TAIWANESE JOURNAL OF MATHEMATICS
Vol. 11, No. 1, pp. 267-275, March 2007
This paper is available online at http://www.math.nthu.edu.tw/tjm/

REDUCED AND $p.q.$-BAER MODULES

Muhittin Başer and Abdullah Harmanci

Abstract. In this paper, we study $p.q.$-Baer modules and some polynomial extensions of $p.q.$-Baer modules. In particular, we show: (1) For a reduced module M_R, M_R is a $p.p.$-module iff M_R is a $p.q.$-Baer module. (2) If M_R is an α-reduced module where α is an endomorphism of R, then M_R is a $p.q.$-Baer module iff $M[x;\alpha]_{R[x;\alpha]}$ is a $p.q.$-Baer module. (3) For an arbitrary module M_R, M_R is a $p.q.$-Baer module if and only if $M[x]_{R[x]}$ is a $p.q.$-Baer module.

1. Introduction

Throughout this work all rings R are associative with identity and modules are unital right R-modules and $\alpha : R \longrightarrow R$ is an endomorphism of the ring R. In [6] Clark called a ring R *quasi-Baer ring* if the right annihilator of each right ideal of R is generated (as a right ideal) by an idempotent. Recently, Birkenmeier et al. [3] called a ring R *right* (resp. *left*) *principally quasi-Baer* [or simply *right* (resp. *left*) $p.q.$*-Baer*] if the right (resp. left) annihilator of a principal right (resp. left) ideal of R is generated by an idempotent. R is called $p.q.$*-Baer* if it is both right and left $p.q.$-Baer. A ring R is called a *right* (resp. *left*) $p.p.$*-ring* if the right (resp. left) annihilator of every element of R is generated by an idempotent. R is called a $p.p.$*-ring* if it is both a right and left $p.p.$-ring. A ring is called *reduced ring* if it has no nonzero nilpotent elements and M_R is called α*-reduced module* by Lee-Zhou [12] if, for any $m \in M$ and $a \in R$, (1) $ma = 0$ implies $mR \cap Ma = 0$, (2) $ma = 0$ iff $m\alpha(a) = 0$, where $\alpha : R \longrightarrow R$ is a ring endomorphism with $\alpha(1) = 1$. The module M_R is called ***a reduced module*** if M is 1_R-reduced. It is clear that R is a reduced ring iff R_R is a reduced module.

We write $R[x], R[[x]], R[x, x^{-1}]$ and $R[[x, x^{-1}]]$ for the polynomial ring, the power series ring, the Laurent polynomial ring and the Laurent power series ring over R, respectively.

Received January 4, 2005, accepted March 21, 2005.
Communicated by Shun-Jen Cheng.
2000 *Mathematics Subject Classification*: 16D80, 16S36.
Key words and phrases: Reduced module, $p.q.$-Baer module.

In [12] Lee-Zhou introduced the following notation. For a module M_R, we consider

$$M[x;\alpha] = \left\{ \sum_{i=0}^{s} m_i x^i \ : \ s \geq 0, m_i \in M \right\},$$

$$M[[x;\alpha]] = \left\{ \sum_{i=0}^{\infty} m_i x^i \ : m_i \in M \right\},$$

$$M[x, x^{-1};\alpha] = \left\{ \sum_{i=-s}^{t} m_i x^i \ : \ s \geq 0, t \geq 0, m_i \in M \right\},$$

$$M[[x, x^{-1};\alpha]] = \left\{ \sum_{i=-s}^{\infty} m_i x^i \ : \ s \geq 0, m_i \in M \right\}.$$

Each of these is an Abelian group under an obvious addition operation. Moreover $M[x;\alpha]$ becomes a module over $R[x;\alpha]$ under the following scalar product operation: For $m(x) = \sum_{i=0}^{s} m_i x^i \ \in M[x;\alpha]$ and $f(x) = \sum_{i=0}^{t} a_i x^i \ \in R[x;\alpha]$

$$m(x)f(x) = \sum_{k=0}^{s+t} \left(\sum_{i+j=k} m_i \alpha^i(a_j) \right) x^k.$$

Similarly, $M[[x;\alpha]]$ is module over $R[[x;\alpha]]$. The modules $M[x;\alpha]$ and $M[[x;\alpha]]$ are called the *skew polynomial extension* and the *skew power series extension of* M respectively. If $\alpha \in Aut(R)$, then with a similar scalar product, $M[[x, x^{-1};\alpha]]$ (resp. $M[x, x^{-1};\alpha]$) becomes a module over $R[[x, x^{-1};\alpha]]$ (resp.$R[x, x^{-1};\alpha]$). The modules $M[x, x^{-1};\alpha]$ and $M[[x, x^{-1};\alpha]]$ are called the *skew Laurent polynomial extension* and the *skew Laurent power series extension of* M, respectively. First we recall the following theorem.

Theorem 1. [12, Theorem 1.6] *The following are equivalent for a module* M_R*;*

(1) M_R *is* α*-reduced;*

(2) $M[x;\alpha]_{R[x;\alpha]}$ *is reduced;*

(3) $M[[x;\alpha]]_{R[[x;\alpha]]}$ *is reduced. If* $\alpha \in Aut(R)$*, then the conditions* $(1)-(3)$ *are equivalent to each of* (4) *and* (5):

(4) $M[x, x^{-1};\alpha]_{R[x,x^{-1};\alpha]}$ *is reduced;*

(5) $M[[x, x^{-1};\alpha]]_{R[[x,x^{-1};\alpha]]}$ *is reduced.*

According to Lee-Zhou [12] a module M_R is called *α-Armendariz* if the following conditions (1) and (2) are satisfied, and module M_R is called *α-Armendariz of power series type* if the following conditions (1) and (3) are satisfied:

(1) For $m \in M$ and $a \in R$, $ma = 0$ if and only if $m\alpha(a) = 0$.

(2) For any $m(x) = \sum_{i=0}^{s} m_i x^i \in M[x;\alpha]$ and $f(x) = \sum_{i=0}^{t} a_i x^i \in R[x;\alpha]$, $m(x)f(x) = 0$ implies $m_i \alpha^i(a_j) = 0$ for all i and j.

(3) For any $m(x) = \sum_{i=0}^{\infty} m_i x^i \in M[[x;\alpha]]$ and $f(x) = \sum_{i=0}^{\infty} a_i x^i \in R[[x;\alpha]]$, $m(x)f(x) = 0$ implies $m_i \alpha^i(a_j) = 0$ for all i and j.

The module M_R is *Armendariz* iff M_R is 1_R-Armendariz; we call M_R *Armendariz of power series type* if M_R is 1_R-Armendariz of power series type. If M_R is α-reduced then M_R is α-Armendariz of power series type. If M_R is α-Armendariz of power series type then M_R is α-Armendariz.

For a subset X of a module M_R, let $r_R(X) = \{r \in R : Xr = 0\}$. In [12] Lee-Zhou introduced Baer modules, quasi-Baer modules and *p.p.*-modules as follows.

(1) M_R is called *Baer* if, for any subset X of M, $r_R(X) = eR$ where $e^2 = e \in R$.

(2) M_R is called *quasi-Baer* if, for any submodule N of M, $r_R(N) = eR$ where $e^2 = e \in R$;

(3) M_R is called *principally projective* (or simply *p.p.*) if, for any $m \in M$, $r_R(m) = eR$ where $e^2 = e \in R$.

In this paper, we study $p.q.$-modules and the some polynomial and power series extensions of $p.q.$-modules. In particular, we show: (1) For a reduced module M_R, M_R is a $p.p.$-module iff M_R is a $p.q.$-Baer module. (2) If M_R is an α-reduced module where α is an endomorphism of R, then M_R is a $p.q.$-Baer module iff $M[x;\alpha]_{R[x;\alpha]}$ is a $p.q.$-Baer module. (3) For an arbitrary module M_R, M_R is a $p.q.$-Baer module if and only if $M[x]_{R[x]}$ is a $p.q.$-Baer module.

We begin with the following definition which is defined in [10].

Definition 2. Let M_R be a module. M_R is called *principally quasi-Baer* (or simply *$p.q.$-Baer*) module if, for any $m \in M$, $r_R(mR) = eR$ where $e^2 = e \in R$.

It is clear that R is a right $p.q.$-Baer ring iff R_R is a $p.q.$-Baer module. If R is a $p.q.$-Baer ring, then for any right ideal I of R, I_R is a $p.q.$-Baer module. Every submodule of a $p.q.$-Baer module is $p.q.$-Baer module. Moreover, every quasi-Baer module is $p.q.$-Baer, and every Baer module is quasi-Baer.

If R is commutative then M_R is $p.p.$-module iff M_R is $p.q.$-Baer module.

The following examples show that there exists a $p.q.$-Baer module that is not a $p.p.$-module.

Example 3. [7, Example 2(1)] Let $\mathbb{Z}$ be the ring of integers and $M_2(\mathbb{Z})$ the 2×2 full matrix ring over $\mathbb{Z}$. We consider the ring

$$R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{Z}) \mid a \equiv d, b \equiv 0 \text{ and } c \equiv 0 \pmod 2 \right\}.$$

Then the module R_R is $p.q.$-Baer, but it is not a $p.p.$-module.

Theorem 4. *Let M_R be a module such that for any $m \in M$ and $a \in R$, $ma = 0$ implies $mRa = 0$. Then M_R is a $p.p.$-module if and only if M_R is a $p.q.$-Baer module.*

Proof. Let $m \in M$. If $a \in r_R(m)$ then $ma = 0$ and by assumption, $mRa = 0$ and so $a \in r_R(mR)$. Then $r_R(m) \subseteq r_R(mR)$. But $r_R(mR) \subseteq r_R(m)$ obviously holds. Consequently, $r_R(mR) = r_R(m) = eR$. Hence the claim follows. ■

Our next result extends [7, Lemma 1].

Corollary 5. *Let M_R be a reduced module. Then M_R is a $p.p.$-module if and only if M_R is a $p.q.$-Baer module.*

Proof. Assume M_R is a reduced module. Then $m \in M$, $a \in R$, $ma = 0$ implies $mRa = 0$ by [12, Lemma 1.2]. The claim follows from Theorem 4. ■

Corollary 6. [7, Lemma 1] *Let R be a reduced ring. Then R is a right $p.p.$-ring if and only if R is a right $p.q.$-Baer ring.*

Theorem 7. *Let $\alpha : R \longrightarrow R$ be an endomorphism of R and assume that, for $m \in M$ and $a \in R$, $ma = 0 \Leftrightarrow m\alpha(a) = 0$. Then the following hold:*

(1) (a) *If $M[x;\alpha]_{R[x;\alpha]}$ is a p.q.-Baer module, then M_R is a p.q.-Baer module. The converse holds if in addition M_R is $\alpha-$reduced.*

(b) *If $M[[x;\alpha]]_{R[[x;\alpha]]}$ is p.q.-Baer, then M_R is p.q.-Baer.*

(2) *Let $\alpha \in Aut(R)$.*

(a) *If $M[x,x^{-1};\alpha]_{R[x,x^{-1};\alpha]}$ is a p.q.-Baer module, then M_R is a p.q.-Baer module. The converse holds if in addition M_R is $\alpha-$reduced.*

(b) *If* $M[[x,x^{-1};\alpha]]_{R[[x,x^{-1};\alpha]]}$ *is a p.q.-Baer module, then* M_R *is a p.q.-Baer module.*

Proof. (1) (a) Similar to the proof of (1)(b).
Converse of (1) (a): Assume that M_R is an α-reduced module and M_R is $p.q.$-Baer module. For any $m \in M$ and $a \in R$, $ma = 0$ implies $mRa = 0$. Then by Theorem 4, M_R is a $p.p.$-module. Since M_R is an α-reduced module, M_R is α-Armendariz. By [12, Theorem 2.11.(1)(a)], $M[x;\alpha]_{R[x;\alpha]}$ is $p.p.$-module. Since M_R is α-reduced, $M[x;\alpha]_{R[x;\alpha]}$ is reduced by Theorem 1. By Corollary 5, $M[x;\alpha]_{R[x;\alpha]}$ is a $p.q.$-Baer module.

(1)(b) Suppose $M[[x;\alpha]]_{R[[x;\alpha]]}$ is a $p.q.$-Baer module. For $m \in M$ we have $r_{R[[x;\alpha]]}(mR[[x;\alpha]]) = f(x)R[[x;\alpha]]$ where $f(x)^2 = f(x) \in R[[x;\alpha]]$. Thus $f(x)R[[x;\alpha]] \subseteq r_{R[[x;\alpha]]}(mR) = r_R(mR)[[x;\alpha]]$. For $g(x) = \sum_{j=0}^{\infty} b_j x^j \in r_R(mR)$ $[[x;\alpha]]$, $mRb_j = 0$ for all $j \geq 0$ and hence $mR\alpha^k(b_j) = 0$ for all $j \geq 0$ and all $k \geq 0$, by assumption. For any $u(x) = \sum_{i=0}^{\infty} u_i x^i \in (mR)[[x;\alpha]]$, $u(x)g(x) = \sum_i \sum_j u_i \alpha^i(b_j) x^{i+j} = 0$. So $g(x) \in r_{R[[x;\alpha]]}((mR)[[x;\alpha]])$. Thus $r_R(mR)[[x;\alpha]] = f(x)R[[x;\alpha]]$. Write $f(x) = \sum_{i=0}^{\infty} a_i x^i$, where all $a_i \in r_R(mR)$. Then, for any $a \in r_R(mR)$, $a = f(x)h(x)$ for some $h(x) \in R[[x;\alpha]]$ so $f(x)a = f(x)f(x)h(x) = f(x)h(x) = a$. It follows that $a = a_0 a$ for all $a \in r_R(mR)$. Thus $r_R(mR) = a_0 R$ with ${a_0}^2 = a_0$. So M_R is $p.q.$-Baer module. Now the rest is clear

(2) Similar to the proof of (1). ∎

Corollary 8. *The following hold for a module* M_R*:*
(1) *If any one of* $M[x]_{R[x]}$,$M[[x]]_{R[[x]]}$,$M[x,x^{-1}]_{R[x,x^{-1}]}$ *and* $M[[x,x^{-1}]]_{R[[x,x^{-1}]]}$ *is a p.q.-Baer module, then so is* M_R.
(2) *Let* M_R *be reduced. If* M_R *is a p.q.-Baer, then both* $M[x]_{R[x]}$ *and* $M[x,x^{-1}]_{R[x,x^{-1}]}$ *are p.q.-Baer.*

Corollary 9. *The following hold for a ring* R*:*
(1) *If any one of* $R[x]$,$R[[x]]$, $R[x,x^{-1}]$ *and* $R[[x,x^{-1}]]$ *is a right p.q.-Baer ring, then so is* R.
(2) *Let* R *be a reduced ring. If* R *is right p.q.-Baer, then both* $R[x]$ *and* $R[x,x^{-1}]$ *are* $p.q.$*-Baer ring.*

Example 10. There is a reduced $p.q.$-Baer module M_R such that $M[[x]]_{R[[x]]}$ is not a $p.q.$-Baer module.

Proof. Let F be a field and R be the ring

$$R = \left\{ (a_n) \in \prod_{n=1}^{\infty} F_n \mid a_n \text{ is eventually constant} \right\},$$

which is the subring of $\prod_{n=1}^{\infty} F_n$, where $F_n = F$ for $n = 1, 2, \ldots$. Let M_R denote the module R_R. We claim M_R is a $p.q.$-Baer module and reduced. But $M[[x]]_{R[[x]]}$ is not $p.q.$-Baer module. It is well known that M_R is a $p.q.$-Baer module and reduced. Let e_i denote the "i^{th} unit vector " $(0, ..., 0, 1, 0, ...)$ and let $X = \{e_1, e_3, e_5, ...\}$. Let $m(x) = e_1 x + e_3 x^3 + \cdots \in M[[x]]_{R[[x]]}$. Assume that $M[[x]]_{R[[x]]}$ is a $p.q.$-Baer module. Then $r_{R[[x]]}(m(x)R[[x]]) = f(x)R[[x]]$ for some idempotent $f(x)^2 = f(x) \in R[[x]]$. Since R is commutative ring, every idempotent in the ring $R[[x]]$ belongs to R by Lemma 8 in [9]. Hence $f(x)$ belongs to R, say $f(x) = f_0 \in R$. Now it is easy to check that $r_{R[[x]]}(m(x)R[[x]]) = f_0 R[[x]]$ implies $r_R(X) = f_0 R$. This is not possible by Example 7.54 in [11]. Thus $M[[x]]_{R[[x]]}$ is not $p.q.$-Baer module. Since M_R is reduced $M[[x]]_{R[[x]]}$ is reduced by Theorem 1. Therefore $M[[x]]_{R[[x]]}$ is not $p.q.$-Baer module by Corollary 5.

■

Recall from [4], an idempotent $e \in R$ is *left* (resp. *right*) *semicentral* in R if $exe = xe$ (resp. $exe = ex$), for all $x \in R$. Equivalently, $e^2 = e \in R$ is left (resp. right) semicentral if eR (resp. Re) is an ideal of R. If M_R is a $p.q.$-Baer module and $m \in M$, then $r_R(mR)$ is generated by a left semicentral idempotent because $r_R(mR)$ is an ideal. We use $\mathcal{S}_l(R)$ for the set of all left semicentral idempotents.

The next theorem improved Corollary 8 for the polynomial extension case.

Theorem 11. *M_R is a $p.q.$-Baer module if and only if $M[x]_{R[x]}$ is a $p.q.$-Baer module.*

Proof. Assume M_R is a $p.q.$-Baer module. Let $m(x) = m_0 + m_1 x + \ldots + m_n x^n \in M[x]$. There exists $e_i \in \mathcal{S}_l(R)$ such that $r_R(m_i R) = e_i R$, for $i = 0, 1, \ldots, n$. Let $e = e_0 e_1 \ldots e_n$. Then $e \in \mathcal{S}_l(R)$ and $eR = \bigcap_{i=0}^{n} r_R(m_i R)$. Hence $eR[x] \subseteq r_{R[x]}(m(x)R[x])$. Observe $r_{R[x]}(m(x)R[x]) \subseteq r_{R[x]}(m(x)R)$. Let $h(x) \in r_{R[x]}(m(x)R)$ and $g(x) = b_0 + b_1 x + \ldots + b_k x^k \in R[x]$. Then

$$\begin{aligned} m(x)g(x)h(x) &= m(x)b_0h(x) + m(x)b_1xh(x) + \ldots + m(x)b_kx^kh(x) = \\ &= m(x)b_0h(x) + m(x)b_1h(x)x + \ldots + m(x)b_kh(x)x^k = 0. \end{aligned}$$

Hence $h(x) \in r_{R[x]}(m(x)R[x])$. Consequently, $r_{R[x]}(m(x)R[x]) = r_{R[x]}(m(x)R)$.

Now, let $h(x) = a_0 + a_1x + \ldots + a_tx^t \in r_{R[x]}(m(x)R)$. Since $m(x)Rh(x) = 0$, we have the following system of equations where d is an arbitrary element of R:

(0) $m_0da_0 = 0$;
(1) $m_1da_0 + m_0da_1 = 0$;
(2) $m_2da_0 + m_1da_1 + m_0da_2 = 0$;
(3) $m_3da_0 + m_2da_1 + m_1da_2 + m_0da_3 = 0$;
...
(l) $m_lda_0 + m_{l-1}da_1 + \ldots + m_1da_{l-1} + m_0da_l = 0.$

By first equation, $a_0 \in r_R(m_0R) = e_0R$, where $e_0 \in \mathcal{S}_l(R)$. Let $s \in R$ and take $d = se_0$ in equation (1). Then $m_1se_0a_0 + m_0se_0a_1 = 0$. But $m_0se_0a_1 = 0$, so $m_1se_0a_0 = m_1sa_0 = 0$. Hence $a_0 \in r_R(m_1R) = e_1R$, where $e_1 \in \mathcal{S}_l(R)$. Thus $a_0 \in e_0e_1R$. Since $m_1da_0 = 0$, then equation (1) yields $m_0da_1 = 0$. Hence $a_1 \in r_R(m_0R) = e_0R$. Take $d = se_0e_1$ in equation (2). Then $m_2se_0e_1a_0 + m_1se_0e_1a_1 + m_0se_0e_1a_2 = 0$. But $m_1se_0e_1a_1 = 0 = m_0se_0e_1a_2$. Hence $0 = m_2se_0e_1a_0 = m_2sa_0$, so $a_0 \in r(m_0R) \cap r(m_1R) \cap r(m_2R) = e_0e_1e_2R$, and so we have by equation (2)

(2$'$) $m_1da_1 + m_0da_2 = 0$

In equation (2$'$) substitute se_0 for d to obtain $m_1se_0a_1 + m_0se_0a_2 = 0$. But $m_0se_0a_2 = 0$, so $m_1sa_1 = m_1se_0a_1 = 0$. Thus $a_1 \in r(m_0R) \cap r(m_1R) = e_0e_1R$. Since $a_1 \in r_R(m_1R)$, then equation (2$'$) yields $m_0da_2 = 0$. Hence $a_2 \in r(m_0R) = e_0R$. Summarizing at this point, we have $a_0 \in e_0e_1e_2R$, $a_1 \in e_0e_1R$ and $a_2 \in e_0R$. Continuing this procedure yields $a_i \in eR$ for all $i = 0, 1, 2, ..., t$. Hence $h(x) \in eR[x]$. Consequently $eR[x] = r_{R[x]}(m(x)R[x])$. Conversely, if $M[x]_{R[x]}$ is a p.q.-Baer, then M_R is p.q.-Baer by Corollary 8 (2). ∎

Corollary 12. *Assume that R is a commutative ring. Then M_R is a $p.p.$-module if and only if $M[x]_{R[x]}$ is a $p.p.$-module.*

Proof. This is an immediate consequence of Theorem 11, since if R is commutative then M_R is a $p.p.$-module if and only if M_R is a $p.q.$-Baer module and R is commutative if and only if $R[x]$ is a commutative. ∎

Corollary 13. [4, Theorem 3.1] *R is a right $p.q.$-Baer ring if and only if $R[x]$ is a right $p.q.$-Baer ring.*

Corollary 14. [8, Theorem 1.2] *Let R is a commutative ring. Then R is a $p.p.$-ring if and only if $R[x]$ is a $p.p.$-ring.*

Corollary 15. [1, Theorem A] *Let R be a reduced ring. Then R is a $p.p.$-ring if and only if $R[x]$ is a $p.p.$-ring.*

Corollary 16. *If $M[[x]]_{R[[x]]}$ is a $p.q.$-Baer module, then so is M_R.*

Proof. This result follows from [4, Proposition 2.5] and a proof similar to that used in Theorem 11. ■

Corollary 17. [4, Proposition 3.5.] *If $R[[x]]$ is a right $p.q.$-Baer ring, then so is R.*

Acknowledgment

We would like to thank Professor Yiqiang Zhou from Memorial University of Newfoundland for his valuable suggestions and comments during his visit to the Module Theory Group, Hacettepe University, Ankara. We also thank to Turkish Scientific and Research Council for support Zhou's visit.

References

1. E. P. Armendariz, A note on extensions of Baer and $p.p.$-Rings, *J. Australian Math. Soc.*, **18** (1974), 470-473.
2. G. F. Birkenmeier, Idempotents and Completely Semiprime Ideals, *Comm. Algebra*, **11** (1983), 567-580.
3. G. F. Birkenmeier, J. Y. Kim, J. K. Park, On extensions of quasi-Baer and principally quasi-Baer rings, Preprint.
4. G. F. Birkenmeier, J. Y. Kim, J. K. Park, On Polynomial extensions of principally quasi-Baer rings, *Kyungpook Math. J.*, **40** (2000), 247-253.
5. A. W. Chatters, C. R. Hajarnavis, *Rings with Chain Conditions*, Pitman, Boston, (1980).
6. W. E. Clark, Twisted matrix units semigroup algebras, *Duke Math. J.*, **34** (1967), 417-424.
7. C. Y. Hong, N. K. Kim and T. K. Kwak, Ore Extensions of Baer and $p.p.$-Rings, *J. Pure Appl. Algebra*, **151** (2000), 215-226.
8. S. Jøndrup, $p.p.$-Rings and finitely generated flat ideals, *Proc. Amer. Math. Soc.*, **28** (1971), 431-435.

9. N. K. Kim, Y. Lee, Armendariz Rings and Reduced Rings, *J. Algebra*, **223** (2000), 477-488.

10. M. T.Koşan, M. Başer and A. Harmanci, Quasi-Armendariz Modules and Rings, Preprint.

11. T. Y. Lam, *Lectures on Modules and Rings*, Springer-Verlag, New York, 1999.

12. T. K. Lee and Y. Zhou, *Reduced Modules, Rings, modules, algebras and abelian groups*, 365-377, Lecture Notes in Pure and Appl. Math., 236, Dekker, New york, 2004.

13. T. K. Lee and Y. Zhou, Armendariz and Reduced Rings, *Comm. Alg.*, **6** (2004), 2287-2299.

Muhittin Başer
Department of Mathematics,
Faculty of Science and Arts,
Kocatepe University,
ANS Campus TR-03200,
Afyon - Turkey
E-mail: mbaser@aku.edu.tr

Abdullah Harmanci
Department of Mathematics,
Hacettepe University,
Ankara - Turkey
E-mail: harmanci@hacettepe.edu.tr