IONOLAB-RAY: A wave propagation algorithm for anisotropic and inhomogeneous ionosphere

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Abstract: A high-frequency (HF) radio wave propagation model based on ray tracing for anisotropic, inhomogeneous, and time-varying ionosphere is developed to form the basis for a user-friendly modular algorithm. The algorithm IONOLAB-RAY applies the principles of ray tracing through the spherical 3D grid model of the ionosphere. The physical parameters of the ionosphere in each voxel are obtained from the ionosphere model IRI-Plas-G. The refractive index of each voxel is calculated with the Appleton–Hartree formula, which includes electron cyclotron frequency, collision frequency, and anisotropicity due to the geomagnetic field. The developed propagation algorithm allows the input of total electron content (TEC) values to update the ionospheric model to the current conditions. This capability has an important impact on the characterization of the ionosphere under geomagnetic storm conditions. The algorithm can be further developed to represent typical HF transmitter and receiver antenna properties and to calculate the electric field strength, time delay, and polarization rotation at the receiver location.

Key words: High-frequency propagation, ionosphere, ray tracing, total electron content

1. Introduction

High-frequency (HF) radio wave propagation is an important phenomenon in long-range communication through the ionosphere. The ionosphere is the region of the atmosphere that is ionized, primarily by solar radiation. It is an anisotropic, inhomogeneous, time- and space-varying and spatiotemporally dispersive medium [1].

The complex structure of the ionosphere has been investigated since the 1930s. An early and simple electron density profile model was developed by Chapman [2].

It is observed that the most critical parameter in HF wave propagation is the refractive index. The most commonly used model of the refractive index is known as the Appleton–Hartree formula [1,3]. This formula is a very important contribution, because it represents the physical phenomena existing in the structure of the ionosphere.

In the late 1960s, a project was started with the goal of producing an empirical model of the ionosphere. As a result of this project, the International Reference Ionosphere (IRI) software tool was developed and then updated in further studies [4–6]. IRI, based on available and reliable observations, calculates hourly and monthly medians of electron density, ion composition, ion and electron temperatures, and total electron content (TEC) of the ionosphere for time and position at an altitude ranging from 60 to 1500 km [4–6].

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IRI extended to the plasmasphere (IRI-Plas) is a more recently developed version that covers the plasmasphere region up to the Global Positioning System (GPS) orbital height of 20,200 km [7]. IRI-Plas is the international standard in ionosphere modeling as given in [8]. IRI-Plas also enables inputting TEC to update and scale the ionospheric coefficients in the computation of desired outputs.

IRI-Plas-G is a modified version of IRI-Plas, where the user can estimate the ionosphere parameters within the user-defined region, date, time, and height of any chosen resolution, whereas in IRI and IRI-Plas, the height resolution is predetermined. Global Ionospheric Map-TEC (GIM-TEC) data can be assimilated as input [9,10].

Due to the complexity of the ionosphere, it is challenging to solve electromagnetic wave equations to calculate the propagation path of the wave. Ray tracing, which is a geometrical optics approximation, can be applied, since the dimension of the irregularities in the ionosphere is generally larger than the wavelength of the HF wave. When this condition is applied to a wave equation, the eikonal equation is derived [1]. To solve the eikonal equation for the ionosphere, the Haselgrove equation set is generated, which is composed of six differential equations [11,12]. The Haselgrove equation set remains hard to solve without any further simplification [1]. To solve this set with assumptions, Jones and Stephenson developed a code, which is a widely used algorithm [13,14]. Several applications have been developed based on this code with limited ionosphere models [15–17].

Propagation in the inhomogeneous ionosphere is approached using the Wentzel–Kramers–Brillouin (WKB) method, as discussed in various studies, including but not limited to [1,18]. The WKB method is applied only in the vertical direction and it is not sufficient to represent the refraction and reflection of a ray due to the inhomogeneity in the azimuth of the ionosphere.

Another approach is to model the ionosphere as a stratified medium with homogeneous and isotropic layers. For this case, Snell's law is generally applied to solve ray tracing [19].

Some of the analytical ray tracing applications were given in [20,21]. The ray path equations are solved analytically for the ionosphere. In these studies, the geomagnetic field is not taken into account, and the refractive index is calculated only with respect to plasma frequency, which is obtained with a fitting function. Anisotropicity is not represented.

Numerical methods are applied to solve electromagnetic wave equations in the ionosphere. In [22,23], the ionosphere, being anisotropic magnetized plasma, is divided into a global longitudinal and latitudinal mesh, and electric and magnetic field propagation is calculated with the finite difference time domain (FDTD) method. Run time and memory are critical for the applicability of such numerical solution methods.

In [24], the IONORT software is developed to calculate HF wave propagation in the ionosphere. Ray tracing in the 3D geocentric spherical coordinates approach is based on the Haselgrove equation set. This set is applied by using the FORTRAN executable of the Johns and Stephenson code, which limits the modification of the algorithm and extends the run time. The software does not include an integrated ionosphere modeling tool, but it offers an analytical Chapman model. The formula and computation of the refractive index is not provided in an explicit manner. Graphical output options are limited to screen plots. In [25], the parameters of the ionosphere are entered for a specific region of Italy and Greece and are compared with the measurement data of the same region.

The study in [26] calculates the HF channel characteristics with a 3D ray tracing model for the extended region of China. The electron density distribution is used to calculate the refractive index by using the Appleton–

Hartree formula. A geomagnetic field is calculated to represent the anisotropicity of the ionosphere, so that both ordinary and extraordinary wave propagations are modeled. In this study, the IRI model, which provides ionosphere parameters for monthly median, is modified for the extended region of China. Therefore, the algorithm is not applicable to other distributed regions.

It can be deduced from the above studies in the literature that as the complexity of the wave propagation computation increases, the ionosphere model needs to become simpler to make it possible to perform the calculations with a feasible computational load and runtime. Longer run times may not be acceptable, because the ionosphere is a time-varying medium. Its structure may dramatically change during long run times [27–29].

The Institute for Telecommunication Sciences (ITS) has developed a software called the Ionospheric Communication Analysis and Prediction Program (IONCAP) to predict the parameters of the ionosphere used in HF radio planning and communication. The Ionospheric Conductivity and Electron Density (ICED) profile model was integrated into IONCAP, and the Ionospheric Communications Enhanced Profile Analysis and Circuit Prediction (ICEPAC) program was developed. The ICEPAC software does not calculate ray propagation, but it predicts some parameters of the ionosphere, using hourly-monthly medians with preset formulas that do not represent the variability in the ionosphere due to disturbances or geomagnetic storms. In [30], the coverage area is predicted using IONCAP and ICEPAC due to a transmitter operating at the HF band.

The advantages of IONOLAB-RAY with respect to existing models are listed as follows:

- Both horizontal and vertical inhomogeneity of the ionosphere is modeled with a spherical 3D voxel structure. The voxel structure is defined for a region of interest in longitude, latitude, and height, where resolution can be defined by the user.

- The anisotropicity of the ionosphere is represented by calculation of the refractive index using the Appleton–Hartree formula with all its components, including geomagnetic field [1,3]. Both ordinary and extraordinary wave propagation, which arise due to anisotropicity, are calculated. The geomagnetic field is calculated with IGRF [31].

- Time dependency of the ionosphere is represented in the algorithm, since the parameters of the ionosphere are calculated with respect to any given date and time.

- The parameters of the ionosphere are calculated with the IRI-Plas-G software tool, which is already embedded in the algorithm as a module. GIM-TEC data assimilation is provided using the algorithm to update the ionosphere to its current state. Advanced interpolation techniques are applied to increase the resolution [32,33].

- In this algorithm, the refractive index is calculated using the Appleton–Hartree formula with plasma frequency, geomagnetic field, collision frequency, and cyclotron frequency components. The needed parameters of the ionosphere to calculate these components are density, mass, and temperatures of electrons, ions, and neutral molecules. These parameters are calculated automatically with embedded modules. Therefore, the user can perform any scenario without any input data obtained by other tools.

The IONOLAB-RAY algorithm calculates wave propagation with ray tracing with the geometrical optics approach. Snell's law is applied to reduce the computational complexity of the algorithm. The base for the IONOLAB-RAY algorithm was formed in [34]. An improvement to the algorithm is performed on the method

of application of Snell's law. It is applied both to the radial and lateral surfaces of the voxels, which makes the algorithm better represent the inhomogeneity of the ionosphere.

In its present state, the IONOLAB-RAY algorithm is a unique and novel tool to investigate parameters of HF wave propagation with respect to all possible input parameters.

2. Ionosphere model in IONOLAB-RAY algorithm

The ionosphere is an inhomogeneous medium; thus, the values of its physical parameters and refractive index change with respect to position. To represent this, the ionosphere is modeled as a combination of a 3D spherical grid, made up of voxels that are in the shape of spherical sections between constant radius surfaces, as shown in Figure 1. In the IONOLAB-RAY algorithm, the user can specify the resolution of the grid. In the region of interest, the size of voxels can vary with respect to the trend of inhomogeneity.
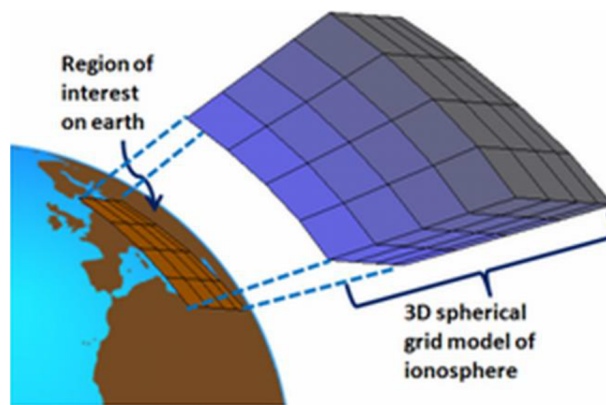

Figure 1. The 3D spherical grid model of the ionosphere.

All parameters that are necessary for the computation of the refractive index are calculated for each voxel for a given date and time using IRI-Plas-G. Hence, the inhomogeneity of the ionosphere is represented in the algorithm, both in vertical and horizontal directions.

TEC is the most critical parameter for representing the ionosphere. TEC is a measure of the total number of electrons along a path of the radio wave and it can be used to investigate the ionospheric variability. Dual frequency GPS receivers worldwide are used for estimating TEC [35,36]. Since GPS-TEC can be estimated only for a limited and mostly sparsely distributed number of receiving stations, a suitable interpolation both in space and time representing the nature of the ionosphere is necessary. Th global ionospheric map (GIM) is the hourly spatial interpolation map of TEC, with a resolution of $2.5°$ in latitude and $5°$ in longitude. Maps of ionospheric parameters and electron density profiles can be obtained with assimilation of GIM-TEC into IRI-Plas-G. GIM-TEC data can be interpolated for the desired grid intervals [32,33]. GIM-TEC data assimilation into IONOLAB-RAY enables a more realistic representation of the ionosphere, especially for the times when the ionosphere deviates from the generalized models, such as during geomagnetic storms.

The ionosphere is an anisotropic medium due to the geomagnetic field. In IONOLAB-RAY, the magnitude and the direction of the geomagnetic field are calculated by the IGRF module [31].

3. Ray tracing in IONOLAB-RAY

In this study, Snell's law is applied as a simplified solution of the eikonal equation [1]. Snell's law is a fast and easy approach to solve the ray tracing problem step by step through the 3D grid structure of the ionosphere,

which includes the variation of all critical parameters using the appropriate model. The refractive index in Snell's law can include information about the physical structure of the ionosphere through the application of the Appleton–Hartree formula, and, at the same time, a short run time in the numerical solution is guaranteed. The application of Snell's law is enforced by the assumption of the spatial scale of the inhomogeneity being larger than the voxel dimensions. A generic representation of voxels as components of the 3D spherical grid model and ray tracing through the voxels are shown in Figure 2 to describe the methodology of the application of Snell's law in the IONOLAB-RAY algorithm.
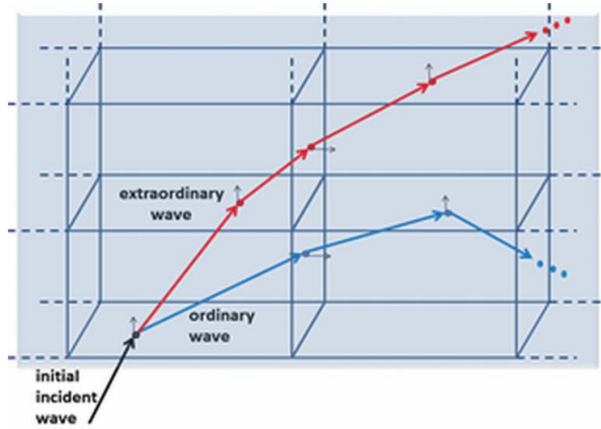

Figure 2. Ray tracing through the voxels.

The refractive index in the ionosphere can be calculated with the Appleton–Hartree formula, which is the most detailed and widely used representation [1,20]. The relation is given in Eq. (1). It can be seen that most of the physical phenomena of the ionosphere are represented in Eq. (1), where the square of the refractive index, n, is given by:

$$n^2 = 1 - \frac{X}{1 - jZ - \frac{Y^2(\sin\theta)^2}{2(1-X-jZ)} \pm \left[\frac{Y^4(\sin\theta)^4}{4(1-X-jZ)^2} + Y^2(\cos\theta)^2\right]^{\frac{1}{2}}} \tag{1}$$

In the above equation,

$$X = N_e e^2/\varepsilon_0 m\omega^2 = f_N^2/f^2$$

$$Y = eB/m\omega = f_h/f \tag{2}$$

$$Z = f_v/f$$

where n^2 is the square of the refractive index, N_e is electron density, f_N is plasma frequency, f is operation frequency, B is magnitude of geomagnetic field, θ is angle between the propagation vector and the direction of the geomagnetic field, f_h is cyclotron frequency, and f_v is collision frequency. e is electron charge, m is electron mass, and ε_0 is free space dielectric constant. The incident wave that enters anisotropic media is divided into two waves, referred to as ordinary and extraordinary waves. The plus sign of the operator '$\pm$' denotes the refractive index for ordinary waves and the minus sign is used for extraordinary waves [1].

4. IONOLAB-RAY algorithm

The algorithm developed in this study, namely IONOLAB-RAY, provides a simulation of wave propagation in a 3D spherical grid model of the ionosphere in the HF band for both ordinary and extraordinary ray components. The algorithm is made up of a preprocessor and main processor. A flowchart of the algorithm is given in Figure 3.
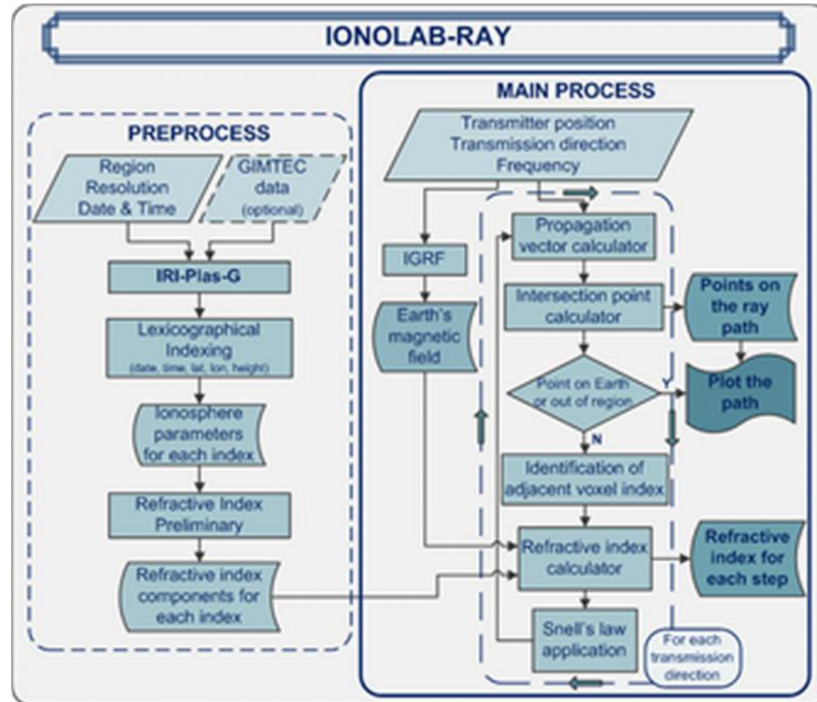

Figure 3. Flowchart of the IONOLAB-RAY algorithm.

In the preprocessor, the user defines the region of interest on the earth by providing the latitude and longitude of the boundary corners. The spatial resolution of the voxels in the ionosphere, date(s), and time(s) are also defined by the user as an input to the algorithm. The user can choose the automatic inclusion of GIM-TEC. For each voxel, date, and time, IRI-Plas-G runs with or without GIM-TEC data. All critical parameters calculated in preprocessing are saved in a lexicographical order (LO). Electron density is the output of IRI-Plas-G [4]. Collision frequency depends on the densities and temperatures of ion and neutral molecules. These parameters are calculated using the NRLMSISE-00 model. Cyclotron frequency is calculated using the magnitude of the geomagnetic field, which is obtained from the IGRF module [31].

In the main processor, the user defines the position of the transmitter, azimuth, and elevation direction of the transmitted wave, operation frequency of the transmitter, and operation date and time as input. In the initial step of the algorithm, the geomagnetic field is calculated at the position of the transmitter. The wave propagates as a ray in the given direction, with the assumption of free space, and reaches the ionosphere. The intersection point between the ray path and the spherical ionospheric boundary layer is calculated. Then the ray splits into ordinary and extraordinary waves. Snell's law is applied and ray propagation for both ordinary and extraordinary waves is calculated voxel by voxel. For each voxel through which the ray will propagate, the angle between the geomagnetic field direction and the incident ray propagation direction is calculated. Then the refractive index is calculated with parameter values stored in the preprocessing phase. The ray will either

exit the ionosphere or will be reflected towards the earth. If the ray is reflected back to the earth, it will be propagating under free space conditions after the exit from the ionospheric boundary surface.

It takes less than 15 s to generate the data set for the 3D spherical grid structure with 10° latitude, 10° longitude, and 90 height steps for a given date and time. The main process runs for approximately 1 s to simulate both ray paths of the ordinary and extraordinary waves for a single scenario, depending on the length of the path, on a PC with processor Intel Core i5-2400 CPU @ 3.10 GHz, and 4 GB of memory.

5. Results

Preliminary analyses are performed with respect to the variation of frequency, elevation angle, and azimuth angle of the transmitted wave. It is observed that the results are compatible with the studies in the literature and physical expectations. These analyses are excluded so as not to increase the length of the paper. Instead, more detailed analyses are given through example scenarios to demonstrate the versatility and reliability of the IONOLAB-RAY algorithm.

In the following example simulations, the resolution of the grid structure is chosen as 1° longitude, 1° latitude, and 10 km in height, which are decided to be optimum for the purposes of these analyses considering the resolution of IRI-Plas-G. Plots of ray paths are given in the local ENU coordinate system with the origin located at the transmitter position.

5.1. Ordinary and extraordinary waves with respect to location, date, and time

One of the unique features of IONOLAB-RAY is the tracing of ordinary and extraordinary components of the propagating wave. An example of the ray paths is provided in Figure 4, where ordinary and extraordinary wave ray paths are obtained for subarctic, midlatitude, and equatorial regions, on 21 June 2011 and 21 December 2011, at midday and midnight at 6 MHz. St Petersburg, Ankara, and Kampala are selected as transmitter locations in the examples given in Figure 4 to demonstrate the latitudinal dependence of the ionosphere. These locations are at high latitude, midlatitude, and equatorial region, respectively. Transmitter elevation is 45° and the azimuth is towards the east.

Figure 4a is the output for St Petersburg, which is in a high latitude region, located at 30.30°E and 59.93°N. The ray paths do not reflect back for the winter ionosphere for midnight hours. For 21 December, midday time, both rays reflect back to the earth. For the summer solstice time of 21 June, both rays reflect at midday and night time; however, during the day the ionosphere is highly ionized and the layer of reflection for 6 MHz is at a lower height compared to midnight. As a result, midday propagation reaches closer ranges, whereas night propagation travels longer distances. In Figure 4b, the propagation paths for Ankara, which is in a midlatitude region located at 32.87°E and 39.93°N, are provided. The ordinary and extraordinary ray paths arrive almost at the same location with very similar traces for summer solstice midday time. During night hours, the propagation paths differ and the ordinary ray path travels longer in the ionosphere, although the reflection height for both paths is very similar. For winter solstice propagation, during the night hours, both rays transmit through the ionosphere due to the lack of sufficient ionization. During the midday hours, the extraordinary rays travel longer in the ionosphere. In Figure 4c, propagation paths for Kampala are provided for summer and winter solstice midday and midnight hours. Uganda is in the equatorial region, located at 0.31°E and 32.60°N. Due to higher ionization levels in equatorial regions, rays reflect back to the earth's surface. The interesting feature of ray propagation for this region is that the ray paths are dominated by the diurnal cycle more than seasonal variations. Ray paths did not differ much between summer or winter solstices, and arrival positions were similar for midday/midnight hours for both seasons.
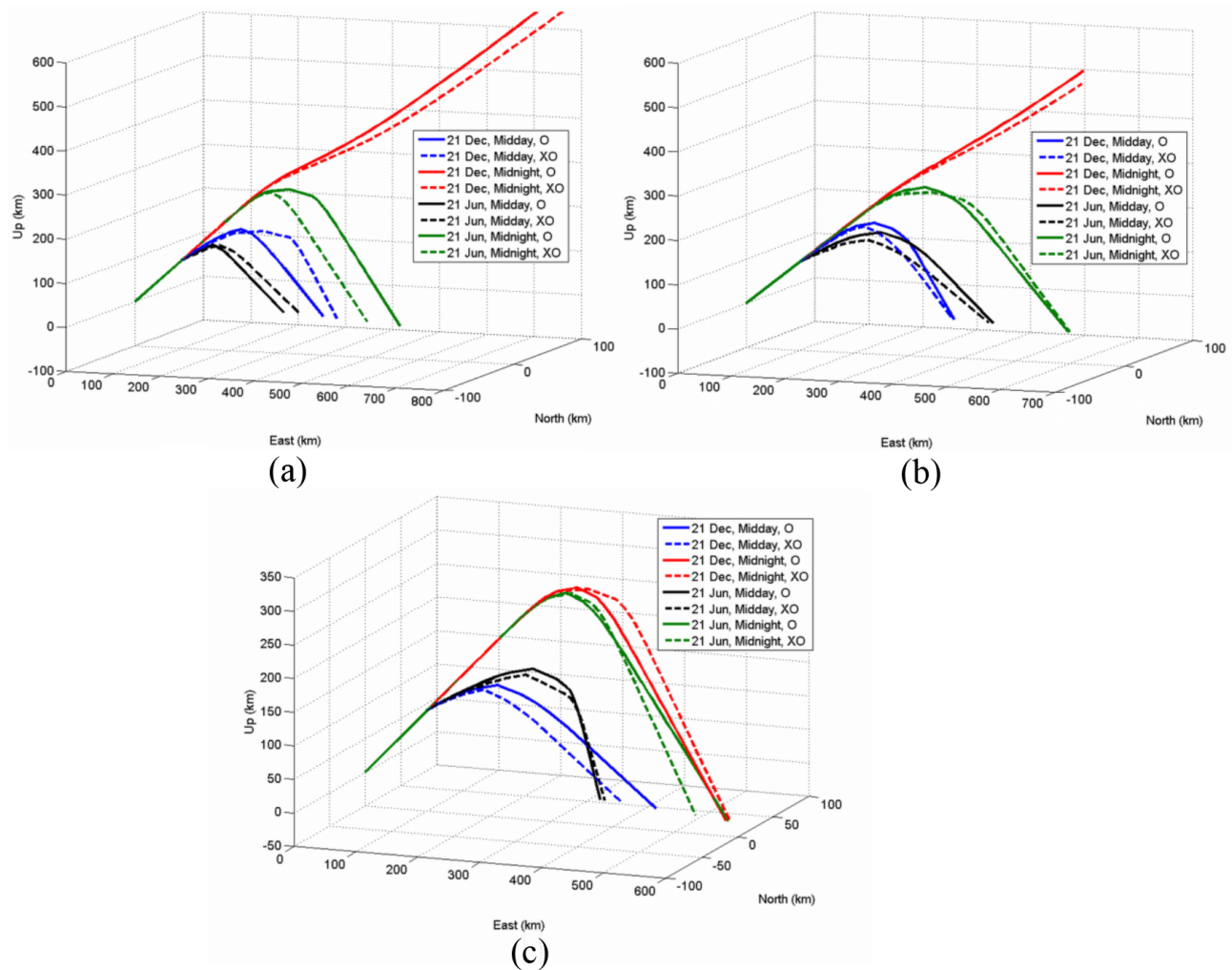
Figure 4. Ordinary (O) and extraordinary (XO) waves with respect to the variation of transmitter location and time: (a) St Petersburg; (b) Ankara; (c) Kampala.

5.2. Effect of GIM-TEC data assimilation

The most important aspect of IONOLAB-RAY is the assimilation of GIM-TEC to update the ionosphere model. An example comparison between the ray paths obtained with and without GIM-TEC data is provided for quiet and storm days in Figure 5.

25 March 2011 is a quiet date, and the wave propagation is provided in Figure 5a, while 6 August 2011 is a day of geomagnetic storm, whose wave propagation paths are shown in Figure 5b. The propagation path is modeled for a transmitter location in Ankara, Turkey (32.87° E, 39.93° N), at 12:00 UT, and transmission is towards the east with 30° elevation. The meshed surface plot represents the earth's surface in the region of interest.

For both subfigures, the ray paths are calculated for the transmitted wave towards the east with a 30° elevation angle and at 9 MHz, where the transmitter is located in Ankara. The propagation was evaluated with (squares in red) and without (circles in blue) GIM-TEC. For the operating frequency of 9 MHz, GIM-TEC assimilation improves the reflection height of the ionosphere, although the arrival positions are similar for a
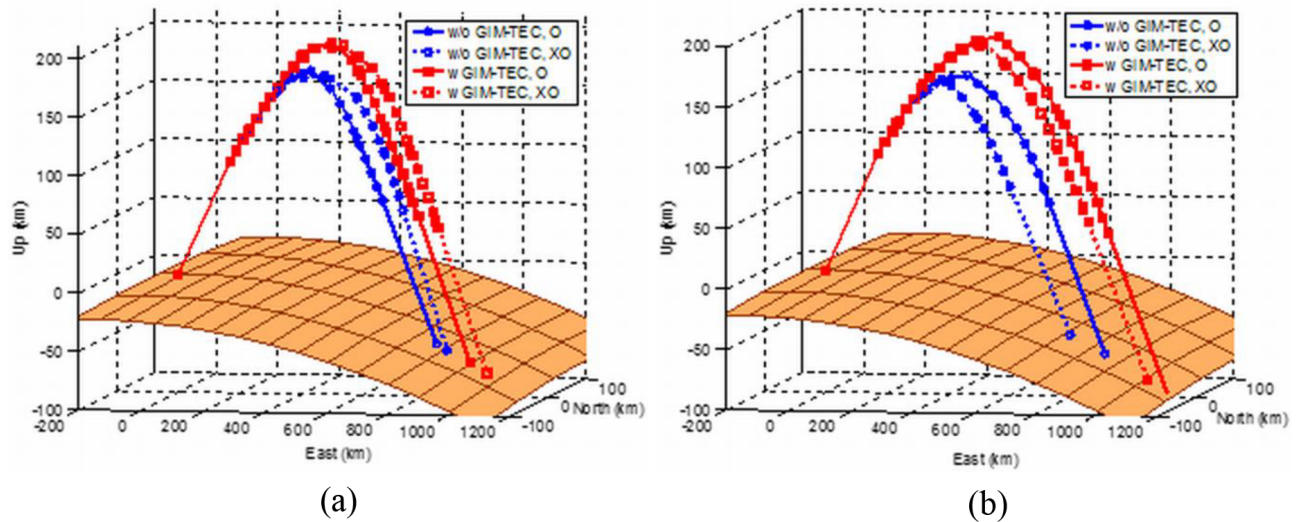
Figure 5. Effect of GIM-TEC data assimilation at Ankara at 12:00 UT: (a) 25 March 2011 quiet day; (b) 06 August 2011 stormy day.

quiet day for both ordinary and extraordinary modes. However, significant differences in reflection heights, propagation paths, and arrival positions of ordinary and extraordinary waves are observed on the stormy day, which cannot be reflected by a model of the ionosphere.

For the quiet day scenario given in this subsection, the differences between the arrival points of paths calculated with and without GIM-TEC data for ordinary and extraordinary waves are 36 and 56 km, respectively. For the stormy day scenario given in this subsection, the ranges between the arrival points calculated with and without GIM-TEC data for ordinary and extraordinary waves are 113 and 63 km, respectively. When quiet and stormy condition scenarios are compared, the differences of the arrival points calculated with storm scenarios are twice the results obtained with quiet scenarios. Amounts of arrival point errors may be tolerated by the HF receivers for quiet scenarios, but GIM-TEC data assimilation is critical, especially for stormy scenarios.

5.3. Comparison with IONORT software

IONORT is a software tool that calculates the HF ray tracing in the ionospheric medium [24]. Ray path results of the IONORT and IONOLAB-RAY are compared. Although these tools have similar purposes, they have major differences; one difference between IONORT and IONOLAB-RAY concerns their ray tracing techniques. IONORT solves Haselgrove equations using an executable FORTRAN code, which is derived from Jones and Stephenson's code [14]. IONOLAB-RAY applies Snell's law using the full version of the Appleton–Hartree formula to represent the physical properties of the ionosphere. Another difference is in the characterization of the ionosphere. In IONORT, the ionosphere is characterized through input text files with electron density specification, which should be prepared beforehand. In IONOLAB-RAY, the ionosphere can be characterized automatically with a detailed set of parameters using IRI-Plas-G. One of the example files of IONORT is selected and the ionosphere is characterized by IONOLAB-RAY for the same date, time, and region of interest. The selected example input file is for 21 October 2010, 12:00 UT, $31°$ to $49°$ latitude, and $-9°$ to $39°$ longitude region of interest. The user interface of IONORT gives the ray paths on a 3D earth image and projects ray paths in 2D. The ray paths of the ordinary and extraordinary waves transmitted from $41°$N, $12°$E, with $45°$ elevation, $0°$ azimuth angles, and with 9 MHz under the defined date time and region of interest are obtained by using

IONORT and IONOLAB-RAY, compared in Figure 6. The outputs are similar in behavior and arrival positions for ordinary and extraordinary waves. Slight differences exist regarding the differences of the algorithms and the representation of the ionosphere. IONORT represents the ionosphere with data of electron density given in a text file, and IONOLAB-RAY simulates the ionosphere using IRI-Plas-G; hence, the electron density profiles are different. Even the ionosphere parameters and ray tracing algorithms are different, but both IONORT and IONOLAB-RAY give consistent results.
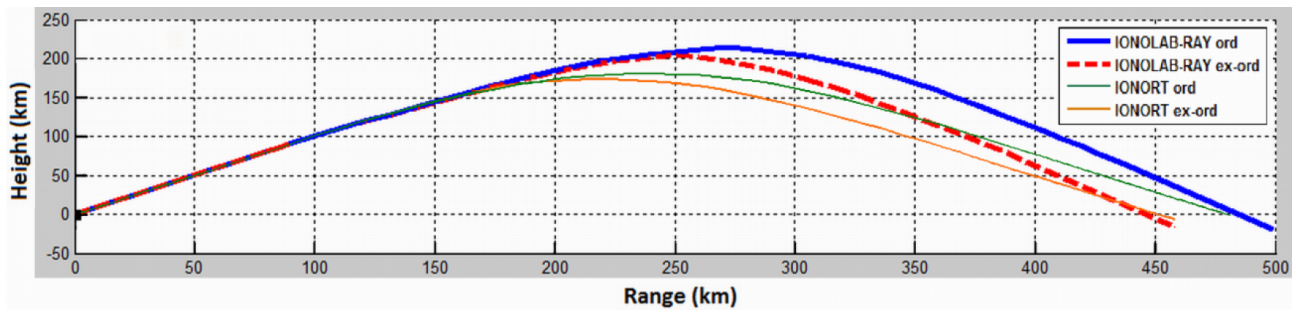

Figure 6. Comparison of IONORT and IONOLAB-RAY paths.

6. Conclusion

Due to the increasing importance of communication, direction finding, positioning, and guidance for both HF and satellite-based systems, the ionosphere has become a major challenge in wave propagation. In this study, a HF wave propagation algorithm, namely IONOLAB-RAY, is developed, which represents the physical complexity of the ionosphere. IONOLAB-RAY has a unique advantage of assimilation of GIM-TEC, which brings an important update for ionospheric conditions.

The IONOLAB-RAY algorithm is designed to accommodate the comparison of wave propagation for a set of dates, propagation times, frequencies, and/or specific transmission angles. The algorithm is computationally fast and the outputs can be presented as a graphical representation of ray paths and arrival positions.

In its current state, the algorithm provides a time-effective and computationally efficient tool that can be further developed to accommodate HF communication and direction-finding scenarios. A natural extension of the IONOLAB-RAY algorithm will be the comparison of the propagation model and outputs with measurements of ionosondes. With the input of GIM-TEC, the analysis will provide an important insight in developing a more realistic model of the ionosphere and interpreting risks during geomagnetic storms. Finally, the models in IONOLAB-RAY can also serve for satellite communication.

Acknowledgments

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