Hacettepe University Graduate School of Social Sciences

Department of Economics

Master of Arts Programme in Economics

A SIMPLE EVOLUTIONARY MODEL OF INVENTION AND GROWTH TAKEOFFS

Farhan Kurnia MAYENDRI

Master's Thesis

Ankara, 2020

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ABSTRACT

MAYENDRI, Farhan Kurnia. *A Simple Evolutionary Model of Invention and Growth Takeoffs,* Master Thesis, Ankara, 2020.

This thesis studies a simple evolutionary model of invention to understand how different behavioral limitations would have affected the transition to modern growth. In neoclassical models of invention, "entrepreneur-inventor"s are fully informed, fully rational, and able to solve mathematical optimization problems. In contrast, this thesis assumes that they are not fully informed, they are subject to bounded rationality constraints, and they cannot solve mathematical optimization problems. Within this structure, the thesis uses a genetic algorithm to model how a society—where the norm (or the status quo) is initially not to spend valuable resources to invention—can learn that invention is actually optimal. In other words, this thesis studies how a technologically stagnant society can converge to a growth equilibrium. Three exogenous factors are mutation (how tolerant the society is to deviant entrepreneurs), elite persistence (how effective the knowledge transmission is across generations), and the size of population. These potentially affect two model outcomes, i.e., how long the transition is and whether the society can converge to the neoclassical benchmark exactly. Results show the following: (i) Mutation is not very strongly correlated with the model outcomes, but higher mutation rates are observed along with faster convergence in some specifications. (ii) Elite persistence does not have a monotone effect on the duration of convergence. (iii) Societies generally exhibit variation around the neoclassical optimum in terms of equilibrium values. (iv) There is a very strong scale effect of population size; larger populations converge significantly faster, and the degree of variation around the neoclassical equilibrium is much smaller for larger populations. Even in the best-case scenario—i.e., high mutation rates, high elite counts, and large populations—the evolutionary model converges to the neoclassical equilibrium in 51 generations.

Key Words

Invention, Innovation, Entrepreneurship, Industrial Revolution, Genetic Algorithm, and Evolutionary Economics

ÖZET (TURKISH ABSTRACT)

MAYENDRI, Farhan Kurnia. *İcat ve Büyüme Kalkışlarının Basit bir Evrimsel Modeli*, Yüksek Lisans Tezi, Ankara, 2020.

Bu tez, farklı davranışsal kısıtlamaların, bu geçişi nasıl etkileyeceğini anlamak için basit bir evrimsel model çalıştırmaktadır. Neoklasik icat modellerinde, "girişimci mucitlerin" tam bilgi sahibi olduğu, tamamen rasyonel olduğu ve matematiksel optimizasyon problemlerini çözebildiği varsayılır. Bunun aksine, bu tez onların tam bilgi sahibi olmadıklarını, sınırlı rasyonellik kısıtlamalarına tabi olduklarını ve matematiksel optimizasyon problemleri çözemediklerini varsaymaktadır. Bu yapı içerisinde tez, icat yapmak için değerli kaynaklar harcamamanın başlangıçta norm (veya statüko) olduğu bir toplumun, buluşun gerçekten optimal olduğunu nasıl öğrenebileceğini modellemek için bir genetik algoritma kullanmaktadır. Başka bir deyişle, bu tez teknolojik olarak durgun bir toplumun bir büyüme dengesine nasıl yaklaşacığını incelemektedir. Üç dışsal faktör, mutasyon (toplumun "sapkın" girişimcilere ne kadar hoşgörülü olduğu), elit kalıcılık (bilgi aktarımının nesilden nesile ne kadar etkili olduğu) ve nüfusun büyüklüğüdür. Bunlar potansiyel olarak iki model çıktısını, yani geçişin ne kadar sürdüğünü ve toplumun neoklasik dengeye tam olarak ulaşıp ulaşamayacağını etkilemektedir. Sonuçlar şöyledir: (i) Mutasyon, model sonuçlarıyla çok güçlü bir şekilde ilişkili değildir, ancak bazı spesifikasyonlarda, daha hızlı yakınsama, daha yüksek mutasyon oranları ile birlikte gözlenmektedir. (ii) Elit kalıcılığın yakınsama süresi üzerinde tek yönlü bir etkisi yoktur. (iii) Toplumlar genellikle denge değerleri açısından neoklasik optimum etrafında farklılık göstermektedir. (iv) Nüfus büyüklüğünün çok güçlü bir ölçek etkisi vardır; daha büyük nüfuslar, önemli ölçüde daha hızlı yakınsamaktadır ve neoklasik denge çevresindeki değişim derecesi daha büyük nüfuslar için çok daha küçüktür. En iyi durumda, yani yüksek mutasyon oranları, yüksek elit sayıları ve büyük nüfusta, evrimsel model 51 nesilde neoklasik dengeye ulaşmaktadır.

Anahtar Sözcükler

İcat, Yenilik, Girişimcilik, Sanayi Devrimi, Genetik Algoritma ve Evrimsel İktisat

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CHAPTER 1

INTRODUCTION

A significant historical break occurred in the mid-eighteenth century with the first Industrial Revolution. In previous centuries, growth rates across the world had remained minuscule. Pre-industrial societies had faced poor standards of living with very low per capita income levels throughout centuries. The event turned the stagnant society into a society of creativity and innovation. The movement of this "reform" began in England, where the first steam engine was invented and became one of the fundamental bases of the transformation. The invented engine succeeded in replacing the need for human resources in many industries.

Maddison (2001) data illustrate that earlier civilizations had lived in a state of stagnation for centuries. His monumental project continued and is known as the Maddison Historical Statistics Project. The newly updated database shows that rate of growth in GDP per capita was nearly zero for the world from 500AD to 1500 AD. In that period, there was no impressive progress in technological development.

Before the historical change in the 18^{th} century, the world economy did not generate the characteristics of growth that describe the world economy of the 19^{th} and 20^{th} centuries (Clark, 2007). Over the past centuries, there was a dramatic change in the world economy through technology and industry. Since the early 19^{th} century, the world economy's real income per capita has risen from $667 in 1820 to $1525 in 1913, and increased to $6,012 in 2000. The Maddison data measures that the industrialized world brought an income per capita increase of 15 to 20 times in the current two centuries after 1820.

Human knowledge diffuses across individuals of a single generation and accumulates from one generation to another. Naturally, humans have the ambition to be creative with their material endowments and with their time. Coupled with curiosity, all pre-modern societies exhibited signs of technological advancement, learned new useful knowledge, and invented new things. One of the most significant roles of knowledge is that it is essential for humans to nurture innovation. New useful knowledge creates for each generation a chance to escape from the ongoing economic stagnation and enter into a new technological civilization with more variation through invention. However, earlier advances did not pave the way into a new civilization in terms of economic growth. The way toward exploiting useful knowledge and creating a sustained growth regime through continuous innovation is not as simple as it sounds.

The development of a civilization of sustained growth does not depend solely on the existence of useful knowledge, but directed efforts are required to exploit useful knowledge. Entrepreneurship skills are required to integrate knowledge, management, and innovation. The role of the entrepreneur in the innovative phase is to allocate an "optimal" level of time and resources for research in order to deliver advanced technological improvements in an effective manner. This outcome has an impact on productivity and the welfare of society.

There is a sizable literature on the roles of entrepreneurship and technology for sustained growth. A large part of this literature intersects with the literature on endogenous growth since Romer (1986) and Lucas (1988). Another part overlaps with it is the literature on UGT and growth takeoffs developed since the pioneering works of Galor and Weil (2000). Both of these literatures commonly presume that stagnation and sustained growth can be understood within (general) equilibrium frameworks where agents are rational decision makers that have full information. However, there is also an evolutionary literature where some of the main assumptions include (i) people have bounded rationality, (ii) learning at the societal level takes time and is subject to inertia, (iii) society may be locked in a particular technological paradigm for long episodes of time, and so on. The intellectual foundations of evolutionary growth models are typically associated with Schumpeter (1934), and formal modeling dates back to Nelson and Winter (1982). Finally, there is also a small literature that applies some of the evolutionary foundations into unified growth models with entrepreneurship where there are different types of agents, and their group sizes change endogenously (Galor and Michalopoulos, 2012) and where preferences are not fixed, and people develop certain characteristics that promote creativity (Doepke and Zilibotti, 2008).

This study aims to understand the transition process from stagnation to growth with an evolutionary model of invention. The so-called entrepreneur-inventor is at the main actor of the analysis that carries the society from technological stagnation to sustained innovation. The evolutionary model the thesis uses works as an agent-based model where "entrepreneur-inventor"s (or simply entrepreneurs or inventors) are subject to behavioral limitations. More specifically, there is an inherent inertia since the population of entrepreneurs need to learn the optimal level of inventive effort through generations. In the remainder of this chapter, (i) the research questions, (ii) the model, (iii) the main results, and (iv) the outline of the thesis are explained briefly.

1.1. QUESTIONS

Investigating the process of a growth takeoff leads to the question of how an evolutionary model of invention can be useful to examine the Industrial Revolution. Endogenous growth models developed in the neoclassical tradition assumed that the optimal level of research effort is known and can be solved by entrepreneurs in the model. In this case, fully informed decision-makers are assumed to be rational and to be able to solve mathematical optimization problems.

In an evolutionary economics context, these assumptions are replaced with behavioral biases, the delayed transmission of information, and learning processes that are subject to inertia. One behavioral bias is known as *the status-quo bias*, leading the individuals to follow what the earlier generations did without realizing that the status-quo behavior is non-optimal. Under the status-quo, every agent may be running her/his firm with a standard feature, using ancient knowledge and old technology to remain in their comfort zone. This behavior of not investing in knowledge and technology is not necessarily the optimal strategy. Agents may not be realizing that spending valuable resources into invention can be a way for them to increase their productivity, market share, and profit.

Building on these notions, this thesis investigates how a society that remains in the status-quo of no invention in the initial period can learn that allocating resources to invention is in fact optimal. In other words, the thesis attempts to understand the dynamics of an industrial revolution and a growth takeoff when the society is initially subject to the status-quo of no inventive effort.

In particular, the thesis tries to measure how the duration of this learning process is affected by

- The rate of mutation "μ" (that determines the fraction of individuals in each generation who would try invention by opposing to the status-quo)
- The elite count "E" (that determines the number of best-performing individuals whose behavior is ensured to survive to the next generation).

In the initial period, mutated entrepreneurs are randomly selected to experiment with invention. Contrary to the status-quo, they spend resources to inventive activity. In later periods, mutated individuals (randomly chosen for each generation) would still be the ones that experiment with random levels of resources allocated to inventive activity.

Clearly, the rate of mutation must be one of the central factors that affect the transition. If the rate of mutation is higher in a society, then this society is more tolerant to deviant behavior in matters of technology. In the real world, the degree of tolerance from this perspective is determined both by *informal institutions* such as peer pressure and social exclusion and *formal institutions* such as property rights laws and oppressive regulations that characterize deviant behaviors as criminal acts.

The elite count is another and distinct dimension of the evolution. Regardless of whether the rate of mutation is large or small, the society needs to transmit the type of behavior that works best in creating productivity growth. When the elite count is larger, the society learns and transmits the behavior of a larger fraction of entrepreneurs that remain closer to the optimality.

The elite count, relative to the total population, is thus a measure of how effective the transmission of useful knowledge within a generation and from one generation to the next. Hence, the elite count may be a central factor in explaining how fast the society would converge to the state of continuous innovation and sustained growth. In the real world, cultural and institutional situations that affect the speed and scope of knowledge dissemination and knowledge codification are determining the elite count. For instance, a society with a larger elite count may be imagined to have better communication and knowledge storage devices and, therefore, to have higher connectivity.

This thesis also investigates whether learning the optimal strategy through evolution is perfect or not in the following sense: If entrepreneurs in the society eventually converge exactly to the optimal level of inventive effort, then this learning is perfect. However, evolutionary phenomena and evolutionary models are always subject to the possibility of heterogeneity in the cross-section dimension. Hence, learning may stop in a state of equilibrium where some of the entrepreneurs spend resources larger than or smaller than the exact optimum.

Finally, the thesis investigates whether the total population of entrepreneurs has a scale effect or not. In other words, the thesis studies whether the number of generations that completes convergence is affected by the total number of entrepreneurs in the society. Such scale effects have been studied by endogenous growth theorists with reference to whether more people implies higher growth rates. Here, the central issue is the length of the transition period, and population size may have an effect in this respect. Within this agent-based framework, this question is of prime importance because the Law of Large Numbers does not apply, and successful entrepreneurs cannot be a remedy for failed entrepreneurs in the calculation of average productivity.

In summary, this thesis investigates the following four questions concerning the growth takeoff:

- Whether higher mutation rates cause convergence to be faster in terms of research activity,
- Whether higher elite counts cause convergence to be faster in terms of research activity,
- Whether the society of entrepreneurs converges to the optimum in an exact manner or they exhibit heterogeneity around the optimum, and, finally,
- Whether the total number of population has a scale effect on the number of generations that pass before the learning (growth takeoff) is completed.

1.2. THE APPROACH

This thesis uses an evolutionary approach to answer the questions posed above. Since the thesis focuses on independent entrepreneur-inventors that compete for each other to have higher market shares and profits, it belongs to the Schumpeterian growth theory in the particular meaning of the term within the mainstream growth economics (Schumpeter, 1934). On the other hand, one can also describe the approach followed in this thesis as a neo-Schumpeterian one in the realm of evolutionary economics. More specifically, the thesis focuses on the role of technology in understanding how a stagnant economy transits from a no-invention no-growth state of equilibrium to another equilibrium state where average productivity across production plants grow over time.

More specifically, the methodology of this thesis falls within the strand of evolutionary growth literature emphasizing the role of mutation, selection, learning, etc. Nelson and Winter (1982) developed a Schumpeterian-evolutionary understanding of firm behavior by focusing on how technology evolves in a dynamic economy. In such evolutionary models, behavioral limitations and biases, heterogeneities, routines, and rules-of-thumb, and the endogenous market structures may all play fundamental roles in determining economic outcomes at the firm, sector, and national economy level.

In this thesis, an economy's evolution from a pre-industrial state of low and stagnant productivity to a modern state with growth and prosperity is modeled at the level of entrepreneur-inventors each owning and managing a firm. In this respect, the industrial organization, market structures, and the firm behavior are extremely simplified. In fact, the employment of workers is also assumed away from the model without altering the main results.

There are three distinct stages of the methodology. The thesis first develops a neoclassical benchmark model of invention and entrepreneurship (Chapter 3). This is necessary to define an entrepreneur-inventor's profit as an appropriate concept of individual fitness. The thesis then develops an evolutionary version of the model (Chapter 4). The evolutionary model features a discrete set of entrepreneur-inventors and specifies mutation, elite persistence, and crossover dynamics. In the third stage, the thesis designs and implements the genetic algorithm by setting values to various parameters, both economic and evolutionary (Chapter 5). The genetic algorithm is then executed for several times for different constellations of parameter values to calculate the number of generations that completes the learning and the level of research time chosen by the entrepreneur-inventors.

The main endowment of individual entrepreneur-inventors is time that is allocated between routine management and inventive research. If an individual decides on spending their potential of the time endowment to management only, then these individuals work with the fixed level of technology. In this case, these agents have no productivity growth. On the other hand, individuals who decide to spend some of their time endowment to research, their productivity is larger as a result of micro-inventions. In evolutionary terms, higher productivity means higher fitness.

To understand the simple mechanics of entrepreneurial invention in this way, the thesis constructs a very simple model. This model is inspired by Attar's (2015) formulation of perfectly competitive innovation that builds upon the model developed by Hellwig and Irmen (2001). In its neoclassical version, the individual is fully informed and rational, and he/she can solve a convex optimization problem to determine the optimal level of research time. Under appropriate normalizations, this is a model of independent entrepreneur-inventors whose research activity explains endogenous productivity growth.

The evolutionary version of the same model assumes that entrepreneur-inventors have bounded rationality, their information set is limited, and they do not have cognitive powers to solve a mathematical problem. Instead, the behavior is assumed to be non-optimal in general, and the evolutionary mechanism (here, a genetic algorithm) allows the society of entrepreneur-inventors to learn the "optimal" behavior in time.

In any generation, entrepreneur-inventors are separated into three groups. The first group of them, the mutated entrepreneurs, is formed by those randomly selected to try random levels of research time. As mentioned above, the rate of mutation is exogenously given. The second group of entrepreneur-inventors, the elite entrepreneurs, is formed by those that replicate the best practice in the previous generation. As mentioned above, how many of the entrepreneur-inventors are in the elite group is also determined exogenously. Finally, remaining entrepreneur-inventors of any generation become the crossover entrepreneurs, whose behavior is jointly determined by any two entrepreneurs in the previous generation.

Within the scope of this thesis, the evolution here is surprisingly both Lamarckian and Darwinian. It is Lamarckian because an acquired characteristic, a particular level of research time, is passed on to the next generation through the elites. But it is also Darwinian because of (i) variation that originates from mutations and (ii) selection that originates from crossovers.

The approach of this thesis can now be summarized as follows:

1. Develop a very simple model of invention as a neoclassical benchmark (this model allows us to calculate the optimum).

2. Develop a simple evolutionary version of the same mechanism (with the rate of mutation, the elite count, and the size of population as exogenous givens), and choose the profit of an entrepreneur-inventor as his/her fitness function.

3. Run the genetic algorithm for different values of model inputs to calculate (i) the length of the learning process (the growth takeoff) in terms of generations and (ii) the resulting level of research time chosen by entrepreneur-inventors.

1.3. RESULTS

This subsection summarizes the main results in a way structured by the research questions.

1. For the effects of the rate of mutation on the duration of transition, there is no clear and strong relationship. Controlling for other model inputs, i.e., *ceteris paribus*, societies with differing values of the rate of mutation may converge at similar numbers of generations. However, there are some cases where the maximum number of generations is observed for lower rates of mutation.

2. For the effects of the elite count, there is no relationship running from the elite count to the duration of transition. Again controlling for other model

inputs, societies with high and low levels of the elite count are observed to achieve similar durations, and societies with the same level of the elite count are observed to converge in differing durations.

3. Regarding the question of whether convergence to the neoclassical optimality is exact in terms of the time endowment allocated to inventive activity, we find that there is generally heterogeneity. In other words, societies generally exhibit variation around the neoclassical optimum. Importantly, the degree of variation is smaller for larger populations.

4. There are strong scale effects concerning the size of population. Societies with larger populations of "entrepreneur-inventor"s converge in significantly fewer generations. Hence, there is a Curse of Small Numbers. The best case among different societies is recorded as 51 generations. If one takes the lifetime of a generation as 25 years, the transition from no-invention to a regime of continuous technological progress takes 1275 years in the best possible scenario.

1.4. OUTLINE

Chapter 2 presents overviews of different related literatures. These include the endogenous growth theories with Marshallian externalities and Schumpeterian innovations, unified growth theory extended with the works of economic historians working on the Industrial Revolution and growth takeoffs, and evolutionary economics and evolutionary growth models.

Chapter 3 is concerned with the neoclassical benchmark model of invention used in the formulation of evolutionary analysis. The chapter develops a simple model where fully-informed and rational "entrepreneur-inventor"s solves an invention problem to determine what fraction of their time endowment is allocated to research and what fraction is allocated to routine management. Hence, the neoclassical benchmark specifies how the profit is determined by inventive activity.

Chapter 4 introduces the evolutionary foundations and the logic of the genetic algorithm. The chapter explains the status-quo bias, mutation, elite persistence, and the crossover mechanism. The final part of the chapter presents the implications of the evolutionary model.

Chapter 5 presents the design of experiments (numerical simulations) and the results originating from these experiments. Through the help of tables and figures, results of the thesis are explained. More specifically, the number of generations necessarily passing to complete the transition and the resulting levels of time allocated to research are calculated for various levels of mutation rates and elite counts under different levels of population.

Finally, Chapter 6 concludes the thesis with a non-technical summary of the findings and a discussion of further research avenues related with this thesis.

CHAPTER 2

RELATED LITERATURE

This thesis is related with three distinct research programs and literatures. First, since the thesis builds upon models with entrepreneurial invention and endogenous productivity, it is centrally related with endogenous growth literature. Second, it is related with unified growth models and the works of economic historians that focused on the transition from stagnation to growth. Finally, the thesis is centrally related with evolutionary models in general and evolutionary growth models in particular.

2.1. ENDOGENOUS GROWTH THEORY

The origin of modern economic growth theory was pioneered by Solow (1956, 1957). His models indicate that productivity expansion explains long-term growth in GDP per capita. Solow (1956, 1957) shows that there is instability in the Harrod-Domar because they use a production function that does not allow for input substitution.

One problem with the Solow model is that saving is exogenous. In Ramsey (1928) – Cass (1965) – Koopmans (1965) model, saving is endogenous through optimal control of capital accumulation. Productivity growth, however, remains as an exogenous determinant. The Ramsey-Cass-Koopmans model became an aggregate growth model as the basis of economists' thought about long-run growth. The main problem of both Solow and the Ramsey-Cass-Koopmans models is that productivity growth is exogenous. This has led economists to develop models that endogenize productivity.

In general, there two classes of endogenous growth models, i.e., models that build upon Marshallian externalities and models that build upon Schumpeterian innovation.

The Marshallian externality is originated by Marshall (1890); the idea emphasizes knowledge spillovers that create external benefits to the firm in a specific location. Then, this idea is formally developed by Romer (1986) within a general equilibrium growth model. This model assumed that firms do not internalize the positive productivity externality of mechanization. More specifically, when the aggregate capital stock increases, all firms benefit from this type of knowledge creation because machines embed useful technological knowledge. Technically, this creates increasing returns in production; knowledge accumulation results in an increase in the marginal product. Romer's (1986) model was heavily inspired by Arros's (1962) learning-by-doing. Hence, in the literature, this type of externality is sometimes called Marshall-Arrow-Romer externality.

Lucas (1988) developed a similar model but focused on human capital. Average human capital increases the productivity of all workers. Hence, once again, there may be increasing returns that are not internalized by private sector decision makers. Lucas (1988) was building upon Uzawa's (1965) model.

Externalities in such frameworks allowed researchers to construct general equilibria that feature balanced growth in the long run. Growth rates in such models, both in the short- and long-run, are endogenous. Besides, competitive equilibrium growth rates are smaller than social planner growth rates because the social planner can intervene to correct for externalities.

Marshallian Externality and Schumpeterian Innovation models have a fundamental difference: While growth is endogenous in Marshallian externality models, the mechanism is not purposeful research. Growth is endogenous in Schumpeterian growth models due to the profit-seeking actions of entrepreneurs and firms.

Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) developed the *first type* of Schumpeterian innovation models, i.e., the First Generation Models. Growth is endogenous and originates from firms and entrepreneurs' competition with each other through research and development (R&D). Entrepreneurs and firms spend scarce resources on inventive activity and this results in increased productivity, increased market share, and, hence, increased profit. These models presented the role of policies affecting growth in the long run and sometimes called the first generation of Schumpeterian innovation theory.

In such models, there is a scale effect of population. If population grows, the total labor supply allocated to R&D also grows. This increases growth rates in theory. But, in reality, growth rates in the US and the UK, for instance, were largely stable after 1950s. The solutions of this scale effect have been the subject of intense debate since Jones (1995). Jones (1995) and Kortum (1997) developed the *second type* of Schumpeterian Semi-Endogenous Growth models. These sterilized the scale effects of earlier models by changing the knowledge production function (by imposing decreasing returns in the production of technology). But these models have the wrong prediction that economic growth is explained by population growth. In this case, policies were not useful to raise growth rates.

Peretto (1998), Young (1998), Dinopoulos and Thompson (1998), and Howitt (1999) developed the *third type* of models, known as the Second Generation of the Schumpeterian model. These models sterilized scale effects without changing knowledge production; they correctly identified that there must be more than one sector or firm that innovate. More specifically, these models argue that, as long as population grows, product variety changes accordingly.

This thesis is related with the endogenous growth theory because it uses an individual level story of technological progress. In the models studied in this thesis, there are “entrepreneur-inventor”s how manages their own firms and who allocate time to invention. This inventive activity results in micro-inventions that raise their firms’ productivity and profit.

2.2. UNIFIED GROWTH THEORY

Endogenous growth models are designed to explain growth and cannot explain poverty. There were efforts on that trajectory where several models were developed to explain prolonged poverty.

The poverty trap can be characterized as an unintentional condition where poverty remains due to the cycle of self-reinforcing mechanism that initially causes poverty. Multiple factors can lead the economy to a poverty trap.

As we discuss below, the very long-run development patterns are strongly related with demographic patterns, especially with the historical fertility decline. Few studies that establish poverty trap models of high fertility and low education must be credited here (Azariadis and Drazen, 1990; Becker et al., 1990).

In Becker et al. (1990) framework, for instance, there are two steady-states. In one of these, fertility is high and human capital is fixed without educational investment. In other steady-state, fertility is low and human capital grows. Because of differences in returns to education, the decentralized equilibrium of economies with relatively low initial human capital stocks is converging to high fertility without steady-state education.

We also need models that describe poverty and growth at the same time, as well as the presence of historical continuity within the model. There are some initial thoughts of non-unified models that reflect on this transition, i.e., earlier studies by Goodfriend and McDermott (1995), Tamura (1996), Acemoglu and Zilibotti (1997), and Arifovic et al. (1997). These models have provided a range of steady-states in which economies are either trapped in an equilibrium of stagnation or converge in an equilibrium of growth. However, these models explain only the existence of a poverty equilibrium without accounting for the transition to growth. In other words, these models cannot explain how the transition starts and are generally not consistent with the reality of demographic transition.

Galor and Weil's (2000) was the first unified growth model that explains both poverty and growth where the transition is gradual and endogenous. Besides, it was consistent with the historical fertility decline, and more generally with the timing of broad historical transitions such as industrialization, urbanization, and the rise of public education.

UGT was designed to provide a consistent model that explains the entire development path of the economy from ancient times to modernity. It does so by developing an endogenous growth and demography model that is consistent with history.

Galor (2005) shows that sustained growth has started with the first Industrial Revolution and diffused to Western Europe and Offshoots, i.e., U.S, Canada, New Zealand, and Australia. The industrial revolution was a turning point from a stagnation era to sustained economic growth as described by others including economic historians (Mokyr, 2002; Clark, 2007; Allen, 2009). In England, a small minority of people created and utilized useful knowledge during the industrial revolution (Mokyr, 2002; Attar, 2015). However, the process of industrial enlightenment was strong enough to cause the rise of a knowledge society where technological innovation and entrepreneurship eventually led to modern economic development. The new technology and the new culture of creating new technology then diffused to the rest of Europe and to North America and Oceania (Lucas, 2009).

The theory of unified growth is not only to build a growth model that merely forecasts several equilibria representing various phases of economic development. Galor and Weil (2000) have formulated a way that it is genuinely unified since the transition is endogenous and gradual: The key advance of philosophy is the study of Malthusian stagnation as a pseudo-state equilibrium that gradually and endogenously disappears in the process of economic development.

There are three regimes in the UGT. The *first regime* is the Malthusian era, sometimes called zero growth ages, stagnation ages, and the poverty era. This is almost a typical Malthusian trap where fertility is endogenous, land is fixed, and increasing population depresses labor productivity. In a typical Malthusian model, the economy returns to fixed population and fixed income (Ashraf and Galor, 2011). It is not exactly the Malthusian trap because population and productivity growth is positively related. This is due to the Boserup effect; more people means that (i) there is a need to develop new technologies (demand side) and (ii) that more useful ideas are developed by more minds (supply side) (Kremer, 1993; Attar, 2015). This enriched Malthusian system eventually hits an endogenous threshold where production becomes sufficiently productive (albeit at a very low level) and people become sufficiently rich (albeit at a very low level). With this endogenous threshold, the equilibrium moves to the second regime.

The *second regime* is called the post-Malthusian era in which population and productivity feed each other and increase slowly. Here, increased productivity is directed to the quantity of children because the quality-quantity tradeoff is not active yet; the return to skill accumulation is still low. However, as long as productivity increase continues, there is another endogenous threshold where people started investing in their children's education.

The *third regime* is called the Modern Growth era where the fertility rates show a significant decrease. This occurs simultaneously with increased education, and increasing education sizably increases productivity growth rates. These lead the society to more human capital with increased incomes. The evidence show that economic growth rose from nearly zero to 2% per year (Madsen et al., 2010). In England, the Post-Malthusian regime starts roughly in 1650s, and the modern growth regime in 1870s.

There are other studies that focus on the transition from different perspectives. There are, for instance, models that endogenize growth through innovations, not through human capital (e.g., Strulik, 2014; Peretto 2015; Attar, 2015). Invention and the process of entrepreneurship play important roles in such models. In Figure 2.1, the equilibrium regimes of Attar's (2015) model are shown with respect to population growth rate (n) and invention intensity (a). Here, the economy leaves the Malthusian trap at $t = t1$, and the industrial revolution starts at $t = t2$.

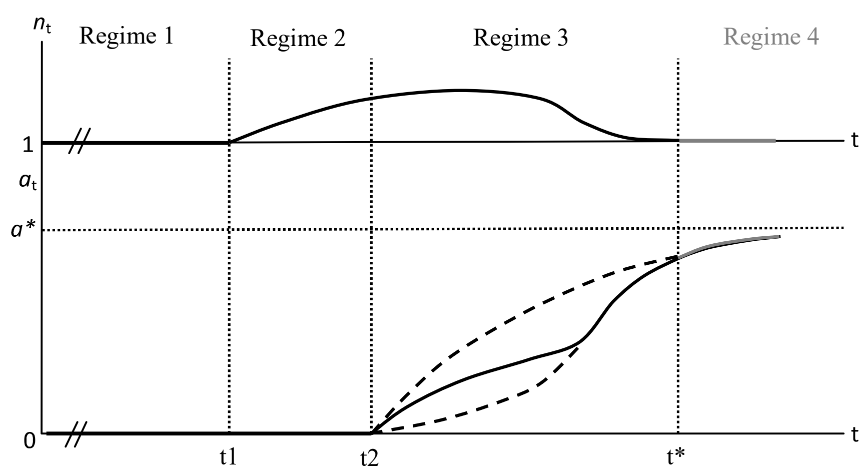

Figure 2.1: The Equilibrium Path from Stagnation to Growth (Attar, 2015)

The causal mechanisms behind the transition of a technologically stagnant, pre-modern economy to a modern type of industrial organization with technological dynamism has been studied by Peretto (2015). In his neoclassical analysis, there are both entrepreneurial activity where (i) small (startup) firms enter the market after inventing a blueprint and (ii) existing big businesses continue to perform in-house R&D to develop better versions of their products.

This thesis is centrally related with the UGT because it studies the process of growth takeoffs. More specifically, the evolutionary model studied here tries to understand what happens between $t = 0$ and $t = t*$ when "entrepreneur-inventor"s are not fully informed, fully rational, and able to solve mathematical optimization problems.

2.3. EVOLUTIONARY ECONOMICS AND EVOLUTIONARY GROWTH

2.3.1. Evolutionary Thinking in Biology and Economics

Based on the Oxford Dictionary, etymologically, the word 'evolution' is derived in the early 17^{th} century from Latin word '*evolutio*(n)' which means unrolling, and from the verb, '*evolvere*' means evolve. Today, in biology, it is centrally associated with Darwin (1859) even though Darwin himself did not use the term. More generally, the concept applies to many natural and social science disciplines and topics including the history of the earth, economic history, development, and culture and society more generally.

The term 'evolution' was first used by German biologist Albrecht von Haller for natural phenomena in 1744. In biology, there are three important founding fathers of evolutionary thinking: Lamarck, Darwin, and Spencer.

In the Lamarckian evolution, characteristics acquired during individuals' life are transmitted to the genes of the individuals in the next generation. The research study by Morgan (1896) has rejected Lamarckism in the biological sphere and raised some questions such as whether human beings had developed only a little in the genetic sphere.

Darwin's theory of natural selection builds on the notion that genetic characteristics that give the highest fitness in an environment are transmitted to the next generations. His theory has three principles: *variation, selection*, and *inheritance*. Variation means that population has an initial genetic heterogeneity partly explained by random mutations. Selection means that individuals that have genes that have the highest fit with the environment live longer. Inheritance means that those individuals that live longer reproduce more.

Spencer tried to apply Darwin's ideas not only in the physical world but also in social, economic, and political domains. He also benefitted from Lamarckian thinking in the social/cultural transmission of acquired traits. He is associated with

Social Darwinism, a naïve but erroneous application of natural selection into social and political domain, sometimes to justify unethical or unjust practices or socio-economic systems. Spencer is famous for using the notion of *survival of the fittest*. In contrast to Darwin, he believed that evolution of human development has a final equilibrium point of higher fitness physically and socially.

Fisher's (1930) replicator dynamics model was an important milestone in formalizing the evolutionary concepts such as selection and fitness. For a population of distinct interacting individuals, fitness and relative population frequencies are tied mathematically. Evolution, or selection more specifically, works when the frequencies of individuals with above-average fitness increase, and those with below-average fitness decrease.

Hodgson (1993) discusses that the development of evolutionary thinking in economics builds on Malthus (predator-prey relationship), Smith (invisible hand, equilibrium), Marx (progress of history, evolution of socio-economic systems), Marshall (organic change, variation), Veblen (cumulative causation, institutional evolution, status, habits), Schumpeter (development, creative destruction, technology), and Hayek (spontaneous order, equilibrium).

Among these economists, the most directly related ones with this thesis are Veblen and Schumpeter. Thorstein Veblen, in his 1898 essay, was known as the first man to use the term 'evolutionary economics.' Veblen has argued that economics would be the next 'post-Darwinian' science to reflect the main principles of Darwinian theory of evolution (Veblen, 1898). This masterpiece, explored socio-economic evolution, with the evolution of individual agents and changing nature of institutions and structures.

Veblen's ideas are important because he envisions a framework where individuality and institutions/society co-evolve by affecting each other in a web of causation flows. Here, in this thesis, the individual "entrepreneur-inventor" evolves but his/her evolution is subject to societal restrictions such as tolerance to deviant behavior. Besides, the society is affected by each and every individual's actions in turn.

Schumpeter uses the word evolution as an entity and takes the notion of development as the same as evolution. Schumpeter (1934) shows that how, in the past, the value of evolution is discredited. Schumpeter critically focuses on entrepreneurs as well as technical change and overall pace and pattern of economic development through innovation. This work provided a valuable opportunity to understand the dynamic transformation a society experiences in the long run. Schumpeter (1934) has defined economic evolution as a process through which knowledge grows and the wealth of the nation increases. This is through new markets, new products, and new organizational forms. The "entrepreneur-inventor" working in a competitive capitalism and manages his/her own firm is the central actor in the early stages of capital accumulation (before the big business and monopolization enter the picture). These "entrepreneur-inventor"s obtains profits through firm ownership as infra-marginal rents (since they manage their own firms). For this reason, they have incentive to increase their firm's productivity through inventions. Once they are successful in creating new technologies or new products, they destruct profits of previously successful "entrepreneur-inventor"s (Attar, 2015).

There have been some works in the 20th century on evolutionary thinking in economics (Alchian, 1950; Downie, 1955; Steindl, 1952). However, a significant break is associated with the development of Nelson and Winter's (1982) model. Since the early 1980s, the evolutionary economic theory has broadened its scope and direction, now featuring a field journal and attracting numerous economists. Silverberg and Verspagen (2005) point out two bold reasons to enhance the application of an evolutionary approach to economics. The first reason is based on a biology analogy in terms of competition, innovation, variation, and selection. The second reason is that the society in modern industrial stage of human evolution is just another discrete stage of a single socio-economic evolution.

Throughout the evolutionary context, the growth models follow the Nelson and Winter (1982) approach by introducing microeconomic fundamentals. The key task is to broaden the initial Nelson and Winter set-up by adding more practical technological postulates or to interpret evolutionary concepts with behavioral approaches that add realism.

According to Araz Takay and Özel's (2008) classification, the evolutionary economics can be categorized into (i) *traditional evolutionary economics* of Veblen where evolution is gradual and either Lamarckian or Darwinian, (ii) *neo-*

Schumpeterian evolutionary economics of Nelson, Winter, and others where evolution is either gradual or radical and Lamarckian, and (iii) *new evolutionary economics* of Dopfer, Witt, and others where evolution is either radical or punctuated and neither Darwinian nor Lamarckian. The approach in this thesis is located somewhere between traditional and neo-Schumpeterian approaches since both the individual and the society evolves. Evolution has both Lamarckian and Darwinian features; acquired technological strategies can be transmitted to next generation's entrepreneurs but the selection process resembles the biological evolution where higher productivity entrepreneurs have higher profits (i.e., higher fitness).

2.3.2. Endogenous Technology

As the pioneering work in evolutionary growth models, Nelson and Winter (1982) emphasize to understand endogenous technology characterized by the behavior of the firm's experiences in the quest for further sophisticated techniques. The Nelson and Winter Model (NWM) uses an approach based on computer simulations to find the different search behavior of the firms that have access to different levels of technology.

Building on the NWM model, Silverberg and Verspagen (2005) indicate that heterogeneous firms employ production methods that differentiate between fixed labor (aK) and capital coefficients (aL). Technological change can be distorted over time such that there may also be a pattern that resembles labor-capital substitution. Research is being carried out within a pool of existing techniques. Some of the strategies that have been used are actually in progress, including those that need to be discovered in the future.

In the beginning, firms are still looking for new techniques that have not yet been discovered. The mutation or quest method may take various forms, either local search or imitation. Furthermore, Nelson and Winter (1982) emphasize that the sense of accuracy given by conventional models is exaggerated. The causal relations between the key parameters in these models are not so clear until a microeconomic structure is implemented in practice. The key factors are heterogeneous firms, disequilibrium, and bounded rationality. An evolutionary growth model almost completely depends on stochastic technological change as the motivating factor underlying economic growth.

In terms of the evolutionary process, Nelson (2004) stresses that the evolutionary economics theory has a core focus to explain economic change. Economic growth depends centrally on the accumulation process of technological advances. Moreover, Metcalfe (2005) investigates the evolutionary concepts of the adaptation process to involve the selection unit, such as the technological approach and operational practices, and that there are replication of and interaction with the individual business entity. Technological advances are thus made in an endogenous way through a resource-efficient pursuit embraced by a variety of individuals (Faggiolo and Dosi, 2003).

Neoclassical models developed by, e.g., Desmet and Parente (2012), Peretto (2015), and Attar (2015), also represent the concept of technology as a single efficiency term that is an endogenous scale parameter that drives the production limit upwards. Attar (2015) discovered that the optimum point of entrepreneurial invention is difficult to achieve if the productivity of the invention process is based solely on the limited (or narrow) stock of useful knowledge. In this thesis, the model used is a simplified version of Attar's (2015) invention model that itself builds upon Hellwig and Irmen's (2001) model of perfectly competitive innovation.

2.3.3. Social Inertia

Investigating cultural inertia as the origin of the industrial revolution is discovered by Crouzet (1986). Spencer's (1880) concept of the rate of socio-economic growth is important in this respect; the speed of the fundamental progress of human life. Most notably, social progress depended on the rate of natural change in individuals. Nelson and Sampat (2001) have suggested that this factor characterized social technologies that appear differently from physical technologies. Social technologies, while typically assumed away from economic growth models, have the potential to enable the utilization of new physical technologies. Simultaneously, the evolution of social technology is more robust and less driven than the evolution of physical technologies. In his research, Nelson (2004) suggests that the life-cycle of an industry or a technology should be interpreted as the co-evolution of both the social and the physical.

In understanding innovation in pre-industrial times and during the Industrial Revolution, Kelly (2005) has suggested that new useful knowledge and skills were the results of social processes with limited technical knowledge and education. The existing stocks of knowledge and skills of every individual innovator are combined with the social network interaction. Successful innovation produces innovators with higher skills, and technological progress stimulates investment in the allocation of time for learning. In such a framework, copying the behaviors of others and how many others each inventor is connected to are significant rivers of change. Hence, the social network and knowledge diffusion themselves become social technologies. A similar argument is made by Mokyr (2002) in his theory of useful knowledge. Mokyr (2002) distinguishes the role of useful knowledge from the role of inventions. Useful knowledge does not have a direct commercial application, but the invention can increase productivity and profit in producing goods and services. In such an environment, collective knowledge is more important for innovation than what each individual member of the society knows.

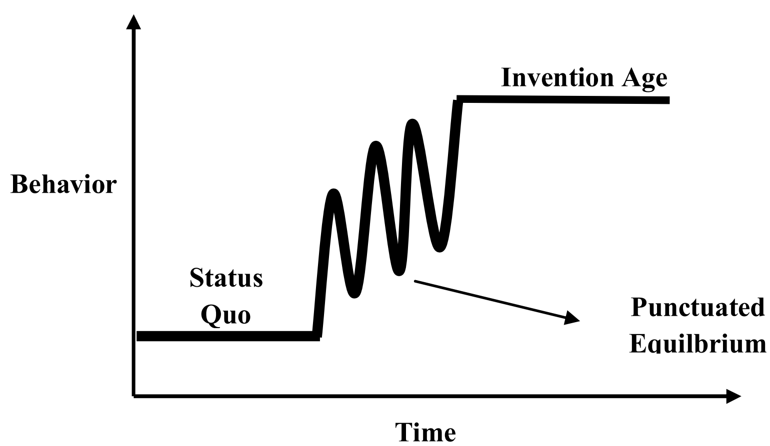

Figure 2.2: Social Inertia

Exploring the role of different biases (e.g., the status quo bias) or types of social inertia is essential for deciphering the complexity of historical process such as the Industrial Revolution as illustrated in Figure 2.2. This thesis studies exactly this type of inertia where the society is initially in a status quo of no invention even though it is optimal to spend resources into invention from a neoclassical point of view.

CHAPTER 3

THE NEOCLASSICAL BENCHMARK

This thesis first uses a simple neoclassical model to understand inventive activity where a single entrepreneur and inventor manages his/her time allocation for the firm to do research and development. This process is trying to identify how the individual optimally uses a scarce resource (time endowment) through research to increase production possibilities.

3.1. THE MODEL

Consider a set $\mathcal{E}$ of entrepreneurs: $\mathcal{E} \equiv [0, N]$ where $N > 0$ is the total measure (or population) of entrepreneurs. Let $i \in \mathcal{E}$ be an index variable for these entrepreneurs.

Suppose that each entrepreneur has access to a production technology that satisfies

$$y_i = f(X_i, m_i) = \theta X_i m_i \tag{3.1}$$

where $y_i > 0$ is output, $X_i > 0$ is productivity, and $m_i \in [0,1]$ is the fraction of time spent on management by entrepreneur i. Here, $\theta > 0$ is a fixed parameter that is common to all entrepreneurs.

Suppose that entrepreneur i has a unit endowment of time allocated to management and research.

$$m_i + r_i = 1 \tag{3.2}$$

If r_i units of time is allocated to research, entrepreneur i's productivity grows as a result of micro-inventions and is equal to

$$X_i = \exp(\beta r_i)\,\bar{X} \tag{3.3}$$

where $\bar{X} > 0$ is some given, baseline level of productivity, and $\beta > 0$ is a fixed parameter; it simply determines marginal contribution of research effort (research productivity).

Suppose that entrepreneurs maximize their output (and profit). After eliminating m_i by using (3.2), the problem of maximizing profit is written as

$$\max_{r_i} \quad \pi(r_i) = \theta \exp(\beta r_i) \bar{X}(1 - r_i) \tag{3.4}$$

Hence, the main tradeoff here is between the management time that increases the profit through labor input (tangible) and the research time that increases the profit through productivity (intangible).

3.2. OPTIMUM

This section describes the optimality properties. The solution to the above problem satisfies the first-order conditions

$$\beta\theta \exp(\beta r_i) \bar{X}(1 - r_i) + (-1)\theta \exp(\beta r_i) \bar{X} \leq 0; \tag{3.5}$$

$$r_i[\beta\theta \exp(\beta r_i) \bar{X}(1 - r_i) + (-1)\theta \exp(\beta r_i) \bar{X}] = 0 \tag{3.6}$$

$$r_i \geq 0 \tag{3.7}$$

In an interior solution ($r_i > 0$), these first-order conditions imply

$$\beta\theta \exp(\beta r_i) \bar{X}(1 - r_i) = \theta \exp(\beta r_i) \bar{X} \tag{3.8}$$

$$r_i = 1 - \frac{1}{\beta} \tag{3.9}$$

Since we have $r_i > 0$, the optimality also requires $\beta > 1$ as seen in (3.9). This condition also implies that $r_i < 1$ at the interior solution.

In a boundary (or corner) solution ($r_i = 0$), the first-order condition in (3.5) implies that

$$\beta \leq 1 \tag{3.10}$$

The optimal behavior of entrepreneurs is thus characterized by a policy function $r(\beta)$ satisfying

$$r_i = r(\beta) = \begin{cases} 0, & if\ \beta \leq 1 \\ 1 - \frac{1}{\beta}, & otherwise \end{cases} \tag{3.11}$$

This point implies that entrepreneurs in the neoclassical benchmark care only about the research productivity β. If their research effort is sufficiently productive, the marginal return of their research effort is sufficiently large. Hence, they choose a positive level of r. Besides, for increasing values of research productivity β, the optimal level of research effort is increasing. Figure 3.1 pictures this policy function.

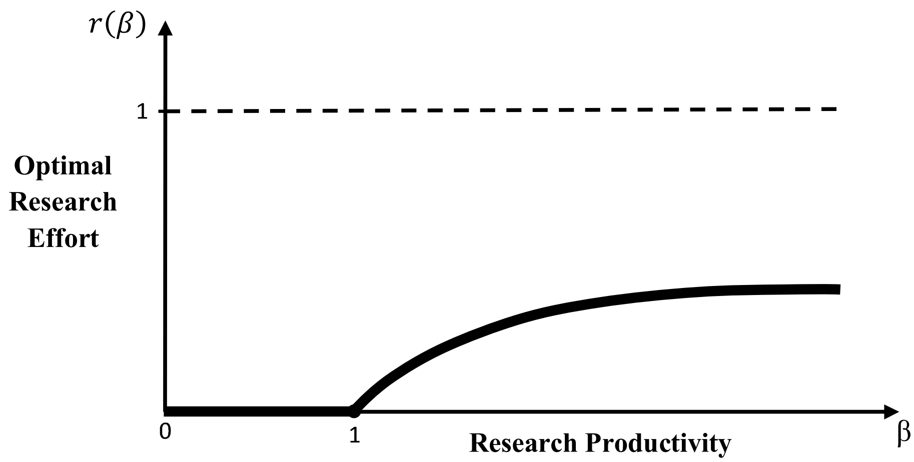

Figure 3.1: Optimal Research Time and Research Productivity

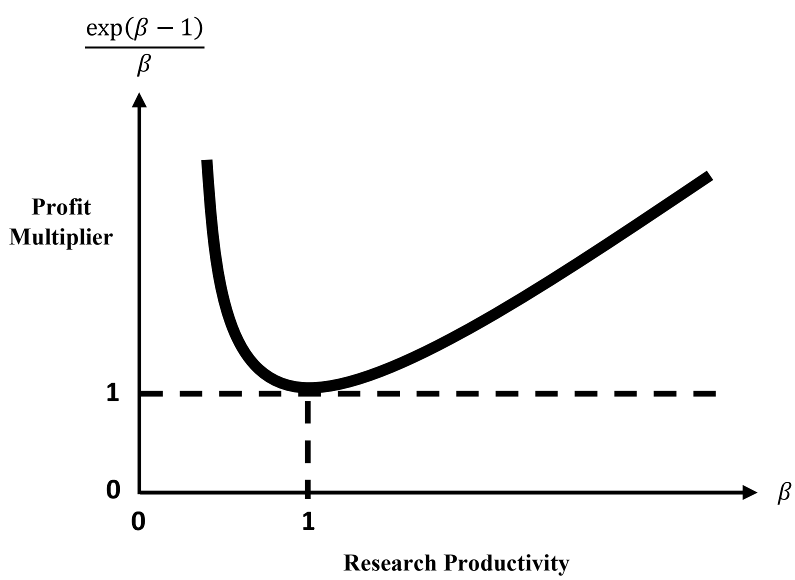

Figure 3.2: Profit Multiplier and Research Productivity

Under this optimal behavior, the optimal level π^* of profit can be written as in

$$\pi^* = \begin{cases} \theta\bar{X}, & if\ \beta \leq 1 \\ \left[\frac{\exp(\beta-1)}{\beta}\right]\theta\bar{X}, & otherwise \end{cases} \tag{3.12}$$

where $\frac{\exp(\beta-1)}{\beta}$ is greater than unity (see below). Hence, when the solution is interior with $r_i > 0$ and $\beta > 1$, optimal profit is larger than the baseline level of $\theta\bar{X}$. Put differently, entrepreneurs that spend a positive amount of time for research attain larger profit levels. Figure 3.2 pictures how this profit multiplier term changes with β.

3.3. IMPLICATIONS

What are the messages originating from this simple model of invention regarding the historical evolution of actual economies? As the answer to this question, let the historical time be denoted by $t \in \{0,1,\dots\}$. Suppose that the research productivity term is not fixed but time-dependent as in

$$\beta_t : \{0,1,\dots\} \to \Re_{++}. \tag{3.13}$$

In other words, research productivity is a function of time but is still exogenously given.

For the simple model to make sense for the economic history of today's developed economies, it must be the case that

$$\beta_0 < 1 \tag{3.14}$$

That is, in the initial period, research productivity must be sufficiently small so that entrepreneurs of the neoclassical model do not spend time on an invention. Hence, initially, the economy is in a state of technological stagnation where all production plants work with the baseline productivity

$$X_{it} = X_t = \bar{X} \tag{3.15}$$

The only way through which this economy starts inventing new technologies is that β_t grows in time. Eventually, there is a particular date $\hat{t} > 0$ such that

$$\beta_{\hat{t}} \le 1 \quad \textit{and} \quad \beta_{\hat{t}+1} > 1 \tag{3.16}$$

Hence, at time $\hat{t}+1$, research is sufficiently productive to incentivize inventive effort. Starting this date, then, X_t starts growing, and the economy industrializes.

To understand the mechanics of such a growth takeoff, assume that, at each t, the baseline level of productivity is equal to the average of the previous period's productivity levels:

$$\bar{X}_t = \frac{1}{N}\int_0^N X_{it-1} di \quad (3.17)$$

Then, after substituting the optimal level of r_{it}, the growth rate of average productivity can be described as in

$$X_{it} = \exp(\beta_t r_{it})\,\bar{X}_t = \exp(\beta_t - 1)\,\bar{X}_t \quad \Rightarrow$$
$$\bar{X}_{t+1} = \frac{1}{N}\int_0^N X_{it-1} di = \frac{1}{N}\int_0^N \exp(\beta_t - 1)\,\bar{X}_t di \quad \Rightarrow$$
$$\frac{\bar{X}_{t+1}}{\bar{X}_t} = \exp(\beta_t - 1) \quad (3.18)$$

In the literature, there exist papers that endogenize the evolution of research productivity through useful knowledge by building upon Mokyr's (2002) theory (Strulik, 2014; Attar, 2015). However, along with the purposes of this thesis, it is assumed that the evolution of β_t is exogenous; this work is interested in how the economy travels to the point where productivity $\bar{X}_t$ starts growing even if $\beta_t > 1$ but entrepreneurs are not fully-informed and rational.

More specifically, in the evolutionary model, research productivity will be fixed in all periods at $\beta_0 > 1$. The work then focuses on how a society that is initially subject to the status quo bias of "no inventive effort" can learn that invention is optimal.

CHAPTER 4

THE EVOLUTIONARY MODEL

4.1. LEARNING THROUGH THE GENETIC ALGORITHM

Once again, this thesis defines a fixed set of entrepreneurs as in the neoclassical benchmark, but the set of entrepreneurs is now discrete:

$$\mathcal{E} \equiv \{1,2,\dots,N\} \tag{4.1}$$

Hence, even though N could be large, the growth rate of average productivity cannot be simplified using the integral formula used in the previous chapter.

For clarity, the model in this section explicitly introduces time as in

$$t \in \{0,1,\dots\} \tag{4.2}$$

so that time starts at an initial period $t = 0$ and diverges to the future.

In the evolutionary model, the production and invention technologies are the same technologies introduced in the previous chapter. Hence, entrepreneur i is still subject to the problem of choosing a time allocation $r_{it} \in [0,1]$ to achieve the highest output and profit.

4.1.1. Status Quo and Mutation

From the perspective of the entrepreneur, however, it is not possible to use optimization theory to find r_{it}. It is assumed that entrepreneurs do not know the parameters of the model. More specifically, they cannot calculate how output and profit changes with r_{it}, and they do not know their research productivity β.

Instead, they use information that they achieve through observation of what earlier generations of entrepreneurs did before t. Since the purpose is to understand how an economy transits from a state of no invention ($r_{it} = 0$ for all i) to the state of positive invention effort ($r_{it} > 0$ for all i), assume that the initial generation of entrepreneurs spends all the available time to management:

$$r_{i0} = 0 \qquad \forall i \in \{1,2,\dots,N\} \tag{4.3}$$

Then, in period $t = 1$, a fraction of entrepreneurs deviates from the status quo by spending positive amounts of time to the invention. They are called "mutants" or mutated entrepreneurs, and the fraction of those deviating from the status quo is determined by the exogenous mutation rate denoted by $\mu \in (0,1)$. Hence, only in $t = 1$ and after mutation, there are two subsets of entrepreneurs:

- $\mathcal{E}_t^{\mathbf{M}}$: the set of mutated entrepreneurs
- $\mathcal{E}_t^{\mathbf{SQ}}$: the set of status quo entrepreneurs

Clearly, the number of elements in $\mathcal{E}_t^{\mathbf{M}}$ is equal to μN, and the number of status quo entrepreneurs after mutation is equal to $(1-\mu)N$ in $t = 1$.

The question is, of course, how the mutated entrepreneurs choose the fraction $r \in (0,1)$ of time that they invest in research. Since they are not endowed with the capabilities of agents in the neoclassical benchmark, they each choose a randomly drawn level according to the uniform distribution.

More specifically, suppose that

$$r_{i1} = \begin{cases} 0, & if\ i \in \mathcal{E}_t^{\mathbf{SQ}} \\ \hat{r}_{i1}, & otherwise \end{cases} \tag{4.4}$$

where $\hat{r}_{i1}$ is the realization of a random variable that follows a uniform distribution with support $[0,1]$.

This situation is explained by the help of a numerical example and a figure. Imagine an economy with $N = 10$ entrepreneurs and a mutation rate of $\mu = 0.3$. Then, three entrepreneurs are in the set of mutated entrepreneurs, and the remaining seven of them are in the set of status quo entrepreneurs after mutation in $t = 1$. More specifically, suppose that the three mutated entrepreneurs are the 2nd, 5th, and 9th ones, and the realizations of their research time drawn from the uniform distribution are $\hat{r}_{2,1} = 0.234$, $\hat{r}_{5,1} = 0.102$ and $\hat{r}_{9,1} = 0.923$, respectively. Then, the population distribution of r_{i1} *after mutation* is illustrated in Figure 4.1.

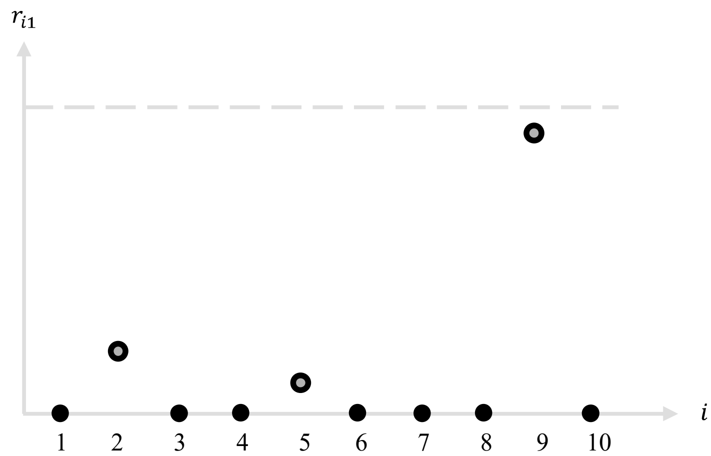

Figure 4.1: The Population Distribution after Mutation in $\boldsymbol{t = 1}$

4.1.2. Profit

As mentioned earlier, profit is the main fitness measure throughout the analysis. The output and profit levels of mutated and status quo entrepreneurs are different. For the status quo entrepreneurs who do not attempt invention, it can be inferred from the neoclassical benchmark that profit is equal to

$$\pi_i^{\mathbf{SQ}} = \theta \exp(\beta r_i)\,\bar{X}(1 - r_i) = \theta \exp(\beta \times 0)\,\bar{X}(1 - 0)$$

$$= \theta\bar{X} = \pi^{\mathbf{SQ}} \qquad (4.5)$$

since they spend their time entirely on management with the old technology (the one that corresponds to the baseline productivity $\bar{X}$).

For the mutated entrepreneurs, profit levels are generally different as determined by their differing r_i levels:

$$\pi_i^{\mathbf{M}} = \theta \exp(\beta\hat{r}_i)\,\bar{X}(1 - \hat{r}_i) \qquad (4.6)$$

Recall that, in the evolutionary model, we assume that $\beta > 1$. This model focuses the analysis on how a society that is initially subject to the status quo bias eventually converges to the neoclassical benchmark (if it does).

The relative magnitudes of $\{\pi_i^{\mathbf{M}}\}_i$ and $\pi^{\mathbf{SQ}}$ are of course important. Notice that the term governs the profit ratio between a mutated entrepreneur and a status quo entrepreneur.

$$\frac{\pi_i^{\mathbf{M}}}{\pi^{\mathbf{SQ}}} = \exp(\beta\hat{r}_i)\,(1-\hat{r}_i) \tag{4.7}$$

As in the neoclassical benchmark, this term achieves its maximum at $r^* = 1 - (1/\beta)$. Since the randomly drawn level $\hat{r}_i$ of research time can be greater or less than r^*, however, this study has

$$\pi_i^{\mathbf{M}} > \pi^{\mathbf{SQ}} \tag{4.8}$$

for some of the mutated entrepreneurs, and

$$\pi_i^{\mathbf{M}} < \pi^{\mathbf{SQ}} \tag{4.9}$$

for the other mutated entrepreneurs. As a result, it is shown that not all mutations contribute to the fitness level of the mutated entrepreneur.

From the initial period to the next, there exists a distribution of profits across entrepreneurs once the mutation is completed. First, one group of entrepreneurs have a profit level that is equal to $\pi^{\mathbf{SQ}}$. Second, one group of mutated entrepreneurs have a profit larger than $\pi^{\mathbf{SQ}}$. Finally, the third group have a profit level smaller than $\pi^{\mathbf{SQ}}$.

Returning to the numerical example of $\{N = 10, \mu = 0.3\}$, we can calculate the $\pi_i^{\mathbf{M}}/\pi^{\mathbf{SQ}}$ ratio for the three mutated entrepreneurs for a given level β of research productivity. For instance, if this model has $\beta = 2$, the $\pi_i^{\mathbf{M}}/\pi^{\mathbf{SQ}}$ ratios for the 2nd, 5th, and 9th entrepreneurs satisfy

- $\pi_{2,1}^{\mathbf{M}}/\pi^{\mathbf{SQ}} = 1.2231$
- $\pi_{5,1}^{\mathbf{M}}/\pi^{\mathbf{SQ}} = 1.1012$
- $\pi_{9,1}^{\mathbf{M}}/\pi^{\mathbf{SQ}} = 0.4878$

4.1.3. Selection: Elite Persistence and Crossover

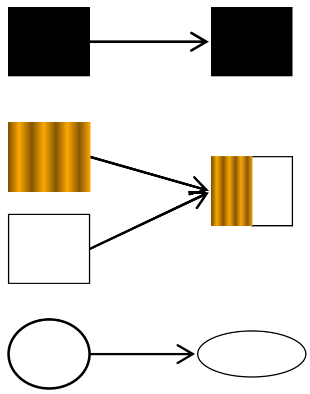

Figure 4.2: The Three Types of "Children"
(MATLAB, 2017)

At the end of any period $t \geq 1$, the genetic selection takes place. This phase occurs through two distinct mechanisms. The first mechanism is called elite persistence, and the other is a crossover.

Elite persistence determines the best performing entrepreneurs in any given period/generation and ensures that their behavior is copied exactly by the same number of entrepreneurs in the next period/generation. The number of elite entrepreneurs is an exogenous input of the model and denoted by E where it naturally has the restriction $E < N$.

Elite persistence in the numerical example works as follows: Suppose that $E = 1$. Hence, only the behavior of exactly one of the best entrepreneurs is going to be replicated in the next generation; his/her survival being ensured by elite persistence. Then this entrepreneur would be the 2nd whose profit ratio of $\pi_i^{\mathbf{M}}/\pi^{\mathbf{SQ}}$ is the largest. If instead, this model has $E = 2$, then both the 2nd and 5th entrepreneurs would be elite.

Since a fraction μ of the entrepreneurs are the mutated entrepreneurs in any given period/generation, the second mechanism called crossover determines the research effort of the remaining

$$(1 - \mu)N - E \qquad (4.10)$$

entrepreneurs in the next period/generation. In the terminology of the genetic algorithm, these agents called the crossover entrepreneurs.

The behavior of crossover entrepreneurs is determined as follows: The research effort of a crossover entrepreneur in period $t+1$ is a *convex combination* of any two randomly chosen entrepreneurs in period t. Formally, for the crossover entrepreneur i in period $t+1$, the model has

$$r_{i,t+1}^{\text{Crossover}} = \lambda \times r_{i,t}^{\text{Parent 1}} + (1-\lambda) \times r_{i,t}^{\text{Parent 2}} \quad (4.11)$$

where $r_{i,t}^{\text{Parent 1}}$ and $r_{i,t}^{\text{Parent 2}}$ are the research times of two randomly chosen entrepreneurs from the previous period/generation, and $\lambda \in (0,1)$ is the weight associated with Parent 1.

The role of the initial generation must be emphasized at this point. In the economic interpretation of such an evolutionary model, the initial generation is completely subject to the status quo bias. Hence,

$$r_{i0} = 0 \qquad \forall i \in \{1,2,\dots,N\} \quad (4.12)$$

is imposed. Then, starting in period $t=1$ and for all future generations, the new generation of entrepreneurs can be of three types:

1. Elite entrepreneurs, whose total number is E.
2. Crossover entrepreneurs whose total number is $(1-\mu)N - E$.
3. Mutated entrepreneurs whose total number is μN

Figure 4.3 pictures the mechanics of this evolution for

$$N = 10, \mu = 0.3, E = 1 \quad (4.13)$$

where the initial population is restricted to be at $r = 0$, i.e., the status quo. The green dashed line represents the optimal level r^* of the research time in the neoclassical benchmark.

In $t=0$, the figure shows that no entrepreneur has a positive level of research time. Hence, the black dot represents all 10 of the entrepreneurs. In $t=1$, the empty circles show the three mutated entrepreneurs with different positive values of research time. Notice that, since the best-performing entrepreneur in the previous period has $r=0$ by construction, the elite agent at $t=1$ is among the entrepreneurs indicated by a black dot at $r=0$. At $t=2$, the elite is represented by the red circle that exactly copies the best performing entrepreneur at $t=1$.

More specifically, a mutated entrepreneur's behavior in the period $t = 1$ is represented as the elite in period $t = 2$. Since the early periods are affected by the restriction that the initial generation is at the status quo, some of the crossover entrepreneurs are also choosing $r = 0$ at $t = 2$.

In the figure, one can see that four crossover entrepreneurs are still represented by the black dot at $r = 0$. It must also be noted that, at period $t = 2$, the best performing entrepreneur that becomes the elite is a crossover entrepreneur, not a mutated one. The figure also shows us that, for large enough t, there must not be any entrepreneur that sticks to the status quo since the genetic algorithm gradually increases the relative fitness of entrepreneurs that choose a strictly positive level r of research time.

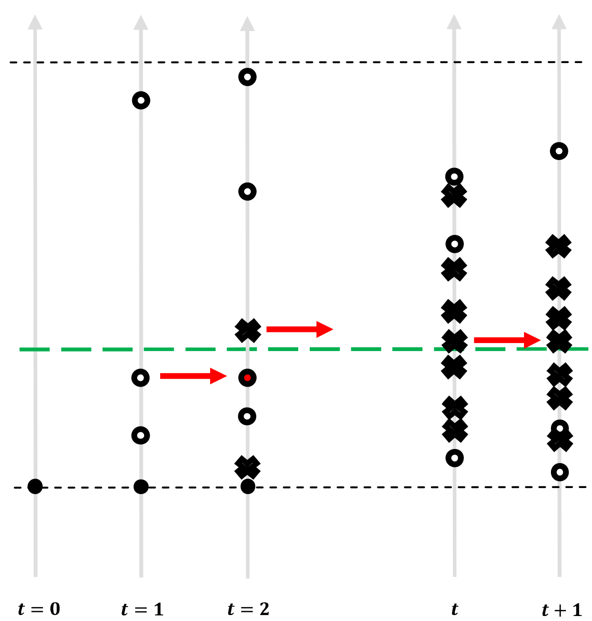

Figure 4.3: The Mechanics Evolution of Entrepreneur

The same example is represented with respect to the entrepreneur identities, as shown in the table below. Here, SQ denotes status quo, M denotes mutated entrepreneurs, E denotes elite entrepreneurs, and CO denoted crossover entrepreneurs.

Table 4.1: Entrepreneur Identities

i	*t=0*	*t=1*	*t=2*	**…**	*t*	*t+1*
1	SQ	SQ	M	…	CO	CO
2	SQ	M	E	…	CO	CO
3	SQ	SQ	SQ	…	CO	M
4	SQ	SQ	SQ	…	M	CO
5	SQ	M	CO	…	CO	E
6	SQ	SQ	M	…	E	M
7	SQ	SQ	M	…	CO	CO
8	SQ	SQ	CO	…	M	CO
9	SQ	M	SQ	…	M	M
10	SQ	SQ	SQ	…	CO	CO

4.2. IMPLICATIONS

What are the outcomes of the evolutionary model of the invention outlined above? First of all, by the very nature of evolution, a new generation of entrepreneurs can observe and copy the behavior of past generations. The critical difference between the evolutionary model and the neoclassical benchmark is that entrepreneurs in the evolutionary model are not capable of solving the profit maximization problem. Instead, they use their bounded rationality in a way to increase their profit by replicating the behavior of entrepreneurs that were successful in earlier periods.

In the example given above and in the numerical application of the model studied in the next chapter, however, this study forces the initial population to be unaware of the profit opportunities through invention. Hence, the initial generation will be subject to the status quo bias. In such a context, the only way by which the learning can start is mutation. Hence, in each period, a randomly chosen subset of the population is allowed to try random values of research time. In every generation, there are also crossover entrepreneurs whose behavior is an imperfect copy of others in the previous generation. Finally, the model allows for the society to exactly copy the behavior of best-performing entrepreneurs through elite persistence.

There are thus two important mechanisms that determine how and when society truly learns invention. First, the rate of *mutation* denoted by $\mu \in (0,1)$ determines how open the society is to the idea of deviating from the status quo. Hence, a society with a larger mutation rate is a society that is more creative because

- a larger fraction of entrepreneurs is tolerated to attempt invention in period $t = 1$
- a larger fraction of entrepreneurs is tolerated to spend diverse amounts of time on the invention in later periods.

That is, in the initial periods, the status quo is "no invention" but, in later periods where every entrepreneur spends a positive amount of time to research, mutation still works to create entrepreneurs that spend too much or too little time to the invention.

The second mechanism, *elite persistence*, allows the society to carry the best performing behavior through time to future generations. It is determined by the number $E < N$ of elite entrepreneurs: a society with a larger value of E (relative to N) is a society that transmits technological knowledge more effectively from one generation to the next.

Consequently, these two distinct mechanisms determine the speed of convergence of the genetic algorithm. This study expects that a society of entrepreneurs that is more open to deviant behavior (higher μ) and a society of entrepreneurs that is more effective in transmitting valuable information from one generation to the next (higher E relative N) should learn that invention is optimal at a faster speed. Hence, the diffusion of the "industrial revolution" among the entrepreneurs is expected to be faster in societies with higher mutation rates and stronger elite persistence.

On the other hand, it must be noted that since the genetic algorithm is subject to randomization at several levels, it is not *a priori* ensured that the evolutionary model always converges exactly to the neoclassical benchmark. Besides, the inherent randomization in the genetic algorithm implies that the relationship between the speed of convergence and the (μ, E) pair is not necessarily a monotonic relationship.

CHAPTER 5

THE EXPERIMENTS AND RESULTS

The purpose of this chapter is to collect the main results originating from the quantitative analyses of the model. There are three sections: First, the parameter values are specified. Second, the roles of mutation and elite persistence are investigated through numerical experiments. Finally, a section focuses on the role of population size.

5.1. PARAMETER VALUES

For the model to be simulated, we need to specify three parameter values that define the profit level for any given level $r_{i,t} \in (0,1)$ of time allocated to research.

Recall from the previous chapter that the profit function is equal to

$$\pi_{i,t} = \theta \exp\left(\beta r_{i,t}\right) \bar{X}\left(1 - r_{i,t}\right) \tag{5.1}$$

Hence, to calculate $\pi_{i,t}$ for a given level of $r_{i,t}$, we need to know $(\beta, \theta, \bar{X})$. As mentioned earlier, we are interested in a situation where the initial generation is subject to the status quo bias, meaning that they choose $r_{i,0} = 0$ even if $\beta > 1$. For this reason, we set a value for β that is strictly greater than unity.

The remaining parameters, $(\theta, \bar{X})$, enter the profit function as level shifters. In other words, they do not alter the curvature of the function. Besides, these parameters are common across entrepreneurs. These allow us to impose any strictly positive values to these parameters. These values do not affect our results, and we normalize each of these to unity.

Consequently, the parameter values we set for the experiments are

$$\beta = 1.5 \qquad \theta = 1 \qquad \bar{X} = 1. \tag{5.2}$$

At this value of research productivity β, the optimal level of research time is equal to

$$r^{*} = 1 - \frac{1}{\beta} = \frac{1}{3} = 0.\bar{3} \tag{5.3}$$

5.2. MUTATION AND ELITE PERSISTENCE

This section presents the results originating from several alternative simulations of the model with various mutation rates (μ) and elite counts (E).

In what follows, $r \in (0,1)$ denotes the fraction of an entrepreneur-inventor's time endowment that is allocated to research under the genetic algorithm. In other words, it is the level of $r \in (0,1)$ at which the genetic algorithm can no longer increase the fitness of the population. Hence, it is the value at which the learning is completed and the algorithm stops.

$G \in \mathbb{N}_{++}$ denotes the number of generations that is necessary to pass for the learning to be completed. In other words, G represents the duration of the transition from an equilibrium of no invention and technological stagnation to an equilibrium of continuing invention and technological progress. A higher value of G means that the society is a comparatively slow learner.

The experiments are implemented using MATLAB's built-in genetic algorithm (the script's name is "ga") with necessary function and run scripts developed for this thesis. All simulations set 250 generations as the maximum number of generations to ensure convergence; this maximum has been adjusted after a couple of runs.

In what follows, results are presented separately for each of the four different levels of population size, denoted by N. Specifically, we collect results for

- $N = 10$,
- $N = 100$,
- $N = 1{,}000$, and
- $N = 10{,}000$.

$T_{(N)}(E, \mu)$ denotes the selected table (T) based on the associated values of population size (N), elite count (E), and mutation rate (μ). These inputs of the genetic algorithm determine the resulting values of r and G.

5.2.1. $N = 10$

Table 5.1: Simulation Data of $T_{(10)}$ (E, μ)

E	9		7		3		1	
μ	r	G	r	G	r	G	r	G
0.1	0.2529	62	0.3521	100	0.3611	54	0.3549	105
0.2	0.3331	80	0.3303	88	0.3095	138	0.2529	62
0.5	0.3402	111	0.343	88	0.3386	77	0.3306	122

Table 5.1 shows the results for a society where the total population of entrepreneur-inventors is 10. Here, there is a wide variation in both r and G. In every constellation of mutation rate and elite count, the resulting r values are generally different than the optimal value of $r^* = 0.\overline{3}$. Different (E, μ) pairs imply a minimum r value of 0.2529 (a value that is about 25% lower than the optimum) and maximum r value of 0.3611 (a value that is about 8% higher than the optimum).

The minimum r value is located in $T_{(10)}(9, 0.1)$ with its value of 0.2529. This society is an example of a trade-off where the r value is less than optimal, but the $G = 62$ is notably small. Hence, this society converges to an equilibrium with continuous invention in a relatively fewer number of generations, but it does not learn to invent at the optimal value of $r^* = 0.\overline{3}$.

The maximum value of r is observed in $T_{(10)}(3, 0.1)$ with a value of 0.3611. This is a society that converges to its invention equilibrium in $G = 54$ generations. This is the lowest level of G recorded for $N = 10$ under alternative (E, μ) pairs. However, this society that converges relatively quickly does not invent at the optimum intensity.

Most notably, there is a considerable variation in G. The highest number of generations for the algorithm to stop is $G = 138$, observed at $T_{(10)}(3, 0.2)$. The r value in this society is 0.3095, being about 8% lower than the optimal value.

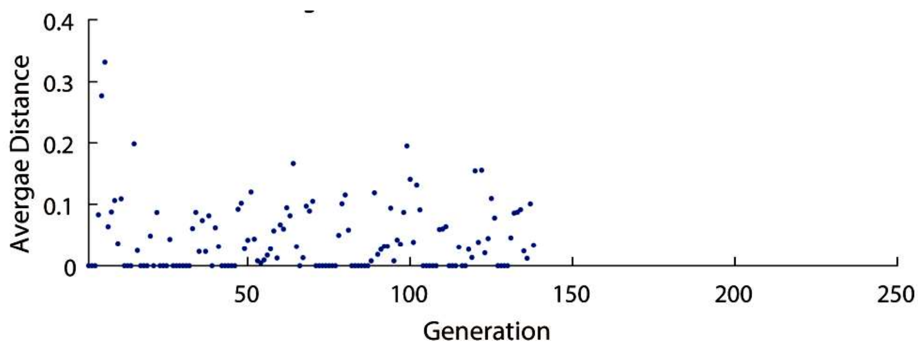

Figure 5.1: Average Distance among Individual $\boldsymbol{r}$ Values for $T_{(10)}(3, 0.2)$

Figure 5.1 shows that the society that needs 138 generations to converge exhibit a large degree of variation among individual r values. The algorithm stops at $G = 138$, but it is far from being a true convergence in the sense that all 10 of the entrepreneur-inventors start spending the same amount of time to research. When judged by the average distance among individual r values, there is no homogeneity across the entrepreneur-inventors.

5.2.2. $N = 100$

Table 5.2: Simulation Data of $T_{(100)}(E, \mu)$

E	**90**		**70**		**30**		**10**	
μ	***r***	***G***	***r***	***G***	***r***	***G***	***r***	***G***
0.01	0.3276	113	0.3552	136	0.3329	122	0.2794	59
0.05	0.3496	93	0.3374	69	0.3427	91	0.3281	56
0.1	0.3354	63	0.3357	58	0.3388	91	0.3343	88
0.2	0.3363	103	0.3331	52	0.3335	60	0.3321	67
0.5	0.3333	54	0.3317	52	0.3338	57	0.3326	68

Table 5.2 summarizes the results for $N = 100$. In terms of (i) the duration G of convergence and (ii) the final value of r, the table shows a better performance compared to $N = 10$. In this set of results with $N = 100$, only one of the r values is found to be significantly less than the $r^* = 0.\overline{3}$ optimum. As in the earlier case, there is still a large gap in the number of generations, but most of the G values here are below 100 generations.

The minimum value of r is 0.2794 at $T_{(100)}(10, 0.01)$; this is about 17% lower than the optimal value. In terms of the number of generations, this society needs $G = 59$ generations to converge to its $r > 0$ equilibrium. The required number of generations is among the smallest of those observed for $N = 100$. Hence, as in the case of $N = 10$, a society may relatively quickly converge to an equilibrium that is far from being the optimal one.

The maximum value of r is observed in $T_{(100)}(70, 0.01)$ is equal to 0.3552 (a value that is about 6% higher than the optimum). This is a society that converges to its invention equilibrium in $G = 136$ generations. Along with r value as the maximum value, this society also records the highest number of G recorded for $N = 100$ under alternative (E, μ) pairs. Hence, this society converges very slowly to reach its invention equilibrium. This maximum value of G is observed along with the lowest mutation rate (μ is 0.01), where only one person is chosen as a candidate for being the mutant for $N = 100$.

The value of r that is closest to the optimal $r^* = 0.\overline{3}$ value is recorded for $E = 70$ and $\mu = 0.2$. This society also records the minimum number of generations at $G = 52$ together with $E = 70$ and $\mu = 0.5$ pair. What is significant here is that, at some particular characterization of mutation and elite persistence, the society converges to its new equilibrium in the shortest duration and highest precision relative to the neoclassical benchmark.

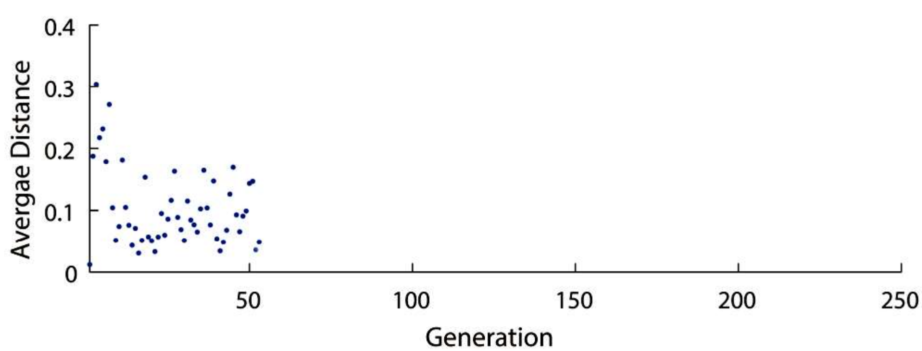

Figure 5.2: Average Distance among Individual $\boldsymbol{r}$ Values for $T_{(100)}$ (70, 0.2)

However, even under these parameter values, there is a large variation across the society in terms of r values as shown in Figure 5.2. When judged by the average distance among individual r values, there are "entrepreneur-inventor"s still trying diverse strategies in terms of inventive effort. Initially, the average distance is quite large, and it exhibits a slow decrease. However, the average distance does not converge to zero under $N = 100$.

5.2.3. $N = 1,000$

Table 5.3: Simulation Data of $T_{(1,000)}(E, \mu)$

E	900		700		300		100	
μ	r	G	r	G	r	G	r	G
0.01	0.3256	68	0.3279	57	0.3324	62	0.336	60
0.05	0.3325	61	0.332	73	0.3334	57	0.3349	59
0.1	0.3338	53	0.3323	58	0.3341	57	0.3333	51
0.2	0.3333	51	0.3336	52	0.3331	56	0.3333	52
0.5	0.3334	52	0.3331	51	0.3334	51	0.3333	51

As shown in Table 5.3, this set of results documents the simulations of a society where the total population of "entrepreneur-inventor"s is $N = 1{,}000$. Unlike in the two previous cases, there is a significant improvement in the number of generations; all the G values here are below 100 generations. This simulation indicates that this population is converging faster compared to the two previous cases. In most of the cases considered with alternative mutation rates and elite counts, the society converges much closer to the optimal value of $r^* = 0.\overline{3}$, and almost no society has a r value below $r^* = 0.\overline{3}$. Different (E, μ) pairs imply a minimum r value of 0.3256 (a value that is about 2% lower than the optimum) and a maximum r value of 0.3349 (a value that is about 0.2% higher than the optimum).

There is also an interesting finding that can be interpreted as a case of multiple equilibria. Some societies with different (E, μ) pairs converge at the same number of generations at $G = 51$. This is observed with $r = r^* = 0.\overline{3}$ in $T_{(1{,}000)}(900, 0.2)$, $T_{(1{,}000)}(100, 0.1)$, and $T_{(1{,}000)}(100, 0.5)$. In the latter two societies, the elite count is $E = 100$, but different mutation rates result in the same (r, G) pair. In other words, two societies that differ only in the rate of mutation can converge to the same equilibrium even though the transition episode can feature different dynamics in terms of average distance across the "entrepreneur-inventor"s.

The minimum r value for $N = 1{,}000$ is located in $T_{(1,000)}(900, 0.01)$ with its value of 0.3256 where the rate of mutation is at its lowest level. This society is the one that has a longer distance to the optimal level among the population, with the $G = 68$ being the second highest number of generations for $N = 1{,}000$.

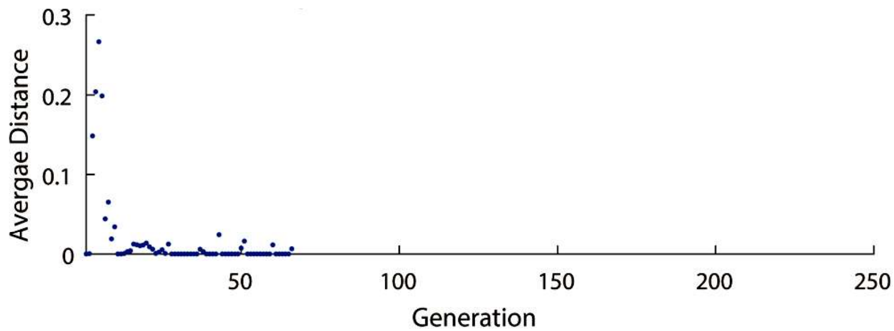

Figure 5.3: Average Distance among Individual $\boldsymbol{r}$ Values for $T_{(1,000)}$ (900, 0.01)

An interesting fact concerning the society in $T_{(1,000)}(900, 0.01)$ is illustrated in Figure 5.3 above. Most of the generations have zero average distance, meaning that individuals have the same r value. Unlike the previous simulations with lower population sizes, the society decreases the average distance very fast. This is truly a convergence to a particular norm or modern society where everybody invents by choosing an inventive effort level very close to the optimal value.

5.2.4. $N = 10{,}000$

Table 5.4: Simulation Data of $T_{(10,000)}$ (E, μ)

E	**9,000**		**7,000**		**3,000**		**1,000**	
μ	***r***	***G***	***r***	***G***	***r***	***G***	***r***	***G***
0.01	0.3332	53	0.3332	53	0.3335	52	0.3332	52
0.05	0.3332	52	0.3333	52	0.3334	51	0.3334	51
0.1	0.3334	51	0.3332	51	0.3334	52	0.3333	51
0.2	0.3333	51	0.3334	51	0.3333	51	0.3334	51
0.5	0.3333	51	0.3333	51	0.3333	51	0.3333	51

While a population of "entrepreneur-inventor"s that is equal to $N = 10{,}000$ is not historically accurate, we include the results for this case to investigate the roles of mutation and elite persistence in such a "modern" society.

Overall, the societies outlined in Table 5.4 have almost identical results in terms of r and G regardless of the elite counts and mutation rates. This result is the example of a fast learner society that relatively quickly converges to an equilibrium value of r that is closest to the neoclassical optimum. There is no longer a significant gap in G. The required number of G is among those observed for $N = 100$, either $G = 51$, $G = 52$, or $G = 53$ generations. This population has the lowest number of generations compared to all previous simulations. In 10,000 populations, the majority of society requires 51 generations to learn how to invent. All the societies in this population have r values very close to the optimal one, $r^* = 0.\overline{3}$.

Yet, again, when the society has a smaller number of people who want to overcome their status quo condition (lower rate of mutation), the creation of the modern society requires more generations than the others. Even with large values of the elite count (9,000 and 7,000 out of 10,000), a society with a very low rate of mutation typically requires a (marginally) larger number of generations to converge.

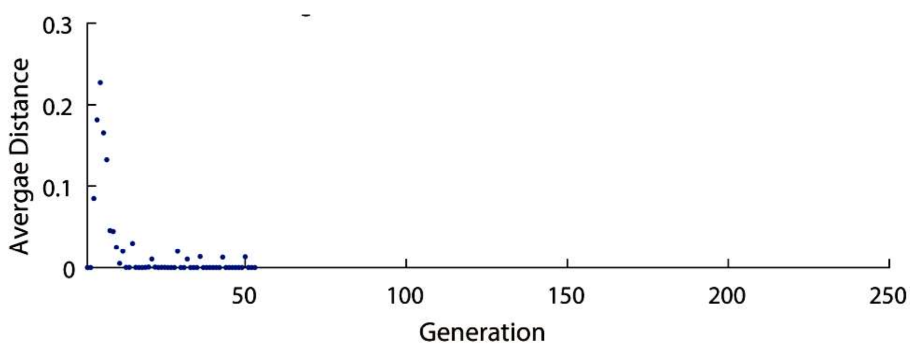

Figure 5.4: Average Distance among Individual $\boldsymbol{r}$ Values for $T_{(10,000)}$ (9000, 0.01)

Figure 5.4 illustrates that the society in $T_{(10,000)}(9000, 0.01)$ needs 53 generations to converge and this converges features a significant degree of variation among individual r values in early generations. However, after around 10 generations, the average distance decreases significantly. As in the previous set of simulations, this figure shows that most of the individuals have zero average distance at several generations.

5.3. THE ROLE OF POPULATION SIZE

This section presents the simulation results for different sizes of population (N). We have already presented above that population size has significant effects on the outcomes. Here, we focus on average values of r and G across different societies characterized by different elite counts and mutation rates. The average values of time allocation in research (r) and the number of generation (G).

Table 5.5: The Role of Population Size

N	**Avg. *r***	**Avr. *G***	**Max. *r***	**Min. *r***	**Max. *G***	**Min. *G***
10	0.307925	88.10	0.3611	0.2529	138	54
100	0.333175	77.60	0.3552	0.2794	136	52
1,000	0.332735	56.60	0.3349	0.3256	73	51
10,000	0.333255	51.45	0.3335	0.3332	53	51

Table 5.5 collects the average, minimum, and maximum r and G values observed for different population sizes. Overall, the data shows that population size is an important determinant of the model outcomes.

For population sizes larger than or equal to 100, average r values are very close to the optimal value of $r^* = 0.\overline{3}$, i.e., the neoclassical benchmark. Hence, if population is larger than a certain level, the genetic algorithm lets the society eventually reach a state of equilibrium that is highly similar to the neoclassical equilibrium.

In terms of average G values, there is a clear decreasing trend as the population size is getting larger. Importantly, even the largest population size of 10,000 requires 51 generations for the learning to be completed. Hence, the initial status quo is not a trivial situation. Any society of “entrepreneur-inventor”s needs to spend considerable amount of time to converge to a modern growth regime with continuous invention.

Another interesting finding is that the maximum number of generations sharply decreases from 136 generations in $N = 100$ to 73 generations in $N = 1{,}000$. Such an effect, however, is not observed for the minimum number of generations. Hence, while there is generally a scale effect of population, some small societies can achieve convergence in relatively smaller number of generations. As discussed above, however, such societies may not get close enough to the neoclassical optimum.

Finally, the size of population size affects the range of r values. For smaller sizes of population, there is a larger range between the maximum and minimum values of r.

What do these results tell us from an historical point of view? The total population of inventors was less than 10,000 inventors throughout the process by which the British nation realized its industrial revolution. Meisenzahl and Mokyr's (2012) inventor database includes a total of 759 British inventors. The birth and death dates of these inventors are 1660 and 1830, respectively. Hence, even if there are missing observations in the mentioned database and even if one must include all firms in the British economy to truly account for the loci of inventive activity, the true population size would be much less than 10,000.

In general, we can say that there is a Curse of Small Numbers as opposed to the Law of Large Numbers. In some mainstream growth models that are used to explain inventive activity of modern economies, e.g., Aghion and Howitt's (2009) textbook models of Schumpeterian growth, the Law of Large Numbers imply that average productivity grows in each generation even if some sectors or innovators fail. However, if the Law of Large Numbers do not apply because of the population size, the society may record lower growth rates as a result of innovation failures.

CHAPTER 6

CONCLUSION

The accumulation of useful knowledge brought the world into an inevitable outcome, an industrial revolution characterized by the initiation of continuous inventions and sustained technological progress. Inventions allowed societies to transit from an equilibrium of nearly zero growth to an equilibrium of sustained growth. For the entire history of *Homo sapiens*, this was a very recent phenomenon, covering a few centuries.

This thesis aims to understand this process of transition (the one from stagnation to growth) with a simple evolutionary model of invention. The evolutionary model used in this study is a proper scientific way to discover the associated patterns of transition to modern growth. If "entrepreneur-inventor"s in the real life are not fully informed, rational and capable of solving complex optimization problems, it takes several generations to realize that spending valuable resources to invention is the optimal response.

The evolutionary framework in this study builds on an intergenerational model where the behavior of earlier generations has an effect on the behavior of later generations. The crucial element here is that, initially, the society is in a status quo of no invention. Hence, there must be a process of learning where the behavior of "entrepreneur-inventor"s that perform better in one generation can be transmitted to the next generation through.

This thesis argues that such a learning process is mainly affected by two things: First, the rate of mutation determines what fraction of "entrepreneur-inventor"s choose a randomly drawn level of research input for invention. This is especially important because all members of the initial generation spend their entire time endowment to routine management. The rate of mutation (μ) measures a society's tolerance or openness to new technological strategies. Second, the elite count (E) determines the number of best-performing "entrepreneur-inventor"s whose behavior is exactly copied in the next generation. Hence, the elite count measures the society's effectiveness in knowledge transmission.

The analysis in this thesis focuses on the roles of mutation and elite persistence on two things: First, the duration of the learning process (G) (i.e., how many generations are necessary for the society to converge to an equilibrium with continuous invention), and, second, the level of research input chosen by the final generation (r) (i.e., what fraction of unit time endowment is spent on inventive effort by the majority of "entrepreneur-inventor"s).

For the question of whether higher mutation rates cause convergence to be faster, we find that there is no strong monotonic relationship between μ and G. For instance, in the smallest society of $N = 10$, larger G values are observed for all levels of mutation rates. On the other hand, in larger societies, we observe that maximum G values are generally recorded for relatively smaller mutation rates. Hence, in summary, we can say that the rate of mutation is not a trivial determinant of the duration of transition even though there is no strong monotonic relationship.

For the question of whether higher elite counts cause faster convergence, we see that there is no clear relationship. For a given population size and a given rate of mutation, G does not systematically change with the elite count E. There are simulations where, for a particular (μ, N) pair, both a large and a small value of E imply very similar G values. Besides, for a particular (μ, N) pair again, there are simulations where the same E value implies differing G values.

For the question of whether the economy converges to the neoclassical optimum in an exact manner or exhibits heterogeneity around the optimum value, we find that there is generally heterogeneity. Importantly, there is no clear pattern on the resulting r values that originate from different (μ, E) values. However, the size of population has an effect; for larger populations, the societies get closer to the neoclassical benchmark in terms of r values.

Finally, for the question of whether the size of population affects the number of generations that pass before the learning (growth takeoff) is completed, we see a strong scale effect: Larger populations of "entrepreneur-inventor"s converge to the neoclassical optimum in a significantly fewer number of generations in general. Specifically, a minimum number of 51 generations is necessary before the pre-industrial, no-invention society to transform itself into a modern economy with sustained technological progress. The significance of this best case can be explained as follows: First, suppose that each generation has a lifetime of 25 years, a common value in the modern growth literature. We can then associate the start of modern growth in England with the starting date of historical fertility decline as in UGT. This means the end of the transition is roughly the year 1875. Then, 51 generations taken to the past sets the initial date as the year 600 AD. If, on the other hand, we look at the worst case of 138 generations, the initial generation 1575 BC.

The model studied in this thesis focuses only on the population of "entrepreneur-inventor"s. In the model, the household side is assumed away for simplicity. In a larger model, the demographic structure could be explicitly specified where some adult agents are workers and the rest of them are "entrepreneur-inventor"s. Besides, the ownership structure in the economy and occupational choice would also be explicitly formulated. Within such an extended model, an interesting question is how to endogenize population growth. The interaction of population growth and mutation may yield interesting results for the dynamics of the growth takeoff. This is left for future research.

It must also be emphasized that the model presented here is truly an abstract model that can be applied to any major historical transition that involves creativity. There are no firms, industries, or sectors. One may imagine the transition process studied in this model as the Neolithic Revolution where inventions are simply the domestication of plants and animals. Developing a fully-fledged evolutionary model of an industrial revolution with different consumption and investment goods and a satisfactorily rich market structure is left for future research.

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