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An asymptotic investigation into the stabilization effect of suction in the rotating-disk boundary layer flow

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Abstract

The suction effects in the three-dimensional boundary layer flow due to a rotating disk are analyzed from the linear stability point of view making use of the asymptotic structure of the suction mean velocity profiles of the boundary layer. The primary interest of the current work is in giving an explanation to the well-known stabilization influence of the suction from an easy to implement asymptotic means. As a consequence of the analysis, the shapes of the linear amplitude functions are derived analytically. There also results a dispersion relation for the eigenvalues existing in the limit of large suction. A comparison is then made between the perturbations obtained from the present work and also from the direct numerical solution of the linear stability equations. The asymptotic approach pursued provides a good indication as to why the large suction in the specific three-dimensional boundary layer should act in favor of the stabilization of the flow by strongly damping the external disturbances received into the suction boundary layer.

Keywords: Rotating-disk flow; Boundary layer suction; Linear stability

1. Introduction

The flow of fluids over boundaries of a rotating disk has many applications in practice such as in boundary layer control and separation. It is widely known that one of the mechanisms of stabilization of the fluid flows is the suction applied through the surface. In fluid dynamics, suction has relevance to many technological applications, e.g., rotating machinery, ships and submarines with a particular significance for the laminar turbulent control of the aircraft wings by delaying the separation. Due to its large usage, boundary layer suction thus has been investigated extensively in the literature on the physical problems mentioned. The present study focuses specifically on the incompressible three-dimensional boundary layer flow over a rotating disk due to [1,2], and the possible influences of strong suction on the existing instability mechanisms are analyzed within an asymptotic approach.

A control approach that is effective in reducing the instability is the flow manipulation through the application of suction. There is an abundance of research in the literature investigating the effects of suction on different flow media; see for instance, the numerical simulations of the development of boundary layer disturbances for a number of different incompressible flow configurations by [3], the optimum suction distribution which gives the longest laminar region

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for a given total suction by [4], the stability of an incompressible swept attachment-line boundary layer flow by [5], the receptivity of a three-dimensional boundary layer flow on a flat surface to a large-scale low frequency periodic external forcing by [6], the optimal steady suction distribution for suppressing convectively unstable disturbances in growing boundary layers on infinite swept wings by [7], the analysis of laminar, two-dimensional flow past heated or cooled bodies with porous walls by [8,9], hydromagnetic two-dimensional flow and heat transfer in an electrically conducting and heat-generating fluid by [10], the laminar boundary layer flow over a porous flat plate by [11] and heat transfer in a porous medium over a stretching surface by [12].

Much work has also been devoted to stability research on the three-dimensional Von Kármán rotating-disk boundary layer flow by means of analytical, numerical and experimental investigations. In the absence of suction, studies of this kind up to now from the linear hydrodynamic stability point of view have been able to explain that besides the well-known convective instability, which may manifest itself in the form of a viscous lower branch or inviscid upper branch, as studied comprehensively in [13–15], the absolute instability of the crossflow vortices may also be a route to transition to turbulence, as investigated in [16–19]. A more rigorous approach based on a large Reynolds number assumption and called the triple-deck theory, see for instance [20–23], was also made use for the stability investigations of rotating-disk boundary layer flow. Ref. [24] used this asymptotic theory first and obtained analytical expressions both for the upper branch and for the lower branch stationary neutral modes in the limit of large Reynolds number. The adherents of [24], such as [25–28], also employed this asymptotic technique to point out several important features of linear as well as non-linear aspects of neutral perturbations.

For when the rotating-disk boundary layer is subjected to suction, the properties of the convective instability were numerically investigated by [29] and those of the absolute instability by [30]. Using the asymptotical analysis of the large Reynolds number limit, the stationary lower branch neutral modes were thoroughly examined by [31,32]. In a recent work by [33], the non-stationary neutral modes of the lower branch were studied when the rotating-disk boundary layer flow was under the influence of suction or injection.

Both the past and recent works, some of which are cited above, point commonly, not surprisingly, to the well-known outcome, due probably to the fact first discovered by the works of [34,35], that the suction is highly stabilizing by suppressing all the modes active in the rotating-disk boundary layer flow. The primary objective of the present study is not yet another investigation of the known stabilizing effects of suction, but mainly to try to elucidate the strong stabilizing effect of suction on all forms of the instability mechanisms, whether convective or absolute, from an analytical aspect. It is thus aimed here to pursue an asymptotic method which is more straightforward to implement as compared to the known asymptotic techniques. With this purpose, the Von Kármán self-similar mean flow is perturbed by infinitesimally small disturbances to get the linearized governing equations. A complete description of the disturbed flow field is then presented based on the large suction asymptotic limit of the steady-state base flow structure as first given by [34]. Analytical derivations are hence constructed for the eigenvalues and eigenfunctions of the linearized problem which match extremely well with detailed, non-asymptotic numerical solutions even for values of the asymptotic small expansion parameter as large as $1/2$. These exact solutions for the eigenfunctions reveal that practically the entire perturbation structure becomes submerged in the viscous boundary layer as the suction increases, thereby giving rise to increasing viscous damping and hence stabilization of the base flow.

The following strategy is pursued in the rest of the paper. In Section 2.1 the formulation of the problem is outlined together with the full governing equations as well as the basic flow field of the incompressible boundary layer flow over a rotating disk in the presence of a uniform suction. Equations are next linearized in Section 2.3 using the conventional hydrodynamic linear stability theory. An asymptotic analysis is later provided in Section 3 which generates analytical expressions for both the eigenvalues and eigenfunctions of the disturbances developing within the sucked boundary layer. Results and comparisons with the numerical stability analysis are presented in Section 4 and finally conclusions are drawn in Section 5.

2. Problem formulation

2.1. Governing equations

We are concerned here with the unsteady flow of an incompressible viscous fluid over an infinite disk rotating with a constant angular velocity Ω about the axial axis z. Having suitably non-dimensionalized the flow variables, in terms of cylindrical polar coordinates (r, θ, z) possessing an orthonormal unit base $(\hat{r}, \hat{\theta}, \hat{k})$ the non-dimensional velocities $\mathbf{u} = (u, v, w)$ and the pressure p are governed by the usual continuity and momentum equations given by

$$\nabla \cdot \mathbf{u} = 0, \tag{2.1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2(\hat{k} \times \mathbf{u}) - r\hat{r} = -\nabla p + \frac{1}{R}\nabla^2 \mathbf{u}. \tag{2.2}$$

Here, R denotes the characteristic Reynolds number defined by $R = \frac{\Omega l^2}{\nu}$, in which l is some reference length, for instance the local radius of the disc, and ν is the free-stream kinematic viscosity of the fluid.

2.2. Basic steady flow

To obtain the steady incompressible flow equations of motion, which are known as the Von Kármán similarity solution in the case of zero suction, Eqs. (2.1)–(2.4) must be supplemented with the no-slip boundary conditions on the wall except with suction/injection permitted on the wall normal velocity through a scaled parameter s such that $s > 0$ is to denote the suction and $s < 0$ the blowing applied to the flow at the surface of the disk. We will only deal with the suction in the present analysis, so $s > 0$ in the rest of the paper. Additionally, no motion far away from the disk should also be allowed.

In what follows the Reynolds number will be taken to be large and, since the boundary layer thickness is $O(R^{-1/2})$, the steady incompressible boundary layer flow over a rotating disk evolves along a boundary layer coordinate of order unity, defined by $y = R^{1/2}z$. Taking this into account, the three-dimensional basic velocity profiles and pressure are represented in terms of Von Kármán's similarity variables in the form

$$(rF(y), rG(y), R^{-1/2}H(y), R^{-1}P(y)), \tag{2.3}$$

where the functions F, G, H and P satisfy the following differential equations:

$$\begin{aligned} &F^2 - (G+1)^2 + HF' - F'' = 0,\\ &2F(G+1) + HG' - G'' = 0,\\ &P' + HH' - H'' = 0,\\ &2F + H' = 0. \end{aligned} \tag{2.4}$$

Eqs. (2.4) are to be solved subject to the boundary conditions

$$\begin{aligned} &H = -s, \quad F = 0, \quad G = 0, \quad \text{at } y = 0,\\ &F = 0, \quad G = -1, \quad \text{as } y \to \infty. \end{aligned} \tag{2.5}$$

2.3. Linear stability equations

The mean flow determined from (2.4) and (2.5) is next perturbed with infinitesimally small disturbances of the form

$$l\Omega\,[f(z), g(z), h(z), p(z)]\,\mathrm{e}^{\mathrm{i}R(\alpha r + \beta\theta - \omega t)},$$

with α and β wavenumbers and ω the frequency of the perturbations, respectively. After substituting the mean flow together with these disturbances into the governing Navier–Stokes equations (2.1) and (2.2), subtracting out the basic field, and linearizing with respect to the perturbations, the evolution of the incompressible disturbed flow will be governed by a system of equations

$$\begin{aligned} &f'' - Hf' - \left[\mathrm{i}R(\alpha F + \beta G - \omega) + \lambda^2 + F\right] f + 2(G+1)g - RF'h - \mathrm{i}\alpha Rp = 0,\\ &g'' - Hg' - \left[\mathrm{i}R(\alpha F + \beta G - \omega) + \lambda^2 + F\right] g - 2(G+1)f - RG'h - \mathrm{i}\beta Rp = 0,\\ &h'' - Hh' - \left[\mathrm{i}R(\alpha F + \beta G - \omega) + \lambda^2 + H'\right] h - Rp' = 0,\\ &\mathrm{i}\bar{\alpha} f + \mathrm{i}\beta g + h' = 0, \end{aligned} \tag{2.6}$$

where $\lambda^2 = \alpha^2 + \beta^2$ and $\bar{\alpha} = \alpha - \frac{i}{R}$. The boundary conditions to be imposed on the perturbations are

$$\begin{aligned} &f = 0, \quad g = 0, \quad h = 0, \quad \text{at } y = 0,\\ &f \to 0, \quad g \to 0, \quad h \to 0, \quad p \to 0, \quad \text{as } y \to \infty. \end{aligned} \tag{2.7}$$

Notice that the pressure perturbation on the wall is unknown, but limited to a finite value. In a usual linear stability approach its prescription on the wall is generally avoided; see for instance [15,18]. Moreover, to search for the eigenvalues within a local stability analysis view, the eigenfunctions are normalized by a constraint on the wall, such as $f'(0) = 1$, as implemented by several researchers.

3. Asymptotic analysis

The asymptotic analysis of [34] showed that for strong suction, the basic flow profile finds itself confined to a layer adjacent to the wall of thickness $O(\frac{1}{s})$. Therefore, in the limit of large suction the velocities are evaluated to the leading order as

$$F = \frac{1}{2s^2}(\mathrm{e}^{-sz} - \mathrm{e}^{-2sz}), \qquad G = -1 + \mathrm{e}^{-sz}, \qquad H = -s. \tag{3.8}$$

Thus for a strong suction, it can be seen that the stability Eqs. (2.6) turn out to be

$$\begin{aligned} &f'' + sf' - af - \mathrm{i}\alpha Rp = 0,\\ &g'' + sg' - ag - \mathrm{i}\beta Rp = 0,\\ &h'' + sh' - ah - Rp' = 0,\\ &\mathrm{i}\bar{\alpha} f + \mathrm{i}\beta g + h' = 0, \end{aligned} \tag{3.9}$$

with the parameter a defined by $a = \lambda^2 - \mathrm{i}R(\beta + \omega)$. The general solution to (3.9) which decays exponentially at large distances from the wall can be written explicitly in the form

$$\begin{aligned} &f = C_1\Lambda_1\mathrm{e}^{\Lambda_1 z} + C\mathrm{i}\alpha R\mathrm{e}^{\Lambda_2 z}, \qquad g = C_2\Lambda_1\mathrm{e}^{\Lambda_1 z} + C\mathrm{i}\beta R\mathrm{e}^{\Lambda_2 z},\\ &h = -\mathrm{i}\bar{\alpha}C_1\mathrm{e}^{\Lambda_1 z} - \mathrm{i}\beta C_2\mathrm{e}^{\Lambda_1 z} + C\Lambda_2 R\mathrm{e}^{\Lambda_2 z}, \qquad p = \Lambda_3^2 C\mathrm{e}^{\Lambda_2 z}, \end{aligned} \tag{3.10}$$

where C and C_1 are the integration constants, and also the parameters Λ_1, Λ_2 and Λ_3 are given by

$$\begin{aligned} &\Lambda_1 = -\frac{s}{2} - \left[\frac{s^2}{4} + a\right]^{1/2},\\ &\Lambda_2^2 = \alpha\bar{\alpha} + \beta^2 = \lambda^2 - \mathrm{i}\frac{\alpha}{R},\\ &\Lambda_3^2 = (\Lambda_2 - \Lambda_1)(\Lambda_1 + \Lambda_2 + s). \end{aligned} \tag{3.11}$$

Due to the decaying character of the perturbations as evaluated in Eq. (3.10), it should be imposed that the real parts of Λ_1 and Λ_2 in (3.11) are strictly less than zero. Additionally, enforcing the no-slip velocity perturbations f and g on the wall, the eigenfunction family (3.10) takes the shape

$$\begin{aligned} &f = \mathrm{i}\alpha RC(\mathrm{e}^{\Lambda_2 z} - \mathrm{e}^{\Lambda_1 z}), \qquad g = \mathrm{i}\beta RC(\mathrm{e}^{\Lambda_2 z} - \mathrm{e}^{\Lambda_1 z}),\\ &h = R\frac{\Lambda_2}{\Lambda_1}C(\Lambda_1\mathrm{e}^{\Lambda_2 z} - \Lambda_2\mathrm{e}^{\Lambda_1 z}), \qquad p = \Lambda_3^2 C\mathrm{e}^{\Lambda_2 z}. \end{aligned} \tag{3.12}$$

It is apparent in Eqs. (3.10) and (3.12) that $\Lambda_1 \neq \Lambda_2$; however, further enforcing the zero wall normal velocity h on the wall in Eq. (3.12) implies clearly that $C = 0$, yielding completely vanished amplitudes of the disturbances, and thus a trivial solution is attained.

In order to obtain a non-trivial solution to (2.6) and (2.7) it is anticipated that

$$\Lambda_1 = \Lambda_2, \tag{3.13}$$

which gives an explicit formula for the eigenrelation. This dispersion relation, on the other hand, suggests that solutions (3.12) to the system (3.9) need to be modified to get, considering also $\Lambda_1 = \Lambda_2 = \Lambda$,

$$\begin{aligned} f &= \mathrm{i}\frac{\alpha R}{2\Lambda + s} C z \mathrm{e}^{\Lambda z}, \qquad g = \mathrm{i}\frac{\beta R}{2\Lambda + s} C z \mathrm{e}^{\Lambda z}, \\ h &= \frac{R}{2\Lambda + s} C(\Lambda z - 1)\mathrm{e}^{\Lambda z} + D, \qquad p = C\mathrm{e}^{\Lambda z}. \end{aligned} \tag{3.14}$$

It can be seen that all the eigenfunctions decay at large distances, as required, if the integration constant D in Eq. (3.14) is assigned as zero. In addition to this, the amplitudes f and g in (3.14) also satisfy the zero-slip condition on the wall for arbitrary values of C, but it appears from the no-slip forcing on the wall of the vertical amplitude h in (3.14) that $C = 0$, generating again a trivial solution to (3.9). Moreover, further imposing the condition $a = 0$, together with the consideration of (3.13), results in similar solutions (3.14) to the system (3.9) too, which gives rise to again a trivial solution as a consequence of the boundary conditions (2.7).

The key to get non-trivial eigenfunctions for large asymptotic suction satisfying (2.6) as well as the boundary conditions (2.7) lies in the form of the fourth equation in (3.14) suggesting that $h'(0)$ be zero. However, the form of the solution as computed in (3.14) never satisfies this demand besides the constraint $h(0) = 0$. This means that the normal perturbation should be sought in the form $h = Az^2\mathrm{e}^{\Lambda z}$, with A being constant. On the other hand, from the differential equation in (3.9), it is seen that for such a solution to h exist, a double root of the characteristic polynomial in (3.9) should occur, resulting in the constraint

$$s^2 + 4a = 0, \tag{3.15}$$

and thus together with (3.13) it reads

$$\Lambda_1 = -\frac{s}{2} = \Lambda_2 = \Lambda. \tag{3.16}$$

Accounting for the restrictions on the eigenvalues as determined in Eqs. (3.15) and (3.16), the following non-trivial solutions to the eigenfunctions are generated:

$$\begin{aligned} p &= C\mathrm{e}^{\Lambda z}, \qquad f = C_1 z\mathrm{e}^{\Lambda z} + \mathrm{i}\frac{\alpha R C}{2} z^2 \mathrm{e}^{\Lambda z}, \\ g &= C_2 z\mathrm{e}^{\Lambda z} + \mathrm{i}\frac{\beta R C}{2} z^2 \mathrm{e}^{\Lambda z}, \qquad h = \frac{R\Lambda C}{2} z^2 \mathrm{e}^{\Lambda z}. \end{aligned} \tag{3.17}$$

The solution (3.17) to the system (3.9) exactly satisfies the boundary conditions (2.7). Moreover, using the fourth equation in (2.6) yields a compatibility condition on the integration constants C_1 and C_2 as

$$\bar{\alpha} C_1 + \beta C_2 = \mathrm{i} R\Lambda C. \tag{3.18}$$

In addition to this, it is possible to normalize the eigenfunctions in (3.17) such that $f'(0) = 1$ as in [14,15], leading to $C_1 = 1$, as a consequence of which the eigenfunctions are uniquely determined. The value of the complex unknown wall pressure C can be substituted from the numerical integration of (2.6) and (2.7).

4. Results and discussion

The mean flow velocity profiles described by the similarity solution (2.3) and calculated at two suction parameters are shown first in Fig. 1(a)–(c). The smooth curves are from the full numerical solution of the mean flow equations (2.4) and (2.5), while the dotted curves on the graphs are from the asymptotic correspondence (3.8) as derived from the large suction limit. Notice that for $s = 5$ the curves in Fig. 1(a)–(c) are almost indistinguishable, confirming the well-known feature of the boundary layer flow exposed to a suction that the three-dimensional Von Kármán basic flow can be replaced by the asymptotic suction boundary layer for large enough suction parameters.

A close inspection of the eigenfunctions obtained in (3.17) taking into the consideration (3.18) demonstrates, for instance, that $|f|$ and $|g|$ have local extrema at the critical points of order of magnitude $z = O(\frac{1}{\Lambda})$, such that the extrema get closer to the wall boundary layer for larger suctions, as a result of which the perturbations f and

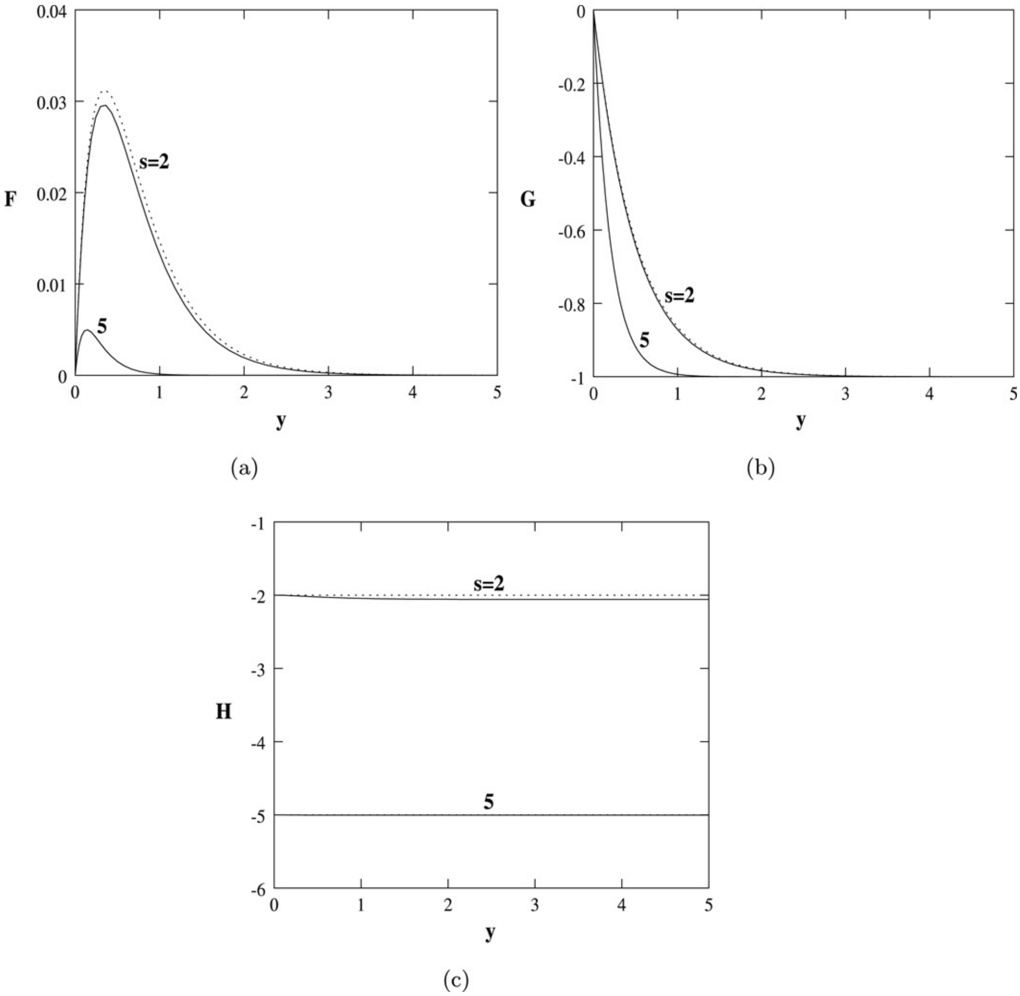

Fig. 1. Mean flow profiles are demonstrated at two selected suction parameters, (a) F, (b) G and (c) H. Smooth curves are from the direct numerical integration of the complete Eqs. (2.4) and (2.5) and the broken curves are from the asymptotic flow variables (3.8).

g become highly damped. A similar trend can also be observed for $|h|$, for which a single extremum occurs at $z = -\frac{2}{\Lambda}$ yielding the same result of attenuation of the vertical eigenfunction. As for the pressure eigenfunction p, it is similarly damped by the presence of exponential multiplier as well as there being numerical evidence that the wall pressure value $|C|$ tends to zero rapidly as the suction parameter is increased. Therefore, the form of the analytic eigenfunctions as found in (3.17) and (3.18) explains clearly why the perturbations introduced into the boundary layer are subjected to a strong damping when the flow is under the influence of a sufficiently large suction.

To further explore (3.15) and (3.16), it can be written that

$$\frac{\alpha}{R} - R(\beta + \omega) = \mathrm{i}\frac{s^2}{2}, \tag{4.19}$$

or writing $\alpha = \alpha_r + \mathrm{i}\alpha_i$, $\omega = \omega_r + \mathrm{i}\omega_i$ and further decomposing (4.19) into real and imaginary parts produces

$$\begin{aligned} &\frac{\alpha_r}{R} - R(\beta + \omega_r) = 0, \\ &\frac{\alpha_i}{R} - R\omega_i = \frac{s^2}{2}. \end{aligned} \tag{4.20}$$

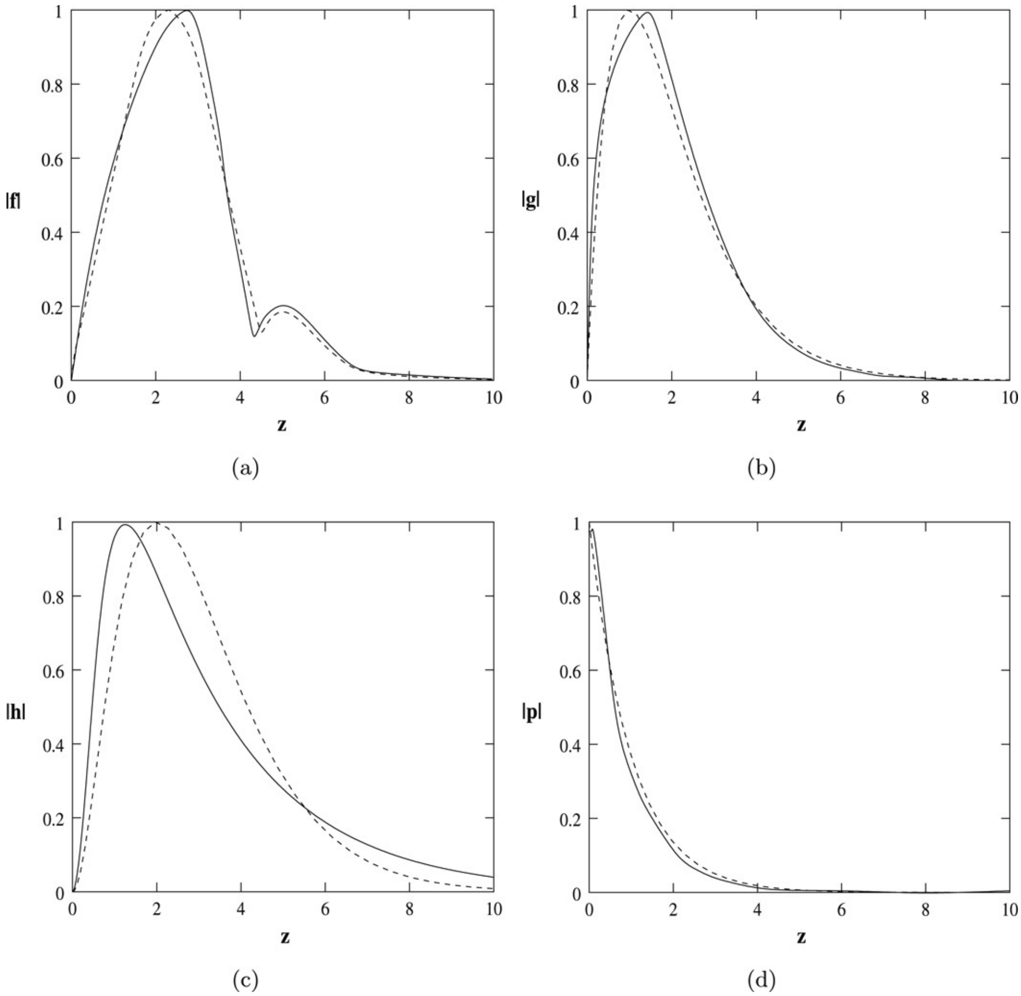

Fig. 2. A comparison between the asymptotically obtained eigenfunctions (straight curves) and the numerically computed eigenfunctions (dashed curves) at the suction $s = 2$ is demonstrated for the particular eigenvalues $\alpha = 0.39$, $\beta = 0.02$ and $R = 5000$. (a) $|f|$, (b) $|g|$, (c) $|h|$ and (d) $|p|$.

A straightforward conclusion then can be drawn from Eq. (4.20) that if the perturbations are allowed to evolve spatially with ω being real, then $\alpha_i = \frac{Rs^2}{2}$, meaning that all the perturbations are spatially damped. However, if the temporal development is considered with α being real, then $\omega_i = -\frac{s^2}{2R}$ from (4.20), resulting in temporally damped perturbations. These results further enable us to conceive a strong stabilization impact of suction. Moreover, for the stationary neutral perturbations it is found from (4.20) that $\alpha = \beta R^2$, leading to the conclusion that perturbations have longer wavelength in the azimuthal direction, being considerably elongated as compared to the small-wavelength structure of the disturbances in the radial direction. Furthermore, in the case of large Reynolds number limit, it can be deduced from (3.15) and (4.20) that $\alpha = O(s)$ and $\beta = O(s^{-1})$, a result that is in line with the recent asymptotic findings of [33]. Finally, differentiating the relation (3.16) yields for the complex group velocity $\frac{\partial \omega}{\partial \alpha} = \frac{2\alpha}{\mathrm{i}R}$, explaining why the absolute instability mechanism should also be stabilized under the influence of large suction, again in compliance with the outcome of [30].

To demonstrate the consistency of our asymptotic approach and results, a comparison between the eigenfunctions obtained analytically in (3.17) and the ones computed from a direct numerical integration of Eqs. (2.6) and (2.7) is targeted next. The numerical method known as the Chebyshev collocation method is based on the Chebyshev polynomials, in which Eqs. (2.6) and (2.7) having transformed into the computational domain were collocated at Gauss and Gauss–Lobatto points. A staggered grid was constructed to avoid the prescription of the pressure boundary condition on the wall. More details of the technique may be found in [18]. Figs. 2 and 3 show the comparison between

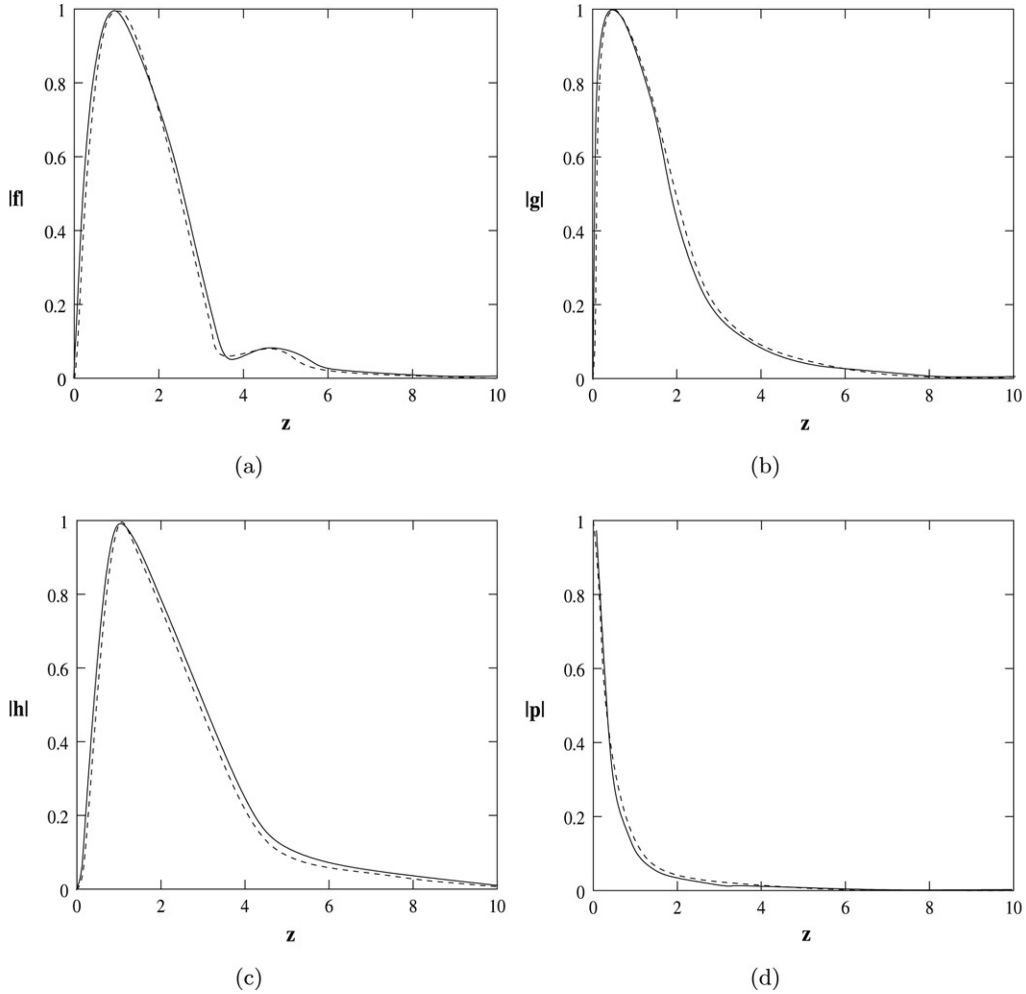

Fig. 3. A comparison between the asymptotically obtained eigenfunctions (straight curves) and the numerically computed eigenfunctions (dashed curves) at the suction $s = 5$ is demonstrated for the particular eigenvalues $\alpha = 0.12$, $\beta = 10^{-4}$ and $R = 10^6$. (a) $|f|$, (b) $|g|$, (c) $|h|$ and (d) $|p|$.

the analytical eigenfunctions (3.17), denoted by straight curves, and the numerically calculated ones, denoted by broken lines, both for the stationary disturbance case. In these figures the absolute values of the eigenfunctions are displayed, which were normalized by their maxima. Fig. 2 is for the suction parameter $s = 2$ and possesses the eigenvalues $\alpha = 0.39$, $\beta = 0.02$ and $R = 5000$, while Fig. 3 is for a larger suction parameter $s = 5$ and has the particular eigenvalues $\alpha = 0.12$, $\beta = 10^{-4}$ and $R = 10^6$. Good agreement between the present solutions and the numerical ones is clearly indicated by the figures. The absorbing feature of basic boundary layer flow, by suction, as predicted in [34] is seen to have great influence on the linear perturbation modes. Particularly for large suction cases, our asymptotic results are good candidates for further research, like initial value problems and direct numerical simulations of the rotating-disk boundary layer flow. The damping effect of the large suction limit is already seen to be captured at even moderate suction parameters as displayed in Figs. 2 and 3.

5. Conclusions

A self-consistent and rigorous asymptotic method which is straightforward to implement has been pursued in the current work to investigate the suction effects on the incompressible three-dimensional Von Kármán boundary layer flow over a disk rotating with a constant axial angular speed. Having linearized the disturbances imposed on the mean flow field within the consideration of hydrodynamic linear stability theory, analytic expressions governing the form of the eigenfunctions as well as the structure of the eigenvalues have been successfully derived, which are shown to be under the strong influence of suction applied through the surface of the disk. A comparison has then been undertaken between the modes obtained from the asymptotic approach as well as the ones calculated by numerically solving the complete linearized stability equations.

The fluid suction in two- or three-dimensional boundary layer flows is now known worldwide to greatly stabilize the linear instability mechanisms acting upon the boundary layer. The approach followed in the present work is not to emphasize this feature of the suction but to try to understand the effects from an analytical point of view. As a result of the analysis, the approximate shapes of the perturbations developing under the strong suction have been derived. The main conclusion to be drawn from the current study hence is that imposition of large suction in the three-dimensional rotating-disk boundary layer considerably reduces the mean velocity profiles, and consequently leads to substantially damped eigenfunctions. Moreover, the eigenvalues determined from an analytic dispersion relationship point clearly to linearly damped perturbations from both spatial and temporal stability viewpoints. The findings outlined here thus jointly offer a good explanation as regards a theoretical view as to why the boundary layer suction acts to stabilize the Von Kármán mean flow.

It is inevitable that the form of the perturbations as computed from the present method will be modified by the presence of non-linearity, but the asymptotic mean adopted here is still applicable to other two- or three-dimensional boundary layers for understanding the linear development of the instabilities whenever a strong suction is active.

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