# A novel design method for underactuated variable oscillation mechanisms 

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#### Abstract

For variable oscillation mechanisms, if output stroke is decreased, output-link force will increase for a same input due to the mechanical advantage. Thus, in order to eliminate input actuator stall phenomena for cases where sudden increase occurs at output load, variable oscillation mechanisms can be employed. In this study, a novel design procedure for seven link two degree-of-freedom variable oscillation mechanisms is proposed. The mechanisms run as a six-link mechanism for a fixed position of a control-link. With this method, a variable oscillation mechanism is designed. It is shown that the design procedure is consistent by performing a kinematic analysis. Also, an approach to switch to second oscillation mode is introduced via underactuation. Finally, it is verified that the theoretical approach proposed is consistent for underactuated switching with Nastran simulation.


Key words: Adjustable mechanisms, Variable oscillation mechanisms, Kinematic synthesis, Multi degree-of-freedom mechanisms, Seven link mechanisms, Underactuation, Mechanical logic element

## 1. Introduction

Commonly used closed-loop mechanisms are single degree-of-freedom and inherently inflexible. Primary advantages of closed-loop mechanisms are cost effectiveness, reliability, repeatability, low inertia, and high-speed capability. When small amount of flexibility is needed, closed-loop mechanisms can also be implemented. However, in this case, degree-of-freedom of the mechanism must be increased. Some multi degree-of-freedom mechanisms are investigated in the literature: Basic principles of six-bar linkages with adjustable output oscillations are discussed (Handra-Luca, 1973). A synthesis technique to design for varieties of synthesis problems for seven link mechanisms with two degrees of freedom is devised (Kohli and Soni 1973). There is a graphical method for synthesis of the general seven link, and two degree-of-freedom planar linkage to generate functions of two variables (Mruthyunjaya, 1975). The method is based on point position reduction and permits synthesis of the linkage to satisfy up to six arbitrarily selected precision positions. A synthesis technique for four bar linkages, having adjustable driven crank pivots, for different motion generation problems is considered (Ahmad and Waldron, 1979). Analytical methods for designing adjustable mechanisms based on synthesis of adjustable dyads are discussed (Chuenchom and Kota, 1997). Kinematic synthesis of adjustable RRSS mechanisms for multi-phase motion generation is studied (Russell and Sodhi., 2001).A study on control of an underactuated manipulator is presented (Mahindakar, et. al, 2006). An underactuated finger operation is studied (Cheng, et. al, 2009). An underactuated variable stroke compliant mechanism is studied (Tanık and Söylemez, 2010). Design method for four-bar mechanisms with variable speeds and length-adjustable driving links is presented (Soong, 2009). Finally, variable novel geared linkage mechanism is studied (Soong, 2014).

A multi degree-of-freedom mechanism will be underactuated if the number of inputs to the mechanisms is less than
its degree-of-freedom. Underactuated mechanisms are simpler and cheaper due to lack of actuators and a controller.
Also, by utilizing one of the freedoms as force control, lack of the actuator can be compensated if an underactuated mechanism is optimized appropriately, free link(s) can be controlled by internal forces. Conventional differential of a wheeled vehicle is a common example to underactuated mechanisms.

For cyclic mechanisms, when oscillation magnitude of output link is decreased, magnitude of output-link force will increase for the same input actuator torque. Therefore, for cases where sudden output load increase takes place, variable oscillation mechanisms can be employed. Basically, a variable oscillation mechanism decreases its output stroke when the amount of output load is increased and increases its stroke back when the output load is decreased to a certain value. Also, if this action is controlled with a mechanical logic element (underactuation), a low cost self-adaptable mechanism can be obtained which will suit variable loading conditions. As an example, a seven link variable oscillation mechanism can be employed to wiper mechanisms: Most of the wipers that are used in the cars actuated by means of a crank-rocker type four-bar mechanism. An electric motor is coupled to the crank and the wiper is connected to the rocker. Thus for a full rotary input, oscillatory output swing of the wiper is achieved. Under normal conditions resistance to the wipers (output load) is approximately constant. However, in snowy winter days when the amount of snow on the windshield is high, wiper mechanism generally stalls. In the case of a variable oscillation mechanism, output stroke will decrease as excessive amount of snow increases output link resistance (torque) and turn back to high stroke when the snow is swept away from the wind shield.

## 2. Design procedure for seven link variable oscillation mechanisms

Seven link variable oscillation mechanisms can easily be derived from combination of cascade attached four-link mechanisms. Two of such mechanisms are shown in Fig. 1. These mechanisms run as a six-link mechanism for fixed position of a control-link. If the input link can fully rotate, an oscillatory motion is obtained at the output for a fixed position of the control-link. Magnitude of oscillation can be controlled via changing position of the control-link since position of this link changes link proportions of the second mechanism while keeping the first one the same. In Fig. 1a for the mechanism shown, position of the control-link changes the fixed link $(D G)$ length of the second four-bar mechanism, thus oscillation magnitude of the output. For the mechanism shown in Fig. 1b, position of the control-link changes eccentricity of the slider-crank mechanism, thus oscillation magnitude of the output.


Fig. 1 Examples of variable oscillation mechanisms

In this section, a design procedure for such kind of underactuated mechanisms is introduced. The design of variable oscillation mechanisms for two different required output oscillations is performed in three steps as follows:
i) Initially, second mechanism (which involves output link) is designed for a larger output link oscillation according to optimum transmission angle, since such variable oscillations mechanisms are generally working most of its running time at its larger oscillation. Free parameters such as initial position of output link, oscillation of input of second mechanism (e.g. oscillation of link CED shown in Fig. 1) can be arbitrarily selected and later be used for optimization.
ii) At the second step, it is assumed that at a fictitious position of control link the smaller required oscillation magnitude is achieved (at this step the length of control link and its position are unknown). Therefore, symbolic values for these parameters are assigned and by using the parameters obtained from the first step, constraint equations are obtained from kinematics of the motion. Generally, these constraint equations are non-linear according to unknown parameters and solved numerically.
iii) The last step is the synthesis of the first mechanism. According to the parameters determined from the first and second steps (oscillation magnitude of link $C E D$ and control-link length $A D$ ), the first mechanism is designed for optimum transmission angle (Fig. 1). Note that, since such seven link mechanisms possess two transmission angles (Balli and Chand, 2001), each of them must be optimized separately during the design steps.

## 3. A variable oscillation mechanism design

In this section, a variable oscillation mechanism (Fig. 2) is designed according to the procedure given in section 2. This mechanism is formed by two slider-crank mechanisms that are connected serially with a common slider. The first slider-crank is an in-line type and the second one has eccentricity. A full rotary input of the mechanism is from $b_{2}$, and $a_{2}$ is the output. The second input of the mechanism is from $b_{1}$ (it is also fixed link of the first slider-crank). Different positions of this link changes eccentricity of the second slider-crank, while link proportions of the first slider-crank thus stroke of the slider remains the same. This mechanism is to be designed for two different required output link oscillations.


Fig. 2 A seven link variable oscillation mechanism
Since the seven link mechanism is composed of two serially connected slider-cranks, transmission angle of each mechanism must be separately taken into consideration. The transmission angle of the first slider-crank mechanism is in between the normal line to slider axis and the connecting $\operatorname{rod}\left(\mu_{1}\right)$ whereas the transmission angle $\left(\mu_{2}\right)$ of the second slider-crank is in between $a_{2}$ and $a_{3}$, since the input to the mechanism is at the common slider and the output is at link $a_{2}$. Minimization of deviation of these transmission angles from $90^{\circ}$ is required to reduce bearing loads. In addition, transmission angle of the second slider-crank mechanism at the second position must also be checked, since the eccentricity and link proportions of the second slider-crank changes.
i) Determination of the link proportions of the second mechanism according to the first required oscillation: Initially, second slider-crank is designed for the larger required output-link oscillation. In Fig. 3 second slider crank is presented in the most general form. The slider works parallel to o $-x_{1}$ axis when switch angle $\alpha=0$ which presents the first mode of the mechanism. The slider works parallel to o- $x_{2}$ axis when $\alpha \neq 0$ which presents the second mode of the mechanism. Now we need the equation that gives us the relationship between the stroke and output link angle for the second slider-crank mechanism. By using geometric relations in Fig. 3, Eqns. (1) and (2) can be written as follows:

$$
\begin{align*}
& a_{3} \sin \beta_{\mathrm{ij}}=a_{2} \sin \left(\theta_{\mathrm{ij}}+\alpha\right)-c_{\mathrm{j}}  \tag{1}\\
& a_{3} \cos \beta_{\mathrm{ij}}=a_{2} \cos \left(\theta_{\mathrm{ij}}+\alpha\right)+s_{\mathrm{ij}} \tag{2}
\end{align*}
$$

Where, $i=e$, f and $j=1,2$. Subscript " $e$ " stands for extended and " $f$ " stands for folded positions of the first slider-crank mechanism (Fig. 2). Subscript " 1 " stands for the first mode and " 2 " stands for the second position of the control-link, where at the first mode the larger and at the second mode the smaller output oscillation is obtained. Switch angle $\alpha=0$ if $\mathrm{j}=1$.

In Eqns. (1) and (2), angle $\beta_{\mathrm{ij}}$ is neither a structure nor an input-output parameter, thus it must be eliminated. If we add square of Eq. (1) to square of Eq. (2) side by side, $\beta_{\mathrm{ij}}$ can be eliminated and the relationship between $s_{\mathrm{ij}}$ and $\theta_{\mathrm{ij}}$ can be obtained:

$$
\begin{equation*}
s_{\mathrm{ij}}^{2}=K_{1} s_{\mathrm{ij}} \cos \left(\theta_{\mathrm{ij}}+\alpha\right)+K_{2} \sin \left(\theta_{\mathrm{ij}}+\alpha\right)-K_{3} \tag{3}
\end{equation*}
$$

Where, $K_{1}=-2 a_{2}, K_{2}=2 a_{2} c_{\mathrm{j}}, K_{3}=a_{2}^{2}-a_{3}^{2}+c_{\mathrm{j}}^{2}$


Fig. 3 Second slider-crank mechanism
For arbitrarily selected initial slider position $s_{\mathrm{e} 1}$ and corresponding output angle $\theta_{\mathrm{e} 1}$ (which will be used as optimization parameter later) the first equation can be obtained from Eq. (3):

$$
\begin{equation*}
s_{\mathrm{e} 1}{ }^{2}=K_{1} s_{\mathrm{e} 1} \cos \theta_{\mathrm{e} 1}+K_{2} \sin \theta_{\mathrm{e} 1}-K_{3} \tag{4}
\end{equation*}
$$

Note that for the first mode of the mechanism, the value of $\alpha$ is zero. Then, taking the stroke of the slider unity $\left(s_{\mathrm{f} 1}=\right.$ $\left.s_{\mathrm{e} 1}+1\right)$ which is the scale factor of the mechanism while dimensioning, for the required larger oscillation $\left(\theta_{\mathrm{fl}}=\Delta \theta_{1}+\theta_{\mathrm{el}}\right)$ the second equation can be obtained from Eq. (3) as:

$$
\begin{equation*}
s_{\mathrm{f} 1}^{2}=K_{1} s_{\mathrm{f} 1} \cos \theta_{\mathrm{f} 1}+K_{2} \sin \theta_{\mathrm{f} 1}-K_{3} \tag{5}
\end{equation*}
$$

Now, we have two equations with three unknowns ( $K_{1}, K_{2}$, and $K_{3}$ ). The free parameter can be used for optimization of the transmission angle ( $\mu_{2 \mathrm{ij}}$ ). The relationship between $s_{\mathrm{ij}}$ and $\mu_{2 \mathrm{ij}}$ can be determined from cosine law:

$$
\begin{equation*}
s_{\mathrm{ij}}^{2}=-c_{\mathrm{j}}^{2}+{a_{3}}^{2}+{a_{2}}^{2}-2 a_{2} a_{3} \cos \left(\mu_{2 \mathrm{ij}}\right) \tag{6}
\end{equation*}
$$

If deviations of $\mu_{2 \mathrm{ij}}$ from $90^{\circ}$ for two extreme slider positions are equated, the third equation for optimum transmission angle can be obtained by using two times Eq. (6) as:

$$
\begin{align*}
& s_{\mathrm{e} 1}^{2}=-c_{1}^{2}+{a_{3}}^{2}+a_{2}^{2}-2 a_{2} a_{3} \cos \left(90^{\circ}-\delta\right)  \tag{7}\\
& s_{\mathrm{f} 1}^{2}=-c_{1}^{2}+{a_{3}}^{2}+{a_{2}}^{2}-2 a_{2} a_{3} \cos \left(90^{\circ}+\delta\right) \tag{8}
\end{align*}
$$

Adding Eq. (7) to Eq. (8) side by side:

$$
\begin{equation*}
s_{\mathrm{e} 1}{ }^{2}+s_{\mathrm{f} 1}^{2}=K_{1}^{2}-2 K_{3} \tag{9}
\end{equation*}
$$

Subtracting Eq. (4) from Eq. (5) a relationship between $K_{2}$ and $K_{1}$ can be determined:

$$
\begin{equation*}
K_{2}=\frac{s_{\mathrm{e} 1}^{2}-s_{\mathrm{f} 1}^{2}+K_{1}\left(s_{\mathrm{f} 1} \cos \theta_{\mathrm{f} 1}-s_{\mathrm{e} 1} \cos \theta_{\mathrm{e} 1}\right)}{\sin \theta_{\mathrm{e} 1}-\sin \theta_{\mathrm{f} 1}} \tag{10}
\end{equation*}
$$

Substituting Eq. (10) into Eq. (4) a relationship between $K_{1}$ and $K_{3}$ can be determined:

$$
\begin{equation*}
K_{1}=\left(K_{3}+u\right) / t \tag{11}
\end{equation*}
$$

Where,

$$
u=s_{\mathrm{e} 1}^{2}+\left(\frac{s_{\mathrm{f} 1}^{2}-s_{\mathrm{e} 1}^{2}}{\sin \theta_{\mathrm{e} 1}-\sin \theta_{\mathrm{f} 1}}\right) \sin \theta_{\mathrm{e} 1} \quad t=s_{\mathrm{e} 1} \cos \theta_{\mathrm{e} 1}+\left(\frac{s_{\mathrm{f} 1} \cos \theta_{\mathrm{f} 1}-s_{\mathrm{e} 1} \cos \theta_{\mathrm{e} 1}}{\sin \theta_{\mathrm{e} 1}-\sin \theta_{\mathrm{f} 1}}\right) \sin \theta_{\mathrm{e} 1}
$$

Substituting $K_{1}$ in Eq. (11) into Eq. (9) a quadratic equation is obtained and solution of this equation is:

$$
\begin{equation*}
K_{3}=t^{2}-u \pm \sqrt{\left(u-t^{2}\right)^{2}-u^{2}+t^{2}\left(s_{\mathrm{e} 1}^{2}+s_{\mathrm{f} 1}^{2}\right)} \tag{12}
\end{equation*}
$$

Once $K_{3}$ is calculated from Eq. (12) then $K_{1}$ and $K_{2}$ can easily be determined from Eqns. (10) and (11) and design of the second slider-crank can be performed for optimum transmission angle.
ii) Determination of the control link position for the second required swing angle: At this moment, eccentricity $c_{2}$ and slider axis $A E$ (Fig. 4) of the second slider-crank are parameters that can only be changed to satisfy the second required oscillation. In order to achieve this oscillation, let the slider axis is rotated by an unknown angle $\alpha$ (Fig. 4). According to the new slider axis (parallel to o- $x_{2}$, Fig. 3), Eq. (3) is used two times to satisfy second required swing angle as follows:

$$
\begin{align*}
& s_{\mathrm{e} 2}^{2}+c_{2}^{2}+2 a_{2}\left[s_{\mathrm{e} 2} \cos \left(\theta_{\mathrm{e} 2}+\alpha\right)-c_{2} \sin \left(\theta_{\mathrm{e} 2}+\alpha\right)\right]=a_{3}^{2}-a_{2}^{2}  \tag{13}\\
& \left(s_{\mathrm{e} 2}+\Delta s\right)^{2}+c_{2}^{2}+2 a_{2}\left[\left(s_{\mathrm{e} 2}+\Delta s\right) \cos \left(\theta_{\mathrm{e} 2}+\Delta \theta_{2}+\alpha\right)-c_{2} \sin \left(\theta_{\mathrm{e} 2}+\Delta \theta_{2}+\alpha\right)\right]=a_{3}^{2}-a_{2}^{2} \tag{14}
\end{align*}
$$



Fig. 4 Stroke relations of the mechanism at two modes
Also, constraint equations due to the kinematic relations at two modes of the mechanism must be determined: A relationship between the second eccentricity $c_{2}$ and the first one as a function of switching angle $\alpha$ and the initial slider position can be obtained from Fig. 4 as:

$$
\begin{equation*}
c_{2}=c_{1} \cos \alpha-\left(p+\Delta s+s_{\mathrm{e} 1}\right) \sin \alpha \tag{15}
\end{equation*}
$$

Similarly, a relationship between second initial slider position $s_{\mathrm{e} 2}$, second eccentricity $c_{2}$ and switching angle $\alpha$, can be determined as:

$$
\begin{equation*}
s_{\mathrm{e} 2}=\left[(p+\Delta s)(1-\cos \alpha)+s_{\mathrm{e} 1}\right] / \cos \alpha+c_{2} \tan \alpha \tag{16}
\end{equation*}
$$

Where, $p=b_{3}-b_{2}$

In Eqns. (15) and (16), there is a new variable $p$ which is the difference between the lengths of connecting rod and the crank of the first slider-crank mechanism. One can eliminate $p$ from Eq. (15) by using Eq. (16), to reduce the number of free parameters. However, in this case, $p$ may take many different values, which is not desired. Because the first slider-crank is in-line type (whose crank length is half of its stroke) the value of $p$ determines the ratio of the connecting rod to the crank, which is the unique parameter that also determines the critical transmission angle of the first slider-crank $\left(\mu_{1}\right)$. (For in line slider-crank mechanism, the maximum value of the transmission angle is $\mu_{1}=\operatorname{acos}\left(b_{2} / b_{3}\right)$ ). Thus, $p$ must be checked after the synthesis, in order to obtain an appropriate transmission angle for the first slider-crank. (As value of $p$ increases transmission angle of the first slider-crank will be better).
iii) Design of the first slider crank: At the last step, the unique design parameter which is the critical transmission angle of the first slider-crank mechanism will be specified (thus $p$ ). Once the critical angle of the inline slider crank is known, proportions of the first slider-crank are also specified. (Unless appropriate transmission angles are obtained for the second slider-crank, $p$ can also be used for the optimization of the mechanism with the other free parameters).

For a given second output-link oscillation $\left(\Delta \theta_{2}\right)$ and known first slider-crank dimensions, in Eqns. (13), (14), (15), and (16) there are four unknown parameters; second initial output-link angle $\theta_{\mathrm{e} 2}$, switching angle $\alpha$, eccentricity $c_{2}$ and second initial slider position $s_{\mathrm{e} 2}$. According to unknown parameters, these non-linear equations can be solved numerically. While determining link proportions according to the first swing angle, initial output-link angle $\theta_{\mathrm{e} 1}$ and initial slider position $s_{\mathrm{e} 1}$ are arbitrarily selected. In addition, critical transmission angle (or $p$ ) of the first slider-crank is determined initially. Optimization of the mechanism according to critical transmission angles can be performed with these free parameters. The design procedure is summarized in Table 1.

Table 1 Design procedure of the variable oscillation mechanism

| Step | Opt. parameters | Requirements | Opt. objectives | Soln. Method. | Results |
| :---: | :---: | :---: | :---: | :---: | :---: |
| i | $\theta_{\mathrm{e} 1,}, s_{\mathrm{e} 1}$ | $\Delta \theta_{1}$ | $\mu_{2}$ | Analytical | $a_{2,} a_{3}, c_{1}$, |
| ii |  | $\Delta \theta_{2}$ |  | Numerical | $\theta_{\mathrm{e} 2}, \alpha$, |
| iii | $p$ |  | $\mu_{1}$ | Analytical | $b_{3}$ |

## Example 1

A seven link variable oscillation mechanism for $50^{\circ}$ and $25^{\circ}$ output link swing angles is required. Let the initial position of the second slider-crank be 1.2 times the stroke and let the initial output-link angle be $78^{\circ}\left(s_{\mathrm{e} 1}=1.2 \Delta s, \theta_{\mathrm{e} 1}=\right.$ $78^{\circ}$ ). Link lengths are determined as $a_{2}=1.153, a_{3}=1.508$ and $c_{1}=0.68$ from Eqns. (10), (11) and (12) for $50^{\circ}$ swing angle. The critical transmission angle deviation for the second slider-crank is $29.3^{\circ}$ at first mode of the mechanism. By solving non-linear Eqns. (13), (14), (15), and (16) numerically for $25^{\circ}$ swing angle, unknown parameters can be determined as $s_{\mathrm{e} 2}=1.197, \alpha=24.2^{\circ}, c_{2}=-0.692, \theta_{\mathrm{e} 2}=113.5^{\circ}$. The critical deviation of the transmission angle from $90^{\circ}$ at second mode is $29.1^{\circ}$. The value of $p$ is taken as unity ( $\Delta s=1$ ), that also causes the critical transmission angle deviation of the first slider-crank mechanism to be $19.5^{\circ}$. The ratio of maximum to minimum link length is 4 . The mechanism for two modes is presented in Fig. 5.


Fig. 5 Variable oscillation mechanism (scaled) for $50^{\circ}$ and $25^{\circ}$ swing angles

Using the same design procedure different variable oscillation mechanisms are synthesized. The results of this optimization are listed in Table 2. From Table 2, it can be concluded that the output oscillation of this type of mechanism is limited to $110^{\circ}-120^{\circ}$. Because at this point, the critical deviation of the transmission angle of the mechanism is over $50^{\circ}$. However, from the second example in Table 2, it is seen that despite a large oscillation magnitude difference $\left(\Delta \theta_{1}-\Delta \theta_{2}=40^{\circ}\right)$, the maximum deviation of transmission angle is still in an acceptable range.

Table 2 Some variable oscillation mechanisms

| $\#$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \theta_{1}, \Delta \theta_{2}$ | $50^{\circ}, 25^{\circ}$ | $70^{\circ}, 30^{\circ}$ | $90^{\circ}, 65^{\circ}$ | $110^{\circ}, 85^{\circ}$ | $120^{\circ}, 90^{\circ}$ |
| $a_{2}$ | 1.15 | 0.86 | 0.707 | 0.57 | 0.56 |
| $a_{3}$ | 1.51 | 1.203 | 1.5 | 2.32 | 1.62 |
| $c_{1}$ | 0.68 | 0.604 | 0.5 | 1.17 | 0.66 |
| $c_{2}$ | -0.69 | -0.29 | -0.5 | -1.16 | -0.66 |
| $\alpha$ | $24.2^{\circ}$ | $19.9^{\circ}$ | $18.6^{\circ}$ | $32.2^{\circ}$ | $24.8^{\circ}$ |
| ctad $^{*} \# 1$ | $29.3^{\circ}$ | $37.1^{\circ}$ | $45^{\circ}$ | $50.4^{\circ}$ | $55.7^{\circ}$ |
| ctad $^{\circ} \# 2$ | $29.3^{\circ}$ | $37.6^{\circ}$ | $45^{\circ}$ | $50.7^{\circ}$ | $56^{\circ}$ |

* ctad stands for maximum deviation of $\mu_{2}$ from $90^{\circ}$
$*^{*} b_{2}=0.5$ and $b_{3}=1.5$ unit for all mechanisms


## 4. Automatic switching with a bistable logic element

The motion of a control-link of a variable oscillation mechanism to another pre-determined position (to satisfy the required second oscillation) can be called "switching". This action can be performed in several ways. In this study, to obtain simple and low cost mechanism, switching is performed without using a second actuator and a controller. Therefore, the mechanism becomes underactuated [the number of inputs (1) is less than the degree-of-freedom of the mechanism (2)]. Actuator stall phenomena can be prevented for increased output load, if such control logic is available: When there is a sudden increase in the output-link torque during forward stroke while the mechanism is working at its first position, the mechanism will switch to low oscillation mode. This type of switching can be called as "automatic switching". In Fig. 6, a bistable mechanical logic element is presented as an example. This element is formed basically from a cam profile, ball and a spring. By adjusting the screw which changes preload of the spring, different threshold values for switching action can be obtained. Such a mechanical logic element can be integrated in between the control-link and fixed-link of the variable oscillation mechanism to perform automatic switching.


Fig. 6 A bistable mechanical logic element
For low-speed operation of such variable oscillation mechanisms, static force analysis can be performed. During switching, the output-link must remain stationary while crank is rotating, since there is no resistance on control-link (unlike the output-link) when it is in between two static positions. The necessary and sufficient conditions that must be satisfied for switching are:

1- Same output-link angle must exist for both position of control-link at the switching position. (During switching the output-link remains stationary while crank is rotating due to underactuation). For example, in Fig. 5 the output-link working range is indicated with dashed lines and it can be observed that there is a certain region available where these areas overlap. If switching starts in a position out of the overlap region, the mechanism will lock and switching process will fail. We call such phenomenon as undesired switching in this study.

2- The resultant torque that applied to the control-link must be along to the required direction of rotation, both at the instant of the start of the switching and at the end of switching.

## Example 2

Let a bistable mechanical logic element is integrated to the control-link of the synthesized mechanism in Example 1. Determine the threshold torque value of the switch key and corresponding crank angles of the mechanism, to switch low oscillation mode when magnitude of the output load is increased by 5 times. Let a resistant torque applied to the output-link is 1 N .unit during the work stroke $\left(180^{\circ}<\phi<360^{\circ}\right)$ and $1 / 5 \mathrm{~N}$. unit during the return stroke $\left(0 \leq \phi \leq 180^{\circ}\right)$ in first mode for normal working condition: $T_{\text {out }}=-1 / 5 \mathrm{~N}$. unit for $0 \leq \phi \leq 180^{\circ}$ and $T_{\text {out }}=1 \mathrm{~N}$. unit for $180^{\circ}<\phi<360^{\circ}$


Fig. 7 Output-link vs. crank position

After kinematic analysis of the mechanism, the output-link vs. input-link position is shown in Fig. 7. Switching is possible only when the output-link angle is in between $113^{\circ}$ and $128^{\circ}$, since except this interval the same output-link angle does not exist for two modes of the mechanism. The corresponding crank angle for this interval is $106.5^{\circ}<\phi<253.5^{\circ}$ at the first mode that is indicated as "switching region" in Fig. 7. After performing static force analysis, the torque applied to the control-link (Fig. 2) can be determined as given in Eq. (17). For two modes of the mechanism control-link torques as a function of the crank angle are calculated by using Eq. (17) and presented in Fig. 8.

$$
\begin{equation*}
T_{\mathrm{c}}=-s_{1} T_{\text {out }}(\sin \beta+\tan \gamma \cos \beta) / a_{2} \sin \mu_{2} \tag{17}
\end{equation*}
$$



Fig. 8 Torque applied to the control-link

While switching from first to second position; the control-link must rotate in clockwise direction (Fig. 5). The requirements for switching to the second mode are; the torque must be negative (CW) at the beginning of the switching as well as during switching, and the crank angle must be in between $106.5^{\circ}<\phi<253.5^{\circ}$. These requirements are simultaneously satisfied when $181^{\circ}<\phi<211^{\circ}$. However, in this interval the torque curve starts from -0.15 N .unit (when $\phi=181^{\circ}$ which is also the start point of the work stroke of the mechanism) and its absolute value always decreases. Therefore, switching can only start at crank angle $\phi=181^{\circ}$. Since all internal forces are linearly increasing with the output-link force, according to the requirement threshold value must be $0.15 \times 5=0.75 \mathrm{~N}$.unit (if another crank angle were selected for the start of switching, e.g. $188^{\circ}$, the threshold value of switch key would be set to 0.5 N .unit). Since the torque applied on the control-link exceeds 0.5 N.unit at $\phi=181^{\circ}(0.75 \mathrm{~N} . u n i t)$, switching again would start when the crank reaches to $181^{\circ}$.

Switching action for the mechanism considered in Example 2 is also analyzed with Nastran simulation to verify the theoretical approach and some snapshots of the simulation results are presented in Fig. 9. It can be observed that control-link (blue) of the mechanism is motionless when crank (red) angle ( $\phi$ ) is smaller than $180^{\circ}$ (Fig. 9a-b).


Fig. 9 Simulation snapshots
Once the crank angle exceeds $180^{\circ}$, the control-link starts to move clockwise (Fig. 9c) and switching action ends when crank angle reaches $\phi=232^{\circ}$ (Fig. 9e). Finally, the mechanism continues its motion in second mode (Fig 9f). The simulation result of switching action is exactly the same as the proposed theoretical approach.

When $316^{\circ}<\phi<360^{\circ}$, the control-link torque at the first mode is greater than the first switching position value ( $T_{\mathrm{c}}>$ 0.15). If the output load increases in this interval, the control-link torque exceeds the threshold value where the corresponding output-link does not exist for the second position. Then, undesired switching (out of the switching region as indicated in Fig. 7) starts and the mechanism locks in this interval. Therefore this mechanism can only switch safely if the increase at the output loading occurs at the beginning of the work stroke ( $\phi=181^{\circ}$ ). In order to overcome this problem, the torque applied to control-link in the region where switching is not required can be decreased under the threshold value by changing the free parameters during the kinematic synthesis. Briefly, obtaining an appropriate torque curve is also an optimization objective. However, many trials showed that it is not always possible to obtain a torque curve as required with appropriate transmission angles and reasonable link proportions.

In order to investigate automatic switching in real world conditions we designed another variable stroke with the theory proposed here and presented in Fig. 10. Unlike the mechanism investigated in Example 2, here the work stroke is in between 0 and $180^{\circ}$ and the crank rotates in clockwise direction. It can be observed that control-link of the mechanism is motionless till a resistance (Fig. 10b) on the output-link (yellow) is available. Once the crank angle is smaller than $180^{\circ}$, the control-link starts to move clockwise (Fig. 10c) and the switching action ends as in Fig. 10e. Finally, the mechanism continues its motion in second mode (Fig. 10f).


Fig. 10 Real model snapshots

## 5. Conclusions

In this paper a general design technique for seven link variable oscillation mechanisms is introduced. An example of such a mechanism is investigated in detail. For the mechanism considered, results of optimization are presented to indicate capability of the mechanism.

Beside the kinematic synthesis of variable oscillation mechanisms, underactuated switching concept for different output-link oscillations is presented. Feasibility of underactuated switching to low oscillation mode is proved with a simulation. Also a real model is built which can automatically switch from high to low oscillation mode.

We believe that underactuated variable oscillation mechanisms may find applications in industry with a cost advantage when compared to mechatronic systems.

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