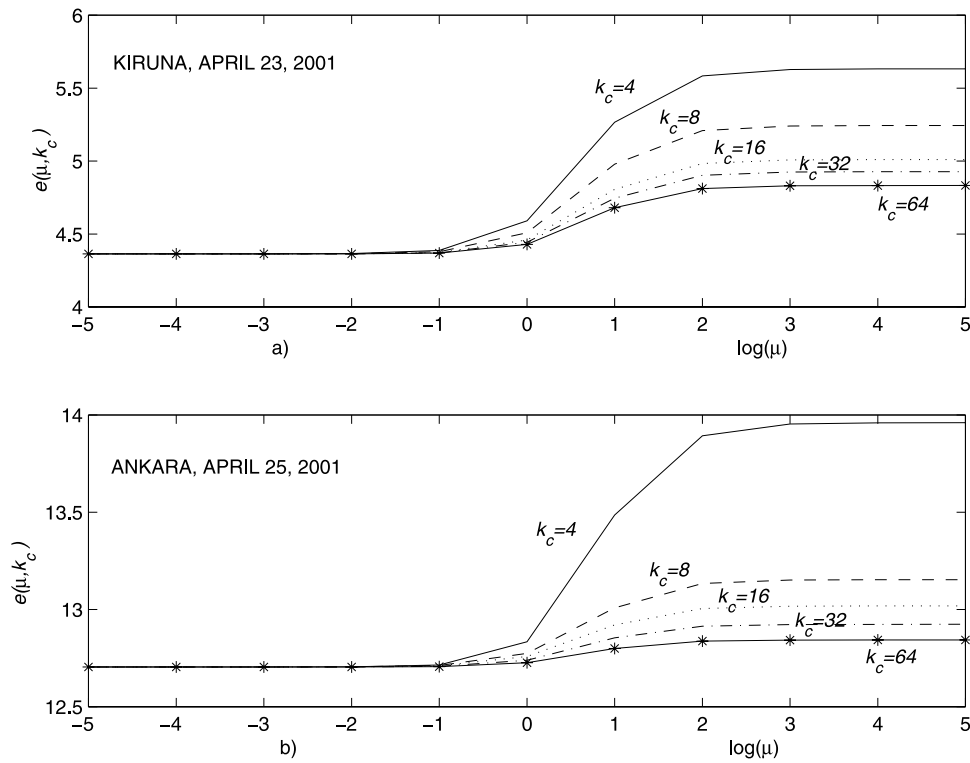


Figure 3. Error function to determine the optimum values of the regularization parameter, μ , and the cutoff frequency, k_c (a) for Kiruna, Norway on 23 April 2001 (high latitude, negatively disturbed day) (b) for Ankara, Turkey on 25 April 2001 (midlatitude, quiet day).

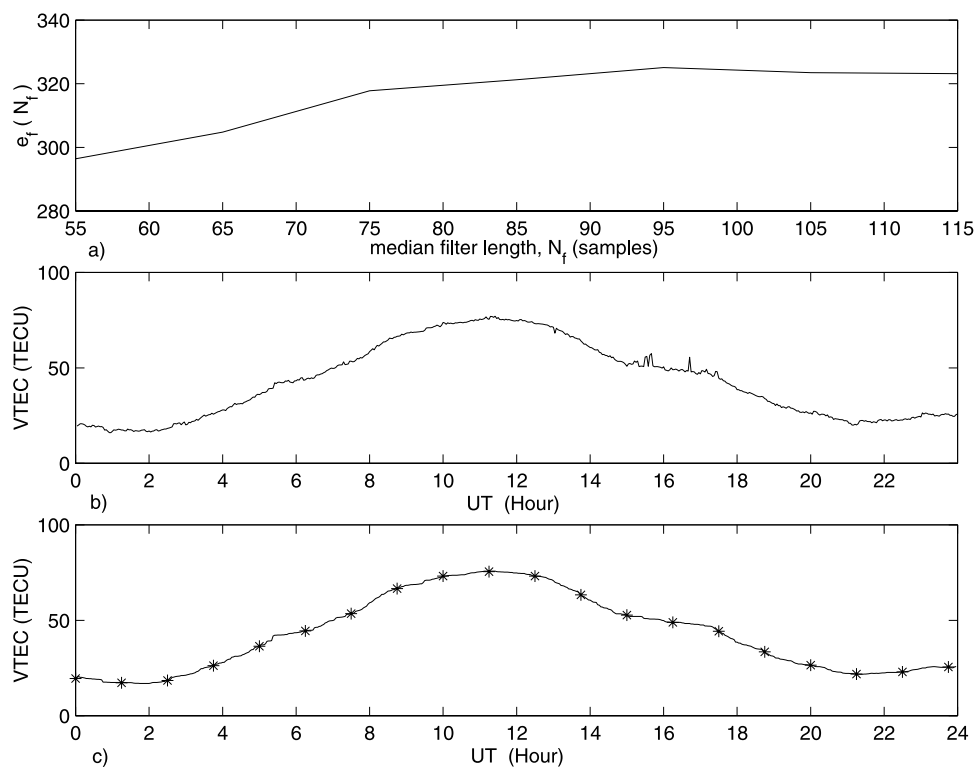


Figure 4. (a) Error function to determine the optimum value of the sliding window median filter length, N_f , for Ankara, Turkey on 25 April 2001. (b) The estimated VTEC values. (c) Median filtered VTEC values with filter lengths $N_f = 85$ (solid line) and $N_f = 105$ (stars).

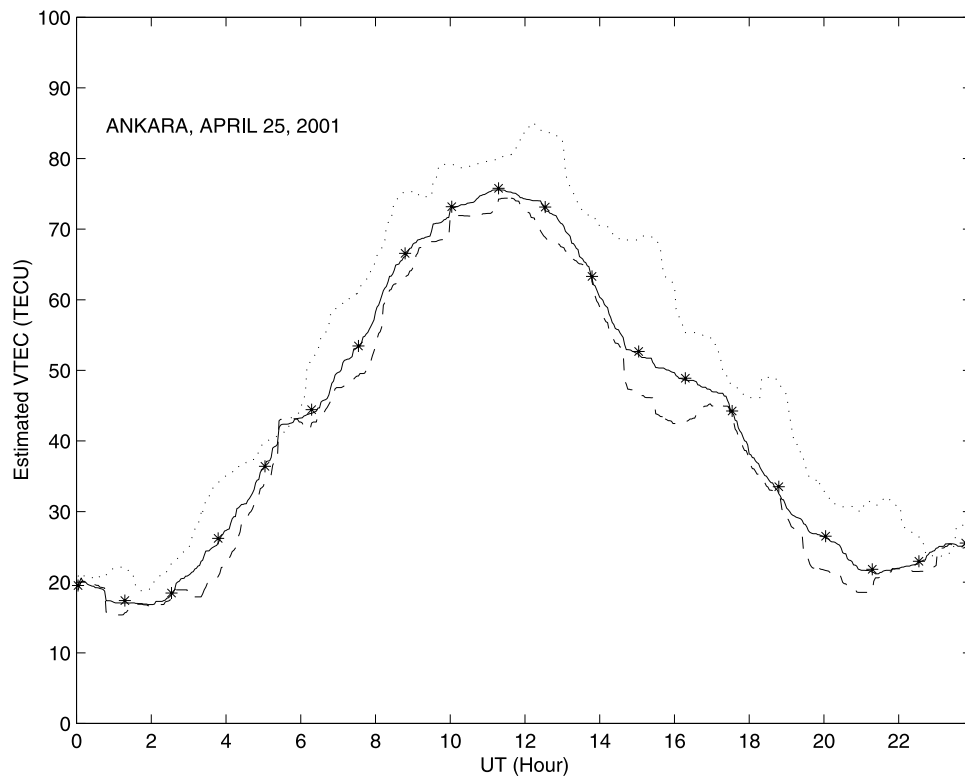


Figure 5. Comparison of optional weighting functions in TEC estimation with the scaling function in equation (6) and 10° cutoff in elevation (stars), 30° cutoff (solid line) with the rectangular function in equation (18) with 60° cutoff (dashed line) and 10° cutoff (dotted line).

[26] In Figure 5, we demonstrate the effect of the optional weighting function that is proposed to scale down the multipath corruption. The standard approach to cancel the multipath contamination is to include satellites which are at high elevation angles only; for example, in the work of Wang *et al.* [1998], the satellites over 60° in elevation are tracked. The dashed line in Figure 5 denotes the estimated TEC by a rectangular weighting function which can be chosen as

$$w_m(n) = \begin{cases} 1, & \text{for } 60^\circ \leq \epsilon_m(n) \leq 90^\circ \\ 0, & \text{for } 10^\circ \leq \epsilon_m(n) < 60^\circ \end{cases} \quad (18)$$

corresponding to the standard approach. If the elevation cutoff angle is reduced from 60° to 10° with the weighting function in equation (18), the dotted line in Figure 5 is obtained. In this curve, the VTEC values from all the satellites are combined with equal weight including the multipath corruption. In some studies, such as Otsuka *et al.* [2002], Fraile [1995], Komjathy and Langley [1996], and Ciraolo and Spalla [1997], lower cutoff angles like 15° , 20° , 25° , and 30° are used. In order to compare the effect of the optional scaling function proposed in equation (6), the estimated TEC values for 10° and 30° cutoff angles are also plotted in Figure 5. With the rectangular weighting function in equation (18), only the satellites which are above 60° elevation are included. Therefore the VTEC estimates are more jagged and generally lower in value than the estimates obtained by using the weighting function in equation (6). Thus with the proposed scaling function, the effect of multipath is reduced, and at the same time, the contribution

from the satellites at lower elevation angles are included in the estimation.

[27] The estimated and median filtered VTEC values for Kiruna, Kiev, Ankara and M. Dragot are provided in Figure 6 for the days of 23, 25, and 28 April 2001. The F region ionospheric storm shows similarities with the magnetic storm in that variations in the maximum electron density and electron content are observed in both cases [Hargreaves, 1992]. As can be observed from Figure 6, during the negative phase of the ionospheric storm on 23 April 2001, the total electron content is reduced in all stations. The effect of the storm is more pronounced for higher-latitude stations like Kiruna and Kiev. The ionosphere gradually returned to its normal state over a period of 2 days on 25 April 2001, which can be labeled as recovery phase. The initial phase of another storm is observed on 28 April 2001. The effect of the irregularities in the TEC values due to positive disturbance can easily be observed in all stations, especially for those in higher latitudes. The enhancement of the total electron content due to the positive disturbance is most visible in Kiev. In Kiruna, a sharp decrease is observed between 1300 UT and 1600 UT. For all the days and stations, the TEC values decrease with increasing latitude. The high time resolution of the regularized estimation algorithm allows the observation of short time scale variations in TEC.

[28] Finally, we would like to compare the estimated TEC values with the output of representative global ionospheric maps for various IGS analysis centers available on the internet, such as RCRU-RAL, CODE, NRCAN, ESA/ESOC, JPL-GNISD, gAGE/UPC, and IRI. Radio Communications

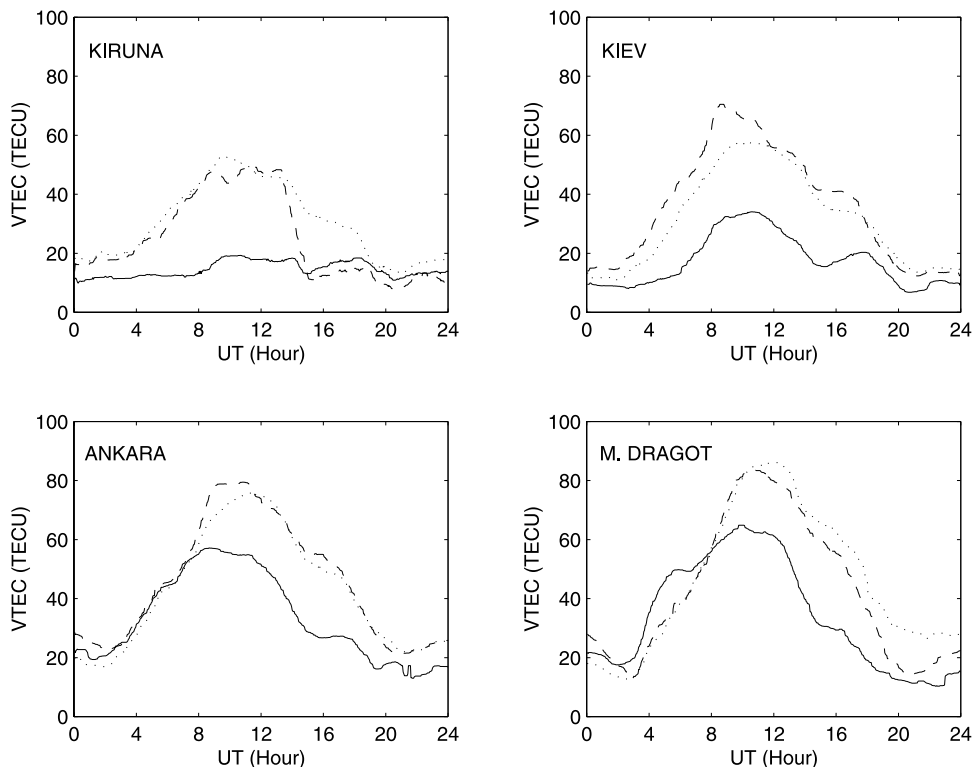


Figure 6. The estimated and median filtered VTEC values with parameters $\mu = 0.1$, $k_c = 8$, and $N_f = 85$ for Kiruna, Kiev, Ankara and M. Dragot. The solid line is for 23 April 2001 (negatively disturbed day), the dotted line is for 25 April 2001 (quiet day), and the dashed line is for 28 April 2001 (positively disturbed day).

Research Unit (RCRU) at Rutherford Appleton Laboratory (RAL) produces the single station VTEC estimates for 24 hour period with 30 s time resolution (available at http://ionosphere.rcru.ral.ac.uk/TECmap_archive.htm). The VTEC values are computed by a two step procedure based on *Ciraolo and Spalla* [1997]. In the first step, TEC and the differential group delay (DGD) biases are obtained from DGD measurements by a least squares method. In the second step, a Kalman filter is implemented in order to estimate corrections for TEC. The dashed line in Figures 7 and 8 denotes RCRU values. CODE provides the global ionospheric maps generated on a daily basis (available at <http://www.aiub.unibe.ch/ionosphere.html>). The TEC is modeled with a spherical harmonic expansion up to degree 12 and order 8 referring to solar-geomagnetic reference frame. The twelve 2-hour sets of 149 ionosphere parameters per day are derived from GPS data of IGS network [Schaer, 1999]. The values of CODE are shown with squares connected with a solid line in Figures 7 and 8. NRCan ionosphere maps are generated daily by the Geodetic Survey Division (GSD) of Natural Resources Canada (NRCan), Ottawa, Ontario, Canada (available at <http://www.nrcan.gc.ca>). The grid point values are mean VTECs estimated in Sun-fixed reference frame. The values of NRCan are given with stars connected with a solid line in Figures 7 and 8. ESA/ESOC model contains two-dimensional TEC maps from the European Space Operations Center (ESOC) of European Space Agency (ESA), Darmstadt, Germany (available at <http://nng.esoc.esa.de/gps/ionmon.html>). The computation is based on vertical inte-

gration of Chapman profile which is modeled as a gaussian extended function [Feltens, 1998]. The values of ESA/ESOC are given with circles connected with a solid line in Figures 7 and 8. JPL-GNISD provides global ionospheric maps generated on hourly and daily basis at Jet Propulsion Laboratory (JPL) Pasadena, California, using data from up to 100 GPS sites of the IGS and other institutions (available at <http://iono.jpl.nasa.gov>). The vertical TEC is modeled in a solar-geomagnetic frame using bicubic splines on a spherical grid. A Kalman filter is used to solve simultaneously for instrumental biases and VTEC on the grid (as stochastic parameters) [Manucci et al., 1998]. The values of JPL-GNISD are given in Figures 7 and 8 with diamonds connected with a solid line. The gAGE/UPC global ionospheric maps are generated by Polytechnical University of Catalonia, Barcelona, Spain (UPC) (available at http://maite152.upc.es/ionex/gAGE_dip/gAGE_dip.html). TEC is modeled independently for each station with a tomographic model, and the estimates are interpolated with radial basis functions assuming a smoothed TEC[GPS]/TEC[IRI] [Hernandez-Pajares et al., 1999]. The values of gAGE/UPC are given in Figures 7 and 8 with triangles connected with a solid line. The TEC values from the International Reference Ionosphere (IRI) model (available at <http://nssdc.gsfc.nasa.gov/space/model/models/iri.html>) are also included in Figures 7 and 8 and denoted by plus signs. The IRI model inputs the geographic location in latitude and longitude and sunspot number of the day of interest. The IRI model produces 24 hour period VTEC estimates by taking 15 min time averages with a path integral limit of 2000 km.

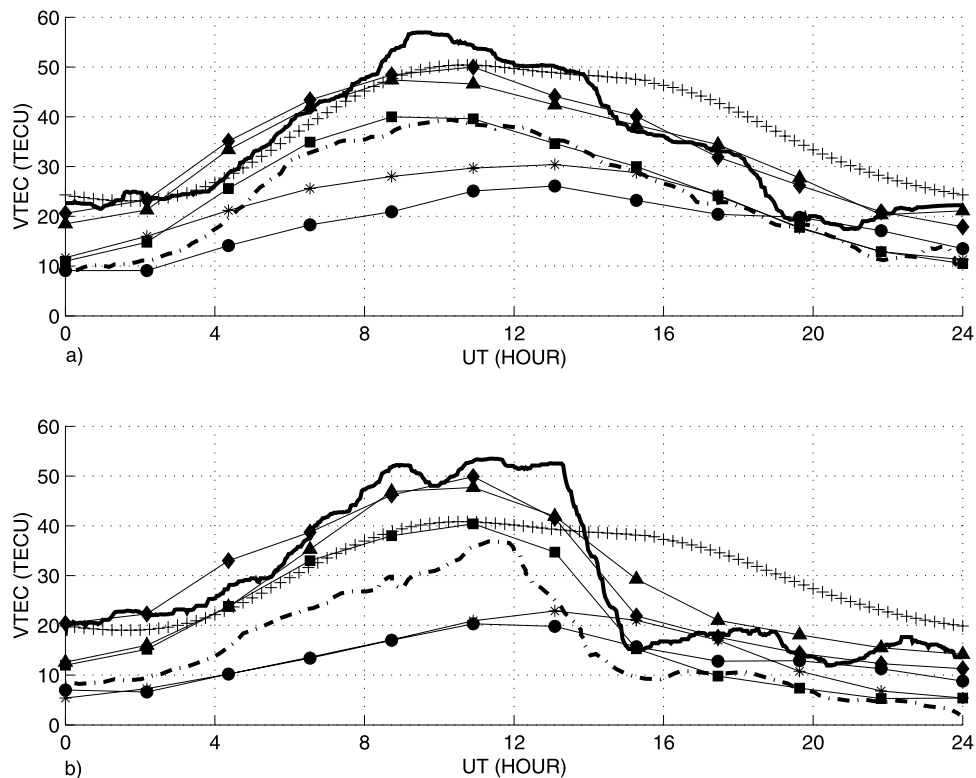


Figure 7. Comparison of the estimated TEC values from the algorithm with other models of TEC for Kiruna, estimated VTEC from the developed algorithm (solid line), RCRU (dashed line), IRI model (pluses), gAGE/UPC (triangles and a solid line), JPL-GNISD (diamonds and a solid line), ESA/ESOC (circles and a solid line), NRCAN (stars and a solid line), CODE (squares and a solid line). (a) 25 April 2001 and (b) 28 April 2001.

The estimated TEC values using the proposed algorithm are indicated with a solid line in Figures 7 and 8.

[29] In Figure 7, for a high latitude station, Kiruna, both a quiet and a positively disturbed day (25 April and 28 April 2001) TEC values obtained from the estimation algorithm are compared with the models mentioned above. Figure 8 contains the same comparison for a midlatitude latitude station, Ankara, for both a quiet and a negatively disturbed day (25 April and 23 April 2001). Except the VTEC from the developed algorithm, all the other methods use a combination of (L1-L2) and (P2-P1) measurements to compute VTEC. Since the inversion algorithms vary, none of the methods totally agree with each other in their VTEC estimates. The differences in the map model estimates are emphasized for midday hours when the Sun is most effective in the ionization process. For the night hours (before the Sun rises and after the Sun sets), almost all the map models provide the same TEC estimates. For the quiet days, TEC estimates from the IRI model follow the general features of the data both for high and midlatitudes especially at night hours. For the disturbed days, the temporally averaged TEC estimates from the map models fail to adopt to the sudden variations in the TEC values. ESA/ESOC model values are in general accordance with IRI estimates especially for midlatitude. For the midlatitude, NRCAN model estimates form a lower bound for TEC values. More general comparisons of these models can be found at internet sites such as <http://www.aoc.nrao.edu/vlba/>

http://MEMOS/SCI/gps_ion/node3.html and http://nng.esoc.esa.de/gps/refs/Beacon_Sat_Symp.pdf.

[30] As can be observed from Figures 7 and 8, the estimated VTEC values from the developed algorithm shows reasonable agreement accordance with RCRU results, especially for Ankara. Due to the 30 s time resolution of both methods, the VTEC data follow the smaller-scale time variations in the ionosphere with a higher accuracy when compared to 2 hour data from the other methods. The VTEC results from JPL, CODE, and UPC are in general accordance with the estimated TEC from the algorithm in terms of magnitude and time variation. Depending on the results provided in this section, the developed VTEC algorithm can successfully estimate vertical total electron content for various latitudes with a robust parameter set for 24 hours and a 30 s time resolution.

5. Conclusions

[31] In this paper, a novel regularization technique which combines signals from all GPS satellites within the vicinity of an Earth-based GPS receiver, is developed to estimate the vertical TEC values for the 24 hour period without missing any important features in temporal or spatial domain. The algorithm is based on the minimization of a cost function including a high pass penalty filter. The developed algorithm is applied to GPS data for four GPS receiver stations in the same time zone located from north to south for the

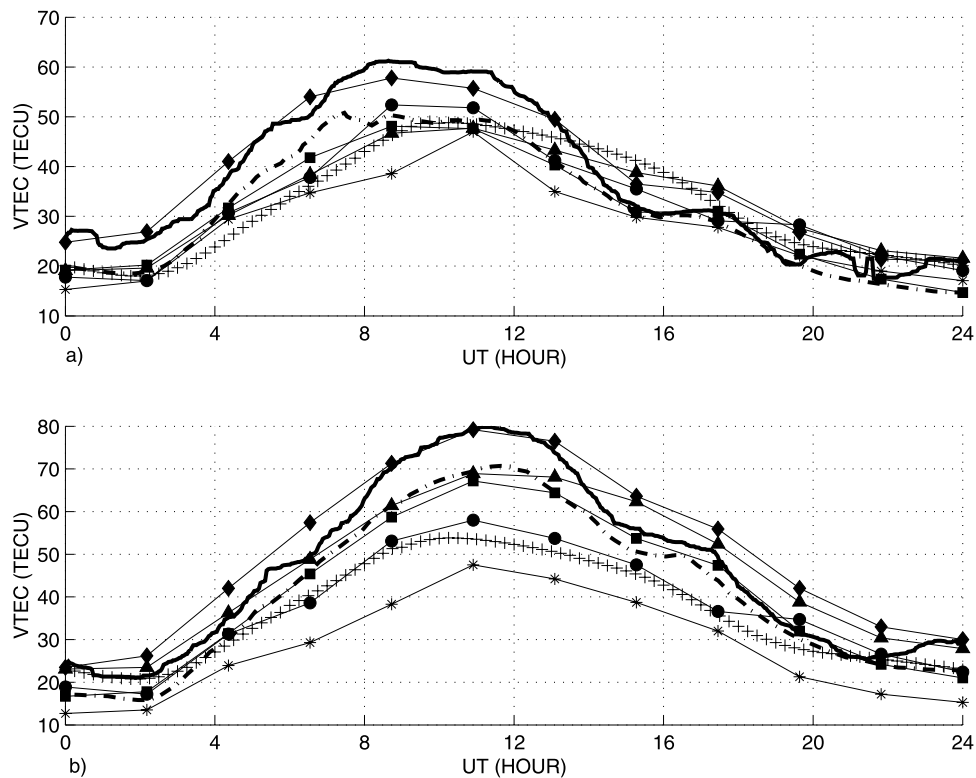


Figure 8. Comparison of the estimated TEC values from the algorithm with other models of TEC for Ankara, estimated VTEC from the developed algorithm (solid line), RCRU (dashed line), IRI model (pluses), gAGE/UPC (triangles and a solid line), JPL-GNISD (diamonds and a solid line), ESA/ESOC (circles and a solid line), NRCAN (stars and a solid line), CODE (squares and a solid line). (a) 23 April 2001 and (b) 25 April 2001.

solar maximum week of 23–28 April 2001. It is observed that the new method is very successful in estimating TEC for both quiet and disturbed days. The robust parameter set required by the regularization and smoothing procedure, can be used for all GPS station locations, for all days both quiet and disturbed. When the estimated VTEC values are compared with the outputs of other available global ionospheric maps, it is observed that the developed estimation algorithm is very accurate in representing the sharp and sudden temporal variations in the ionosphere due to its 30 s time resolution. The best agreement is observed with the RCRU, JPL, CODE, and UPC outputs.

[32] **Acknowledgments.** The authors wish to express their thanks to M. Mendillo of Center for Space Physics, Boston University, Boston, Mass., for his supportive comments on the developed technique and pointing out the important of application of the estimated TEC data in further analysis of structure of ionosphere on disturbed days.

[33] Arthur Richmond thanks M. Bakry El-Arin, Brian D. Wilson, and another reviewer for their assistance in evaluating this paper.

References

- Calais, E., and J. Bernard Minster, GPS, earthquakes, the ionosphere, and the Space Shuttle, *Phys. Earth Planet. Int.*, 105, 167–181, 1998.
- Ciraolo, L., and P. Spalla, Comparison of ionospheric total electron content from the Navy Navigation Satellite System and the GPS, *Radio Sci.*, 32(3), 1071–1080, 1997.
- Conkright, R. O., K. Davies, and S. Musman, Comparisons of ionospheric total electron contents made at Boulder, Colorado, using the Global Positioning System, *Radio Sci.*, 32(4), 1491–1497, 1997.
- Davies, K., and G. K. Hartmann, Studying the ionosphere with the Global Positioning System, *Radio Sci.*, 32(4), 1695–1703, 1997.
- Erol, C. B., F. Arikan, and O. Arikan, A new technique for TEC estimation, paper presented at IEEE IGARSS Symp., Toronto, Canada, 24–28 July 2002a.
- Erol, C. B., F. Arikan, and O. Arikan, An alternate method for local vertical TEC estimation, paper presented at PIERS'02, Cambridge, Mass., 1–5 July 2002b.
- Feltens, J., Chapman profile approach for 3-D global TEC representation, paper presented at IGS Anal. Cent. Workshop, ESOC, Darmstadt, Germany, 9–11 February 1998.
- Frail, J. M., Real-time TEC determination for ionospheric modeling in WADGPS, paper presented at 8th Int. Tech. Meet. of the Satellite Div., Inst. of Nav., Palm Springs, Calif., 12–15 September 1995.
- Hargreaves, J. K., *The Solar-Terrestrial Environment*, Cambridge Univ. Press, New York, 1992.
- Hernandez-Pajares, M., J. M. Juan, and J. Sanz, New approaches in global ionospheric determination using ground GPS data, *J. Atmos. Sol. Terr. Phys.*, 61, 1237–1247, 1999.
- Hocke, K., and A. G. Pavelyev, General aspect of GPS data use for atmospheric science, *Adv. Space Res.*, 27(6–7), 1313–1320, 2001.
- Jakowski, N., Generation of TEC maps over the COST251 area based on GPS measurements, *Proc. of the 2nd COST 251 Workshop, COST251TD(98)005*, Rutherford Appleton Lab., Chilton, Didcot, Oxfordshire, UK, 1998.
- Jakowski, N., E. Sardon, E. Engler, A. Jungstand, and D. Klahn, Relationships between GPS-signal propagation errors and EISCAT observations, *Ann. Geophys.*, 14, 1429–1436, 1996.
- Komjathy, A., and R. Langley, An assessment of predicted and measured ionospheric total electron content using a regional GPS network, paper presented at Nat. Tech. Meet., Inst. of Nav., Santa Monica, Calif., 22–24 January 1996.
- Manucci, A. J., B. D. Wilson, D. N. Yuan, C. H. Ho, U. J. Lindqwister, and T. F. Runge, A global mapping technique for GPS derived ionospheric total electron content measurements, *Radio Sci.*, 33(3), 565–582, 1998.

- Otsuka, Y., T. Ogawa, A. Saito, T. Tsugawa, S. Fukao, and S. Miyazaki, A new technique for mapping of total electron content using GPS network in Japan, *Earth Planets Space*, 54, 63–70, 2002.
- Sardon, E., A. Rius, and N. Zarraoa, Estimation of the receiver differential biases and the ionospheric total electron content from Global Positioning System observations, *Radio Sci.*, 29(3), 577–586, 1994.
- Schaer, S., Mapping and predicting the Earth's ionosphere using the global positioning system, Ph.D. thesis, Univ. of Bern, Bern, Switzerland, 1999.
- Wang, Y., P. Wilkinson, and J. Caruana, Using GPS to monitor ionospheric irregularities in the southern high latitude region, *Proc. of the 2nd COST 251 Workshop, COST251TD(98)005*, Rutherford Appleton Lab., Chilton, Didcot, Oxfordshire, UK, 1998.
- Warnant, R., Reliability of the TEC computed using GPS measurements—The problem of hardware biases, *Acta Geod. Geoph. Hung.*, 32(3–4), 451–459, 1997.
-
- F. Arıkan, Department of Electrical and Electronics Engineering, Hacettepe University, Beytepe, 06532, Ankara, Turkey. (arikan@hacettepe.edu.tr)
- O. Arıkan, Department of Electrical and Electronics Engineering, Bilkent University, Bilkent, 06533, Ankara, Turkey. (oarikan@ee.bilkent.edu.tr)
- C. B. Erol, TUBITAK, UEKAE Ankara Branch, Tunali Hilmi Cad., Binnaz Sok., No: 2 Kat 3, 06100, Kavaklıdere, Ankara, Turkey. (cberol@tubitak.gov.tr)